Effects of *CP* violation from neutral heavy fermions on neutrino oscillations, and the LSND/MiniBooNE anomalies

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Neutrinos may mix with ultralight fermions, which gives flavor oscillations, and with heavier fermions, which yields short distance flavor change. I consider the case where both effects are present. I show that in the limit where a single oscillation length is experimentally accessible, the effects of heavier fermions on short baseline neutrino oscillations can generically be accounted for by a simple formula containing only four parameters, including observable *CP* violation. I consider the anomalous LSND and MiniBooNE results, and show that these can be fit in a model with *CP* violation and two additional sterile neutrinos, one in the mass range between 0.1 and 20 eV, and the other with mass between 32 eV and 40 GeV. I also show that this model can avoid conflict with constraints from existing null short baseline experimental results.

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I. INTRODUCTION

Since the discovery of neutrino flavor change in a variety of long baseline experiments [1–15], a new standard picture has emerged [16–27]. In this picture the neutrino flavor eigenstates e, μ, τ are related to the mass eigenstates $\nu_{1,2,3}$ via a 3-by-3 unitary matrix [28]:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad (1)$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$. Long baseline measurements are consistent with the following values of the three angles θ_{ij}:

$$\begin{aligned} \tan^2\theta_{12} &= 0.45 \pm 0.05 \\ \sin^2\theta_{13} &= 0.020 \pm 0.008 \\ \sin^2 2\theta_{23} &= 1.0^{+0}_{-0.1} \end{aligned} \quad (2)$$

and neutrino mass squared differences:

$$\begin{aligned} \Delta m_{12}^2 &= (8.0 \pm 0.3) \times 10^{-5}\ \text{eV}^2 \\ |\Delta m_{23}^2| &= (2.5 \pm 0.2) \times 10^{-3}\ \text{eV}^2. \end{aligned} \quad (3)$$

These mass eigenstates are so light and nearly degenerate that in all neutrino experiments neutrinos propagate at essentially the speed of light, and the components of the neutrino wave packets with different mass do not separate spatially. The phases of the different mass components oscillate quantum mechanically with different frequencies, and, as the different flavors are different superpositions of mass eigenstates, the flavor composition of a neutrino beam in vacuum will exhibit spatial variation as a function of L/E, where E is the neutrino energy and L is the distance from the source [29]. In propagation through matter, the phases are also altered by forward scattering from the weak interactions, which alters the flavor change probability in a way which depends on both E and L/E, and differs for neutrinos and antineutrinos [30,31]. These matter effects are very small in experiments with baseline much shorter than 1000 km, unless there exist exotic forces [32–38]. Oscillations are now significantly favored over alternatives such as neutrino decoherence or decay [39,40].

The measurement of the small mixing angle θ_{13} is a primary goal of the current generation of long baseline experiments. Also sought is evidence for a nonzero value of the *CP* violating parameter δ, and knowledge of whether the pair of states with the smaller mass squared splitting Δm_{12}^2 are heavier or lighter than the third state.

There have been reports of neutrino flavor change in the short baseline LSND [41] and MiniBooNE [42–44] experiments, which would upset this standard picture, as the values of L/E in both these experiments is of order 1 MeV/m, which is too small for the small mass squared splittings Δm_{12}^2 and Δm_{23}^2 to produce flavor change at the observed level. The LSND and MiniBooNE results favor antielectron neutrino appearance in a muon antineutrino beam, at different energies and distance but similar values of L/E. The MiniBooNE data on neutrinos disfavors electron neutrino appearance in a muon neutrino beam at the values of L/E explored by LSND, but favors an excess of electron neutrinos at higher values of L/E [43]. With at least two additional sterile neutrinos, *CP* violation in oscillations can reconcile the LSND and MiniBooNE antineutrino results with MiniBooNE neutrino results [45–47]. However, reconciling MiniBooNE and LSND with constraints on muon or electron neutrino disappearance from a variety of other short baseline experiments [48–52] is more difficult. Attempts to do so have introduced additional exotic ingredients beyond neutrino mixing [38,45,53–61].

Existing studies of neutrino oscillations generally are not sensitive to mass squared differences larger than 1000 eV^2, as the resulting oscillation length is too short to measure. Furthermore, mixing with such heavy neutrinos has not previously been considered as a mechanism to reconcile LSND and MiniBooNE with short baseline disappearance constraints. In this paper we will consider electron neutrino or antineutrino appearance in a muon neutrino or antineutrino beam in the case where at least one neutrino is so heavy that the associated mass squared difference is larger than 1000 eV^2.

II. NEUTRINO OSCILLATIONS AND MIXING WITH A NEUTRINO ASSOCIATED WITH A MASS SQUARED DIFFERENCE LARGER THAN 1000 eV^2

Previous analyses of neutrino oscillations involving sterile neutrinos have typically considered mass squared differences which are smaller than 1000 eV^2, which is the largest mass squared difference probed by oscillation searches for neutrino disappearance. In addition they have assumed coherence among the different mass components of the neutrino wave function, although this assumption breaks down for mixing with a heavier neutrino, as discussed in, e.g., Ref. [62]. Assuming a unitary mixing matrix and small mass differences, the formula for the probability of electron neutrino appearance in a muon neutrino beam is given by the standard result

$$= \Big| \sum_{i>1} U_{ei} U_{\mu i}^* (e^{-2ix_{i1}} - 1) \Big|^2, \tag{4}$$

where

$$x_{ij} \equiv 1.27 \frac{(m_i^2 - m_j^2)}{\mathrm{eV}^2} \frac{L/E}{\mathrm{m/MeV}}. \tag{5}$$

Specializing to the short baseline case, where the mass differences among the three light states can be neglected, and assuming two additional states, this formula becomes

$$P_{\nu_\mu \to \nu_e} = |U_{e4} U_{\mu 4}^* e^{-2ix_{41}} + U_{e5} U_{\mu 5}^* e^{-2ix_{51}} - U_{e4} U_{\mu 4}^* - U_{e5} U_{\mu 5}^*|^2. \tag{6}$$

For antineutrinos the matrix elements are complex conjugated. This probability can be written as [63]

$$P_{\nu_\mu \to \nu_e} = 4|U_{e4}|^2 |U_{\mu 4}|^2 \sin^2 x_{41} + 4|U_{e5}|^2 |U_{\mu 5}|^2 \sin^2 x_{51} + 8|U_{e5}||U_{e4}||U_{\mu 4}||U_{\mu 5}| \sin x_{51} \sin x_{41} \times \cos(x_{51} - x_{41} - \phi), \tag{7}$$

where

$$\phi \equiv \arg\left(\frac{U_{e5} U_{\mu 5}^*}{U_{e4} U_{\mu 4}^*}\right) \tag{8}$$

is a physically observable CP violating phase and for antineutrinos we must replace $\phi - \to -\phi$. In the limit where the fifth neutrino is heavy x_{51} varies very rapidly and should be averaged over. The appearance probability then becomes

$$P_{\nu_\mu \to \nu_e} 4|U_{e4}|^2 |U_{\mu 4}|^2 \sin^2 x_{41} + 2|U_{e5}|^2 |U_{\mu 5}|^2 + 4|U_{e5}||U_{e4}||U_{\mu 4}||U_{\mu 5}| \sin x_{41} \sin(x_{41} + \phi). \tag{9}$$

This expression is also valid if the fifth mass eigenstate is heavy enough to have decohered with the lighter neutrinos.

We can simplify this expression by defining the CP odd quantity β,

$$\beta \equiv \frac{1}{2} \tan^{-1}\left(\frac{\sin\phi |U_{e5}||U_{\mu 5}|}{|U_{e4}||U_{\mu 4}| + \cos\phi |U_{e5}||U_{\mu 5}|}\right), \tag{10}$$

and the mixing ratio r,

$$r \equiv \frac{|U_{e5} U_{\mu 5}^* + U_{e4} U_{\mu 4}^*|}{|U_{e4} U_{\mu 4}^*|}, \tag{11}$$

so that

$$r e^{2i\beta} = \frac{U_{e5} U_{\mu 5}^* + U_{e4} U_{\mu 4}^*}{U_{e4} U_{\mu 4}^*} \tag{12}$$

and get oscillation probability

$$P_{\nu_\mu \to \nu_e} = |U_{e4}|^2 |U_{\mu 4}|^2 [2(1 - r)^2 + 4r \sin^2\beta + 4r \sin^2(x_{41} + \beta)]. \tag{13}$$

For antineutrinos we replace $\beta \to -\beta$. Note that CP violation remains observable in the limit of heavy m_5. Note also that while with four neutrinos the amplitude of oscillations associated with m_4 is $4|U_{e4}|^2 |U_{\mu 4}|^2$, mixing with a heavy fifth neutrino alters the oscillation amplitude by a factor of r, which may be either larger or smaller than 1. The parameter r may allow for reconciliation of neutrino appearance and neutrino disappearance results.

In contrast with appearance experiments, disappearance experiments are much less sensitive to mixing with heavy neutral fermions. Averaging over the short oscillation length associated with Δm_{51}^2, and neglecting the mass differences among the three light neutrinos, the probability for vacuum electron neutrino or electron antineutrino disappearance is

$$4|U_{e4}|^2 (1 - |U_{e4}|^2 - |U_{e5}|^2) \sin^2 x_{41} + 2|U_{e5}|^2 (1 - |U_{e5}|^2) \tag{14}$$

and the probability for muon neutrino or muon antineutrino disappearance is obtained by replacing $e \to \mu$ in the preceding formula. Typically, stringent bounds on disappearance are obtained by canceling systematic errors using the L/E dependence. Thus, U_{e5} and $U_{\mu 5}$ are only weakly constrained by disappearance searches in the limit where the oscillation length associated with Δm^2_{51} is too short to give any measurable dependence on L/E.

The tension between electron antineutrino appearance at LSND and MiniBooNE and short baseline electron and muon neutrino disappearance experiments may be reduced by allowing both nonzero β and allowing r to be greater than 1. Because of the constant term in neutrino appearance formula Eq. (13), the ratio r is constrained by very short baseline electron neutrino appearance searches. The strongest such constraints come from the NOMAD and E776 experiments [64,65]. The NOMAD constraint on ν_e appearance at $L/E < 0.025$ m/MeV implies

$$|U_{e4}|^2|U_{\mu 4}|^2[(1-r)^2 + 4r\sin^2\beta] < 0.0007. \tag{15}$$

Therefore the existence of the heavy fifth neutrino makes it is possible to obtain much larger electron neutrino or antineutrino appearance probabilities at nonzero L/E than would be possible in a 3 + 1 model. For instance, for $\beta = 0$ and $|U_{e4}|^2|U_{\mu 4}|^2 = 2 \times 10^{-4}$, the maximum allowed value of r is 3. For a 3 + 1 model with $\delta = 0$ and $r = 1$, the maximum probability of electron neutrino appearance as a function of L/E is 8×10^{-4}. In contrast, for $r = 3$, the probability of electron neutrino appearance maximizes at a much larger 4×10^{-3}. In the next section we will find even larger values of r are possible when neutrinos mix with fermions which are too heavy to be produced.

III. A VERY HEAVY NEUTRINO

In this section we consider in detail a model with two additional sterile neutrinos, one of which is heavy enough that it is possible to kinematically distinguish it from the others. In principle, it is produced with a reduced phase space, or, if sufficiently heavy, it is not produced at all. Mixing with a heavy neutrino will be constrained from loop contributions to charged lepton flavor violation, such as $\mu \to e\gamma$ [66], but as long as it is lighter than about 40 GeV such a neutrino could contribute substantially to the LSND/MiniBooNE anomalies while the charged lepton flavor violation rate will be sufficiently suppressed by the Glashow-Iliopoulos-Maiani [67] mechanism. In the case of neutrino mixing with such a heavy neutrino, the beam will not initially be in a pure flavor eigenstate, and the situation can be described in terms of a nonunitary mixing matrix for the light states [66]. Without the unitarity constraint, *CP* violation is possible even in two neutrino oscillations [68]. In this section we will consider the effect of an additional state which is significantly heavier than the other neutrinos, but light enough so that charged lepton flavor violation constraints on unitarity violation are not constraining.

Neglecting the mass differences among the three light eigenstates, but assuming there is a fourth neutrino whose mass squared difference with the others is not negligible, the formula for the probability of electron neutrino appearance in a muon neutrino beam is

$$P_{\nu_\mu \to \nu_e} = |U_{e4}U^*_{\mu 4}e^{-2ix_{14}} - U_{e4}U^*_{\mu 4} - U_{e5}U^*_{\mu 5}|^2 + a|U_{e5}|^2|U_{\mu 5}|^2 \tag{16}$$

$$= |U_{e4}|^2|U_{\mu 4}|^2|e^{-2ix_{14}} - re^{2i\beta}|^2 + a|U_{e5}|^2|U_{\mu 5}|^2, \tag{17}$$

and get electron neutrino appearance probability

$$|U_{e4}|^2|U_{\mu 4}|^2\{(1-r)^2 + a[(1-r)^2 + 4r\sin^2\beta] + 4r\sin^2(x_{41} + \beta)\} \tag{18}$$

with $\beta \to -\beta$ for antineutrinos. Here a is a phase space factor associated with production of the heavy state, which depends on the way in which the beam is produced, the heavy neutrino mass, and the neutrino energy. In general a is less than 1, and is 0 if the state is heavier than the available energy. Assuming that $a = 0$, as for instance would be the case for a muon neutrino beam produced from pion decay in the limit where the heavy neutrino is heavier than the pion-muon mass difference, the constraint on r from very short baseline electron neutrino appearance is correspondingly weakened, to

$$|U_{e4}|^2|U_{\mu 4}|^2\left[\frac{(1-r)^2}{2} + 2r\sin^2\beta\right] < 0.0007. \tag{19}$$

For instance, for $\beta = 0$ and $|U_{e4}|^2|U_{\mu 4}|^2 = 2 \times 10^{-4}$, the maximum allowed value of r is 3.8, and the probability of electron neutrino appearance maximizes at 8×10^{-3}. In contrast, in a 3 + 1 model with $|U_{e4}|^2|U_{\mu 4}|^2 = 2 \times 10^{-4}$, and no fifth heavy neutrino, the maximum short baseline electron appearance probability would be only 8×10^{-4}.

Also somewhat constraining will be the muon decay rate, which could be affected from ν_μ and ν_e mixing with a sufficiently heavy neutrino, and which is constrained from lepton universality and from precision electroweak tests [69]. For a fifth neutrino which is heavier than the muon, we must require $|U_{e5}|$ and $|U_{\mu 5}|$ to be smaller than $\sim$0.05.

IV. GENERAL FORMULA FOR ANALYZING NEUTRINO APPEARANCE OSCILLATION EXPERIMENTS IN VACUUM

The results of the previous two sections are easily generalized. Any single neutrino oscillation experiment is typically sensitive to oscillations in a range of L/E which varies by no more than an order of magnitude or so. This suggests a simple generalization of the two flavor dominance formula. Assuming a single oscillation length is comparable to the range of the experiment, much shorter oscillation lengths may be averaged over, and much longer oscillation lengths may be neglected, the probability $P_{a\to b}$ of appearance of flavor a in a beam of flavor b in vacuum may generically be written

$$P_{a\to b} = \sin^2(2\theta_{ab})\sin^2\left(1.27\frac{\Delta m^2 L/E}{\text{eV}^2\ \text{m/MeV}} + \beta\right) + 0.5\cos^2(2\theta_{ab})\sin^2\alpha, \tag{20}$$

where Δm^2 is the relevant mass squared difference, θ_{ab} is an effective mixing angle, β is a CP violating phase difference between the two different components (which is allowed to be nonzero if we do not have two flavor unitarity), and α gives the constant term resulting from averaging over short wavelength oscillations, from any nonunitarity, and from any neutrinos which are too heavy to participate in oscillations. For antineutrinos the oscillation probability in this limit would be

$$P_{a\to b} = \sin^2(2\theta_{ab})\sin^2\left(1.27\frac{\Delta m^2 L/E}{\text{eV}^2\ \text{m/MeV}} - \beta\right) + 0.5\cos^2(2\theta_{ab})\sin^2\alpha. \tag{21}$$

This parametrization is chosen to satisfy the generic constraints for CPT conserving vacuum oscillations $0 \le P_{a\to b} \le 1$ and $0 \le \langle P_{a\to b}\rangle \le 1/2$. For example, in the five neutrino model of the previous section, we would have

$$\sin^2(2\theta_{e\mu}) = 4|U_{e5}U^*_{\mu 5} + U_{e4}U^*_{\mu 4}||U_{e4}U_{\mu 4}| \tag{22}$$

$$0.5\cos^2(2\theta_{e\mu})\sin^2\alpha = |U_{e4}|^2|U_{\mu 4}|^2\{(1-r)^2 + a[(1-r)^2 + 4r\sin^2\beta]\}. \tag{23}$$

The CP odd parameter β would have the same definition as in that model. Note that α may be as small as 0 in the case where $r = 1$ and $a = 0$, for an arbitrary value of β.

V. LSND AND MINIBOONE

The liquid scintillator neutrino detector (LSND) experiment at Los Alamos [41] has reported statistically significant (3.8σ) evidence for electron antineutrinos in a beam produced by the decay of μ^+ at rest, consistent with oscillations of antimuon neutrinos. The MiniBooNE experiment at Fermilab [42–44] which has a similar range of L/E to LSND, has searched for muon to electron neutrino and antineutrino appearance. The MiniBooNE electron neutrino appearance results showed no excess in the preferred analysis region but do show an excess at lower energies. The MiniBooNE antineutrino data shows an excess which is consistent with a neutrino oscillation interpretation of the LSND signal, and which is poorly fit by background. The KARMEN experiment [70] also searched for antielectron neutrino appearance in an antimuon neutrino beam, at values of L/E ranging form 0.36 to 0.74, and saw no excess, giving a 90% C.L. on the oscillation probability in this region of 0.0017. Several experiments have searched for muon to electron neutrino conversion at very short baseline with results consistent with 0 [64,65], with the NOMAD experiment providing the strongest constraint.

In order to test whether Eq. (20) could account for the LSND and MiniBooNE results, I have taken the oscillation probabilities given in Ref. [44] for electron neutrino and antineutrino appearance as a function of L/E to construct a χ^2 function,

$$\chi^2(\theta_{\mu e}, \Delta m^2, \alpha, \beta) = \sum_i \frac{[P_i^{\text{theory}}(\theta_{\mu e}, \Delta m^2, \alpha, \beta) - P_i^{\text{exp}}]^2}{\sigma_i^2}, \tag{24}$$

where P_i^{exp} represents the oscillation probability for bin i extracted from experimental results, $P_i^{\text{theory}}(\theta_{\mu e}, \Delta m^2, \alpha, \beta)$ is given by Eq. (20), averaged over the range of L/E included in bin i, and σ_i is the experimental error. I include eight bins for LSND, and nine bins each for MiniBooNE neutrinos, and for MiniBooNE antineutrinos. I do not use the MiniBooNE data for $E < 400$ MeV ($L/E > 1.37$ m/MeV) because of the large systematic error, which should be correlated, as I do not have access to the correlation data. Inclusion of these points with the systematic error included and treated as uncorrelated makes little difference in the fits, but is not justifiable. I also include in the fit a bin for KARMEN, and a bin for NOMAD, with the experimental errors chosen to correspond to the 90% upper bound on the average oscillation probability. The total number of fit points included is 28, and there are four free parameters.

The best fit point has $\Delta m^2 = 0.40$ eV^2, $\sin^2(2\theta_{\mu e}) = 0.0083$, $\beta = -0.123$, $\alpha = 0$, and a total χ^2 of 24.14 for 24 degrees of freedom. A nearly equally good fit may be obtained for any Δm^2 in the range from 0.02 to 0.60 eV^2. The fit prefers a nonzero value for the CP violating parameter β. In Fig. 1, I show the lowest χ^2 obtainable for a given value of Δm^2, with and without the $\beta = 0$ constraint. The value of χ^2 at $\theta_{\mu e} = \alpha = 0$ (no flavor change) is 45.6.

In Fig. 2, I show the LSND and MiniBooNE electron antineutrino appearance probabilities and the MiniBooNE electron neutrino appearance probabilities as a function of L/E, together with the curves from four points within the preferred region. Also shown are the constraints from the shorter baseline experiments NOMAD and KARMEN, which did not observe any excess. Note that Eq. (20) appears to give a good fit to all the data on electron neutrino or antineutrino appearance at MiniBooNE and LSND, while being compatible with KARMEN and NOMAD, for a wide range of masses. In Fig. 3, I show the region in the Δm^2 and $\theta_{\mu e}$ plane where the χ^2 goodness of fit test is within a factor of 10 of the best value (χ^2 less than 37.7) for four different assumptions about the α and β parameters. Because the data has been extracted from the published plots without including information about correlations these results should be taken as indicative of the

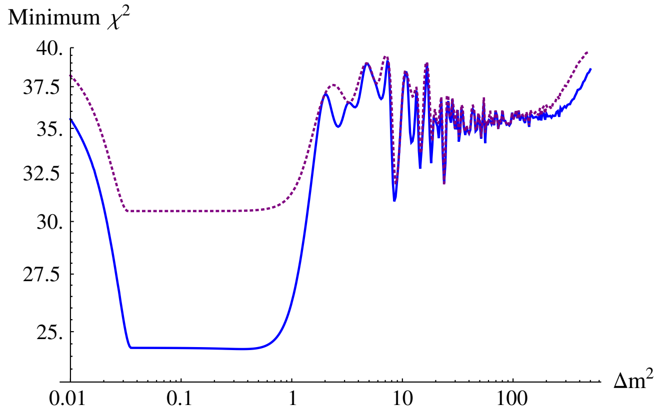

FIG. 1 (color online). The minimum value of the χ^2 function described in the text as a function of the Δm^2_{41} mass squared difference in eV^2. The solid blue line shows the minimum with $\theta_{\mu e}$, α, and β chosen to minimize χ^2. The dotted purple line shows the minimum with $\theta_{\mu e}$, and α chosen to minimize χ^2 and the *CP* violating parameter β set to 0, showing that the best fit region has a mass squared difference between 0.02 and 0.60 eV^2 and nonvanishing *CP* violation.

preferred values rather than as a definitive constraint region.

Note that the inclusion of the α and β parameters has little effect on the best fit values of Δm^2, but greatly increases the preferred region for $\theta_{\mu e}$, allowing the effective mixing angle than can give sufficient $\bar{\nu}_e$ appearance to be much smaller than in a $3+1$ model. I do not show the

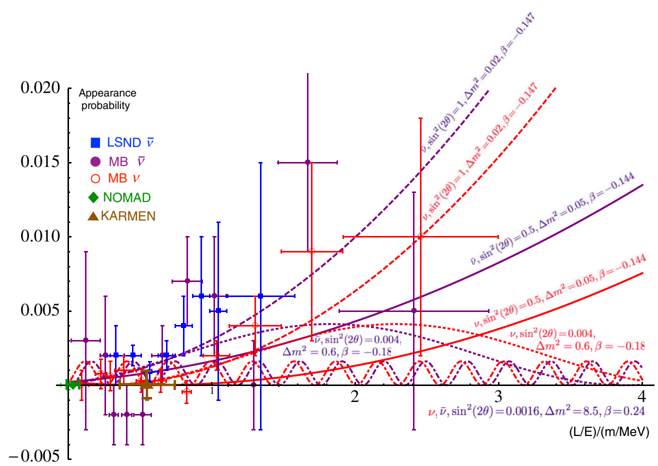

FIG. 2 (color online). Electron flavor appearance probability against L/E in a muon neutrino or antineutrino beam for the four different parameter values indicated, all of which are in the preferred region. In all cases shown the parameter α has been set to zero. The neutrino appearance probabilities are shown in red (medium gray) and the antineutrino probabilities in purple (dark gray). The neutrino and antineutrino probabilities differ for the same parameters due to *CP* violation. Also shown are the probabilities extracted from the MiniBooNE neutrino and antineutrino data, LSND, KARMEN, and NOMAD.

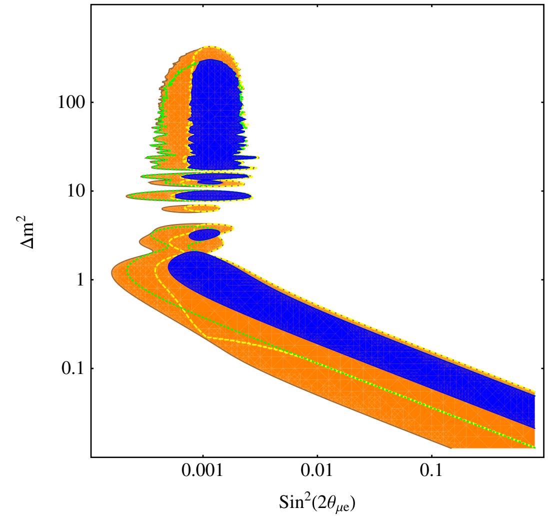

FIG. 3 (color online). The $\chi^2 \leq 37.7$ region for the Δm^2_{41} mass squared difference and effective mixing angle is shown in orange (medium gray), with α and β chosen to minimize χ^2. Also shown in blue (dark gray) is the preferred region with the parameters α and β set to 0, which corresponds to a $3+1$ neutrino model. The green (light gray) dotted line shows the preferred region when $\beta = 0$ (no *CP* violation) and α chosen to minimize χ^2, while the yellow (very light gray) dashed line shows the preferred region when $\alpha = 0$ and β chosen to minimize χ^2.

constraints on the allowed region from disappearance only experiments, since, as discussed in the previous sections, these depend on U_{e5} and $U_{\mu 5}$, which are only weakly constrained for values of m_5 greater than $\sqrt{1000\ \mathrm{eV}^2} \sim 32$ eV. I conclude that the inclusion of *CP* violation and nonunitarity in the $3+1$ dimensional mixing matrix allows for mixing of the three active neutrinos with a sub-eV mass sterile neutrino to fit all the short baseline electron flavor appearance data without necessarily conflicting with muon and electron neutrino disappearance data. Previous attempts [45–47] to fit the short baseline data with oscillations among three active plus two sterile neutrinos were not able to obtain as good a fit because only mass squared differences of less than 1000 eV^2 were considered, for which the values of U_{e5} and $U_{\mu 5}$ are more constrained. As an existence proof of the possibility of a good fit to both appearance and disappearance experiments, consider the point $\Delta m^2 = 0.40\ \mathrm{eV}^2$, $\sin^2(2\theta_{\mu e}) = 0.0083$. The point could result from a fifth neutrino of mass 200 MeV, heavy enough so that it cannot be produced in muon decay or in the decay $\pi \to \mu\nu$. For such a heavy neutrino, for a muon neutrino beam produced in pion decay, the factor a is 0. Taking mixing parameters $|U_{e4}| = |U_{\mu 4}| = |U_{e5}| = |U_{\mu 5}| = 0.032$, and $U_{e5}U^*_{\mu 5}/(U_{e4}U^*_{\mu 4}) = e^{-0.245i}$, gives parameters $r = 1.99$, $\beta = -0.123$, and $\alpha = 2.3 \times 10^{-6}$. These values give a total χ^2 of 24.14 for 24 degrees of freedom for electron neutrino or electron antineutrino short baseline appearance experiments. The effective mixing angle θ_d for the oscillating term in either muon or for electron neutrino disappearance is only $\sin^2(2\theta_d) = 0.004$, well outside any of the exclusion regions for a mass squared difference of 0.40 eV^2 in electron or muon neutrino disappearance. There is also be a small constant decrease in the flux and an even smaller increase in the muon lifetime, however both these effects are too small to cause disagreement between experiment and theory.

VI. SUMMARY

Neutrino oscillation experiments offer an unparalleled window into exotic physics beyond the standard model. A simple extension of the standard model is to add ''sterile'' fermions which are neutral under all gauge interactions. The theoretical motivations for such fermions include grand unified theories, Dirac neutrino masses, the seesaw model of neutrino mass, supersymmetric models, dark matter theories, and exotic hidden sectors. Such fermions could mix with neutrinos, and the mixing angles are not necessarily correlated with neutrino mass. If these exotic fermions are light they can appear in neutrino oscillation experiments as a new state, providing a an additional oscillation length. Even if they are not light, they can affect neutrino oscillations by allowing the mixing matrix among the light states to be nonunitary.

In this paper, I considered the existence of neutral fermions of a wide range of masses, which mix significantly with neutrinos, and showed that in the limit of sensitivity to a single oscillation length the usual two parameter oscillation formula should be generalized to a four parameter formula, which accounts for *CP* violation and for a distance and energy independent component to flavor change.

I also fit the electron neutrino and antineutrino short baseline appearance data to the new formula, and found a good fit to the anomalous LSND and MiniBooNE results. The preferred parameter region has an additional sterile neutrino with a mass squared difference between 0.02 and 0.60 eV^2, and a *CP* violating mixing matrix, which could result from mixing with a second state, with mass between 32 eV and 40 GeV. *CP* violation reconciles the MiniBooNE neutrino results with the MiniBooNE and LSND antineutrino results. Constructive interference with a short distance flavor changing term can enhance the amplitude of the oscillatory term in appearance experiments, allowing for reconciliation of the evidence for neutrino flavor change at LSND and MiniBooNE with the lack of evidence from short baseline disappearance searches.

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