

## World data of $J/\psi$ production consolidate nonrelativistic QCD factorization at next-to-leading order

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We calculate the cross sections of inclusive  $J/\psi$  production in photoproduction and two-photon scattering, involving both direct and resolved photons, and in  $e^+e^-$  annihilation at next-to-leading order within the factorization formalism of nonrelativistic quantum chromodynamics, including the full relativistic corrections due to the intermediate  $^1S_0^{[8]}$ ,  $^3S_1^{[8]}$ , and  $^3P_J^{[8]}$  color-octet states. Exploiting also our previous results on hadroproduction, we perform a combined fit of the respective color-octet long-distance matrix elements to all available high-quality data of inclusive  $J/\psi$  production, from KEKB, LEP II, RHIC, HERA, the Tevatron, and the LHC, comprising a total of 194 data points from 26 data sets.

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The factorization formalism of nonrelativistic quantum chromodynamics (NRQCD) [1] provides a rigorous theoretical framework for the description of heavy-quarkonium production and decay. This implies a separation of process-dependent short-distance coefficients, to be calculated perturbatively as expansions in the strong-coupling constant  $\alpha_s$ , from supposedly universal long-distance matrix elements (LDMEs), to be extracted from experiment. The relative importance of the latter can be estimated by means of velocity scaling rules; i.e., the LDMEs are predicted to scale with a definite power of the heavy-quark ( $Q$ ) velocity  $v$  in the limit  $v \ll 1$ . In this way, the theoretical predictions are organized as double expansions in  $\alpha_s$  and  $v$ . A crucial feature of this formalism is that it takes into account the complete structure of the  $Q\bar{Q}$  Fock space, which is spanned by the states  $n = {}^{2S+1}L_J^{[a]}$  with definite spin  $S$ , orbital angular momentum  $L$ , total angular momentum  $J$ , and color multiplicity  $a = 1, 8$ . In particular, this formalism predicts the existence of color-octet (CO) processes in nature. This means that  $Q\bar{Q}$  pairs are produced at short distances in CO states and subsequently evolve into physical, color-singlet (CS) quarkonia by the nonperturbative emission of soft gluons. In the limit  $v \rightarrow 0$ , the traditional CS model (CSM) is recovered in the case of  $S$ -wave quarkonia. In the case of  $J/\psi$  production, the CSM prediction is based just on the  $^3S_1^{[1]}$  CS state, while the leading relativistic corrections, of relative order  $\mathcal{O}(v^4)$ , are built up by the  $^1S_0^{[8]}$ ,  $^3S_1^{[8]}$ , and  $^3P_J^{[8]}$  ( $J = 0, 1, 2$ ) CO states. The CSM is not a complete theory, as may be understood by noticing that the next-to-leading order (NLO) treatment of  $P$ -wave quarkonia is plagued by uncanceled infrared singularities, which are, however, properly removed in NRQCD.

The test of NRQCD factorization has been identified to be among the most exigent milestones on the roadmap of quarkonium physics at the present time [2]. While, for  $J/\psi$  polarization, comparisons of HERA and Tevatron data with NRQCD predictions, which are not yet fully known at NLO, unravel a rather confusing pattern, the situation is

eventually clearing up for the  $J/\psi$  yield, which is now fully known at NLO in NRQCD for direct photoproduction [3] and hadroproduction [4,5]. In fact, it has been demonstrated [4] that the set of CO LDMEs fitted to transverse-momentum ( $p_T$ ) distributions measured by H1 at HERA I [6] and II [7] and by CDF at Tevatron II [8] also lead to very good descriptions of distributions in the  $\gamma p$  c.m. energy  $W$  and the inelasticity  $z$ , which measures the fraction of  $\gamma$  energy passed on to the  $J/\psi$  meson in the  $p$  rest frame, from HERA [6,7] and of  $p_T$  distributions from RHIC [9] and the LHC [10]. On the other hand, the Tevatron II [8] data alone can only pin down two linear combinations of the three CO LDMEs [5,11], and the fit results of Ref. [5] are incompatible with Ref. [4]. It is the purpose of this Letter, to overcome this highly unsatisfactory situation jeopardizing the success of NRQCD factorization by performing a global fit to all available high-quality data of inclusive unpolarized  $J/\psi$  production, comprising a total of 194 data points from 26 data sets. Specifically, these include  $p_T$  distributions in hadroproduction from PHENIX [9] at RHIC, CDF at Tevatron I [12] and II [8], ATLAS [13], CMS [10], ALICE [14], and LHCb [15] at the LHC;  $p_T^2$ ,  $W$ , and  $z$  distributions in photoproduction from ZEUS [16] and H1 [6] at HERA I and H1 [7] at HERA II; a  $p_T^2$  distribution in two-photon scattering from DELPHI [17] at LEP II; and a total cross section in  $e^+e^-$  annihilation from Belle [18] at KEKB.

Incoming photons participate in the hard scattering either directly or via partons into which they fluctuate (resolve) intermittently, and both modes of interaction contribute at the same order of perturbation theory. Furthermore, the factorization of the collinear singularities associated with the direct photon produces at NLO an explicit logarithmic scale dependence, which is formally compensated by the evolved parton distribution functions (PDFs) of the resolved photon, so that only the combined contribution is physically meaningful. Therefore, we need to extend the theoretical ingredients available from Refs. [3,4] by also treating  $\gamma p \rightarrow J/\psi + X$  with the photon being resolved

and  $\gamma\gamma \rightarrow J/\psi + X$  with none [19], one, or both of the photons being resolved at NLO in NRQCD. We repeat the analysis of Ref. [19], in which the Coulomb singularities were regularized by  $\nu$ , using dimensional regularization as in Refs. [3,4] in order to obtain analytic expressions sufficiently compact for our purposes. We also find it necessary to revisit  $e^+e^- \rightarrow J/\psi + X$  at NLO in NRQCD because the results of Ref. [20] have not yet been verified by an independent calculation, are only available in numerical form, and lack the  ${}^3S_1^{[8]}$  contribution, which comes both with  $X = q\bar{q}$  [21] and  $gg$ . Higher-order corrections to the CSM process  $e^+e^- \rightarrow c\bar{c}[{}^3S_1^{[1]}]gg$  [22], which enters our analysis at NLO, are beyond the order considered here.

The additional analytic calculations proceed along the lines of Refs. [3,4] and are not described here in detail for lack of space. We merely present our master formula based on the factorization theorems of the QCD parton model and NRQCD [1]:

$$d\sigma(AB \rightarrow J/\psi + X) = \sum_{i,j,k,l,n} \int dx_1 dx_2 dy_1 dy_2 f_{i/A}(x_1) f_{k/i}(y_1) f_{j/B}(x_2) \times f_{l/j}(y_2) \langle \mathcal{O}^{J/\psi}[n] \rangle d\sigma(kl \rightarrow c\bar{c}[n] + X), \quad (1)$$

where  $f_{i/A}(x_1)$  is the PDF of parton  $i = g, q, \bar{q}$  in hadron  $A = p, \bar{p}$  or the flux function of photon  $i = \gamma$  in charged lepton  $A = e^-, e^+$ ,  $f_{k/i}(y_1)$  is  $\delta_{ik}\delta(1-y_1)$  or the PDF of parton  $k$  in the resolved photon  $i$ ,  $d\sigma(kl \rightarrow c\bar{c}[n] + X)$  are the partonic cross sections, and  $\langle \mathcal{O}^{J/\psi}[n] \rangle$  are the LDMEs. In the fixed-flavor-number scheme, we have  $q = u, d, s$ . In the case of  $e^+e^-$  annihilation, all distribution functions in Eq. (1) are delta functions. As in Refs. [3,4],  $X$  always contains one hard parton at leading order (LO) and is void of heavy flavors, which may be tagged and vetoed experimentally.

We now describe our theoretical input for our numerical analyses. We set  $m_c = 1.5$  GeV, adopt the values of  $m_e, \alpha$ , and the branching ratios  $B(J/\psi \rightarrow e^+e^-)$  and  $B(J/\psi \rightarrow \mu^+\mu^-)$  from Ref. [23], and use the one-loop (two-loop) formula for  $\alpha_s^{(n_f)}(\mu_r)$ , with  $n_f = 4$  active quark flavors, at LO (NLO). As for the proton PDFs, we use set CTEQ6L1 (CTEQ6M) [24] at LO (NLO), which comes with an asymptotic scale parameter of  $\Lambda_{\text{QCD}}^{(4)} = 215$  MeV (326 MeV). As for the photon PDFs, we employ the best-fit set AFG04\_BF of Ref. [25]. We evaluate the photon flux function using Eq. (5) of Ref. [26], with the upper cutoff on the photon virtuality  $Q^2$  chosen as in the considered data set. As for the CS LDME, we adopt the value  $\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}) \rangle = 1.32$  GeV<sup>3</sup> from Ref. [27]. Our default choices for the renormalization, factorization, and NRQCD scales are  $\mu_r = \mu_f = m_T$  and  $\mu_\Lambda = m_c$ , respectively, where  $m_T = \sqrt{p_T^2 + 4m_c^2}$  is the  $J/\psi$  transverse mass. The bulk of the theoretical uncertainty is due to the

lack of knowledge of corrections beyond NLO, which are estimated by varying  $\mu_r, \mu_f$ , and  $\mu_\Lambda$  by a factor of 2 up and down relative to their default values.

We exclude from our fit all data points of photoproduction and two-photon scattering with  $p_T < 1$  GeV and of hadroproduction with  $p_T < 3$  GeV, which cannot be successfully described by our fixed-order calculations as expected. This leaves a total of 194 data points. The fit results for the CO LDMEs obtained at NLO in NRQCD with default scale choices are collected in Table I. They depend only feebly on the precise locations of the  $p_T$  cuts. In the following, we use the values of Table I throughout.

In Figs. 1 and 2(a), all data sets fitted to, except the single data point from Belle [18], are compared with our default NLO NRQCD results (solid lines). For comparison, also the default results at LO (dashed lines) as well as those of the CSM at NLO (dot-dashed lines) and LO (dotted lines) are shown. The yellow and blue (shaded) bands indicate the theoretical errors on the NLO NRQCD and CSM results. We observe from Fig. 1 that the experimental data are nicely described by NLO NRQCD, being almost exclusively contained within its error bands, while they overshoot the NLO CSM predictions typically by 1–2 orders of magnitude for hadroproduction and a factor of 3–5 for photoproduction. The description of the  $z$  distributions in photoproduction by NLO NRQCD significantly benefits from two features, rendering it considerably more favorable than in Refs. [3,4]. On the one hand, as conjectured in Refs. [3,4], resolved photoproduction usefully enhances the cross section in the low- $z$  range, being dominant for  $z \lesssim 0.25$ , as is evident from Fig. 2(b). On the other hand, owing to the negative value of  $\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]}) \rangle$  in Table I, the  ${}^1S_0^{[8]}$  and  ${}^3P_J^{[8]}$  contributions interfere destructively thus attenuating the familiar rise in cross section in the limit  $z \rightarrow 1$ , as may be seen from Fig. 2(c). As for the  $p_T^2$  and  $W$  distributions in photoproduction, the cut  $z > 0.3$  (0.4) applied by H1 (ZEUS) greatly suppresses resolved photoproduction, to the level of 1%. In contrast to the LO analysis of Ref. [28], the DELPHI [17] data tend to systematically overshoot the NLO NRQCD result, albeit the deviation is by no means significant in view of the sizeable experimental errors. As is evident from Fig. 2(d), this may be attributed to the destructive interference of the  ${}^1S_0^{[8]}$  and  ${}^3P_J^{[8]}$  contributions mentioned above, which is a genuine NLO phenomenon. We have to bear in mind, however, that the DELPHI measurement comprises only 16 events with  $p_T > 1$  GeV and has not been confirmed by any of the other three LEP II experiments. In two-photon scattering at

TABLE I. NLO fit results for the  $J/\psi$  CO LDMEs.

$\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}) \rangle$	$(4.97 \pm 0.44) \times 10^{-2}$ GeV <sup>3</sup>
$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \rangle$	$(2.24 \pm 0.59) \times 10^{-3}$ GeV <sup>3</sup>
$\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]}) \rangle$	$(-1.61 \pm 0.20) \times 10^{-2}$ GeV <sup>5</sup>

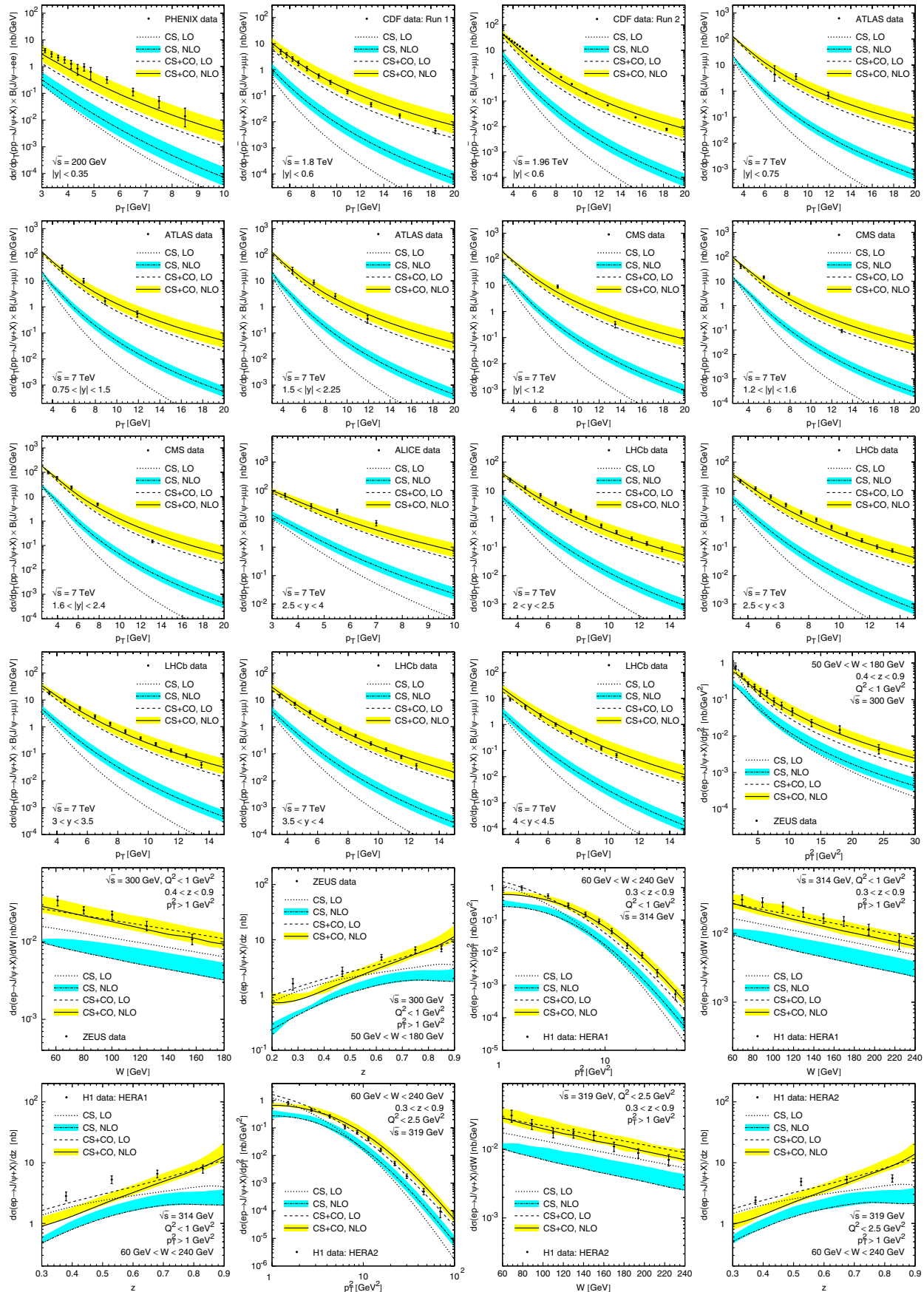


FIG. 1 (color online). NLO NRQCD fit compared to RHIC [9], Tevatron [8,12], LHC [10,13–15], and HERA [6,7,16] data.

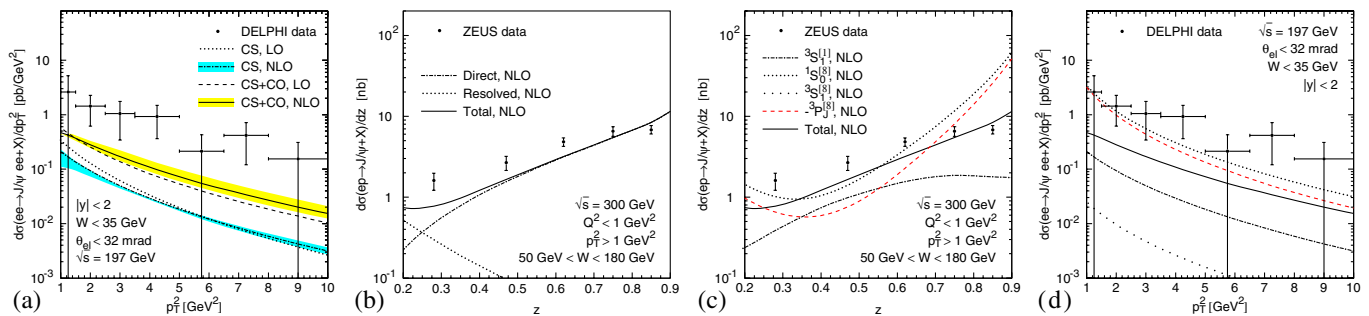


FIG. 2 (color online). (a) NLO NRQCD fit compared to LEP II [17] data and (d) its decomposition to  $c\bar{c}[n]$  channels. Decomposition of  $z$  distribution at HERA I [16] (b) to contributions due to direct and resolved photoproduction and (c) to  $c\bar{c}[n]$  channels.

LEP II, the single-resolved contribution vastly dominates over the direct and double-resolved ones, as was already observed for the LO case in Ref. [28]. The Belle measurement,  $\sigma(e^+e^- \rightarrow J/\psi + X) = (0.43 \pm 0.13)$  pb, is compatible both with the NLO NRQCD and CSM results,  $(0.70^{+0.35}_{-0.17})$  pb and  $(0.24^{+0.20}_{-0.09})$  pb, respectively; at LO, where  $X = g$ , we are dealing with a pure CO process, with a total cross section of 0.23 pb. The overall goodness  $\chi^2_{d.o.f.} = 857/194 = 4.42$  of our NLO NRQCD fit, which we quote for completeness, is of limited informative value, since the theoretical uncertainties exceed most of the experimental errors.

Our theoretical results refer to direct  $J/\psi$  production, as the data from Tevatron I [12] do, while the data from KEKB [18], Tevatron II [8], and LHC [10,13–15] comprise prompt events and those from LEP II [17], HERA [6,7,16], and RHIC [9] even nonprompt ones. However, the resulting error is small against our theoretical uncertainties and has no effect on our conclusions. In fact, the fraction of  $J/\psi$  events originating from the feed-down of heavier charmonia only amounts to about 36% [12] for hadroproduction, 15% [7] for photoproduction at HERA, 9% for two-photon scattering at LEP II [19], and 26% for  $e^+e^-$  annihilation at KEKB [22], and the fraction of  $J/\psi$  events from  $B$  decays is negligible RHIC, HERA [7], and LEP II [19] energies. Refitting the data with the estimated feed-down contribution subtracted yields  $\langle\mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle = (3.04 \pm 0.35) \times 10^{-2} \text{ GeV}^3$ ,  $\langle\mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle = (1.68 \pm 0.46) \times 10^{-3} \text{ GeV}^3$ , and  $\langle\mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle = (-9.08 \pm 1.61) \times 10^{-3} \text{ GeV}^5$  with a slightly reduced value  $\chi^2_{d.o.f.} = 725/194 = 3.74$ .

In summary, we carried out the first complete NLO NRQCD calculations of the inclusive  $J/\psi$  yields in photoproduction and two-photon scattering, including also resolved photons, so as to establish factorization scale invariance, and in  $e^+e^-$  annihilation, including also the  $^3S_1^{[8]}$  channel. Exploiting also our previous results [3,4], we then performed the first NLO NRQCD analysis of the world's high-quality data of inclusive unpolarized  $J/\psi$  production, from KEKB [18], LEP II [17], RHIC [9], HERA I [6,16] and II [7], Tevatron I [12] and II [8], and the LHC [10,13–15], comprising a total of 194 data points from 26 data sets. The fit values of the CO LDMEs in Table I agree with our previous ones [4], extracted just from the  $p_T$  distributions of Refs. [6–8], within the errors of the latter, but the new errors are about 40% smaller, rendering the disagreement with the fit results of Ref. [5] more pronounced. In compliance with the velocity scaling rules of NRQCD [1], these values are approximately of order  $\mathcal{O}(v^4)$  relative to  $\langle\mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle$ . In the case of photoproduction, the inclusion of resolved photons reconciles HERA data [6,7,16] with NLO NRQCD in the low- $z$  regime [see Fig. 2(b)]. Furthermore, the notorious  $z \rightarrow 0$  crisis is greatly mitigated thanks to a strong cancellation between the  $^1S_0^{[8]}$  and  $^3P_0^{[8]}$  contributions [see Fig. 2(c)]. Our findings manifestly consolidate the verification of NRQCD factorization for charmonium and provide rigorous evidence for LDME universality and the existence of CO processes in nature.

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