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# Higgs-strahlung and pair production in the $e^+e^-$ collision in the noncommutative standard model

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The Higgs-strahlung process $e^+e^- \to ZH$ and pair production process $e^+e^- \to HH$ are studied in the framework of the minimal noncommutative (NC) standard model. In particular, the Feynman rules involving all orders of the noncommutative parameter $\theta$ are derived using reclusive formation of the Seiberg-Witten map. It is shown that the total cross section and angular distribution can be significantly affected because of space-time noncommutativity when the collision energy exceeds to 1 TeV. It is found that in each process, there is an optimal collision energy ($E_{\rm oc}$) for achieving the greatest noncommutative effect, and $E_{\rm oc}$ varies linearly with the NC scale $\Lambda_{\rm NC}$. A brief discussion on the process $e^+e^- \to \mu^+\mu^-$ is also given.

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## I. INTRODUCTION

In string theory, noncommutative (NC) space-time appears naturally in $D$-brane dynamics in the low-energy limit [1–3]. It is generally believed that the stringy effect can only be observed at the Plank scale $M_P$, which is at far from detectable. However, given the possibility [4,5] that the large hierarchy between the gravitational scale $M_P$ and the weak scale $M_W$ can be narrowed down to a few TeV, one can expect to see the NC effect predicted by the noncommutative field theory (NCQFT) at around 1 TeV. The noncommutative space-time can be characterized by the coordinate operators satisfying

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu} = \frac{ic_{\mu\nu}}{\Lambda^2_{\rm NC}}, \tag{1}$$

where the matrix $\theta_{\mu\nu}$ in Eq. (1) is constant, antisymmetric and real. The elements of the dimensionless constant matrix $c_{\mu\nu}$ are assumed to be of order unity and $\Lambda_{\rm NC}$ represents the NC scale, having the dimension of inverse mass. NCQFT can be constructed through Weyl correspondence, where the ordinary product of fields is replaced by the Moyal-Weyl star product [3]

$$(f \star g)(x) = \exp\left(\frac{1}{2}\theta_{\mu\nu}\partial_{x^\mu}\partial_{y^\nu}\right)f(x)f(y)|_{y=x}. \tag{2}$$

Using this method, high-energy processes of quantum electrodynamics in noncommutative space-time (NCQED) have been extensively studied [5,6]. An interesting consequence is the raising of triple and 4-point photon vertex in NCQED analogous to the Yang-Mills gauge theory. However, some obstructions such as charge quantization [7] and no-go theorem [8] must be considered if one in tends to build an arbitrary gauge theory. Up to now there are two versions of the noncommutative standard model (NCSM). One is that the gauge group is restricted to $U(3) * U(2) * U(1)$ [9]. In this case, however, additional heavy gauge bosons and a delicate Higgs mechanism have to be introduced in order to remove two extra $U(1)$ factors. Another is a minimal version of the noncommutative standard model (mNCSM) [10], in which the group closure property is still valid when one generalizes the $SU(3) * SU(2) * U(1)$ Lie algebra gauge theory to the enveloping algebra value using the Seiberg-Witten map (SWM) method [3].

The SWM means that both the matter fields $\hat{\psi}$ and the gauge fields $\hat{A}_\mu$ in noncommutative space-time can be expanded in terms of the commutative ones as power series in $\theta$,

$$\hat{\psi}(x, \theta) = \psi(x) + \theta\psi^{(1)} + \theta^2\psi^{(2)} + \dots \tag{3}$$

$$\hat{A}_\mu(x, \theta) = A_\mu(x) + \theta A^{(1)}_\mu + \theta^2 A^{(2)}_\mu + \dots \tag{4}$$

The striking feature of mNCSM is that it predicts new physics which are not only the noncommutative correction of particle vertices but also new interactions beyond the SM in ordinary space-time. This attracts many authors to focus attention on the noncommutative phenomenology of particles based on mNCSM. Recently, several high-energy processes such as $e^-e^- \to e^+e^-$ (Moller), $e^+e^- \to e^+e^-$ (Babaha) [11], $e^+e^- \to \gamma\gamma$ [12], $e^+e^- \to \mu^+\mu^-$ [13] and neutrino-photon scattering [14] have been investigated in the context of mNCSM. The possibility to detect the NC effect though SM forbidden decay such as $Z \to \gamma\gamma$ [15], $J/\psi \to \gamma\gamma$ and $K \to \pi\gamma$ [16] has also been explored by many authors in order to obtain a lower $\Lambda_{\rm NC}$ constraint.

Most of the existing analysis is only up to the first $\theta$ order. It is necessary to examine higher-order contributions since in future colliders the center mass energy can be comparable or even exceed the NC scale. In a recent work [17], the authors pointed out that an incorrect $\Lambda_{\rm NC}$ lower bound could be obtained from ultra-high-energy cosmic ray experiments if one simply expands the noncommutative interaction term to the linear order. To overcome this, the $\theta$-exact expression of SWM was derived by

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directly solving the gauge equivalence relation and applied to ultra-high-energy neutrino processes [18].

On the other hand, the Higgs boson, although not yet observed, can play an important role in electroweak spontaneous symmetry breaking (SSB) through which the gauge boson can have mass. The LEP2 experiment gives a lower bound of 114.4 GeV [19] and if we take the global electroweak fit into account, the Higgs mass should be no more than 200 GeV [20]. Recently, the CDF and D0 Collaborations at the Tevatron excluded the Higgs boson in the range between 158 GeV and 175 GeV at 95% confidence level [21]. It is believed that colliders such as the LHC and the planned International Linear Collider (ILC) will help people to prove or exclude the existence of the Higgs boson.

It is interesting to see if new physics can appear along with the Higgs boson. The possibility has already been extensively discussed in many theories beyond the SM. In this paper, we explore the Higgs-strahlung process: $e^+e^- \to HZ$, and the pair production process: $e^+e^- \to HH$ in the framework of the mNCSM. The later channel is forbidden in the ordinary SM and has been studied recently [22] in the linear $\theta$ order. However, the results of Ref. [22] are not valid when the on-shell condition is applied. In Sec. II, the $n$-th order SWM solution is given as a recursive formulation from the Seiberg-Witten differential equation. Although the resulting expression is lengthy, most terms are not relevant to the interaction considered, thus allowing us to derive the full-$\theta$ expression for the fermion, gauge boson, and Higgs boson. In Sec. III, we give the scattering amplitudes of $e^+e^- \to HZ$ and $e^+e^- \to HH$. We shall also briefly discuss the process $e^+e^- \to \mu^+\mu^-$. Numerical analysis of the total cross section and azimuthal angular distribution of the cross section are presented in Sec. IV. Finally, we summarize and discuss our results in Sec. V.

## II. SEIBERG-WITTEN MAPS AND NONCOMMUTATIVE STANDARD MODEL

SWM relates the noncommutative fields to their counterpart in ordinary space-time. When the limit $\theta \to 0$ is taken, the noncommutative fields reduce to the ordinary ones in commutative space-time. SWM can be derived as perturbative solutions of the gauge equivalence relation order by order. It is shown in Ref. [23] that the $n$-th order SWM can also be obtained from a differential equation introduced by Seiberg and Witten [3]. The SW-differential equation of the gauge field $\hat{V}_\mu$ is [3,13]

$$\delta\theta^{\kappa\lambda}\frac{\partial \hat{V}_\mu}{\partial\theta^{\kappa\lambda}} = -\frac{1}{4}\delta\theta^{\kappa\lambda}\{\hat{V}_\kappa, \partial_\lambda \hat{V}_\mu + \hat{F}_{\lambda\mu}\}_*, \quad (5)$$

and that of the fermion fields $\hat{\Psi}$ is [23]

$$\delta\theta^{\kappa\lambda}\frac{\partial \hat{\Psi}}{\partial\theta^{\kappa\lambda}} = -\frac{1}{4}\delta\theta^{\kappa\lambda}\hat{V}_\kappa * (\partial_\lambda\hat{\Psi} + \hat{D}_\lambda\hat{\Psi}), \quad (6)$$

which can be derived by changing $\theta$ to $\theta + \delta\theta$. After inserting Eqs. (5) and (6) to the Taylor expansions of the NC fields, the $n$-th order solution can be obtained. Here we list the results given in Refs. [23,24],

$$\hat{V}_\mu^{(n+1)} = -\frac{1}{4(n+1)}\theta^{\kappa\lambda}\sum_{\alpha+\beta+\gamma=n}\{\hat{V}_\kappa^{(\alpha)}, \partial_\lambda\hat{V}_\mu^{(\beta)} + \hat{F}_{\lambda\mu}^{(\beta)}\}_{*^{(\gamma)}}. \quad (7)$$

$$\hat{\Psi}^{(n+1)} = -\frac{1}{4(n+1)}\theta^{\kappa\lambda} \times \sum_{\alpha+\beta+\gamma=n}\hat{V}_\kappa^{(\alpha)} *^{(\gamma)} (\partial_\lambda\hat{\Psi}^{(\beta)} + (D_\lambda\hat{\Psi})^{(\beta)}). \quad (8)$$

Following Ref. [10], the fermion, and Higgs and Yukawa sectors of mNCSM are

$$S_{\rm fermions} = \int d^4x\sum_{i=1}^{3}(\bar{\hat{l}}_L^{(i)} * (i\hat{\not{D}}\hat{l}_L^{(i)}) + \bar{\hat{Q}}_L^{(i)} * (i\hat{\not{D}}\hat{Q}_L^{(i)}) + \bar{\hat{l}}_R^{(i)} * (i\hat{\not{D}}\hat{l}_R^{(i)}) + \bar{\hat{u}}_R^{(i)} * (i\hat{\not{D}}\hat{u}_R^{(i)}) + \bar{\hat{d}}_R^{i} * (i\hat{\not{D}}\hat{d}_R^{(i)})) \quad (9)$$

$$S_{\rm Higgs} = \int d^4x[(\hat{D}_\mu\hat{\Phi})^\dagger * (\hat{D}^\mu\hat{\Phi}) - \mu^2\hat{\Phi}^\dagger * \hat{\Phi} - \lambda\hat{\Phi}^\dagger * \hat{\Phi} * \hat{\Phi}^\dagger * \hat{\Phi}] \quad (10)$$

$$S_{\rm Yukawa} = -\int d^4x\sum_{i,j=1}^{3}[C_l^{(ij)}(\bar{\hat{l}}_L^{(i)} * \hat{\Phi}_l * \hat{e}_R^{(j)}) + C_l^{\dagger(ij)}(\bar{\hat{e}}_R^{(i)} * \hat{\Phi}_l^\dagger * \hat{l}_L^{(j)}) + C_u^{(ij)}(\bar{\hat{Q}}_L^{(i)} * \hat{\Phi}_u^c * u_R^{(j)}) + C_u^{\dagger(ij)}(\bar{\hat{u}}_R^{(i)} * \hat{\Phi}_u^{c\dagger} * \hat{Q}_L^{(j)}) + C_d^{(ij)}(\bar{\hat{Q}}_L^{(i)} * \hat{\Phi}_d * \hat{d}_R^{(j)}) + C_d^{\dagger(ij)}(\bar{\hat{d}}_R^{(i)} * \hat{\Phi}_d^\dagger * \hat{Q}_L^{(j)})] \quad (11)$$

with

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}, \qquad l = \begin{pmatrix} \nu \\ e \end{pmatrix}, \qquad \Phi^c = i\tau_2\Phi^*, \quad (12)$$

where $\nu$, $e$, $u$, $d$, $l$ and $Q$ stand for the neutrinos, charged leptons, up-type quarks, down-type quarks, lepton doublets and quark doublets, respectively, for three generations. (To avoid confusion, we denote electron by $e^-$), and the subscripts $L$ and $R$ stand for the left- and right-hand, respectively.) The expression given above is the same as the SM in ordinary space-time, except for the replacement of the ordinary fields by corresponding NC fields and substitution of the ordinary product by the star products [10]. Note that $\hat{\Phi}$ and $\hat{\Phi}_Y$ ($Y = l, u, d$) are the noncommutative Higgs fields in the free and Yukawa sectors, respectively. The NC Higgs field $\hat{\Phi}_Y$ transforms under two different gauge groups. The corresponding gauge potentials $\hat{V}_\mu$ and $\hat{V}'_\mu$ inherited from the fermions on the right and left of the Higgs fields in Yukawa sector. Thus, the SWM of $\hat{\Phi}_Y$ has a hybrid feature and is given by

$$\hat{\Phi}_Y \equiv \hat{\Phi}[\Phi.V, V'] = \Phi + \frac{1}{2}\theta^{\kappa\lambda}V_\lambda\Big(\partial_\kappa\Phi - \frac{i}{2}(V_\kappa\Phi - \Phi V'_\kappa)\Big) + \frac{1}{2}\theta^{\kappa\lambda}\Big(\partial_\kappa\Phi - \frac{i}{2}(V_\kappa\Phi - \Phi V'_\kappa)\Big)V'_\lambda + \mathcal{O}(\theta^2). \tag{13}$$

The hybrid SWM guarantees the equivalence of covariant transformation between the noncommutative and ordinary fields, which means

$$\delta_{\lambda,\lambda'}\hat{\Phi}_Y[\Phi_Y, V, V'] = i\hat{\Lambda} * \hat{\Phi}_Y - i\hat{\Phi}_Y * \hat{\Lambda}', \tag{14}$$

where $\hat{\Lambda}$, $\hat{\Lambda}'$ are noncommutative gauge parameters corresponding to their ordinary counterparts ($\lambda$ and $\lambda'$). In Ref. [10], the representation of SMW for the NC Higgs field $\hat{\Phi}$ in the Higgs kinetic sector Eq. (10) is $\hat{\Phi} \equiv \hat{\Phi}[\Phi, V_\mu, 0]$, which is chosen to be of the same representation as the standard model. From the point of gauge invariance, however, there is no *a priori* requirement that we must take this simplest representation. In order to explore as much new physics as possible, here we choose a more general SWM expression

$$\hat{\Phi} \equiv \hat{\Phi}[\Phi, V_\mu, V'_\mu] \tag{15}$$

where

$$V_\mu = xg'B_\mu + gW^a_\mu\frac{\sigma^a}{2}, \tag{16}$$

$$V'_\mu = -\Big(\frac{1}{2} - x\Big)g'B_\mu, \tag{17}$$

and $B_\mu$, $W^a_\mu$ and $g'$, $g$ are gauge fields and coupling constants of the $U(1)$ and $SU(2)$ groups, respectively, in the usual space-time. The parameter $x$ introduced here represents the ambiguity of noncommutative $U(1)$ gauge transform which means that the covariant derivative for $\hat{\Phi}$ thus is given by

$$\hat{D}_\mu\hat{\Phi} = \partial_\mu\hat{\Phi} - i\hat{V}_\mu * \hat{\Phi} + i\hat{\Phi} * \hat{V}'_\mu. \tag{18}$$

Clearly, this formulation reduces to the commutative one with right hypercharge in SM if one sets $\theta \to 0$. Now we derive the $n$-th order SWM for the Higgs field. Following Refs. [23,24], we get the SW-differential equation

$$\delta\theta^{\kappa\lambda}\frac{\delta\hat{\Phi}}{\delta\theta^{\kappa\lambda}} = -\frac{1}{2}\delta\theta^{\kappa\lambda}\Big[\hat{V}_\kappa * \Big(\partial_\lambda\hat{\Phi} - \frac{i}{2}(\hat{V}_\lambda * \hat{\Phi} - \hat{\Phi} * \hat{V}'_\lambda)\Big) - \Big(\partial_\kappa\hat{\Phi} - \frac{i}{2}(\hat{V}_\kappa * \hat{\Phi} - \hat{\Phi} * \hat{V}'_\kappa)\Big) * \hat{V}'_\lambda\Big] \tag{19}$$

which can be written as

$$\begin{aligned}\frac{\delta\hat{\Phi}}{\delta\theta^{\kappa\lambda}} = &-\frac{1}{4}\hat{V}_\kappa * \Big(\partial_\lambda\hat{\Phi} - \frac{i}{2}(\hat{V}'_\lambda * \hat{\Phi} - \hat{\Phi} * V'_\lambda)\Big) \\ &+ \frac{1}{4}\hat{V}_\lambda * \Big(\partial_\kappa\hat{\Phi} - \frac{i}{2}(\hat{V}_\kappa * \hat{\Phi} - \hat{\Phi} * \hat{V}'_\kappa)\Big) \\ &- \frac{1}{4}\Big(\partial_\lambda\hat{\Phi} - \frac{i}{2}(\hat{V}_\lambda * \hat{\Phi} - \hat{\Phi} * \hat{V}'_\lambda)\Big) * \hat{V}'_\kappa \\ &+ \frac{1}{4}\Big(\partial_\kappa\hat{\Phi} - \frac{i}{2}(\hat{V}_\kappa * \hat{\Phi} - \hat{\Phi} * \hat{V}'_\kappa)\Big) * \hat{V}'_\lambda.\end{aligned} \tag{20}$$

On the other hand, $\hat{\Phi}$ can be Taylor expanded to up to the $(n+1)$ th order in $\theta$

$$\hat{\Phi}^{n+1} = \hat{\Phi}^{(0)} + \hat{\Phi}^{(1)} + \ldots + \hat{\Phi}^{(n+1)} = \Phi + \sum_{k=1}^{n+1}\frac{1}{k!}\theta^{\mu_1\nu_1}\ldots\theta^{\mu_\kappa\nu_\kappa}\Big(\frac{\partial^k}{\partial\theta^{\mu_1\nu_1}\ldots\partial\theta^{\mu_\kappa\nu_\kappa}}\hat{\Phi}^{(n+1)}\Big)_{\theta=0}. \tag{21}$$

Inserting Eq. (20) into Eq. (21), one can get the recursive solution up to the $n+1$ order

$$\begin{aligned}\hat{\Phi}^{(n+1)} = &-\frac{1}{4(n+1)}\theta^{\kappa\lambda}\sum_{\alpha+\beta+\gamma=n}[\hat{V}^{(\alpha)}_\kappa *^{(\beta)} (\partial_\lambda\hat{\Phi})^{(\gamma)} \\ &+ \hat{V}^{(\alpha)}_\kappa *^{(\beta)} (\hat{D}_\lambda\hat{\Phi})^{(\gamma)} + (\partial_\lambda\hat{\Phi})^{(\alpha)} *^{(\beta)} \hat{V}^{(\gamma)}_\kappa \\ &+ (\hat{D}_\lambda\hat{\Phi})^{(\alpha)} *^{(\beta)} \hat{V}'^{(\gamma)}_\kappa],\end{aligned} \tag{22}$$

where

$$(\hat{D}_\lambda\hat{\Phi})^{(\gamma)} \equiv (\partial_\lambda\hat{\Phi})^{(\gamma)} - i\sum_{m+n+t=\gamma}(\hat{V}^{(m)}_\lambda *^{(n)} \hat{\Phi}^{(t)} - \hat{\Phi}^{(t)} *^{(n)} \hat{V}'^{(m)}_\lambda). \tag{23}$$

The all-expanded expressions of Eqs. (7), (8), and (22) are rather lengthy, so that in many existing works they are limited to the lowest $\theta$ order. We note, however, that for the high-energy process discussed in this paper, the number of the gauge and matter fields taking part in each particle vertex is no more than two. The terms with three or more gauge fields can thus be set to zero and the solutions of SWM can then be written in a compact form

$$\hat{V}^{(\mathrm{eff})}_\mu = V_\mu - \frac{1}{4}\theta^{\kappa\lambda}V_\kappa(\hat{O} + \hat{O}')(\partial_\lambda V_\mu + F_{\lambda\mu}), \tag{24}$$

$$\hat{\Psi}^{(\mathrm{eff})} = \psi - \frac{1}{2}\theta^{\kappa\lambda}V_\kappa\hat{O}(\partial_\lambda\psi), \tag{25}$$

$$\begin{aligned}\hat{\Phi}^{(\mathrm{eff})} = &\Phi - \frac{1}{2}\theta^{\kappa\lambda}V_\kappa\hat{O}(\partial_\lambda\Phi) - \frac{1}{2}V'_\kappa\hat{O}'(\partial_\lambda\Phi) \\ &+ \frac{i}{4}\theta^{\kappa\lambda}(V_\kappa\hat{O}V^\Phi_\lambda)\Phi + \frac{i}{4}\theta^{\kappa\lambda}(V'_\kappa\hat{O}'V^\Phi_\lambda)\Phi \\ &+ \hat{\Phi}(V, V', \partial\Phi),\end{aligned} \tag{26}$$

where

$$\hat{O} \equiv \frac{e^{i/2\theta^{\alpha\beta}\overleftarrow{\partial}_\alpha\vec{\partial}_\beta} - 1}{\frac{i}{2}\theta^{\alpha\beta}\overleftarrow{\partial}_\alpha\vec{\partial}_\beta}, \tag{27}$$

$$\hat{O}' \equiv \hat{O}|_{\theta\to-\theta}, \tag{28}$$

$$V^{\Phi}_\mu \equiv V_\mu - V'_\mu. \tag{29}$$

The superscript "eff" means that we only keep the terms taking part in the process $e^+e^- \to HZ$ and $e^+e^- \to HH$. The last term on the right-hand side of Eq. (26) contains two gauge fields and derivatives of $\Phi$. Inserting Eqs. (24)–(26) into Eqs. (9) and (10) and imposing spontaneous symmetry breaking under the unitary gauge

$$\Phi = \begin{pmatrix} \Phi^\dagger \\ \Phi^0 \end{pmatrix} \xrightarrow{\text{SSB}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} \tag{30}$$

where $v$ is the vacuum expectation value, we derive the relevant vertex and Feynman rules. We cannot give an nonperturbative expression for the term $\hat{\Phi}(V, V', \partial\Phi)$. However, it is easy to verify that the contribution of this term to the interaction under consideration is zero. We show the vertex needed for processes $e^+e^- \to HZ$ and $e^+e^- \to HH$ in Figs. 1–5 where all the gauge boson momenta are ingoing except that for $p_1$ in Fig. 5. The relative Feynman rules are

$$V_1^\mu(p_1, k, p_2) = ie\gamma^\mu e^{i/2p_1\theta p_2} \tag{31}$$

for the photon-charged lepton vertex,

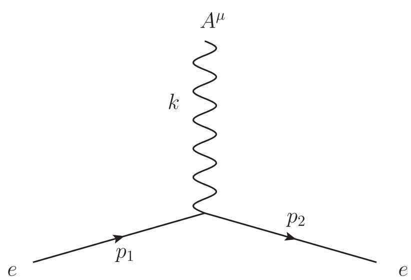


FIG. 1. $\gamma$-$e$-$e$.

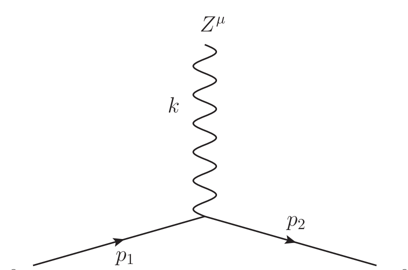


FIG. 2. $Z$-$e$-$e$.

$$V_2^\mu(p_1, k, p_2) = -\frac{ie}{\sin 2\theta_W}\gamma^\mu(C_V - C_A\gamma_5)e^{i/2p_1\theta p_2} \tag{32}$$

for the $Z$ boson-charged lepton vertex,

$$V_3^\mu(p_1, k, p_2) = 2e\left(x - \frac{1}{2}\right)(p_2 - p_1)^\mu \sin\left(\frac{1}{2}p_1\theta p_2\right) \tag{33}$$

for the photon-Higgs-Higgs vertex,

$$V_4^\mu(p_1, k, p_2) = 2e\left[\left(x - \frac{1}{4}\right)\tan\theta_W + \frac{1}{4}\cot\theta_W\right] \times (p_1 - p_2)^\mu \sin\left(\frac{1}{2}p_1\theta p_2\right) \tag{34}$$

for the $Z$ boson-Higgs-Higgs vertex, and

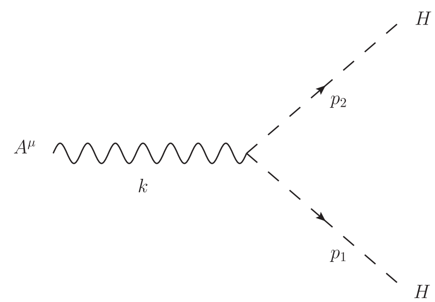


FIG. 3. $\gamma$-$H$-$H$.

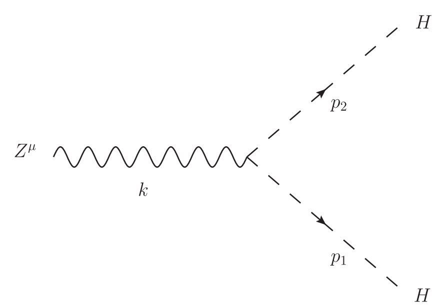


FIG. 4. $Z$-$H$-$H$.

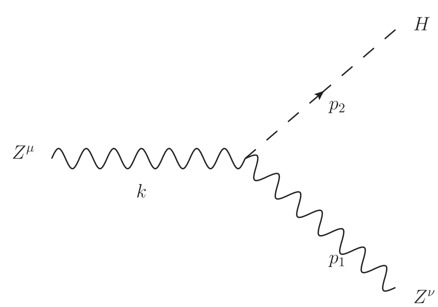


FIG. 5. $Z$-$H$-$Z$.

$$V_5^\mu(p_1, k, p_2) = \frac{iem_Z}{\sin 2\theta_W}\Big[2\cos\Big(\frac{1}{2}p_1\theta p_2\Big)g_{\mu\nu} + \frac{1}{4}((\theta p_2)_\mu p_{1\nu} + (\theta p_2)_\nu k_\mu).\Big(\frac{\cos(\frac{1}{2}p_1\theta p_2) - 1}{p_1\theta p_2}\Big)\Big] \quad (35)$$

for the $Z$ boson-$Z$ boson-Higgs vertex. Here, $C_V = -\frac{1}{2} + 2\sin^2\theta_W$, $C_A = -\frac{1}{2}$, and $\theta_W$ is the Weinberg angle. The masses of the Higgs, $Z$, and $W$ bosons can be written as

$$m_H^2 = -2\mu^2 = 2v^2\lambda, \qquad m_W^2 = \frac{1}{4}v^2g^2, \qquad m_Z^2 = \frac{1}{4}v^2(g^2 + g'^2) = \frac{m_W^2}{\cos^2\theta_W}. \quad (36)$$

Since we are only concerned with the lowest tree-level process, we apply the equations of motion to the particles in the external line, and ignore the terms vanishing due to on-shell condition. It should be mentioned that the Feynman rule for $Z$-$H$-$H$ above is different from the one given in Ref. [22] even at the $\theta$ order. The detailed calculation is in the Appendix. It is found that the Feynman rules in Ref. [22] are not complete, and cannot work for the on-shell condition.

## III. SCATTERING AMPLITUDES IN NCSM

### A. $e^+e^- \to ZH$

The tree-level Feynman diagram is shown in Fig. 6, where the momenta $p_1$ of $Z$ boson external line is outgoing. The process is $s$-channel and proceeds though mediated $Z$ bosons. Using the Feynman rules in Sec. II, the relative amplitude is given by

$$M = \frac{ie^2m_Z}{\sin^2 2\theta_W}\bar{v}(k_2)\gamma_\mu(C_V - C_A\gamma_5)u(k_1) \times \frac{i}{s - m_Z^2 + i\Gamma_Z}\Gamma^{\mu\nu}(p_1, p_2)\epsilon_\nu^*(p_1)e^{i/2k_2\theta k_1}, \quad (37)$$

in which

$$\Gamma^{\mu\nu}(p_1, p_2) = 2\cos\Big(\frac{1}{2}p_1\theta p_2\Big)g^{\mu\nu} + \frac{1}{4}\Big[\frac{\cos(\frac{1}{2}p_1\theta p_2) - 1}{p_1\theta p_2}\Big] \times [(\theta p_2)^\mu p_1^\nu + (\theta p_2)^\nu k^\mu], \quad (38)$$

where $k_1$, $k_2$, $p_1$, and $p_2$ are the four momentums of the electron, positron, Higgs boson, and outgoing $Z$ boson; $s_1$, $s_2$ are spin indices, $s = (k_1 + k_2)^2 = (p_1 + p_2)^2$, and $\Gamma_Z$ is the decay width of the $Z$ boson. We omit the electron (positron) mass in a high energy limit.

### B. $e^+e^- \to HH$

We now give the scattering amplitude of neutral Higgs boson pair production. The corresponding Feynman diagrams of the process are shown in Fig. 7. Different from Higgs-strahlung, this process is forbidden in the ordinary SM. Using the Feynman rules given in Sec. II, we give the following amplitudes,

$$M_\gamma = \frac{2e^2}{s}\Big(x - \frac{1}{2}\Big)\bar{v}(k_2)\gamma^\mu u(k_1)(p_1 - p_2)_\mu \times \sin\Big(\frac{1}{2}p_1\theta p_2\Big)e^{i/2k_2\theta k_1} \quad (39)$$

for $\gamma$ mediated and

$$M_Z = \frac{2e^2}{\sin 2\theta_W}\Big[\Big(x - \frac{1}{4}\Big)\tan\theta_W + \frac{1}{4}\cot\theta_W\Big]\bar{v}(k_2) \times \gamma^\mu(C_V - C_A\gamma_5)u(k_1)\frac{1}{s - m_Z^2 + i\Gamma_Z m_Z} \times (p_1 - p_2)_\mu \sin\Big(\frac{1}{2}p_1\theta p_2\Big)e^{i/2k_2\theta k_1} \quad (40)$$

for $Z$ mediated. The total amplitude is

$$M = M_\gamma + M_Z. \quad (41)$$

### C. $e^+e^- \to \mu^+\mu^-$

Using the SWM expanding to the $\theta^2$ order, the squared-amplitude for $e^+e^- \to \mu^+\mu^-$ up to the $\theta^4$ order was studied in Ref. [13]. An interesting result is that all the contribution from $\theta$, $\theta^2$, and $\theta^3$ terms to the cross section

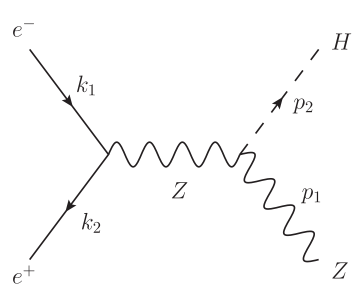


FIG. 6. Feynman diagrams for process $e^+e^- \to ZH$.

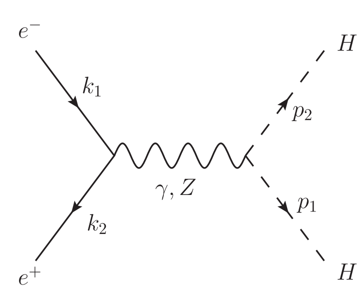


FIG. 7. Feynman diagrams for process $e^+e^- \to HH$.

cancelled out. Can such cancellation also occur in higher-order SWM? Now we can say yes. Using the Feynman rules, the amplitude of $e^+(k_2)e^-(k_1) \to \mu^+(p_1)\mu^-(p_2)$ can be written as

$$\begin{aligned} M &= M_\gamma + M_Z \\ &= \frac{ie^2}{s}\bar{v}(k_2)\gamma_\mu u(k_1)\bar{u}(p_2)\gamma^\mu v(p_1)e^{i/2(k_2\theta k_1+p_2\theta p_1)} \\ &\quad + \frac{ie^2}{\sin^2(2\theta_W)s}\bar{v}(k_2)\gamma_\mu(C_V - C_A\gamma_5)u(k_1)\bar{u}(p_2) \\ &\quad \times \gamma^\mu(C_V - C_A\gamma_5)v(p_1)e^{i/2(k_2\theta k_1+p_2\theta p_1)} \\ &= M_{\rm SM}e^{i/2(k_2\theta k_1+p_2\theta p_1)} \end{aligned} \tag{42}$$

where $M_{\rm SM}$ is the amplitude in the SM. Since the contributions from SWM alone vanish due to the on-shell condition, the NC correction merely appears as phase factors from the Moyal-Weyl product, leading to no net noncommutative effect.

## IV. NC CROSS SECTION AND NUMERICAL ANALYSIS

The differential cross section for the two-body process is given by

$$\frac{d\sigma}{d\cos\theta d\phi} = \frac{1}{64\pi^2 s}|M|^2 \tag{43}$$

where $\theta$ and $\phi$ are polar and azimuthal angles, respectively. Then the NC correction is

$$(\Delta\sigma)_{\rm NC} = \sigma - \sigma_0 \tag{44}$$

where $\sigma_0$ is the total scattering cross section in ordinary space-time. We are also interested in the relative correction:

$$\delta_r = \frac{(\Delta\sigma)_{\rm NC}}{\sigma_0}. \tag{45}$$

In the following analysis, we decompose $c_{\mu\nu}$ into electriclike parts $\vec{\theta}_E = (\theta_{01}, \theta_{02}, \theta_{03})$ and magneticlike parts $\vec{\theta}_B = (\theta_{23}, \theta_{31}, \theta_{12})$, where the vectors $\vec{\theta}_E$ and $\vec{\theta}_B$ are given in Refs. [11,13], i.e., $\vec{\theta}_E = \frac{1}{\sqrt{3}}(\vec{i} + \vec{j} + \vec{k})$, $\vec{\theta}_B = \frac{1}{\sqrt{3}}(\vec{i} + \vec{j} + \vec{k})$.

### A. Cross section and angular distribution of $e^+e^- \to ZH$ in NCSM

In Fig. 8, we show the ordinary total cross section $\sigma_0$ and the NC corrected cross section $\sigma$ as function of the collision energy $E_c(=\sqrt{s})$ for $m_H = 135$ GeV and NC scale $\Lambda_{\rm NC} = 600$ GeV. We can see from the figure that the NC effect significantly suppresses the ordinary total cross section when $E_c$ is high enough. In Table I, we present the relative correction for $E_c = 1000$ GeV and 1500 GeV for different parameters. The $(\Delta\sigma)_{\rm NC}$ as a function of the collision energy is presented in Fig. 9. The curve shows a negative kurtosis distribution that has a maximum correction if the NC scale is fixed. For $\Lambda_{\rm NC} = 600$ GeV, 800 GeV, and 1000 GeV, the $(\Delta\sigma)_{\rm NC}$ reaches its largest correction when the collision energy is at 1500 GeV, 2000 GeV, and 2500 GeV, respectively. It is useful to obtain a relation between the NC scale energy $\Lambda_{\rm NC}$ and the optimal collision energy $E_{\rm oc}$, as shown in Fig. 10. We have

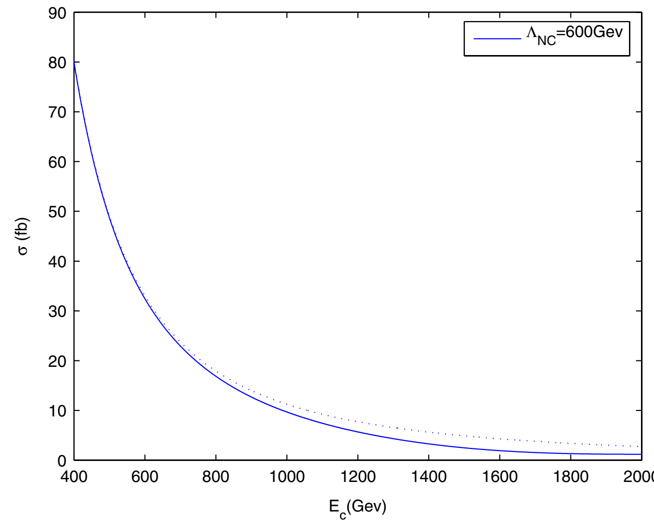


FIG. 8 (color online). The total cross section for $e^+e^- \to ZH$ as a function of $E_c$ in the ordinary SM (dotted line) and mNCSN with $\Lambda_{\rm NC} = 600$ GeV (solid line), $m_H = 135$ GeV.

$$E_{\rm oc} = 2.4986\Lambda_{\rm NC} + 7.9642 \qquad (m_H = 135\ \text{GeV}) \tag{46}$$

$$E_{\rm oc} = 2.4789\Lambda_{\rm NC} + 44.2820 \qquad (m_H = 200\ \text{GeV}). \tag{47}$$

When the Higgs boson mass is accurately measured in the LHC or other devices, the relations given here can provide an effective method for indirectly estimating the NC scale value since it is much easier to determine the peak point of a curve than its inflexion point. Similar relations were obtained in the context of NC QED [25].

TABLE I. The relative correction for the process $e^+e^- \to ZH$ with collision energy $E_c = 1000$ GeV and 1500 GeV; $m_H = 135$ GeV and 200 GeV; $\Lambda_{\rm NC} = 600$ GeV, 800 GeV, and 1000 GeV, respectively.

| $E_C$ (GeV) | $\Lambda_{\rm NC}$ (GeV) | $\delta_r(m_H = 135\ \text{GeV})$ | $\delta_r(m_H = 200\ \text{GeV})$ |
|---|---|---|---|
| 1000 | 600 | 0.1389 | 0.1332 |
| | 800 | 0.0468 | 0.0447 |
| | 1000 | 0.0195 | 0.0186 |
| 1500 | 600 | 0.4913 | 0.4863 |
| | 800 | 0.2161 | 0.2125 |
| | 1000 | 0.0968 | 0.0950 |

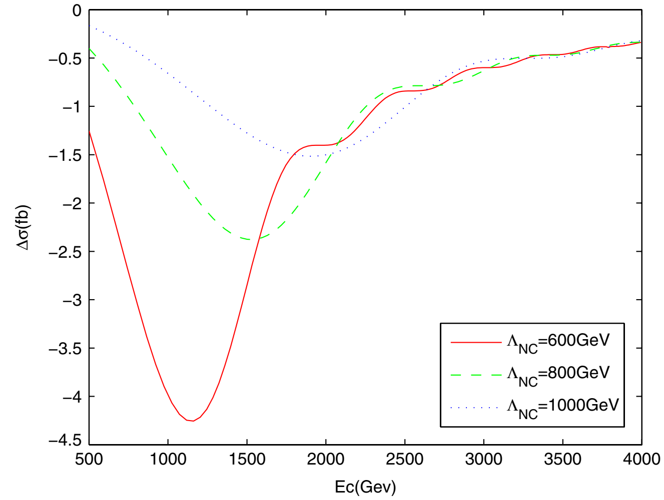


FIG. 9 (color online). The NC correction $(\Delta\sigma)_{\rm NC}$ as a function of $E_c$ for $e^+e^- \to ZH$ with $m_H = 135$ GeV, $\Lambda_{\rm NC} = 600$ GeV, 800 GeV and 1000 GeV, respectively.

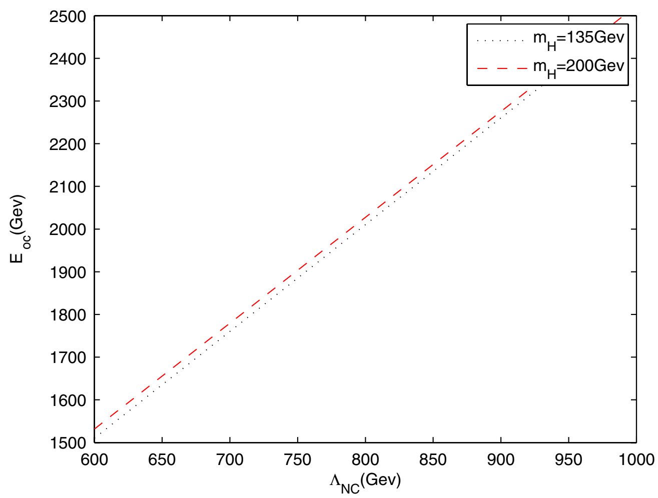


FIG. 10 (color online). The optimal collision energy $E_{\rm oc}$ as a function of NC scale energy $\Lambda_{\rm NC}$ for $e^+e^- \to ZH$ for $m_H =$ 135 GeV and 200 GeV, respectively.

In the mNCSM scenario, one cannot get such a linear relation by simply expanding the Lagrangian to the $\theta$ order. As shown in many papers, in this case the NC scattering cross section changes monotonously when the collision energy is gradually increased.

We show the azimuthal angular distribution $\frac{d\sigma}{d\phi}$ in Fig. 11. Here the collision energy $E_c$ is 1.5 TeV. One can see from the figure that $\frac{d\sigma}{d\phi}$ is anisotropic. This is due to an inherent characteristic of NC space-time. The curves reach their maxima at $\phi = 2.37$ rad and $\phi = 5.51$ rad. The two minima are located at $\phi = 0.80$ rad and $\phi = 3.95$ rad. This unique feature can help us in identifying the NC effect from the other effects.

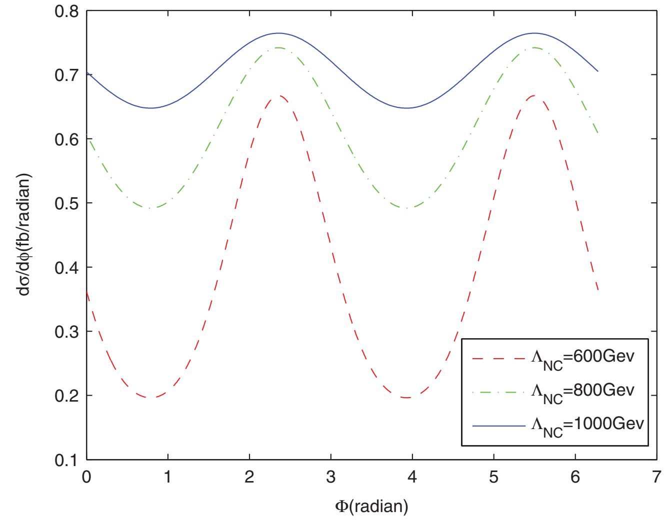


FIG. 11 (color). $\frac{d\sigma}{d\Phi}$ as a function of $\Phi$ for $e^+e^- \to ZH$ for $m_H = 135$ GeV, $\Lambda_{\rm NC} = 600$ GeV, 800 GeV and 1000 GeV.

### B. Cross section and angular distribution of $e^+e^- \to HH$ in NCSM

The neutral Higgs pair production $e^+e^- \to HH$ is forbidden at the tree level in the ordinary standard model. Thus the correction in the cross section is just the cross section itself. The reason why we are particularly interested in the process is that any signal of a SM forbidden process will imply new physics.

For simplicity, we first set $x = \frac{1}{2}$ in Eqs. (16) and (17), which is corresponding to the case as that in Ref. [10], i.e., the process is only $Z$ mediated. The total cross section $\sigma$ as a function of collision energy $E_c(= \sqrt{s})$ is shown in Fig. 12. Here we set $m_H = 135$ GeV with NC scale $\Lambda_{\rm NC} = 600$ GeV, 800 GeV and 1000 GeV. As expected,

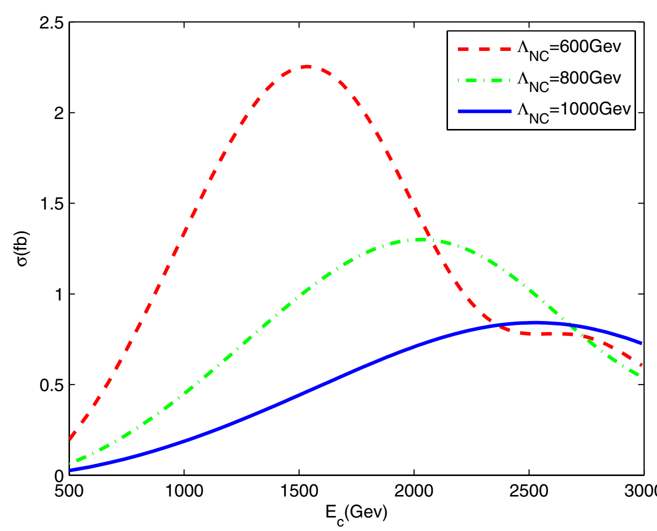


FIG. 12 (color online). The total cross section for $e^+e^- \to HH$ as a function of $E_c$, for $m_H = 135$ GeV, $\Lambda_{\rm NC} = 600$, 800 and 1000 GeV.

a maximum cross section appears. The relative optional collision energy is located at about 1500 GeV, 2000 GeV, and 2500 GeV for the cross sections 2.25 fb, 1.30 fb, and 0.84 fb, respectively.

The relation between $E_{oc}$ and $\Lambda_{NC}$ is given by

$$E_{oc} = 2.4728\Lambda_{NC} + 52.9257 \quad (m_H = 135\,\text{GeV}) \tag{48}$$

$$E_{oc} = 2.4414\Lambda_{NC} + 110.2084 \quad (m_H = 200\,\text{GeV}) \tag{49}$$

as shown in Fig. 13.

The azimuthal angular distribution $\frac{d\sigma}{d\phi}$ is given in Fig. 14 for $m_H = 135$ GeV. The curves are for $\Lambda_{NC} = 600$ GeV, 800 GeV, and 1000 GeV, respectively. The maxima (minima) are at $\phi = 2.36$ rad, 5.50 rad (0.79 rad, 3.93 rad), respectively.

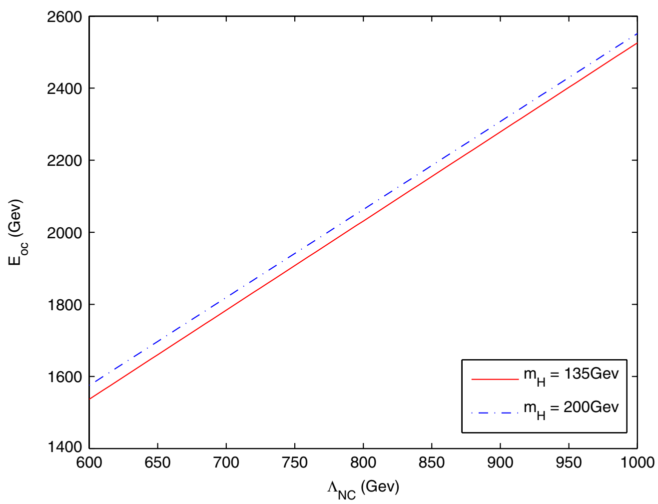


FIG. 13 (color online). The optimal collision energy $E_{oc}$ as a function of NC scale energy $\Lambda_{NC}$ for $e^+e^- \to HH$ for $m_H =$ 135 GeV and 200 GeV.

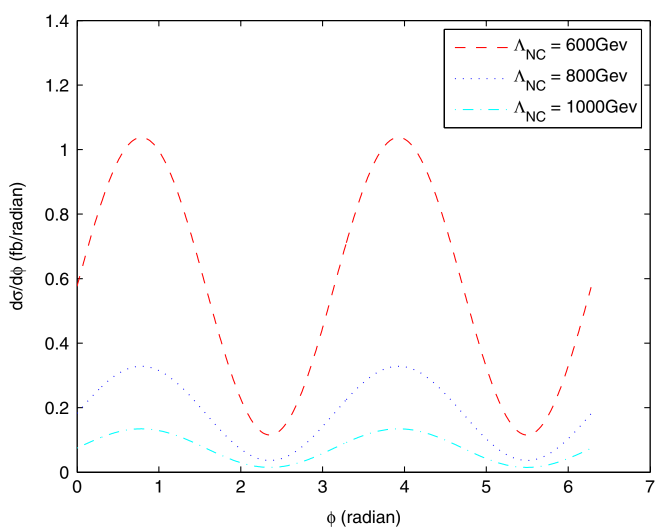


FIG. 14 (color online). The $\frac{d\sigma}{d\Phi}$ as a function of $\Phi$ for $e^+e^- \to HH$ for $m_H = 135$ GeV, $\Lambda_{NC} = 600$ GeV, 800 GeV and 1000 GeV.

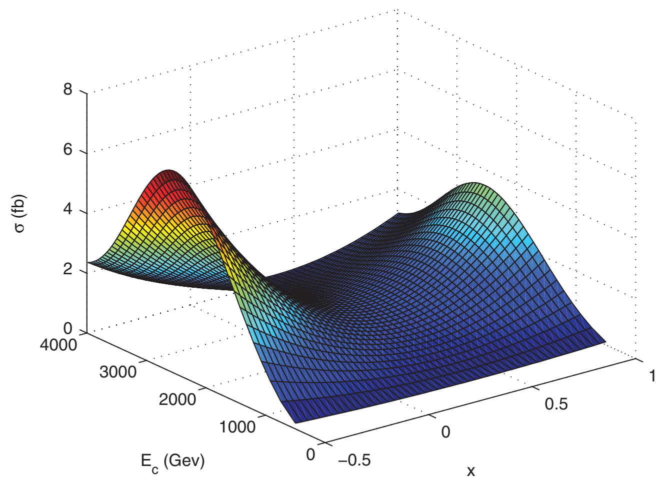


FIG. 15 (color). The total cross section for $e^+e^- \to HH$ as a function of $E_c$ and $x$ for $m_H = 135$ GeV, $\Lambda_{NC} = 1000$ GeV.

Now we consider the impact of the $U(1)$ gauge ambiguity discussed in Sec. II, which does not contribute to $e^+e^- \to ZH$. In this case, the contribution from the photon-Higgs-Higgs diagram must be considered. Using Eqs. (40) and (41) we obtain and show in Fig. 15 the total cross section as a function of $E_c$ and $x$ for $m_H = 135$ GeV and $\Lambda_{NC} = 1000$ GeV. Here we assume that $x$ varies between $-0.5$ and 1. One can see that the cross section shows a parabolic dependence on $x$ when the collision energy is fixed. The saddle point in Fig. 15 is at $x = 0.4$. When $x$ is located at [0.5, 1] or [$-0.5$, 0.3], the total cross section is greatly enhanced. However, if $x$ is in [0.3, 0.5], the cross section will be slightly suppressed.

## V. CONCLUSION AND DISCUSSION

In this paper we have explored the NC effect in the Higgs boson production process $e^+e^- \to ZH$ and SM forbidden process $e^+e^- \to HH$. Several new results are obtained. First, the $n$-th order Seiberg-Witten map for complex scalar fields is given. Despite the lengthy expression, for the processes discussed we can still obtain enough information to get the complete Feynman rules. Second, it is found that the NC effect can significantly reduce the cross section of the process $e^+e^- \to ZH$ when the collision energy exceeds 1 TeV. For $e^+e^- \to HH$, we obtained the total cross section and angular distribution using the simplest representation of SWM given by Ref. [10]. Moreover, we can also include more complicated representation, as well as photon-Higgs-Higgs interaction which does not arise in Ref. [10]. It is shown that although the process $e^+e^- \to ZH$ is independent from this changing, the total cross section of $e^+e^- \to HH$ cross section can be enhanced. This increases our confidence for detecting the NC signal associated with the Higgs boson in the future

International Linear Collider. For each process we can find an optimal collision energy as a function of the NC scale $\Lambda_{NC}$ in order to get the largest NC correction, which can help us to determine $\Lambda_{NC}$ effectively. Finally, we briefly comment on the process $e^+e^- \to \mu^+\mu^-$ studied in Ref. [13]. Using the $n$-th order Seiberg-Witten map, we show that the NC scattering amplitude differs from the ordinary one by only a phase factor, without NC effect.

The SWMs given in Sec. II are not general. One can add an homogeneous solution of the Seiberg-Witten equation to obtain another solution. As is well known, the degrees of freedom play an essential role in the renormalization of NCQFT [24,26]. For the process $e^+e^- \to HH$ and $e^+e^- \to \mu^+\mu^-$, all these ambiguities vanish because of the on-shell condition, thus the physical results are freedom independent. For the process $e^+e^- \to ZH$, the contributions from the homogeneous solutions containing two gauge fields cancel or vanish when the on-shell condition is applied. Thus the contribution from these degrees of freedom is limited to that containing one gauge field. Until now the phenomenological modification of these homogeneous solutions has not been considered, except for the pure gauge sector [15]. This is because we still do not have enough information on the renormalizability of NCQFT. We expect that further progress on the renormalizability of the noncommutative Higgs sector can finally remove this ambiguity and provide a more solid foundation for the phenomenological study, as has been done in the pure gauge sector. It should be noted that a compromising and more practical method is given in Ref. [27] where all possible deformed terms were considered. In any case, here we have demonstrated the rich phenomenological correlation between the Higgs physics and noncommutative space-time, and other important production processes such as $e^+e^- \to \bar{\nu}_e \nu_e H$ are being investigated.

## ACKNOWLEDGMENTS

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## APPENDIX: THE FEYNMAN RULE FOR *Z-H-H* INTERACTION

The Feynman rule for the $Z(k)$-$H(p_1)$-$H(p_2)$ vertex given in Ref. [22] is

$$\frac{g m_H^2 (k\Theta)_\mu}{4\cos\theta_W}. \tag{A1}$$

Since the expression (A1) is proportional to $m_H^2$, we need only to investigate

$$\int d^4x((\partial_\mu \hat{\Phi}^\dagger) * (\partial^\mu \hat{\Phi}) - \mu^2 \hat{\Phi} * \hat{\Phi} - \lambda(\hat{\Phi}^\dagger * \hat{\Phi})^2) \tag{A2}$$

in the Higgs sector.

Following Ref. [10], we take the SWM representation

$$\hat{\Phi} = \Phi - \frac{1}{2}\theta^{\alpha\beta} V_\alpha \partial_\beta \Phi - \frac{i}{4}\theta^{\alpha\beta} V_\beta (V_\alpha \Phi - \Phi V'_\alpha), \tag{A3}$$

where

$$\begin{aligned} V_\mu &= \frac{1}{2} g' B_\mu + g W_\mu^a \frac{\sigma^a}{2} \\ &= \begin{pmatrix} eA_\mu + \frac{g}{2\cos\theta_W}(1-2\sin^2\theta_W) Z_\mu & \frac{g}{\sqrt{2}} W_\mu^+ \\ \frac{g}{\sqrt{2}} W_\mu^- & -\frac{g}{2\cos\theta_W} Z_\mu \end{pmatrix}. \end{aligned} \tag{A4}$$

The last term of the right-hand side in Eq. (A3) contains two gauge fields, which is not related to the $Z$-$H$-$H$ interaction. Excluding this term, one has

$$\hat{\Phi} = \Phi - \frac{1}{2}\theta^{\alpha\beta} V_\alpha \partial_\beta \Phi. \tag{A5}$$

When

$$\Phi \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix},$$

we have

$$\begin{aligned} \hat{\Phi} &\to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} - \frac{1}{2}\theta^{\alpha\beta} V_\alpha \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \partial_\beta h \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{g}{2\sqrt{2}}\theta^{\alpha\beta} W_\alpha^+ \partial_\beta h \\ v + \hat{h} \end{pmatrix} \end{aligned} \tag{A6}$$

where

$$\hat{h} = h + \frac{\theta^{\alpha\beta} g}{4\cos\theta_W} Z_\alpha \partial_\beta h. \tag{A7}$$

For simplicity we rewrite it as

$$\hat{h} = h + \theta f. \tag{A8}$$

Inserting Eq. (A6) into (A2) and ignoring the unrelated terms, we obtain

$$\int dx^4 \left( \frac{1}{2}(\partial_\mu \hat{h}^\dagger)(\partial^\mu \hat{h}) - \frac{\lambda}{4} v^2 (\hat{h}^\dagger \hat{h}^\dagger + 2\hat{h}^\dagger \hat{h} + \hat{h}\hat{h}) \right). \tag{A9}$$

Using (A8) and taking partial integration, the corresponding NC correction up to the $\theta$ order is given by

$$-\theta \int dx^4 f(\partial_\mu \partial^\mu h + m_H^2 h). \tag{A10}$$

Obviously, when the Higgs boson is on-shell, this term does not contribute to the $Z$ vertex, thus leaving no NC effect.

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