Simple model for the dynamical Casimir effect for a static mirror with time-dependent properties

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We consider a real massless scalar field in 1 + 1 dimensions satisfying time-dependent Robin boundary condition at a static mirror. This condition can simulate moving reflecting mirrors whose motions are determined by the time dependence of the Robin parameter. We show that particles can be created from vacuum, characterizing in this way a dynamical Casimir effect.

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I. INTRODUCTION

The phenomenon of particle creation from quantum vacuum by moving boundaries or due to time-dependent properties of materials, commonly referred to as the dynamical Casimir effect (DCE) [1,2], has been investigated since the pioneering works of Moore [3] and DeWitt [4] (see also the subsequent works carried out in Refs. [5]) in a wide variety of situations and with the aid of quite different approaches (see Refs. [6] for excellent reviews on the subject). Particularly, perturbative and numerical approaches were applied for single mirrors [7–10] and cavities [11]. Initial field states different from vacuum were also considered for single mirrors [12,13] and cavities as well [14]. The first experimental observation of this phenomenon was recently announced in Ref. [15].

Taking into account the difficulties in generating appreciable mechanical oscillation frequencies (of the order of GHz) to obtain a detectable number of photons, recent experimental schemes focus on simulating moving boundaries by considering material bodies with time-dependent electromagnetic properties. These possibilities were first proposed by Yablonovitch [1] and have been further developed in theoretical works that considered materials with time-dependent permittivities and time-dependent surface conductivities [16–18] (see the nice compilation done in Ref. [17]). For instance, in Ref. [18] the DCE for a massless scalar field within a cavity containing a thin semiconducting film with time-dependent conductivity and centered at the middle of the cavity was studied. The coupling of such a film to the quantum scalar field was modeled by a delta potential with time-dependent strength. A generalization to the case of an electromagnetic field was carried out in [17]. Very promising and ingenious experimental setups to simulate nonstationary boundaries include the changing of the reflectivity of a semiconductor by the incidence of a periodic sequence of short laser pulses [19,20] or by using a coplanar waveguide terminated by a superconducting quantum interference device (SQUID). Applying a variable magnetic flux on the SQUID, a single moving mirror can be simulated [23,24]. A first step toward the experimental verification of the DCE was recently made in [25] using this approach. Moreover, the same group recently claimed to have observed the DCE [15]. Considering moving mirrors it is expected that the created photons can be indirectly detected through the phenomenon of superradiance [26,27].

Key ingredients in the predictions of the DCE are the boundary conditions (BCs) under consideration and naturally the quantum field submitted to those BCs. Quite general BCs are the so-called Robin ones which, for the case of a scalar field in $1 + 1$ dimensions and a single mirror fixed at $x = a$, are defined by $\phi(t, x = a) = \gamma[\partial_x \phi(t, x)]_{x=a}$, where γ is a real parameter (called hereafter as Robin parameter). For the case of a moving boundary, the previous relation is imposed in the comoving frame and the corresponding BC in the laboratory frame is obtained after an appropriate Lorentz transformation.

This BC has the nice feature of interpolating continuously Dirichlet ($\gamma \to 0$) and Neumann ($\gamma \to \infty$) ones and occurs in several areas of physics and mathematics. For instance, in classical mechanics they will appear if one considers a vibrating string coupled to a spring that satisfies Hooke's law and is localized at one of its edges [9,10,28]. In nonrelativistic quantum mechanics, Robin BCs occur as the most general BCs imposed by a wall ensuring the Hermiticity of the Hamiltonian as well as a null probability flux through it [29]. Regarding the static Casimir effect [30], it was shown that the Casimir force between two parallel plates that impose Robin BCs on a real scalar field may have its sign changed if appropriate choices are made for the corresponding Robin parameters of each mirror [31]. Such kind of repulsive Casimir force was also predicted, in the case of parallel plates, by Boyer in the 1970s, who considered a pair of perfectly conducting and infinitely permeable plates [32]. Further investigations on the influence of Robin BCs in the static Casimir effect,

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including thermal corrections and the case of Casimir piston setups, were carried, for instance, in Refs. [33]. See also Refs. [34] for the influence of this BC on the structure of quantum vacuum.

Only recently Robin BCs were considered in the context of the DCE. For a massless scalar field in $1+1$ dimensions submitted to a Robin BC at a single moving mirror the radiation reaction force on the moving mirror and the particle creation rate were computed in Refs. [9,10]. Interestingly, for Robin BCs, the radiation reaction force acquires a dispersive component, in sharp contrast with Dirichlet and Neumann cases where the force is purely dissipative. It was also shown that, for a given Robin parameter, there exists a mechanical frequency of motion that dramatically reduces the particle creation effect [10]. Finally, and of crucial importance for the present work, Robin BCs can also be useful to describe phenomenological models for penetrable surfaces and under certain conditions they simulate the plasma model for real metals [35]. In these situations, for frequencies ω much smaller than the plasma frequency ω_P, the Robin parameter γ can be identified as the plasma wavelength λ_P (see the Appendix). In other words, the Robin parameter γ gives us an estimate of the penetration length of the mirror under consideration [36].

Since to simulate a motion of a reflecting mirror is equivalent to simulate a real metal with time-dependent plasma wavelength, the above interpretation of γ leads naturally to the consideration of time-dependent Robin parameters. Specifically speaking, it is quite natural to simulate the motion of a reflecting mirror by considering the quantum field submitted to a Robin BC at a static mirror but with a time-dependent Robin parameter $\gamma(t)$. The kind of boundary motion that is being simulated is determined by the kind of time dependence of $\gamma(t)$. The purpose of this paper is precisely to analyze this situation for a massless scalar field in $1+1$ dimensions. Particularly, we shall compute explicitly the particle creation rate for a natural choice of time dependence for $\gamma(t)$ that is directly related to recent experimental proposals. This paper in organized as follows: In Sec. II the Bogoliubov transformation between the in and out creation/annihilation operators are obtained, allowing us to find the spectral distribution of the created particles and the particle creation rate in Secs. III and IV, respectively. Finally, in Sec. V we present our conclusions and final remarks. Throughout this work we consider $\hbar = c = 1$.

II. THE BOGOLIUBOV TRANSFORMATION

We start considering a real massless scalar field ϕ in $1+1$ dimensions that satisfies the Klein-Gordon equation, $\partial^2\phi = 0$, and is submitted to a time-dependent Robin BC at a mirror fixed at the origin, namely, $\gamma(t)\partial\phi/\partial x|_{x=0} - \phi(0,t) = 0$. For simplicity, we assume that $\gamma(t)$ departs only slightly from a positive constant γ_0, so that we can write $\gamma(t) = \gamma_0 + \delta\gamma(t)$, where $\delta\gamma(t)$ is a smooth time-dependent function satisfying the condition $\max|\delta\gamma(t)| \ll \gamma_0$, for every t. Under these assumptions in the limit $\gamma_0 \to \infty$ we recover the Neumann BC. On the other hand, to reobtain the Dirichlet BC ($\gamma_0 \to 0$), because of condition $\max|\delta\gamma(t)| \ll \gamma_0$, we must also take $\delta\gamma(t) \to 0$. If we consider only $\delta\gamma(t) = 0$ we reobtain the usual time-independent Robin BCs. Moreover, we shall also impose that $\delta\gamma(t) \to 0$ for $t \to \pm\infty$. The BC satisfied by $\delta\gamma(t)$ then reads

$$\gamma_0\left[\frac{\partial\phi(x,t)}{\partial x}\right]_{x=0} - \phi(0,t) + \delta\gamma(t)\left[\frac{\partial\phi(x,t)}{\partial x}\right]_{x=0} = 0. \quad (1)$$

Also for the field, a perturbative approach will be adopted. Following Ford and Vilenkin [37] we write

$$\phi(x,t) = \phi_0(x,t) + \delta\phi(x,t), \quad (2)$$

where, by assumption, ϕ_0 satisfies the Klein-Gordon equation, $\partial^2\phi_0 = 0$, and the time-independent Robin BCs,

$$\gamma_0\left[\frac{\partial\phi_0(x,t)}{\partial x}\right]_{x=0} - \phi_0(0,t) = 0. \quad (3)$$

The small perturbation $\delta\phi$ takes into account the contribution to the total field ϕ caused by the time dependence of the Robin parameter, described by the function $\delta\gamma(t)$. Since both ϕ and ϕ_0 satisfy the Klein-Gordon equation, so does $\delta\phi$, namely, $\partial^2\delta\phi = 0$. The BC satisfied by $\delta\phi$ is obtained, up to first order terms, by substituting (2) into Eq. (1), which leads to

$$\gamma_0\left[\frac{\partial\delta\phi(x,t)}{\partial x}\right]_{x=0} - \delta\phi(0,t) = -\delta\gamma(t)\left[\frac{\partial\phi_0(x,t)}{\partial x}\right]_{x=0}, \quad (4)$$

where Eq. (3) was used. Hereafter it will be convenient to work in the Fourier domain, such that

$$\Phi(x,\omega) = \int dt\phi(x,t)e^{i\omega t}; \quad \Phi_0(x,\omega) = \int dt\phi_0(x,t)e^{i\omega t};$$
$$\delta\Phi(x,\omega) = \int dt\delta\phi(x,t)e^{i\omega t}; \quad \delta\Gamma(\omega) = \int dt\delta\gamma(t)e^{i\omega t}. \quad (5)$$

It is worth emphasizing at this moment that, by assumption, $\delta\gamma$ is a prescribed function of t, so that $\delta\Gamma(\omega)$ is known, in principle. Since $\phi_0(x,t)$ is the solution with time-independent Robin BCs, this field is already known, and as is its Fourier transform, which is given by (for the region $x > 0$),

$$\Phi_0(x,\omega) = \sqrt{\frac{4\pi}{|\omega|(1+\gamma_0^2\omega^2)}}[\sin(\omega x) + \gamma_0\omega\cos(\omega x)] \times [\Theta(\omega)a(\omega) - \Theta(-\omega)a^\dagger(-\omega)], \quad (6)$$

where $\Theta(\omega)$ is the Heaviside step function. The operators $a(\omega)$ and $a^\dagger(\omega)$ satisfy the usual bosonic commutation relation $[a(\omega), a^\dagger(\omega')] = 2\pi\delta(\omega - \omega')$.

In order to obtain $\Phi(x.\omega) = \Phi_0(x,\omega) + \delta\Phi(x,\omega)$ we need to compute $\delta\Phi(x,\omega)$, which satisfies the Helmholtz equation,

$$(\partial_x^2 + \omega^2)\delta\Phi(x, \omega) = 0, \tag{7}$$

and is submitted to the BC below, obtained by Fourier transforming Eq. (4),

$$\gamma_0\left[\frac{\partial\delta\Phi(x, \omega)}{\partial x}\right]_{x=0} - \delta\Phi(0, \omega) = -\int\frac{d\omega'}{2\pi}\left[\frac{\partial\Phi_0(x, \omega')}{\partial x}\right]_{x=0}\delta\Gamma(\omega - \omega'). \tag{8}$$

A further condition that must be imposed to the solution of Eq. (7) for $x > 0$ is that it will lead to a solution for $\phi(x, t)$ that must travel to the right, since $\delta\phi(x, t)$ must describe a contribution coming from the mirror, and not going towards the mirror. The desired solution can be written in terms of Green functions. Following the procedure given in [10] it can be shown that the in and out fields, denoted, respectively, as $\Phi_{\rm in}$ and $\Phi_{\rm out}$, are related to each other according to

$$\Phi_{\rm out}(x, \omega) = \Phi_{\rm in}(x, \omega) + \frac{1}{\gamma_0}[G_{\rm R}^{\rm ret}(0, x, \omega) - G_{\rm R}^{\rm adv}(0, x, \omega)] \times\left\{\gamma_0\left[\frac{\partial\delta\Phi(x, \omega)}{\partial x}\right]_{x=0} - \delta\Phi(0, \omega)\right\}, \tag{9}$$

where $G_{\rm R}^{\rm ret}(0, x, \omega)$ ($G_{\rm R}^{\rm adv}(0, x, \omega)$) is the retarded (advanced) Robin Green function, satisfying the time-independent Robin BC at $x = 0$. These Green functions are given, respectively, by

$$G_{\rm R}^{\rm ret}(0, x, \omega) = \left(\frac{\gamma_0}{1 - i\gamma_0\omega}\right)e^{i\omega x}, \tag{10}$$

and

$$G_{\rm R}^{\rm adv}(0, x, \omega) = \left(\frac{\gamma_0}{1 + i\gamma_0\omega}\right)e^{-i\omega x}. \tag{11}$$

Inserting Eqs. (6) (appropriately relabeled as $\Phi_{\rm out}$ and $\Phi_{\rm in}$), (8), (10), and (11), into Eq. (9), we can readily obtain the Bogoliubov transformation between $a_{\rm out}$ and $a_{\rm in}$ and its Hermitian conjugates:

$$a_{\rm out}(\omega) = a_{\rm in}(\omega) - 2i\sqrt{\frac{\omega}{1 + \gamma_0^2\omega^2}} \times\int_{-\infty}^{+\infty}\frac{d\omega'}{2\pi}\sqrt{\frac{\omega'}{1 + \gamma_0^2\omega'^2}}[\Theta(\omega')a_{\rm in}(\omega') - \Theta(-\omega')a_{\rm in}^\dagger(-\omega')]\delta\Gamma(\omega - \omega'). \tag{12}$$

Noting that the annihilation operator $a_{\rm out}(\omega)$ is given in terms of the annihilation and creation operators $a_{\rm in}(\omega)$ and $a_{\rm in}^\dagger(\omega)$, respectively, we conclude that the state $|0_{\rm in}\rangle$ is not annihilated by the $a_{\rm out}(\omega)$ operators. Consequently, we can state that particles were created from an initial vacuum state due only to the time dependence of $\delta\gamma(t)$ in the BC (1) imposed on the field by the static mirror. In fact, for $\delta\gamma(t) = 0$ for all times, which corresponds to a static mirror imposing the standard time-independent Robin BC on the field, we have $a_{\rm out}(\omega) = a_{\rm in}(\omega)$ and no particles will be created, as expected. The particle creation effect will be further investigated in the next sections, where we will choose a specific time-dependent expression for $\gamma(t)$ in order to compute explicitly the corresponding spectral distribution of the created particles as well as the respective particle creation rate.

III. SPECTRAL DISTRIBUTION OF THE CREATED PARTICLES

We start by writing the spectral distribution of the created particles as

$$\frac{dN(\omega)}{d\omega}d\omega = \frac{1}{2\pi}\langle 0_{\rm in}|a_{\rm out}^\dagger(\omega)a_{\rm out}(\omega)|0_{\rm in}\rangle d\omega, \tag{13}$$

where $dN(\omega)/d\omega$ is the number of created particles with frequency between ω and $\omega + d\omega$ ($\omega \geq 0$) per unit frequency. From the previous definition for $dN(\omega)/d\omega$, it follows immediately that the total number of created particles from $t = -\infty$ to $t = +\infty$ is given by

$$N = \int_0^\infty\frac{dN(\omega)}{d\omega}d\omega. \tag{14}$$

From Eq. (12) and its Hermitian conjugate $a_{\rm out}^\dagger$, it is straightforward to show that

$$\frac{dN(\omega)}{d\omega} = \frac{2}{\pi}\left(\frac{\omega}{1 + \gamma_0^2\omega^2}\right) \times\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\omega'}{1 + \gamma_0^2\omega'^2}|\delta\Gamma(\omega - \omega')|^2\Theta(\omega'). \tag{15}$$

In what follows we will obtain the spectral distribution for a particular case of $\delta\Gamma(\omega)$. With this purpose in mind, let us consider the following expression for $\delta\gamma(t)$,

$$\delta\gamma(t) = \epsilon_0\cos(\omega_0 t)e^{-|t|/T}, \tag{16}$$

with $\omega_0 T \gg 1$. This choice of $\delta\gamma(t)$ may simulate, for instance, the changing magnetic flux through a SQUID fixed at the extreme of a unidimensional transmission line, as in Ref. [23], where a Robin-like BC arises naturally from quantum network theory applied to the system under consideration.

The expression of $\delta\Gamma(\omega)$, obtained by Fourier transforming Eq. (16), contains, in the limit of $\omega_0 T \gg 1$, two sharped peaks around $\omega = \pm\omega_0$, which can be approximated by Dirac delta functions, leading to the result

$$|\delta\Gamma(\omega)|^2 \approx \frac{\pi}{2}\epsilon_0^2 T[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]. \tag{17}$$

Substituting the above result into Eq. (15), we finally obtain the desired spectral distribution,

$$\frac{dN(\omega)}{d\omega} = \left(\frac{\epsilon_0^2 T}{2\pi}\right)\frac{\omega(\omega_0 - \omega)}{(1 + \gamma_0^2\omega^2)[1 + \gamma_0^2(\omega_0 - \omega)^2]}\Theta(\omega_0 - \omega), \tag{18}$$

for this particular situation.

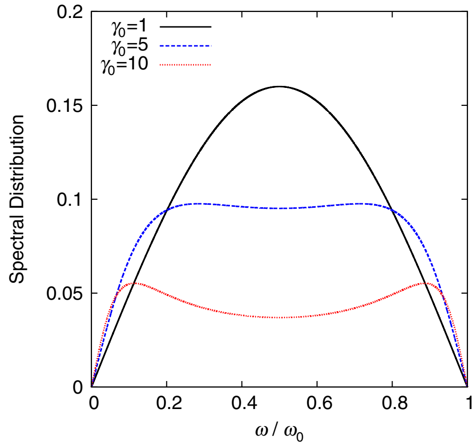

FIG. 1 (color online). The spectral distribution of the created particles $[(2\pi)/(\epsilon_0^2 T)]dN/d\omega$ as a function of ω/ω_0 for several values of γ_0. Notice the reflection symmetry around $\omega/\omega_0 = 0.5$: a signature of the fact that particles are created in pairs. The full line corresponds to $\gamma_0 = 1$; the dashed line to $\gamma_0 = 5$ and $20 \times [(2\pi)/(\epsilon_0^2 T)]dN/d\omega$; and the dotted line to $\gamma_0 = 10$ and $100 \times [(2\pi)/(\epsilon_0^2 T)]dN/d\omega$.

A few comments are in order. First, observe [see Fig. 1 and Eq. (18)] that $dN(\omega)/d\omega$ vanishes for $\omega > \omega_0$, which means that no particles are created with frequencies larger than ω_0—the characteristic frequency of the time-dependent BC. We also notice that the spectrum is left invariant under the replacement $\omega \to \omega_0 - \omega$. This is a signature of the fact that particles are created in pairs: for each particle created with frequency ω there is a twin particle created with frequency $\omega_0 - \omega$. Second, note that for $\epsilon_0 \to 0$, where a Robin BC with a time-independent parameter γ_0 is reobtained, the spectrum of created particles vanishes, as expected (recall that the mirror that imposes the BC on the field is at rest). Further, for a fixed (finite) value of ω_0, the limit $\gamma_0 \to \infty$ (Neumann BC imposed on the field at a static mirror) also leads to a vanishing spectrum of created particles. Finally, since we assumed $\epsilon_0 \ll \gamma_0$, the limit $\gamma_0 \to 0$ (Dirichlet BC imposed on the field by a static mirror) necessarily leads to a vanishing spectrum as well.

IV. PARTICLE CREATION RATE

The total number of created particles is obtained by substituting Eq. (18) in (14), namely,

$$N = \left(\frac{\epsilon_0^2 T}{2\pi}\right)\int_0^\infty \frac{\omega(\omega_0 - \omega)\Theta(\omega_0 - \omega)}{(1 + \gamma_0^2\omega^2)[1 + \gamma_0^2(\omega_0 - \omega)^2]} d\omega$$
$$= \left(\frac{\epsilon_0^2 \omega_0^3 T}{2\pi}\right) F(\xi), \tag{19}$$

where $\xi = \gamma_0\omega_0$ and the function $F(\xi)$ is given by

$$F(\xi) = \frac{(2 + \xi^2)\ln(1 + \xi^2) - 2\xi\arctan(\xi)}{\xi^4(4 + \xi^2)}. \tag{20}$$

As N is proportional to T—as expected for an open cavity—the physical meaningful quantity is the particle creation rate defined as $R = N/T$, that is

$$R = \left(\frac{\epsilon_0^2\omega_0^3}{2\pi}\right) F(\xi). \tag{21}$$

In the limits $\gamma_0\omega_0 \ll 1$ and $\gamma_0\omega_0 \gg 1$, the particle creation rate is approximately given by

$$R \approx \left(\frac{\epsilon_0^2\omega_0^3}{12\pi}\right) \quad \text{for } \gamma_0\omega_0 \ll 1, \tag{22}$$

$$R \approx \left(\frac{\epsilon_0^2\omega_0^3}{2\pi}\right)\frac{2\ln(\xi)}{\xi^4} \quad \text{for } \gamma_0\omega_0 \gg 1. \tag{23}$$

For the sake of comparison with Eq. (21), we recall the total particle creation rates for moving mirrors with Dirichlet [8] (or equivalently for Neumann BCs as proved in [13])

$$R_{\mathrm{D/N}} = \frac{\delta q_0^2\omega_0^3}{12\pi}, \tag{24}$$

and for time-independent Robin BCs [10]

$$R_{\text{ti-R}} = \left(\frac{\delta q_0^2\omega_0^3}{2\pi}\right) G(\gamma_0\omega_0), \tag{25}$$

where

$$G(\xi) = \frac{\xi[4\xi + \xi^3 + 12\arctan(\xi)] - 6(2 + \xi^2)\ln(1 + \xi^2)}{6\xi^2(4 + \xi^2)}. \tag{26}$$

The formulas above were obtained assuming a nonrelativistically small amplitude oscillatory law of motion for the mirror. For both cases δq_0 is the amplitude and ω_0 is the frequency of oscillation. We remark that for $\gamma_0\omega_0 \ll 1$ the particle creation rate in our model is exactly the same as that for a moving mirror [8] with Dirichlet BCs where ϵ_0 plays the role of the amplitude of oscillation of the motion. This reinforces the possibility of simulating moving boundaries through a static mirror with time-dependent Robin BC. The three particle creation rates are compared in Fig. 2.

It is worth noting that the particle creation rate shown in Fig. 3 starts growing with ω_0 until it achieves a maximum value for a given value of ω_0 and then it approaches monotonically to zero as ω_0 goes to infinity. This behavior should be compared with that obtained for a moving mirror that imposes on the field a Robin BC with a time-independent parameter, where the particle creation rate after passing through one maximum and one minimum grows indefinitely as ω_0 goes to infinity (see Ref. [10]). Naively, we could expect similar behaviors for these two

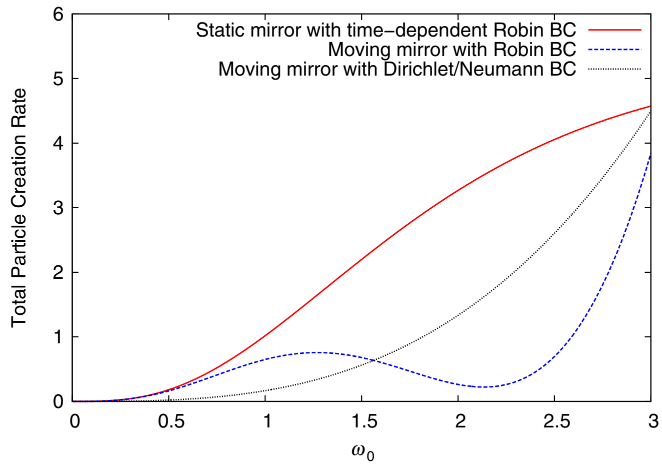

FIG. 2 (color online). Comparison between the total particle creation rates given by Eqs. (21), (24), and (25). The full line corresponds to scaled creation rate $10 \times [(2\pi\gamma_0^3)/(\epsilon_0^2)]R$. The dashed line corresponds to $10 \times [(2\pi\gamma_0^3)/(\delta q_0^2)]R_{\text{ti-R}}$. Finally, the dotted line corresponds to $[(2\pi)/(\delta q_0^2)]R_{\text{D/N}}$. In both curves involving Robin BCs we considered $\gamma_0 = 1$.

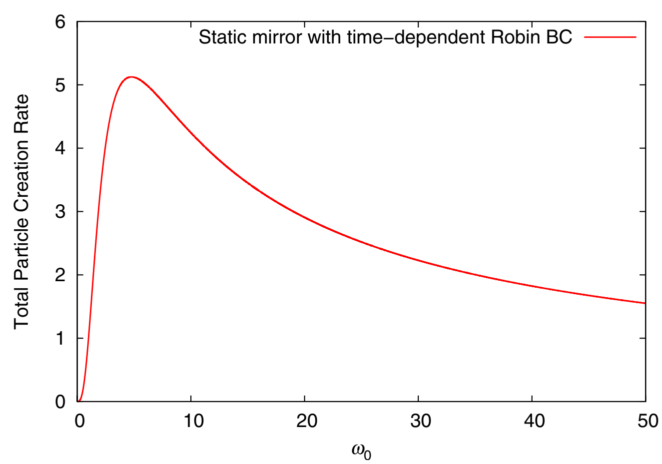

FIG. 3 (color online). Behavior of Eq. (21) for a larger range of ω_0, assuming $\gamma_0 = 1$. The creation rate approaches smoothly to zero as $\omega_0 \to \infty$. This does not happen for the other cases shown in Fig. 2: for *moving* mirrors the particle creation rate increases unlimitedly for larger values of ω_0.

problems, after all, a time-dependent Robin parameter should simulate, in principle, a moving mirror so that a high frequency oscillating $\gamma(t)$ should mean a high frequency oscillating mirror. However, the interpretation of the Robin parameter γ as an estimate of the penetration depth of the material boundary is rigorously proved only for static mirrors. Even in this case, this identification is valid only for the field modes whose frequencies are much smaller than the plasma frequency (but this condition is easily achieved since the plasma frequency is much higher than the mechanical frequencies we want to simulate). It is plausible that such an interpretation remains valid for *slowly* time-varying $\gamma(t)$, but not for high-frequency oscillating $\gamma(t)$. In fact, our results show that this interpretation for $\gamma(t)$ fails for high values of ω_0.

V. CONCLUSIONS AND FINAL REMARKS

Exploring the peculiar properties of Robin BCs, particularly, the interpretation of the Robin parameter, we presented a simple and yet instructive theoretical model where a single static mirror with time-dependent properties described by a time-dependent Robin parameter simulates a moving boundary. We used this model to study analytically the dynamical Casimir effect of a system that may be of some value for further understanding of an ongoing experiment based on a one-dimensional transmission line terminated by a SQUID. In this setup a time-dependent magnetic flux through the SQUID gives rise to a particle creation phenomenon. Employing a perturbative approach, we showed that particles can be created due to the time dependence of the Robin parameter γ. We obtained explicitly the spectrum of the created particles as well as the total particle creation rate for a particular choice of $\gamma(t)$ that has a practical interest concerning the experiment just described. Our model can also be used as a theoretical model to investigate other experimental setups suggested for measuring the dynamical Casimir effect, as, for example, the promising experimental proposal of the Padua group [19]. All we have to do is to choose appropriately the time dependence of $\gamma(t)$ to simulate correctly the physical situation under consideration.

We emphasize that the particle creation phenomena due to a time-dependent Robin BC imposed on the field at a static mirror has similarities and differences with the case where a time-independent Robin BC is imposed on the field at a moving mirror, as discussed by Mintz *et al.* [10]. The main difference being the respective behaviors of the total particle creation rate for high values of ω_0 [in the former case, where ω_0 means the mechanical frequency of the moving mirror, this rate grows indefinitely as $\omega_0 \to \infty$, while in the latter case, where ω_0 gives a measure of how quick the time-dependent Robin parameter $\gamma(t)$ varies, this rate goes to zero, as $\omega_0 \to \infty$]. In the appropriate limits of the usual time-independent Dirichlet ($\gamma_0 \to 0$), Neumann ($\gamma_0 \to 0$) and Robin ($\delta\gamma(t) = 0$) BCs no particles are created, as expected.

The generalization of the present work for $3 + 1$ dimensions and cavities are also expected to have induced photon creation. The latter case is particularly relevant since it is known that the particle creation can be parametrically amplified in this situation. Considering a $1 + 1$-dimensional cavity with time-dependent Robin BCs at $x = 0$ and Dirichlet BCs at $x = L$ we can expect that the particle production can be intensified for an adequate choice on how $\gamma(t)$ varies. We can also conjecture that it is possible to generate squeezed states of light in such configuration. For cavities with moving mirrors, considering Dirichlet BCs, this was first predicted in [38]. These problems are under investigation and will be discussed elsewhere.

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APPENDIX: CONNECTION BETWEEN THE ROBIN PARAMETER AND THE PLASMA WAVELENGTH

For the sake of completeness, here we briefly review how the interpretation of the Robin parameter γ as the plasma wavelength arises in $1+1$ dimensions. Let us consider a monochromatic wave, with frequency ω and wave number k, propagating in negative direction of the x-axis towards a mirror imposing Robin BC at $x=0$. For $x>0$, the superposition of the incident and reflected waves reads

$$\phi(t,x) = Ae^{-i(kx+\omega t)} + Be^{i(kx-\omega t)}. \tag{A1}$$

Using the time-independent Robin BC, we can find the following reflection coefficient $r_{\mathrm{R}} = B/A$ for this BC:

$$r_{\mathrm{R}} = -\frac{1+i\gamma\omega}{1-i\gamma\omega} = e^{i\theta(\omega)}, \tag{A2}$$

where $\theta(\omega) = 2\arctan(\gamma\omega)$ is a *phase shift* between the reflected and the incident waves. Notice that for frequencies such that $\omega \ll 1/\gamma$ we obtain

$$r_{\mathrm{R}} \approx -e^{2i\gamma\omega} + \mathcal{O}(\gamma^2\omega^2). \tag{A3}$$

Now let us consider the plasma model for a real metal. The reflection coefficient r_{P} in this case is given by

$$r_{\mathrm{P}} = \frac{n-1}{n+1} = \frac{\sqrt{\epsilon(\omega)}-1}{\sqrt{\epsilon(\omega)}+1}, \tag{A4}$$

where $\epsilon(\omega) = 1-(\omega_{\mathrm{P}}/\omega)^2$ is the plasma permittivity and ω_{P} denotes the plasma frequency. Consequently, for an incident wave with frequency $\omega < \omega_{\mathrm{P}}$ we have

$$r_{\mathrm{P}} = \frac{i\sqrt{(\omega_{\mathrm{P}}/\omega)^2-1}-1}{i\sqrt{(\omega_{\mathrm{P}}/\omega)^2-1}+1}, \tag{A5}$$

which for $\omega \ll \omega_{\mathrm{P}}$ results in

$$r_{\mathrm{P}} \approx -e^{2i\omega/\omega_{\mathrm{P}}} + \mathcal{O}(\omega^2/\omega_{\mathrm{P}}^2). \tag{A6}$$

Comparing Eqs. (A3) and (A6), we conclude that for frequencies $\omega \ll \omega_{\mathrm{P}}$, the Robin parameter γ is equivalent to the plasma wavelength $\lambda_{\mathrm{P}} = 1/\omega_{\mathrm{P}}$. Therefore, in this situation, a real metal described by the plasma model can effectively be emulated using Robin BCs.

[1] E. Yablonovitch, Phys. Rev. Lett. **62**, 1742 (1989).

[2] J. Schwinger, Proc. Natl. Acad. Sci. U.S.A. **89**, 4091 (1992).

[3] G. T. Moore, J. Math. Phys. (N.Y.) **11**, 2679 (1970).

[4] B. S. DeWitt, Phys. Rep. **19**, 295 (1975).

[5] S. A. Fulling and P. C. W. Davies, Proc. R. Soc. A **348**, 393 (1976); P. C. W. Davies and S. A. Fulling, Proc. R. Soc. A **356**, 237 (1977); **354**, 59 (1977); P. Candelas and D. J. Raine, J. Math. Phys. (N.Y.) **17**, 2101 (1976); P. Candelas and D. Deutsch, Proc. R. Soc. A **354**, 79 (1977).

[6] V. V. Dodonov, Adv. Chem. Phys. **119**, 309 (2001); J. Phys. Conf. Ser. **161**, 012027 (2009); D. A. R. Dalvit, P. A. Maia Neto, and F. D. Mazzitelli, in *Casimir Physics*, edited by D. A. R. Dalvit, P. W. Milonni, D. C. Roberts, and F. S. S. Rosa, Lectures Notes in Physics, Vol 834 (Springer, Berlin, 2011) [arXiv:1006.4790v2]; V. V. Dodonov, Phys. Scr. **82**, 038105 (2010).

[7] L. H. Ford and A. Vilenkin, Phys. Rev. D **25**, 2569 (1982); P. A. Maia Neto, J. Phys. A **27**, 2167 (1994); P. A. Maia Neto and L. A. S. Machado, Phys. Rev. A **54**, 3420 (1996); Braz. J. Phys. **25**, 324 (1995).

[8] A. Lambrecht, M.-T. Jaekel, and S. Reynaud, Phys. Rev. Lett. **77**, 615 (1996).

[9] B. Mintz, C. Farina, P. A. Maia Neto, and R. B. Rodrigues, J. Phys. A **39**, 6559 (2006).

[10] B. Mintz, C. Farina, P. A. Maia Neto, and R. B. Rodrigues, J. Phys. A **39**, 11325 (2006).

[11] C. K. Law, Phys. Rev. Lett. **73**, 1931 (1994); Y. Wu, K. W. Chan, M. C. Chu, and P. T. Leung, Phys. Rev. A **59**, 1662 (1999); P. Wegrzyn, J. Phys. B **40**, 2621 (2007); V. V. Dodonov, A. B. Klimov, and D. E. Nikonov, J. Math. Phys. (N.Y.) **34**, 2742 (1993); D. A. R. Dalvit and F. D. Mazzitelli, Phys. Rev. A **57**, 2113 (1998); C. K. Cole and W. C. Schieve, Phys. Rev. A **52**, 4405 (1995); **64**, 023813 (2001); M. Razavy and J. Terning, Phys. Rev. D **31**, 307 (1985); G. Calucci, J. Phys. A **25**, 3873 (1992); C. K. Law, Phys. Rev. A **49**, 433 (1994); V. V. Dodonov and A. B. Klimov, Phys. Rev. A **53**, 2664 (1996); D. F. Mundarain and P. A. Maia Neto, Phys. Rev. A **57**, 1379 (1998); D. T. Alves, C. Farina, and E. R. Granhen, Phys. Rev. A **73**, 063818 (2006); J. Sarabadani and M. F. Miri, Phys. Rev. A **75**, 055802 (2007).

[12] M.-T. Jaekel and S. Reynaud, J. Phys. I (France) **3**, 339 (1993); Phys. Lett. A **172**, 319 (1993); L. A. S. Machado, P. A. Maia Neto, and C. Farina, Phys. Rev. D **66**, 105016 (2002).
[13] D. T. Alves, C. Farina, and P. A. Maia Neto, J. Phys. A **36**, 11333 (2003); D. T. Alves, E. R. Granhen, and M. G. Lima, Phys. Rev. D **77**, 125001 (2008).
[14] V. V. Dodonov, J. Phys. A **31**, 9835 (1998); G. Plunien, R. Schützhold, and G. Soff, Phys. Rev. Lett. **84**, 1882 (2000); J. Hui, S. Qing-Yun, and W. Jian-Sheng, Phys. Lett. A **268**, 174 (2000); R. Schützhold, G. Plunien, and G. Soff, Phys. Rev. A **65**, 043820 (2002); G. Schaller, R. Schützhold, G. Plunien, and G. Soff, Phys. Rev. A **66**, 023812 (2002); D. T. Alves, E. R. Granhen, H. O. Silva, and M. G. Lima, Phys. Rev. D **81**, 025016 (2010).
[15] C. M. Wilson *et al.*, arXiv:1105.4714v1.
[16] E. Yablonovitch, J. P. Heritage, D. E. Aspnes, and Y. Yafet, Phys. Rev. Lett. **63**, 976 (1989); Y. E. Lozovik, V. G. Tsvetus, and E. A. Vinogradov, JETP Lett. **61**, 711 (1995); Phys. Scr. **52**, 184 (1995); T. Okushima and A. Shimizu, Jpn. J. Appl. Phys. **34**, 4508 (1995); M. Uhlmann, G. Plunien, R. Schützhold, and G. Soff, Phys. Rev. Lett. **93**, 193601 (2004).
[17] W. Naylor, S. Matsuki, T. Nishimura, and Y. Kido, Phys. Rev. A **80**, 043835 (2009).
[18] M. Crocce, D. A. R. Dalvit, F. C. Lombardo, and F. D. Mazzitelli, Phys. Rev. A **70**, 033811 (2004).
[19] C. Braggio *et al.*, Europhys. Lett. **70**, 754 (2005); A. Agnesi *et al.*, J. Phys. A **41**, 164024 (2008); J. Phys. Conf. Ser. **161**, 012028 (2009).
[20] See a criticism to this approach [21] and the response [22].
[21] W.-J. Kim, J. H. Brownell, and R. Onofrio, Europhys. Lett. **78**, 21002 (2007).
[22] C. Braggio *et al.*, Europhys. Lett. **78**, 21003 (2007).
[23] J. R. Johansson, G. Johansson, C. M. Wilson, and F. Nori, Phys. Rev. Lett. **103**, 147003 (2009); Phys. Rev. A **82**, 052509 (2010).
[24] P. D. Nation, J. R. Johansson, M. P. Blencowe, and F. Nori, arXiv:1103.0835v1.
[25] C. M. Wilson *et al.*, Phys. Rev. Lett. **105**, 233907 (2010).
[26] W.-J. Kim, J. H. Brownell, and R. Onofrio, Phys. Rev. Lett. **96**, 200402 (2006).
[27] J. H. Brownell, W. J. Kim, and R. Onofrio, J. Phys. A **41**, 164026 (2008).
[28] K. Gustafson, *Introduction to Partial Differential Equations and Hilbert Space Methods* (Dover, New York, 1999); G. Chen and J. Zhou, *Vibration and Damping in Distributed Systems* (CRC Press, Boca Raton, Florida, 1999), Vol. 1.
[29] T. E. Clark, R. Menikoff, and D. H. Sharp, Phys. Rev. D **22**, 3012 (1980); M. Carreau, E. Farhi, and S. Gutmann, Phys. Rev. D **42**, 1194 (1990); V. S. Araujo, F. A. B. Countinho, and J. Fernando Perez, Am. J. Phys. **72**, 203 (2004); V. S. Araujo, F. A. B. Coutinho, and F. M. Toyama, Braz. J. Phys. **38**, 178 (2008); B. Belchev and M. A. Walton, J. Phys. A **43**, 085301 (2010).
[30] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948).
[31] A. Romeo and A. A. Saharian, J. Phys. A **35**, 1297 (2002).
[32] T. H. Boyer, Phys. Rev. A **9**, 2078 (1974).
[33] Z. H. Liu and S. A. Fulling, New J. Phys. **8**, 234 (2006); E. Elizalde, S. D. Odintsov, and A. A. Saharian, Phys. Rev. D **79**, 065023 (2009); L. P. Teo, J. High Energy Phys. 11 (2009) 095; M. Asorey, D. García Álvares, and J. M. Muñoz-Castañeda, J. Phys. A **39**, 6127 (2006).
[34] M. Asorey and J. M. Muñoz-Castañeda, J. Phys. A **41**, 164043 (2008); **41**, 304004 (2008).
[35] V. M. Mostepanenko and N. N. Trunov, Sov. J. Nucl. Phys. **45**, 818 (1985).
[36] The set of references concerning the physical applications of Robin BCs provided in the present paper obviously does not intend to be complete. Our objective is just to give the reader a taste of the richness of physical situations involving this BC.
[37] Ford and Vilenkin, [7].
[38] V. V. Dodonov, A. B. Klimov, and V. I. Man'ko, Phys. Lett. A **149**, 225 (1990).