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Energy extraction and particle acceleration around a rotating black hole in Hořava-Lifshitz gravity

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The Penrose process on rotational energy extraction of the black hole in the original nonprojectable Hořava-Lifshitz gravity is studied. The strong dependence of the extracted energy from the special range of parameters of the Hořava-Lifshitz gravity, such as parameter Λ_W and specific angular momentum a, has been found. Particle acceleration near the rotating black hole in Hořava-Lifshitz gravity has been studied. It is shown that the fundamental parameter of the Hořava-Lifshitz gravity can impose a limitation on the energy of the accelerating particles preventing them from the infinite value.

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I. INTRODUCTION

Hořava proposed a new theory of quantum gravity motivated by the Lifshitz theory in solid state physics. The Hořava-Lifshitz theory is nonrelativistic and power-counting ultraviolet-renormalizable, and should recover general relativity in the infrared limit [1,2]. Since then, many authors have paid attention to this scenario to apply it to black hole (BH) physics [3–8], cosmology [9–15], and observational tests [16]. Here we investigate the Penrose process around rotating BHs in the Hořava-Lifshitz gravity theory. The quantum interference effects [17] and the motion of the test particle around a BH [18] in Hořava-Lifshitz gravity have been also recently studied.

In Ref. [16], the possibility of observationally testing Hořava-Lifshitz gravity at the scale of the Solar System, by considering the classical tests of general relativity (perihelion precession of the planet Mercury, deflection of light by the Sun, and the radar echo delay) for the Kehagias-Sfetsos (KS) asymptotically flat black hole solution of Hořava-Lifshitz gravity has been considered. The stability of the Einstein static universe by considering linear homogeneous perturbations in the context of an infrared (IR) modification of Hořava-Lifshitz gravity has been studied in [19]. Potentially observable properties of black holes in the deformed Hořava-Lifshitz gravity with Minkowski vacuum, the gravitational lensing, and quasinormal modes have been studied in [20]. The authors of Ref. [21] derived the full set of equations of motion and then obtained spherically symmetric solutions for UV completed theory of Einstein proposed by Hořava.

Recently authors of Ref. [22] have studied the particle motion in the spacetime of a KS black hole. In Ref. [23], the authors systematically studied black holes in the Hořava theory in the framework of the kinematic approach.

Black hole solutions and the full spectrum of spherically symmetric solutions in the five-dimensional nonprojectable Hořava-Lifshitz type gravity theories have been recently studied in [24]. Geodesic stability and the spectrum of entropy/area for a black hole in Hořava-Lifshitz gravity via a quasinormal modes approach are analyzed in [25]. Particle geodesics around a Kehagias-Sfetsos black hole in Hořava-Lifshitz gravity are also investigated by the authors of Ref. [26]. Recently observational constraints on Hořava-Lifshitz gravity have been found from the cosmological data [27]. The authors of Ref. [8] have found all spherical black hole solutions for two, four, and six derivative terms in the presence of a Cotton tensor.

Recently Patil and Joshi [28] have shown that the naked singularities that form due to the gravitational collapse of massive stars provide a suitable environment where particles could get accelerated and collide at arbitrarily high center-of-mass energies. Particle acceleration around a black hole is systematically studied in Ref. [29].

Recently the rotating black hole solution in the context of the Hořava-Lifshitz gravity has been obtained in [8]. In this paper we plan to study the energy extraction mechanism through the Penrose process and particle acceleration mechanisms near the rotating black hole in the Hořava-Lifshitz gravity. The authors of Ref. [30] considered Kerr black holes as particle accelerators to arbitrary high energies. The results of Ref. [30] have been addressed in Ref. [31], where the authors concluded that astrophysical limitations on the maximal spin, backreaction effects, and sensitivity to the initial conditions impose severe limits on

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the likelihood of such accelerations. This new proposed solution forces us to study the particle acceleration in the gravity theory of Hořava-Lifshitz.

The paper is organized as follows. The description of the rotating black hole solution and the ergosphere around it is considered in the Sec. II. The Penrose process in the ergosphere of the rotating black hole in Hořava-Lifshitz gravity has been studied in Sec. III. Section IV is devoted to study of the particle acceleration mechanism near the black hole in Hořava-Lifshitz gravity. We conclude our results in Sec. V.

We use a system of units in which $c = G = 1$, a spacelike signature $(-,+,+,+)$, and a spherical coordinate system (t, r, θ, φ). Greek indices are taken to run from 0 to 3 and Latin indices from 1 to 3.

II. EXTREME ROTATING BLACK HOLE IN HOŘAVA-LIFSHITZ GRAVITY

The four-dimensional metric of the spherical-symmetric spacetime written in the Arnowitt-Deser-Misner (ADM) formalism [16,19,20,32] has the following form:

$$ds^2 = -N^2c^2dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (1)$$

where N is the lapse function and N^i is the shift vector to be defined.

The Hořava-Lifshitz action describes a nonrelativistic renormalizable theory of gravitation and is given by (for more details, see Refs. [1,2,16,19,20,32])

$$S = \int dt dx^3 \sqrt{-g} N\bigg[\frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda_g K^2) + \frac{\kappa^2\mu}{2\nu_g^2}\epsilon^{ijk}R_{il}\nabla_j R^l{}_k - \frac{\kappa^2\mu^2}{8}R_{ij}R^{ij} + \frac{\kappa^2\mu^2}{8(3\lambda_g - 1)} \times \left(\frac{4\lambda_g - 1}{4}R^2 - \Lambda_W R + 3\Lambda_W^2\right) - \frac{\kappa^2}{2\nu_g^4}C_{ij}C^{ij}\bigg], \quad (2)$$

where κ, λ_g, ν_g, μ, and Λ_W are constant parameters, the Cotton tensor is defined as

$$C^{ij} = \epsilon^{ikl}\nabla_k\left(R^j{}_l - \frac{1}{4}R\delta^j_l\right), \quad (3)$$

R_{ijkl} is the three-dimensional curvature tensor, and the extrinsic curvature K_{ij} is defined as

$$K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i), \quad (4)$$

where dot denotes a derivative with respect to coordinate t.

If one considers up to second derivative terms in the action (2), one can find the known topological rotating solutions given by [33] for equations of motion in the Hořava-Lifshitz gravity. Because we are considering matter coupled with the metric in a relativistic way, we can consider the metric in Boyer-Lindquist coordinates instead of its ADM form, which is the solution of Hořava-Lifshitz gravity. In the Einstein gravity, this spacetime metric reads in Boyer-Lindquist-type coordinates in the following form (see e.g. [8]):

$$ds^2 = -\frac{\Delta_r}{\Sigma^2\rho^2}[dt - a\sin^2\theta d\varphi]^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta \sin^2\theta}{\Sigma^2\rho^2}[a dt - (r^2 + a^2)d\varphi]^2, \quad (5)$$

where the notations

$$\Delta_r = (r^2 + a^2)\left(1 + \frac{r^2}{l^2}\right) - 2Mr, \qquad \Delta_\theta = 1 - \frac{a^2}{l^2}\cos^2\theta,$$

$$\rho^2 = r^2 + a^2\cos^2\theta, \qquad \Sigma = 1 - \frac{a^2}{l^2}, \qquad l^2 = -2/\Lambda_W$$

are introduced, M is the total mass of the central BH, and a is the specific angular momentum of the BH. Note that metric (5) in ADM form can be written as [33]

$$ds^2 = -\frac{\rho^2\Delta_r\Delta_\theta}{\Sigma^2\Xi^2}dt^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Xi^2\sin^2\theta}{\Sigma^2\rho^2}[d\varphi - \varpi dt]^2, \quad (6)$$

where

$$\Xi^2 = (r^2 + a^2)^2\Delta_\theta - a^2\Delta_r\sin^2\theta,$$

$$\varpi = -\frac{a}{\Xi^2}[-(r^2 + a^2)\Delta_\theta + \Delta_r].$$

The spacetime (5) has a horizon where the four-velocity of a corotating observer tends to zero, or the surface $r = \text{const}$ becomes null. Thus we have

$$r_+ \simeq (1 - 3\delta)\left[M + \sqrt{M^2 - a^2(1 + 3\delta)}\right], \quad (7)$$

where we have introduced a small dimensionless parameter $\delta = a^2/l^2 \ll 1$.

The static limit is defined where the time-translation Killing vector $\xi^\alpha_{(t)}$ becomes null (i.e. $g_{00} = 0$), so the static limit of the BH can be described as

$$r_{st} \simeq (1 - 3\delta)\left\{M + \sqrt{M^2 - a^2(1 + \delta\sin^2\theta)(1 + 3\delta)\cos^2\theta}\right\}. \quad (8)$$

In the recent paper [33], the authors provide the ADM and Boyer-Lindquist forms of the spacetime metric, both in Hořava-Lifshitz gravity. From the expression (6) one can easily see that the results for the radii of the event horizon and static limit will be identical in the ADM and Boyer-Lindquist ones (for more details about these forms of spacetime metric presentation in Hořava-Lifshitz gravity we refer to the papers [8,33]).

Considering only the outer horizon r_+ and static limit r_{st}, it can be verified that the static limit always lies outside the horizon. The region between the two is called the ergosphere, where timelike geodesics cannot remain static but can remain stationary due to corotation with the BH

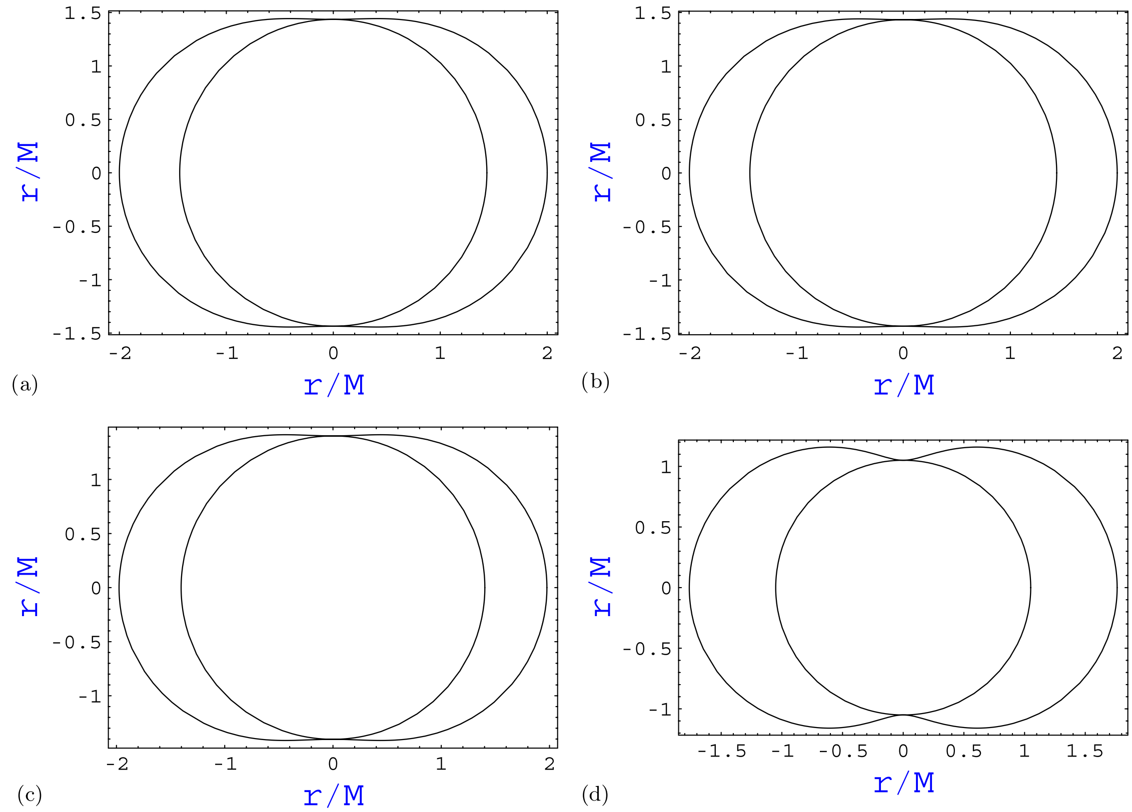

FIG. 1 (color online). The dependence of the shape of the ergosphere from the small dimensionless parameter δ: (a) $\delta = 0$, (b) $\delta = 0.001$ (c) $\delta = 0.01$, (d) $\delta = 0.1$.

with the specific frame dragging velocity at the given location in the ergosphere. This is the region of spacetime where timelike particles with negative angular momentum relative to the BH can have negative energy relative to the infinity. Then, energy could be extracted from the hole by the well-known Penrose process [34].

In Fig. 1 the dependence of the shape of the ergosphere from the small dimensionless parameter δ is shown. From the figure one can see that the relative shape of the ergosphere becomes bigger with increasing the module of the parameter δ. Although in the polar region there is no ergoregion in the presence of the nonvanishing δ parameter, where the Penrose process can be realized, near to the polar zone the ergoregion becomes bigger than that in the Kerr spacetime. It may increase the efficiency of the Penrose process.

III. ENERGY EXTRACTION OF A BLACK HOLE THROUGH THE PENROSE PROCESS

Because of the existence of an ergosphere around the BH, it is possible to extract energy from a black hole by means of the Penrose process. Inside the ergosphere, it is possible to have a timelike or null trajectory with negative total energy. As a result, one can envision a particle falling from infinity into the ergosphere and splitting into two fragments, one of which attains negative energy relative to the infinity and falls into the hole at the pole, while the other fragment would come out by conservation of energy with the energy being greater than that of the original incident particle. This is how the energy could be extracted from the hole by axial accretion of particles with the non-vanishing momentum and δ parameter.

Consider the equation of motion of such a negative energy particle at the equatorial plane ($\theta = \pi/2$, $\dot{\theta} = 0$). Using the Hamilton-Jacobi formalism the energy E and angular momentum L of the particle are given as (see e.g. [35])

$$\begin{aligned} -\tilde{E} &= -\frac{E}{m} \\ &= \left[-\frac{1}{\Sigma^2}\left(1 - \frac{2M}{r} - \frac{r^2 + a^2}{l^2}\right)\right]\dot{t} \\ &\quad + \frac{a}{\Sigma^2}\left(\frac{r^2 + a^2}{l^2} - \frac{2M}{r}\right)\dot{\varphi}, \end{aligned} \tag{9}$$

$$\begin{aligned} \tilde{L} &= \frac{L}{m} \\ &= \frac{1}{\Sigma^2}\left(r^2 + a^2\frac{l^2 - r^2 - a^2}{l^2} + \frac{2Ma^2}{r}\right)\dot{\varphi} \\ &\quad + \frac{a}{\Sigma^2}\left(\frac{r^2 + a^2}{l^2} - \frac{2M}{r}\right)\dot{t}, \end{aligned} \tag{10}$$

From Eqs. (9) and (10) one can easily obtain the equation of motion as

$$\alpha E^2 + \beta E + \gamma + \frac{\rho^2}{\Delta_r}\dot{p}^{r^2} + m^2 = 0, \quad (11)$$

where we have introduced the following notations:

$$\alpha = \frac{1}{\Sigma^2}\left(r^2 + a^2 - a^2\frac{r^2+a^2}{l^2} + a^2\frac{2M}{r}\right)\Gamma^{-1}, \quad (12)$$

$$\beta = \frac{2aL}{\Sigma^2}\left(\frac{r^2+a^2}{l^2} - \frac{2M}{r}\right)\Gamma^{-1}, \quad (13)$$

$$\gamma = -\frac{L^2}{\Sigma^2}\left(1 - \frac{2M}{r} + \frac{r^2+a^2}{l^2}\right)\Gamma^{-1}, \quad (14)$$

and

$$\Gamma = -\frac{1}{\Sigma^4}\left(1 - \frac{2M}{r} + \frac{r^2+a^2}{l^2}\right)\left(r^2 + a^2\frac{l^2-a^2-r^2}{l^2} + a^2\frac{2M}{r}\right) - \frac{a^2}{\Sigma^4}\left(\frac{r^2+a^2}{l^2} - \frac{2M}{r}\right)^2. \quad (15)$$

From Eqs. (9)–(11), one can easily obtain the equations of motion in the following form:

$$\frac{dt}{ds} = \frac{\Sigma^2}{r^2\Delta_r}\{[(r^2+a^2)^2 - \Delta_r a^2]E + (\Delta_r - r^2 - a^2)aL\}, \quad (16)$$

$$\frac{d\varphi}{ds} = \frac{\Sigma^2}{r^2\Delta_r}\{(\Delta_r - a^2)L + (r^2 + a^2 - \Delta_r)E\}, \quad (17)$$

$$\left(\frac{dr}{ds}\right)^2 = E^2 - V_{\rm eff}, \quad (18)$$

$$V_{\rm eff} = \left(1 + \frac{\Delta_r\alpha}{\rho^2}\right)E^2 + \frac{\Delta_r}{\rho^2}(\beta E + \gamma + 1).$$

In the Fig. 2 the radial dependence of the effective potential of radial motion of the massive test particle has been shown for the different values of the parameter δ. Here for the energy and momenta of the particle the following values are taken: $E/m = 0.9$, $L/mM = 6.2$. The presence of the parameter δ slightly shifts the shape of the effective potential upward.

When one of the two produced particles falls into the central BH, the mass of the BH will change by $\Delta M = E$. The change in mass can be made as large as one pleases by increasing the mass m of the infalling particle. However, there is a lower limit on ΔM which could be added to the BH corresponding to $m = 0$ and $\dot{p}^r = 0$ [35]. Evaluating all of the required quantities at the horizon r_+, one can easily get the limit for the change in BH mass as

$$E_{\rm min} = L\frac{\delta(a^2 + r_+^2)/a - 2Ma/r_+}{r_+^2 + a^2 - a^2\delta - r^2\delta + a^2 2M/r_+}. \quad (19)$$

From the expression (19), one may conclude that the Penrose process can be realized if the condition $\delta < 2Ma^2/r_+(a^2 + r_+^2)$ will be satisfied. Since current astrophysical data indicate that parameter δ is much less than 1, one may conclude that the Penrose process is a more realistic process among the energy extraction mechanisms from a BH in the Hořava-Lifshitz scenario. However, it should be mentioned that in the early Universe when the module of the cosmological constant played an important role, energy extraction from the rotating BH could be impossible in Hořava=Lifshitz scenario because of the positivity of the sign of $E_{\rm min}$. This limitation for the Penrose process does not exist in the standard theory gravity and appears in the modified theory gravity such as the Hořava-Lifshitz one.

IV. PARTICLE ACCELERATION NEAR THE BLACK HOLE

Let us find the energy $E_{\rm cm}$ in the center of mass of the system of two colliding particles with energy at infinity E_1

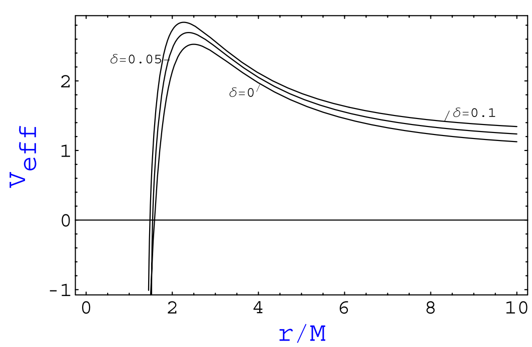

FIG. 2 (color online). The radial dependence of the effective potential of radial motion of the massive test particle for different values of the dimensionless parameter δ.

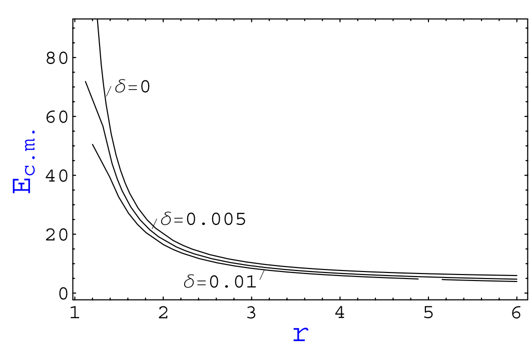

FIG. 3 (color online). The radial dependence of the center of mass energy of two infalling particles for different values of the parameter δ in the case of the extreme black hole ($a = M$).

and E_2 in the gravitational field described by spacetime metric (5). It can be obtained from

$$\left(\frac{1}{\sqrt{-g_{00}}}E_{\rm cm}, 0, 0, 0\right) = m_1 v^\alpha_{(1)} + m_2 v^\alpha_{(2)}, \qquad (20)$$

where $v^\mu_{(1)}$ and $v^\nu_{(2)}$ are the four-velocities of the particles, properly normalized by $g_{\mu\nu}v^\mu v^\nu = -1$ and m_1, m_2 are rest masses of the particles. We will consider two particles with equal mass ($m_1 = m_2 = m_0$) which have the energy at infinity $E_1 = E_2 \simeq 1$. Thus we have

$$E_{\rm cm} = m_0\sqrt{2}\sqrt{1 - g_{\alpha\beta}v^\alpha_{(1)}v^\beta_{(2)}}. \qquad (21)$$

Now using Eqs. (16)–(18) one can obtain the expression for the energy of colliding particles near the Hořava-Lifshitz black hole as

$$E^2_{\rm c.m.} = \frac{2m_0^2\Sigma^2}{r\Delta_r}\Bigg\{\frac{\delta r^5}{a^2} + 2r^2[r(1+2\delta) - 1 - 4\delta] - [2a - \delta ra(1+r^2)](l_1 + l_2) + l_1 l_2\left[2 - r + \delta r\left(1 + \frac{r^2}{a^2}\right)\right] + 2a^2[1 + (1 + 1.5\delta)r] - \sqrt{2(a-l_1)^2 - (a^2\delta - 2a\delta l_1 + l_1^2 + l_1^2\delta + 2r)r - 2\delta + 2\frac{\delta l_1}{a} - \frac{l_1^2 + r^2}{a^2}\delta r^3} \times\sqrt{2(a-l_2)^2 - (a^2\delta - 2a\delta l_2 + l_2^2 + l_2^2\delta + 2r)r - 2\delta + 2\frac{\delta l_2}{a} - \frac{l_2^2 + r^2}{a^2}\delta r^3}\Bigg\}. \qquad (22)$$

In Fig. 3 the radial dependence of the center of mass energy of two particles for the different values of the dimensionless parameter δ has been shown.

From Fig. 3 one can easily see that in the Hořava-Lifshitz gravity the particle can essentially accelerate near the horizon but not to arbitrary high energies. With increasing the parameter δ the maximal value of the center of mass energy is decreasing.

Bañados, Silk, and West [30] have shown that the total energy of two colliding test particles has no upper limit in their center of mass frame in the neighborhood of an extreme Kerr black hole, even if these particles were at rest at infinity in the infinite past. On the contrary we show here that the energy of two colliding particles in the center of mass frame observed from the infinity has an upper limit in the Hořava-Lifshitz gravity.

V. CONCLUSION

We have studied the energetics of the rotating black hole in Hořava-Lifshitz gravity. First, we considered the energy extraction mechanism via the Penrose process and found an exact expression for the limit for the change in BH mass (19) and concluded that the Penrose process can be realized if the condition $\delta < 2Ma^2/r_+(a^2 + r_+^2)$ will be satisfied. Since the parameter δ is much less than 1, it is easy to conclude that energy extraction through the Penrose process is a more realistic process among the energy extraction mechanisms from a BH in the Hořava-Lifshitz scenario. However, it should be mentioned that in the early Universe when the module of the cosmological constant played an important role, energy extraction from the rotating BH could be impossible in the Hořava-Lifshitz scenario because of the positivity of the sign of $E_{\rm min}$. This limitation for the Penrose process does not exist in the standard theory gravity and appears in the modified theory gravity such as the Hořava-Lifshitz one.

In Ref. [30], the authors underlined that a rotating black hole can, in principle, accelerate the particles falling to the central black hole to arbitrary high energies. Because of some mechanisms such as astrophysical limitations on the maximal spin, backreaction effects, and sensitivity to the initial conditions, there appears to be some upper limit for the center of mass energy of the infalling particles. One of the mechanisms offered in this paper is appearing due to the Hořava-Lifshitz gravity correction which prevents the particle from the infinite acceleration.

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