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Bulk matter fields on two-field thick branes

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In this paper, we obtain a new solution of a brane made up of a scalar field coupled to a dilaton. There is a unique parameter b in the solution, which decides the distribution of the energy density and will affect the localization of bulk matter fields. For free vector fields, we find that the zero mode can be localized on the brane. And for vector fields coupled with the dilaton via $e^{\tau\pi}F_{MN}F^{MN}$, the condition for localizing the zero mode is $\tau \geq -\sqrt{b/3}$ with $0 < b \leq 1$, or $\tau > -1/\sqrt{3b}$ with $b > 1$, which includes the case $\tau = 0$. While the zero mode for free Kalb-Ramond fields cannot be localized on the brane, if only we introduce a coupling between the Kalb-Ramond fields and the dilaton via $e^{\zeta\pi}H_{MNL}H^{MNL}$. When the coupling constant satisfies $\zeta > 1/\sqrt{3b}$ with $b \geq 1$ or $\zeta > \frac{2-b}{\sqrt{3b}}$ with $0 < b < 1$, the zero mode for the KR fields can be localized on the brane. For spin half fermion fields, we consider the coupling $\eta\bar{\Psi}e^{\lambda\pi}\phi\Psi$ between the fermions and the background scalars with positive Yukawa coupling η. The effective potentials for both chiral fermions have three types of shapes decided by the relation between the dilaton-fermion coupling constant λ and the parameter b. For $\lambda \leq -1/\sqrt{3b}$, the zero mode of left-chiral fermion can be localized on the brane. While for $\lambda > -1/\sqrt{3b}$ with $b > 1$ or $-1/\sqrt{3b} < \lambda < -\sqrt{b/3}$ with $0 < b \leq 1$, the zero mode for left-chiral fermion also can be localized.

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I. INTRODUCTION

The brane theory [1,2], which considers our four-dimensional Universe as a hypersurface (''brane world'') embedded in more higher dimensional space-time, has received a great deal of renewed attention. In this theory, all matter fields are confined to the brane in a high-dimensional space while only gravity is free to propagate in all dimensions. The extra dimensions can be compact [3–5], or infinite and noncompact [6,7]. This theory has opened up new avenues to explain some questions in particle physics and in astrophysics, such as the hierarchy problem, the cosmological problem, the nature of dark matter and dark energy [4–6,8–11].

Some brane models consider the brane as infinitely thin branes with deltalike localization of matter, which are ideal models [4–6], so some thick brane models are proposed. The thick branes are usually realized naturally by one [12–23] or two [24–26] background scalar fields configuration coupled with gravity. For a comprehensive review on thick brane solutions and related topics, please see Ref. [27].

In this paper, we will investigate the localization of various matter fields on a thick brane generalized by two background scalar fields, i.e., a kink scalar and a dilaton scalar, which is similar to that in Refs. [24,28]. But in our solution of the brane there is a unique parameter b, which makes the solution in Refs. [24,28] only one case of ours. The unique parameter b decides the distribution of the energy density of the bulk, and will affect the localization of various bulk matter fields differently.

The localization of various matter fields on the branes is an important problem in the braneworld theory, which is used in order to build up the standard model. It has been known that massless scalar fields and graviton can be localized on branes of different types [5,6,29] with an exponentially decreasing warp factor. But spin-1 Abelian vector fields can only be localized on the Randall-Sundrum (RS) brane in some higher-dimensional cases [30], or on the thick de Sitter brane and the Weyl thick brane [31,32].

The antisymmetric Kalb-Ramond (KR) tensor field $B_{\mu\nu}$ was first introduced in the string theory, in which it is associated with massless modes. Then it was used to explain the torsion of the space-time in the Einstein-Cartan theory. Moreover in the four-dimensional space-time, by a symmetry known as duality, antisymmetric tensor fields are just equivalent to scalar or vector fields [33]. However, in extra dimensions they will indicate new types of particles. Thus, any observational effect involving the KR fields is a window into the inaccessible world of very high energy physics. The investigation of the KR fields in the context of theories with extra dimensions has been carried out in Refs. [28,34–38].

In Refs. [34–36], the authors proved that in the background of RS space-time both the massless and the massive Kaluza-Klein (KK) modes of the KR fields appear much weaker than the curvature to an observer on the visible RS brane; however, when the KR fields couple with the dilaton fields, the trilinear dilaton-KR couplings may lead to new signals in Tev scale experiments. In Ref. [28], the author also proved that only when the KR fields couple with the dilaton field, the zero mode of the KR fields can be

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localized on a thick brane, but there is only a zero mode and no bound massive KK mode. In our work, we also find that it is necessary to introduce the dilaton-KR coupling in order to obtain a localized zero mode, and we find there are massive bound KK modes.

The localization of the spin $1/2$ fermion fields is also interesting. It has been proved that in order to have normalizable zero modes, the fermion fields should couple with the background scalars. With a different scalar-fermion coupling, there may exist a single bound state and a continuous gapless spectrum of massive fermion KK states [17,39–42], or finite discrete KK states (mass gap) and a continuous gapless spectrum starting at a positive m^2 [32,43–45], or even only bound KK modes [46,47]. In this paper, we will show that with one scalar-fermion coupling to both background scalars the above three cases will exist with different relations between the unique parameter b and the dilaton-fermion coupling constant.

Our paper is organized as follows: In Sec. II, we give a brief review of the braneworld generated by two scalars. Then, in Sec. III, we study the localization and mass spectra of the vector, KR, and fermion fields on the brane by presenting the potentials of the corresponding Schrödinger equations. Finally, a brief discussion and conclusion are given in the last section.

II. REVIEW OF THE BRANE GENERATED BY TWO INTERACTING SCALARS

In this paper, we consider the braneworld generated by two interacting scalars ϕ and π. The action of the system is

$$S = \int d^5x\sqrt{-g}\Big[\frac{1}{2\kappa_5^2}R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\pi)^2 - V(\phi,\pi)\Big] \tag{1}$$

with R the scalar curvature and $\kappa_5^2 = 8\pi G_5$, where G_5 is the five-dimensional Newton constant. Here we set $\kappa_5 = 1$. The line-element of a five-dimensional space-time can be assumed as [24,28,48]

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2B(y)}dy^2, \tag{2}$$

where e^{2A} and e^{2B} are the warp factors and y stands for the extra coordinate. The background scalars ϕ, π are assumed to be only the functions of y, because the brane can be treated as the cross section of the bulk. In this model, the thick brane is realized by the potential $V(\phi,\pi)$. Then the equations of motion generated from the action (1) with the ansatz (2) are given by

$$\frac{1}{2}\phi'^2 + \frac{1}{2}\pi'^2 - e^{2B}V = 6A'^2, \tag{3}$$

$$\frac{1}{2}\phi'^2 + \frac{1}{2}\pi'^2 + e^{2B}V = -6A'^2 - 3A'' + 3A'B', \tag{4}$$

$$\phi'' + (4A' - B')\phi' = e^{2B}\frac{\partial V}{\partial\phi}, \tag{5}$$

$$\pi'' + (4A' - B')\pi' = e^{2B}\frac{\partial V}{\partial\pi}, \tag{6}$$

where the prime stands for the derivative with respect to y.

The solutions of the system can be found by following the superpotential method [12]. With the superpotential function $W(\phi)$ and suppose $V = e^{-2\sqrt{b/3}\pi}[\frac{1}{2}(\frac{\partial W}{\partial\phi})^2 - \frac{4-b}{6}W^2]$, it can be verified that the following first-order differential equations are the solutions of the equations of motion (3)–(6):

$$\begin{aligned} \phi' &= \frac{\partial W}{\partial\phi}, \qquad A' = -\frac{1}{3}W, \\ B &= bA, \qquad \pi = \sqrt{3b}A, \end{aligned} \tag{7}$$

where b is a positive constant. For a specific superpotential $W(\phi)$ [24,28],

$$W(\phi) = va\phi\Big(1 - \frac{\phi^2}{3v^2}\Big), \tag{8}$$

the solutions are found to be

$$\phi(y) = v\tanh(ay), \tag{9}$$

$$A(y) = -\frac{v^2}{9}\Big(\ln\cosh^2(ay) + \frac{1}{2}\tanh^2(ay)\Big), \tag{10}$$

$$\pi(y) = \sqrt{3b}A(y), \tag{11}$$

$$B(y) = bA(y), \tag{12}$$

where v, a are both positive constants. It can be seen that the solution for ϕ is a kink and $\pi = \sqrt{3b}A$ is the dilaton field consistent with the metric and the kink. The solutions in Refs. [24,28] are only one case of the above solutions when we let $v^2/9 = \beta$ and $b = 1/4$. Here we have another parameter b, which leads to new solutions of the brane world. Our solutions for the brane do not amount to a simple coordinate change, because the solutions are decided by the scalar potential $V(\phi)$ with the parameter b, which does not depend on the coordinate systems.

In order to clarify this question more clearly, we would like to discuss the effect of the parameter b on the brane under the physical coordinate $\bar{y}$. To this end, we perform a coordinate transformation $dy = e^{-bA}d\bar{y}$ to translate the different coordinate y to the same physical coordinate $\bar{y}$. Then the metric is read as

$$ds^2 = e^{2A(y(\bar{y}))}\eta_{\mu\nu}dx^\mu dx^\nu + d\bar{y}^2. \tag{13}$$

With the relation of the two coordinate systems, we have

$$\bar{y} = \int_0^y e^{bA}d\tilde{y} \to \int_0^y e^{-(2v^2a/9)b\tilde{y}}d\tilde{y}, \quad \text{for } \tilde{y}\to\infty, \tag{14}$$

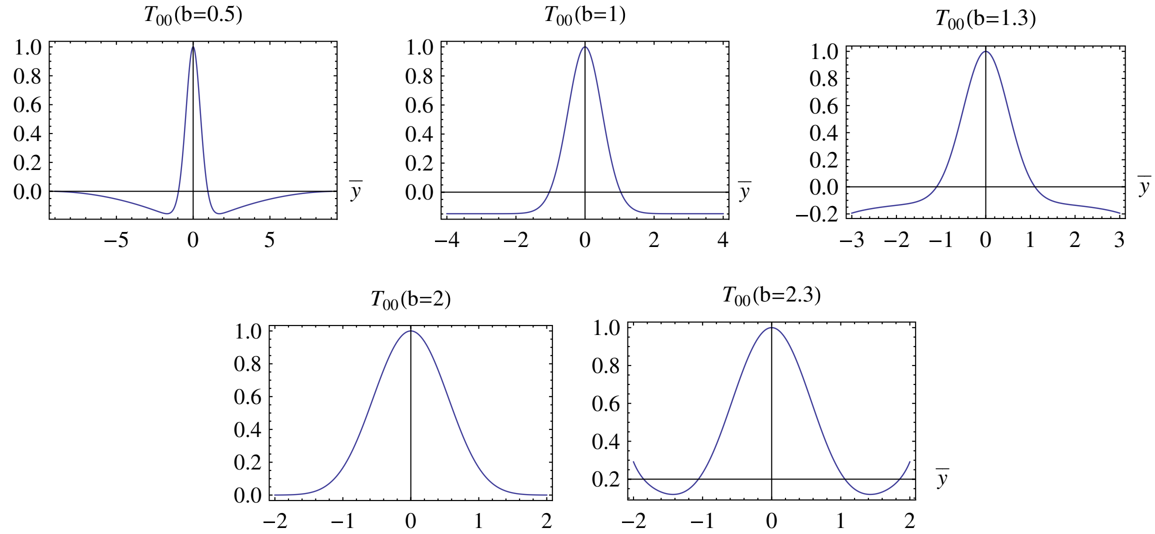

FIG. 1 (color online). The shapes of the energy density $T_{00}(\bar{y})$ for different b. The parameters are set to $a = 1$, $v = 1$.

so it can be seen that in this new coordinate system the extra dimension $\bar{y}$ is finite (with $\bar{y}_{\max} = \frac{9}{2v^2ab}$), which is different from the nonphysical and infinite coordinate y. For different b, the boundary of the extra dimension $\bar{y}$ is also different.

Now we can investigate the energy density of the system $T_{00}(\bar{y})$ in the new coordinate. With the solution (10) and the relation between the two coordinates, we calculate the values of the energy density $T_{00}(\bar{y})$ at $\bar{y} = 0$ and at $|\bar{y}| \to \bar{y}_{\max}$:

$$T_{00}(0) = a^2 v^2, \tag{15}$$

$$T_{00}(|\bar{y}| \to \bar{y}_{\max}) \to \frac{4a^2v^4}{27}(b-2)\left(-\frac{2v^2ab}{9}\bar{y}+1\right)^{-2(b-1)/b}, \tag{16}$$

from which we can see that the behavior of $T_{00}(|\bar{y}| \to \bar{y}_{\max})$ at the boundaries of the extra dimension is

$$T_{00}(|\bar{y}| \to \bar{y}_{\max}) = \begin{cases} 0, & 0 < b < 1 \\ -\frac{4}{27}a^2v^4, & b = 1 \\ -\infty, & 1 < b < 2. \\ 0, & b = 2 \\ \infty, & b > 2 \end{cases} \tag{17a}$$

We plot the energy density $T_{00}(\bar{y})$ for different b using numerical method in Fig. 1.

In the following, we will mainly discuss the effect of the parameter b on the localization of bulk matter fields.

III. LOCALIZATION AND MASS SPECTRA OF VARIOUS BULK MATTER FIELDS ON THE BRANE

In this section, we will investigate the localization and mass spectra of various bulk matter fields in this braneworld by presenting the potentials of the corresponding Schrödinger equations. In order to make sure the solutions of the system obtained before are valid, we treat the bulk matter fields considered below as perturbations around the background [49,50], namely, we neglect the backreaction of bulk matter fields on the background geometry. We will use the conformally flat metric

$$ds^2 = e^{2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2). \tag{18}$$

Comparing it with the metric (2), we find the two coordinate systems are connected by $dz = e^{2(b-1)A}dy$. For the conformally flat space-time, the extra dimension z will be infinite for $0 < b \le 1$ and finite (with $|z| \le z_{\max} = \frac{9}{2v^2a(b-1)}$) for $b > 1$. From the following calculations, we can see that the mass-independent potentials can be obtained conveniently with the conformally flat metric (18), and we will mainly investigate the effect of the parameter b on the zero modes and the spectra for various bulk matter fields.

A. Spin-1 vector fields

First, we investigate the localization of the spin-1 vector fields in five-dimensional space. The action of vector fields coupled with the dilaton is

$$S_1 = -\frac{1}{4}\int d^5x\sqrt{-g}e^{\tau\pi}g^{MR}g^{NS}F_{MN}F_{RS}, \tag{19}$$

where the field strength tensor is given by $F_{MN} = \partial_M A_N - \partial_N A_M$ and τ is the coupling constant between the dilaton and the vector field. The equations of motion can be obtained using the background geometry (18):

$$e^{\tau\pi}\frac{1}{\sqrt{-\hat{g}}}\partial_\nu(\sqrt{-\hat{g}}\hat{g}^{\nu\rho}\hat{g}^{\mu\lambda}F_{\rho\lambda}) + \hat{g}^{\mu\lambda}e^{-A}\partial_4(e^{(A+\tau\pi)}F_{4\lambda}) = 0, \tag{20}$$

$$e^{\tau\pi}\partial_\mu(\sqrt{-\hat{g}}\hat{g}^{\mu\nu}F_{\nu 4}) = 0. \tag{21}$$

Then with the gauge choice $A_4 = 0$ and the decomposition of the vector field $A_\mu(x, z) = \sum_n a_\mu^{(n)}(x)\rho_n(z)e^{-(1+\sqrt{3b}\tau)A/2}$, we find that the KK modes of the vector field satisfy the following Schrödinger-like equation:

$$[-\partial_z^2 + V_1(z)]\rho_n(z) = m_n^2\rho_n(z), \tag{22}$$

with m_n the masses of the four-dimensional vectors, and the effective potential is

$$V_1(z) = \frac{(1+\sqrt{3b}\tau)^2}{4}(\partial_z A)^2 + \frac{1+\sqrt{3b}\tau}{2}\partial_z^2 A. \tag{23}$$

Furthermore, providing the orthonormality condition

$$\int_{-z_b}^{z_b} dz\rho_m(z)\rho_n(z) = \delta_{mn}, \tag{24}$$

we can get the four-dimensional effective action

$$S_1 = \sum_n \int d^4x\sqrt{-\hat{g}}\bigg(-\frac{1}{4}\hat{g}^{\mu\alpha}\hat{g}^{\nu\beta}f_{\mu\nu}^{(n)}f_{\alpha\beta}^{(n)} - \frac{1}{2}m_n^2\hat{g}^{\mu\nu}a_\mu^{(n)}a_\nu^{(n)}\bigg), \tag{25}$$

where $f_{\mu\nu}^{(n)} = \partial_\mu a_\nu^{(n)} - \partial_\nu a_\mu^{(n)}$ is the four-dimensional field strength tensor.

We rewrite the potential (23) as the function of y:

$$V_1(y) = e^{2(1-b)A}\bigg[\frac{1+\sqrt{3b}\tau}{2}\partial_y^2 A + \frac{(1+\sqrt{3b}\tau)^2 + 2(1+\sqrt{3b}\tau)(1-b)}{4}(\partial_y A)^2\bigg]. \tag{26}$$

So, with the relation $dz = e^{(b-1)A}dy$, we can get the values of $V_1(z)$ at $z = 0$ and $|z| \to z_b$:

$$V_1(0) = -\frac{1}{6}a^2 v^2(1+\sqrt{3b}\tau), \tag{27}$$

$$V_1(|z| \to z_b) = \frac{v^4 a^2(1+\sqrt{3b}\tau)(3-2b+\sqrt{3b}\tau)}{81[-\frac{2v^2 a}{9}(b-1)z+1]^2}. \tag{28}$$

In order to get a zero mode, we have to insure that the value of $V_1(z)$ at $z = 0$ is negative, so the condition turns out to be

$$\tau > -1/\sqrt{3b}. \tag{29}$$

From this we can see that if $\tau = 0$, namely, there is no coupling between the vector and the dilaton field, there could also exist a zero mode. Then with the condition (29), the potential $V_1(z)$ is volcanolike and PT-like ones for $0 < b < 1$ and $b = 1$, respectively. For $b > 1$, the potential will be divergent at the boundary of the extra dimension with $\tau \neq \frac{2b-3}{\sqrt{3b}}$, but vanish with $\tau = \frac{2b-3}{\sqrt{3b}}$. We plot the shapes of the potential in Fig. 2.

Under the condition (29), the zero mode for the vector field can be obtained by setting $m_0 = 0$:

$$\rho_0 \propto e^{((1+\sqrt{3b}\tau)/2)A(z)}. \tag{30}$$

In order to check whether the zero mode for the vector field can be localized on the brane, we can investigate whether it

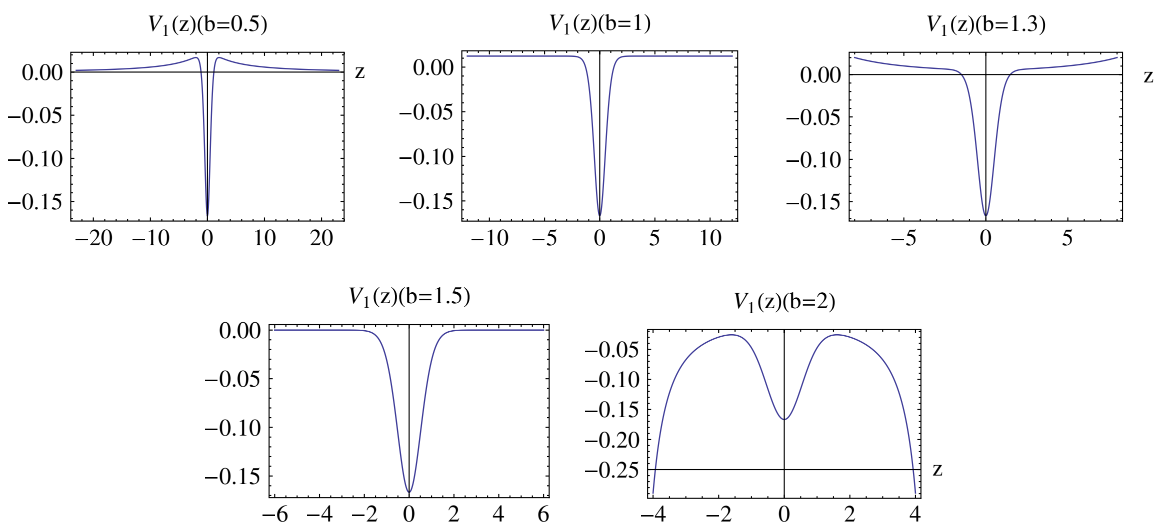

FIG. 2 (color online). The shapes of the potentials of the vector field $V_1(z)$ for different parameter b, and $a = 1$, $v = 1$, $\tau = 0$.

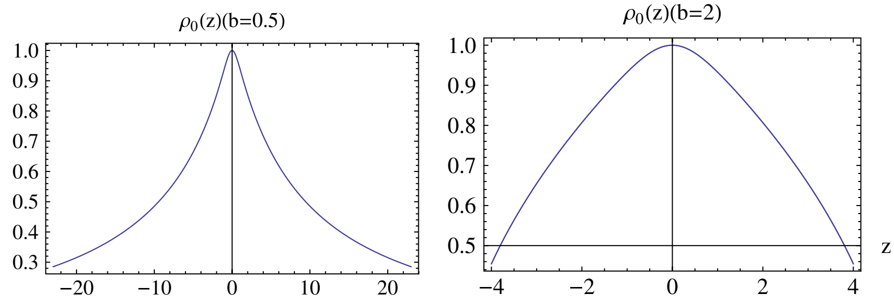

FIG. 3 (color online). The shape of the zero mode for vector field $\rho_0(z)$ with $a = 1$, $v = 1$, $\tau = 0$.

satisfies the orthonormality condition $\int_{-z_b}^{z_b} dz\rho_m(z)\rho_n(z) = \delta_{mn}$ (or $\int_{-\bar{y}_{\max}}^{\bar{y}_{\max}} d\bar{y}e^{-A}\rho_m(\bar{y})\rho_n(\bar{y}) = \delta_{mn}$). With relation $dz = e^{(b-1)A}dy$, we get

$$\int \rho_0^2 dz = \int \rho_0^2 e^{(b-1)A} dy \to \int e^{-(2v^2a/9)(b+\sqrt{3b}\tau)y} dy \quad \text{for } y \to \infty, \tag{31}$$

which is in accord with that in Ref. [24].

So if there is no coupling between the dilaton and the vector fields, the orthonormality condition for the zero mode becomes

$$\int_{-\bar{y}_{\max}}^{\bar{y}_{\max}} d\bar{y}e^{-A}\rho_0^2(\bar{y}) \propto \int_{-\bar{y}_{\max}}^{\bar{y}_{\max}} d\bar{y} < \infty, \tag{32}$$

which is the same as that in Ref. [29]. However, it is clear that the zero mode for the vector field can be localized on the brane, because the extra dimension $\bar{y}$ is finite. The shapes of the zero mode for the vector fields are shown in Fig. 3.

While for the case that there is a coupling, it can be obtained from the relation (31) that the condition for the localization of the zero mode for the vector field is $\tau \geq -\sqrt{b/3}$ for $0 < b \leq 1$ or $\tau > -1/\sqrt{3b}$ for $b > 1$.

The zero mode can be localized on the brane for different b, so there will no unstable KK modes. For $b < 1$ there is no massive bound KK mode, but some resonance may exist. There will be a finite number of the massive bound KK modes for $b = 1$, but infinite modes for $b > 1$.

B. The Kalb-Ramond fields

In this subsection, we investigate the KR fields. The action of a KR field coupled with the dilaton is

$$S_{\rm KR} = -\int d^5x\sqrt{-g}e^{\zeta\pi}H_{MNL}H^{MNL}, \tag{33}$$

where $H_{MNL} = \partial_{[M}B_{NL]}$ is the field strength for the KR field, $H^{MNL} = g^{MO}g^{NP}g^{LQ}H_{OPQ}$ and ζ is the coupling constant. The equations of motion derived from this action and the conformal metric (18) read

$$e^{\zeta\pi}\partial_\mu(\sqrt{-g}H^{\mu\alpha\beta}) + \partial_4(\sqrt{-g}e^{\zeta\pi}H^{4\alpha\beta}) = 0, \tag{34}$$

$$e^{\zeta\pi}\partial_\mu(\sqrt{-g}H^{\mu 4\beta}) = 0. \tag{35}$$

If we choose the gauge $B_{\alpha 4} = 0$ and make a decomposition of the KR field $B^{\alpha\beta}_{(n)}(x^\lambda, z) = \sum_n \hat{b}^{\alpha\beta}_{(n)}(x^\lambda)U_{(n)}(z)e^{(-7-\sqrt{3}\zeta)A/2}$, we will get the following Schrödinger equation for the KK mode $U_n(z)$:

$$(-\partial_z^2 + V_{\rm KR}(z))U_n(z) = m_n^2U_n(z), \tag{36}$$

where the effective potential $V(z)$ takes the following form:

$$V_{\rm KR} = \frac{(1-\sqrt{3b}\zeta)^2}{4}A'^2 + \frac{\sqrt{3b}\zeta - 1}{2}A''. \tag{37}$$

Provided the orthonormality condition $\int_{-z_b}^{z_b} dzU_m(z)U_n(z) = \delta_{mn}$, the action of the KR field (33) is reduced to

$$S_{\rm KR} = -\sum_n \int d^4x\sqrt{-\hat{g}}\Big(\hat{g}^{\mu'\mu}\hat{g}^{\alpha'\alpha}\hat{g}^{\beta'\beta}\hat{h}^{(n)}_{\mu'\alpha'\beta'}\hat{h}^{(n)}_{\mu\alpha\beta} + \frac{1}{3}m_n^2\hat{g}^{\alpha'\alpha}\hat{g}^{\beta'\beta}\hat{b}^{(n)}_{\alpha'\beta'}\hat{b}^{(n)}_{\alpha\beta}\Big) \tag{38}$$

with $\hat{h}^{(n)}_{\mu\alpha\beta} = \partial_{[\mu}\hat{b}_{\alpha\beta]}$ the four-dimensional field strength tensor.

The expression of the effective potential for the KR field $V_{\rm KR}$ can be written as the function of y:

$$V_{\rm KR} = e^{2(1-b)A}\Big[\frac{\sqrt{3b}\zeta - 1}{2}\partial_y^2A + \frac{(1-\sqrt{3b}\zeta)^2 + 2(1-\sqrt{3b}\zeta)(1-b)}{4}(\partial_yA)^2\Big]. \tag{39}$$

So with the braneworld solution (10), the values of the potential $V_{\rm KR}(z)$ at $z = 0$ and at the boundaries are

$$V_{\rm KR}(0) = -\frac{1}{6}a^2v^2(\sqrt{3b}\zeta - 1) \tag{40}$$

and

$$V_{\mathrm{KR}}(|z| \to z_b) = \frac{a^2 v^4 (\sqrt{3b}\zeta - 1)(\sqrt{3b}\zeta + 1 - 2b)}{81(-\frac{2v^2 a}{9}(b-1)z + 1)^2}. \tag{41}$$

In order to get a negative value of the potential $V_{\mathrm{KR}}(z)$ at $z = 0$, the parameter ζ should satisfy

$$\zeta > 1/\sqrt{3b}, \tag{42}$$

which is necessary for the localization of the zero mode. Therefore, it is clear that the zero mode of free KR fields ($\zeta = 0$) cannot be localized on the brane, and the coupling with the dilaton field π is necessary for the purpose of localizing the KR field zero mode. From Eq. (41) it can be seen that, under the condition (42), the potential is a volcanolike one and a PT-like one for $0 < b < 1$ and $b = 1$, respectively, while for $b > 1$, the potential is divergent at the boundary $z = z_{\max}$ with $\zeta \neq \frac{2b-1}{\sqrt{3b}}$, but vanishes with $\zeta = \frac{2b-1}{\sqrt{3b}}$. We plot the shapes of the potential for KR field KK modes in the conformally flat space-time in Fig. 4.

So with the condition (42), we can obtain a zero mode for the KR field by setting $m = 0$:

$$U_0 \propto \mathrm{e}^{(\sqrt{3b}\zeta - 1/2)A(z)}. \tag{43}$$

We should also check whether the zero mode for the KR fields can be localized on the brane through the orthonormality condition $\int_{-z_b}^{z_b} dz U_0^2(z) < \infty$. With the relation $dz = \mathrm{e}^{(b-1)A} dy$, we get

$$\int U_0^2 dz = \int U_0^2 \mathrm{e}^{(b-1)A} dy \to \int \mathrm{e}^{-(2av^2/9)(\sqrt{3b}\zeta - 2 + b)y} dy \quad \text{for } y \to \infty. \tag{44}$$

Thus, only when the coupling constant ζ satisfies $\zeta > 1/\sqrt{3b}$ for $b \geq 1$ or $\zeta > \frac{2-b}{\sqrt{3b}}$ for $0 < b < 1$, the integral $\int_{-z_b}^{z_b} U_0^2 dz$ is finite, i.e., the zero mode for the KR field can be localized on the brane. The shape of the zero mode for KR field is plotted in Fig. 5.

The spectrum structure of KR KK modes under the condition (42) is similar to the case of vector fields.

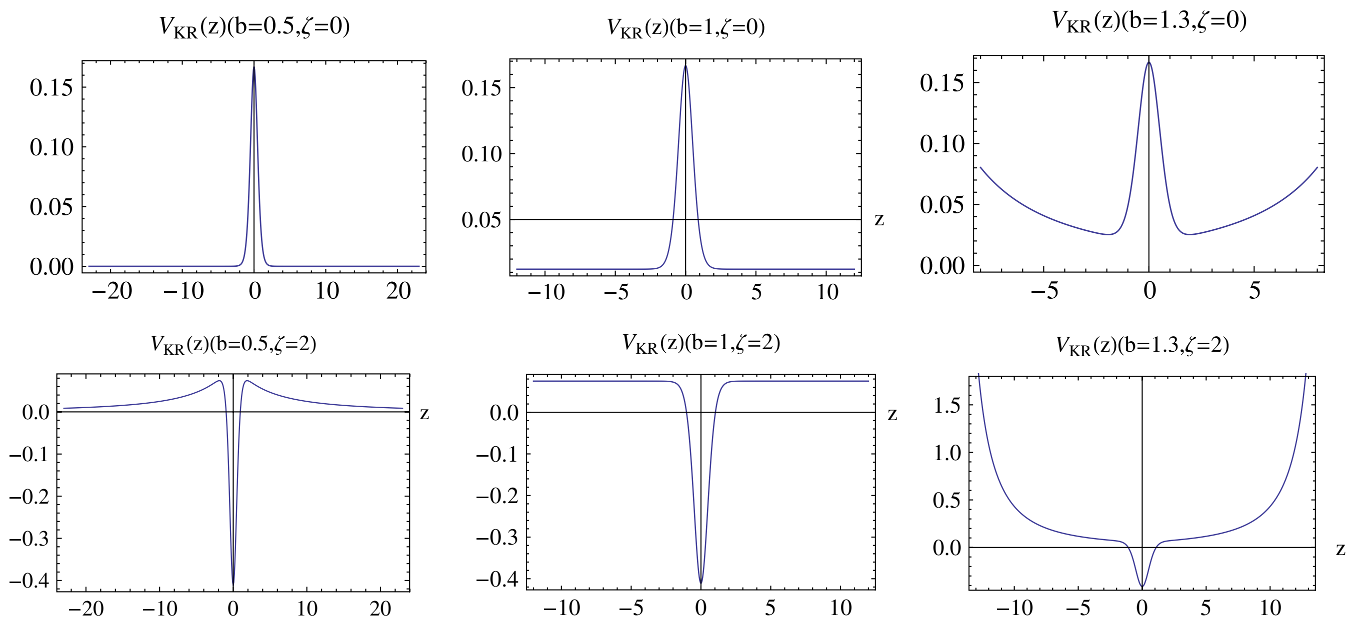

FIG. 4 (color online). The shapes of the potentials of the KR field $V_{\mathrm{KR}}(z)$. The parameters are set to $a = 1$, $v = 1$.

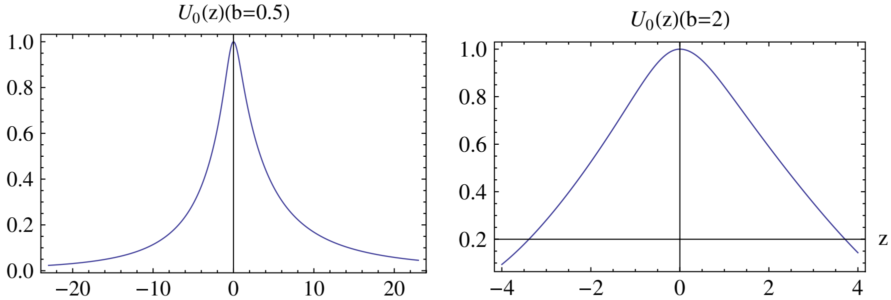

FIG. 5 (color online). The shape of the zero mode for KR field $U_0(z)$ with $a = 1$, $v = 1$, $\zeta = 2$.

C. The spin-1/2 fermion fields

In the last subsection, we investigate the spin-1/2 fermion fields. Consider a massless spin $1/2$ fermion coupled with gravity and the background scalars ϕ and π in five-dimensional space, the Dirac action is

$$S_{1/2} = \int d^5x\sqrt{-g}(\bar{\Psi}\Gamma^M(\partial_M + \omega_M)\Psi - \eta\bar{\Psi}F(\phi,\pi)\Psi) \tag{45}$$

with η the coupling constant and $F(\phi,\pi)$ the type of the coupling. As in Ref. [51,52], we have $\Gamma^M = (\mathrm{e}^{-A}\gamma^\mu, \mathrm{e}^{-A}\gamma^5)$, $\omega_\mu = \frac{1}{2}(\partial_z A)\gamma_\mu\gamma_5$, $\omega_5 = 0$, where γ^μ and γ^5 are the usual flat gamma matrices in the four-dimensional Dirac representation. Then, the five-dimensional Dirac equation is read as

$$\{\gamma^\mu(\partial_\mu + \hat{\omega}_\mu) + \gamma^5(\partial_z + 2\partial_z A) - \eta\mathrm{e}^A F(\phi,\pi)\}\Psi = 0, \tag{46}$$

where $\gamma^\mu(\partial_\mu + \hat{\omega}_\mu)$ is the four-dimensional Dirac operator. Using the general chiral decomposition $\Psi(x,z) = \mathrm{e}^{-2A}\sum_n(\psi_{Ln}(x)f_{Ln}(z) + \psi_{Rn}(x)f_{Rn}(z))$, we can get that $f_{Ln}(z)$ and $f_{Rn}(z)$ satisfy the following coupled equations:

$$[\partial_z + \eta\mathrm{e}^A F(\phi,\pi)]f_{Ln}(z) = m_n f_{Rn}(z), \tag{47a}$$
$$[\partial_z - \eta\mathrm{e}^A F(\phi,\pi)]f_{Rn}(z) = -m_n f_{Ln}(z), \tag{47b}$$

where $\psi_{Ln,Rn}(x)$ satisfy the four-dimensional massive Dirac equations $\gamma^\mu(\partial_\mu + \hat{\omega}_\mu)\psi_{Ln}(x) = m_n\psi_{Rn}(x)$ and $\gamma^\mu(\partial_\mu + \hat{\omega}_\mu)\psi_{Rn}(x) = m_n\psi_{Ln}(x)$. From the above coupled equations, we can get the following Schrödinger-like equations for the KK modes of the left- and right-chiral fermions:

$$(-\partial_z^2 + V_L(z))f_{Ln} = m_n^2 f_{Ln}, \tag{48a}$$
$$(-\partial_z^2 + V_R(z))f_{Rn} = m_n^2 f_{Rn}, \tag{48b}$$

where the effective potentials take the following forms:

$$V_L(z) = (\eta\mathrm{e}^A F(\phi,\pi))^2 - \eta\partial_z(\mathrm{e}^A F(\phi,\pi)), \tag{49a}$$
$$V_R(z) = V_L(z)|_{\eta\to-\eta}. \tag{49b}$$

Moreover, provided the following orthonormality conditions for f_{Ln} and f_{Rn},

$$\int_{-z_b}^{z_b} f_{Lm}(z)f_{Ln}(z)dz = \delta_{mn}, \tag{50}$$

$$\int_{-z_b}^{z_b} f_{Rm}(z)f_{Rn}(z)dz = \delta_{mn}, \tag{51}$$

$$\int_{-z_b}^{z_b} f_{Lm}(z)f_{Rn}(z)dz = 0, \tag{52}$$

we can obtain the standard four-dimensional action for massive chiral fermions:

$$S_{1/2} = \sum_n \int d^4x\sqrt{-\hat{g}}\,\bar{\psi}_n(x)[\gamma^\mu(\partial_\mu + \hat{\omega}_\mu) - m_n]\psi_n(x). \tag{53}$$

From (49a) and (49b), it can be seen that there must exist some kind of scalar-fermion coupling in order to localize the left- and right-chiral fermions. If we demand that $V_{L,R}(z)$ are invariant under the reflection symmetry $z\to -z$, $F(\phi(z),\pi(z))$ should be an odd function of the extra dimension y. Thus, we get $F(\phi(0),\pi(0)) = 0$ and $V_L(0) = -V_R(0) = -\eta\partial_z(F(\phi(0),\pi(0)))$, which results in that at most only one of the massless left- and right-chiral fermions could be localized on the brane. However, the masses of the massive KK modes of both chiral fermions are the same. In the following discussion, we only give the mass spectra for the left-chiral fermions.

In order to investigate the potentials of both chiral fermions, we use the relation $\partial_z = \mathrm{e}^{(1-b)A}\partial_y$ to rewrite the potentials $V_{L,R}$ as the functions of y:

$$V_L(y) = \eta\mathrm{e}^{2A}[\eta F(\phi,\pi)^2 - \mathrm{e}^{-bA}(\partial_y F(\phi,\pi)^2 + F(\phi,\pi)\partial_y A)], \tag{54}$$

$$V_R(y) = V_L(y)|_{\eta\to-\eta}. \tag{55}$$

Here, we consider the case that the scalar-fermion coupling $F(\phi,\pi)$ takes the form $\mathrm{e}^{\lambda\pi}\phi$ with λ the dilaton-fermion coupling constant, then we can get the values of $V_{L,R}(z)$ at $z=0$ and $z\to z_b$ with the coordinate transformation $dz = \mathrm{e}^{(b-1)A}dy$ and the expressions (54) and (55):

$$V_L(0) = -av\eta, \tag{56}$$

$$V_R(0) = av\eta, \tag{57}$$

$$\begin{aligned} V_L(z\to\pm z_b)\to & v^2\eta^2\left[-\frac{2v^2a}{9}(b-1)z+1\right]^{2(1+\sqrt{3b}\lambda)/b-1} \\ & +\frac{4av^3\eta}{9}(1+\sqrt{3b}\lambda) \\ & \times\left[-\frac{2v^2a}{9}(b-1)z+1\right]^{-((1+\sqrt{3b}\lambda)/b-1)-1}, \end{aligned} \tag{58}$$

$$V_R(z\to\pm z_b) = V_L(z\to\pm z_b)|_{\eta\to-\eta}. \tag{59}$$

We can see that for $1+\sqrt{3b}\lambda = 0$, the values of the potentials at the boundaries will tend to be constant. For $1+\sqrt{3b}\lambda > 0$, the values of $V_{L,R}(|z|\to z_b)$ will always vanish for different b, while for $1+\sqrt{3b}\lambda < 0$, it will always divergent. Thus there are three cases in total, which are decided by the relation between λ and b.

1. *Case 1:* $1 + \sqrt{3b}\lambda = 0$

For the case $1 + \sqrt{3b}\lambda = 0$, i.e., $\lambda = -1/\sqrt{3b}$, it can be seen that both $V_L(z \to \pm z_b)$ and $V_R(z \to \pm z_b)$ trend to a constant $v^2\eta^2$, which means the potentials are PT-like ones for left-chiral fermion with $\eta > 0$ and for right-chiral fermion with $\eta > a/v$. So, there exists a zero mode for left-chiral fermions for any positive value of η, and there will be a mass gap between the bound KK modes and continuous modes for both chiral fermions. The shapes of the potentials for a set of parameters are shown in Fig. 6.

From (47a), we can solve the zero mode for the left-chiral fermions:

$$f_{L0}(z) \propto \exp\left(-\eta \int_0^z d\bar{z} e^{A(\bar{z})} F(\phi(\bar{z}), \pi(\bar{z}))\right). \tag{60}$$

In order to check whether the zero mode can be localized on the brane, we should check whether the integral

$$\begin{aligned}\int f_{L0}^2(z)dz &\propto \int \exp\left(-2\eta \int_0^z d\bar{z} e^{A(\bar{z})} F(\phi(\bar{z}), \pi(\bar{y}))\right)dz \\ &= \int \exp\left(-2\eta \int_0^y e^{(b+\lambda\sqrt{3b})A} d\bar{y}\right) e^{(b-1)A} dy\end{aligned} \tag{61}$$

is finite. With the relation $\lambda = -1/\sqrt{3b}$, we have

$$\int f_{L0}^2(z)dz \to \int e^{-2\eta y} dy, \quad \text{for } b = 1, \tag{62}$$

$$\int f_{L0}^2(z)dz \to \int e^{-2(\eta/k(b-1))e^{k(b-1)y}} e^{k(b-1)y} dy, \quad \text{for } b \neq 1, \tag{63}$$

with $k = -2v^2a/9$, which are both finite for different b. So, the orthonormality conditions (50) are always satisfied, namely, the zero mode for the left-chiral fermion can be localized on the brane with the dilaton-fermion coupling constant $\lambda = -1/\sqrt{3b}$.

Because the potentials are PT-like ones, there exists a mass gap for both chiral fermions with large $\eta > 0$.

2. *Case 2:* $1 + \sqrt{3b}\lambda > 0$

For the case $1 + \sqrt{3b}\lambda > 0$, i.e., $\lambda > -1/\sqrt{3b}$, which includes $\lambda = 0$, it can be seen that both $V_L(z \to \pm z_b)$ and $V_R(z \to \pm z_b)$ vanish, which means the potentials are volcanolike ones for left- and right-chiral fermions with large η. So there is only a zero mode for left-chiral fermions for $\eta > 0$, and there does not exist a mass gap, but some resonances may appear for proper values of the parameters. The shapes of the potentials for both chiral fermions are shown in Fig. 7.

Here we also need to check whether the zero mode (60) satisfies the conditions (50), i.e., whether the zero mode can be localized on the brane. From (61), we get that

$$\int f_{L0}^2(z)dz \to \int e^{-(2\eta/k(b+\lambda\sqrt{3b}))e^{k(b+\lambda\sqrt{3b})y}} e^{k(b-1)y} dy. \tag{64}$$

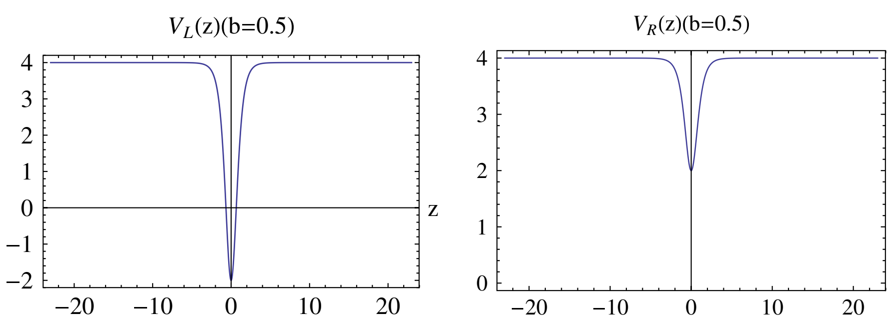

FIG. 6 (color online). The shapes of potentials $V_L(z)$ and $V_R(z)$ with $a = 1$, $v = 1$, $b = 0.5$, $\eta = 2$.

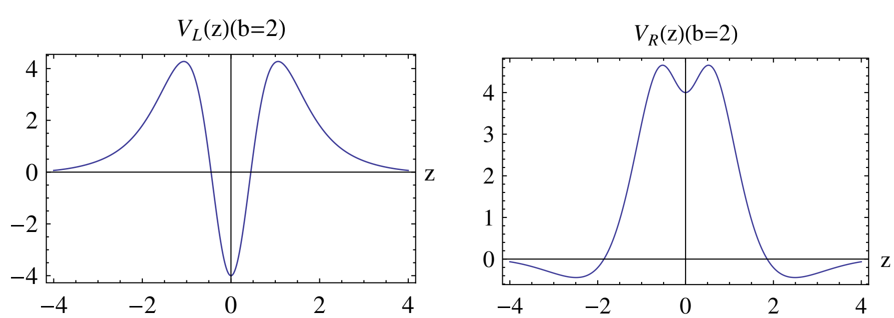

FIG. 7 (color online). The shapes of potentials $V_L(z)$ and $V_R(z)$. The parameters are set to $a = 1$, $v = 1$, $b = 0.5$, $\lambda = 1$, $\eta = 3$.

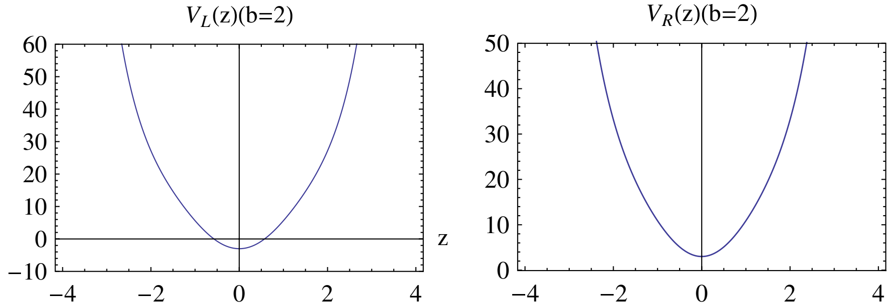

FIG. 8 (color online). The shapes of potentials $V_L(z)$ and $V_R(z)$. The parameters are set to $a = 1$, $v = 1$, $b = 1$, $\lambda = -1$, $\eta = 3$.

So we find that, for $\lambda > -1/\sqrt{3b}$ with $b > 1$ or $-1/\sqrt{3b} < \lambda < -\sqrt{b/3}$ with $0 < b < 1$, the integral is finite, and the zero mode of the left-chiral fermion can be localized on the brane. But for $b = 1$, the integral is infinite.

3. *Case 3:* $1 + \sqrt{3b}\lambda < 0$

For the last case $1 + \sqrt{3b}\lambda < 0$, i.e., $\lambda < -1/\sqrt{3b}$, the potentials $V_{L,R}(z)$ are divergent at $z \to \pm z_b$. So there will be only bound KK modes for both chiral fermions, but there is only the zero mode for the left-chiral fermion for positive value of η. We plot the shapes of the potentials for both chiral fermions in this case in Fig. 8.

From the relation (64), we find that the integral $\int_{-z_b}^{z_b} f_{L0}^2 dz$ is always finite in this case, so the zero mode of the left-chiral fermion (60) can be localized on the brane.

IV. DISCUSSIONS AND CONCLUSIONS

In this paper, by presenting the shapes of the mass-independent potentials of KK modes in the corresponding Schrödinger equations, we investigated the localization and mass spectrum of various bulk matter fields in a braneworld. The braneworld is generated by two interacting scalar fields, i.e., the kink ϕ and the dilaton π. There is a unique parameter b in the solution, which leads to different distributions of the energy density of the system, and will affect the localization of various bulk matter fields.

For spin-1 vector fields coupled with the dilaton via $e^{\tau\pi}F_{MN}F^{MN}$, when $\tau > -1/\sqrt{3b}$, the effective potential is a volcanolike potential for $0 < b < 1$ and a PT-like one for $b = 1$. It will diverge at the boundaries of the extra dimension for $b > 1$ but $\tau \neq \frac{2b-3}{\sqrt{3b}}$, and vanish for $\tau \neq \frac{2b-3}{\sqrt{3b}}$. There is always a localized vector zero mode for $\tau > -\sqrt{b/3}$ with $0 < b \leq 1$, or $\tau > -1/\sqrt{3b}$ with $b > 1$. So, when $\tau = 0$, i.e., the case without coupling with the dilaton, the vector zero mode can also be localized on the brane for any $b > 0$.

For KR fields coupled with the dilaton π, using the conformal metric and the KK decompositions, we also obtained the Schrödinger equations for the KR KK modes. When the coupling constant ζ satisfies $\zeta > 1/\sqrt{3b}$ with $b \geq 1$, or $\zeta > \frac{2-b}{\sqrt{3b}}$ with $0 < b < 1$, there will be a localized zero mode. With $\zeta > 1/\sqrt{3b}$, there are also three types of potentials for different b, which is similar to the vector fields.

While for spin-1/2 fermion fields, in order to localize the left-chiral and right-chiral fermions on the brane, some kind of coupling between the fermions and the background scalars should be introduced. In the paper, we considered the coupling type $\eta\bar{\Psi}e^{\lambda\pi}\phi\Psi$ with positive Yukawa coupling constant η. We found that the relation between the constant λ and b is crucial. With different relations, there will be three types of potentials.

If $\lambda = -1/\sqrt{3b}$, the potentials for left- and right-chiral fermions with larger η are PT-like ones. So there is a zero mode for left-chiral fermion and the zero mode can be localized on the brane for different b. The number of the massive bound KK modes for both chiral fermions is finite for $b > 0$.

For $\lambda > -1/\sqrt{3b}$, the potentials for both chiral fermions become volcanolike potentials. Therefore, there is no massive bound KK mode for this case. The zero mode for the left-chiral fermion can be localized on the brane for $b > 1$ and $\lambda > -1/\sqrt{3b}$, or $b < 1$ and $-1/\sqrt{3b} < \lambda < -\sqrt{b/3}$. However, for $b = 1$, the zero mode for the left-chiral fermion cannot be localized on the brane.

The potentials for left- and right-chiral fermions have infinite potential wells when $\lambda < -1/\sqrt{3b}$, so there will be only bound KK modes for any $b > 0$. The zero mode for the left-chiral fermion can be localized on the brane.

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[1] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125**, 136 (1983); **125**, 139 (1983).
[2] K. Akama, Lect. Notes Phys. **176**, 267 (1983).
[3] I. Antoniadis, Phys. Lett. B **246**, 377 (1990).
[4] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **436**, 257 (1998).
[5] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
[6] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[7] R. Gregory, V. A. Rubakov, and S. M. Sibiryakov, Phys. Rev. Lett. **84**, 5928 (2000).
[8] M. Gogberashvili, Int. J. Mod. Phys. D **11**, 1635 (2002).
[9] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, and R. Sundtrum, Phys. Lett. B **480**, 193 (2000).
[10] S. Kachru, M. B. Schulz, and E. Silverstein, Phys. Rev. D **62**, 045021 (2000).
[11] J. S. Alcaniz, D. Jain, and A. Dev, Phys. Rev. D **66**, 067301 (2002); D.-J. Liu, H. Wang, and B. Yang, Phys. Lett. B **694**, 6 (2010).
[12] O. DeWolfe, D. Z. Freedman, S. S. Gubser, and A. Karch, Phys. Rev. D **62**, 046008 (2000).
[13] M. Gremm, Phys. Lett. B **478**, 434 (2000); Phys. Rev. D **62**, 044017 (2000); M. Cvetic and M. Robnik, Phys. Rev. D **77**, 124003 (2008).
[14] M. Giovannini, Phys. Rev. D **65**, 064008 (2002); S. Kobayashi, K. Koyama, and J. Soda, Phys. Rev. D **65**, 064014 (2002); Y.-X. Liu, K. Yang, and Y. Zhong, J. High Energy Phys. 10 (2010) 069.
[15] O. Arias, R. Cardenas, and I. Quiros, Nucl. Phys. **B643**, 187 (2002).
[16] A. Wang, Phys. Rev. D **66**, 024024 (2002); A. Melfo, N. Pantoja, and A. Skirzewski, Phys. Rev. D **67**, 105003 (2003); K. A. Bronnikov and B. E. Meierovich, Gravitation Cosmol. **9**, 313 (2003); O. Castillo-Felisola, A. Melfo, N. Pantoja, and A. Ramirez, Phys. Rev. D **70**, 104029 (2004); M. Gogberashvili, A. Herrera-Aguilar, and D. Malagon-Morejon, arXiv:1012.4534; N. Barbosa-Cendejas, A. Herrera-Aguilar, M. A. Reyes Santos, and C. Schubert, Phys. Rev. D **77**, 126013 (2008).
[17] H.-T. Li, Y.-X. Liu, Z.-H. Zhao, and H. Guo, Phys. Rev. D **83**, 045006 (2011).
[18] R. Guerrero, A. Melfo, and N. Pantoja, Phys. Rev. D **65**, 125010 (2002).
[19] V. Dzhunushaliev, V. Folomeev, K. Myrzakulov, and R. Myrzakulov, Gen. Relativ. Gravit. **41**, 131 (2008).
[20] D. Bazeia, C. Furtado, and A. R. Gomes, J. Cosmol. Astropart. Phys. 02 (2004) 002; D. Bazeia, F. A. Brito, and A. R. Gomes, J. High Energy Phys. 11 (2004) 070; D. Bazeia, F. A. Brito, and L. Losano, J. High Energy Phys. 11 (2006) 064; A. Herrera-Aguilar, D. Malagon-Morejon, R. R. Mora-Luna, and U. Nucamendi, Mod. Phys. Lett. A **25**, 2089 (2010); N. Barbosa-Cendejas and A. Herrera-Aguilar, Phys. Rev. D **73**, 084022 (2006); **77**, 049901(E) (2008); J. High Energy Phys. 10 (2005) 101.
[21] Y. Shtanov, V. Sahni, A. Shafieloo, and A. Toporensky, J. Cosmol. Astropart. Phys. 04 (2009) 023; K. Farakos, N. E. Mavromatos, and P. Pasipoularides, J. Phys. Conf. Ser. **189**, 012029 (2009); M. Sarrazin and F. Petit, Phys. Rev. D **81**, 035014 (2010); V. Dzhunushaliev, V. Folomeev, and M. Minamitsuji, Phys. Rev. D **79**, 024001 (2009).
[22] D. Bazeia, A. R. Gomes, L. Losano, and R. Menezes, Phys. Lett. B **671**, 402 (2009); Y.-X. Liu, Y. Zhong, and K. Yang, Europhys. Lett. **90**, 51001 (2010).
[23] H. Guo, Y.-X. Liu, S.-W. Wei, and C.-E. Fu, arXiv:1008.3686.
[24] A. Kehagias and K. Tamvakis, Phys. Lett. B **504**, 38 (2001).
[25] D. Bazeia and A. R. Gomes, J. High Energy Phys. 05 (2004) 012.
[26] V. Dzhunushaliev, V. Folomeev, D. Singleton, and S. Aguilar-Rudametkin, Phys. Rev. D **77**, 044006 (2008).
[27] V. Dzhunushaliev, V. Folomeev, and M. Minamitsuji, Rep. Prog. Phys. **73**, 066901 (2010).
[28] M. O. Tahim, W. T. Cruz, and C. A. S. Almeida, Phys. Rev. D **79**, 085022 (2009).
[29] B. Bajc and G. Gabadadze, Phys. Lett. B **474**, 282 (2000); A. Herrera-Aguilar, D. Malagon-Morejon, and R. R. Mora-Luna, J. High Energy Phys. 11 (2010) 015.
[30] I. Oda, Phys. Lett. B **496**, 113 (2000).
[31] Y.-X. Liu, Z.-H. Zhao, S.-W. Wei, and Y.-S. Duan, J. Cosmol. Astropart. Phys. 02 (2009) 003.
[32] Y.-X. Liu, L.-D. Zhang, S.-W. Wei, and Y.-S. Duan, J. High Energy Phys. 08 (2008) 041; Y.-X. Liu, L.-D. Zhang, L.-J. Zhang, and Y.-S. Duan, Phys. Rev. D **78**, 065025 (2008).
[33] S. Krippendorf, F. Quevedo, and O. Schlotterer, arXiv:1011.1491.
[34] B. Mukhopadhyaya, S. Sen, and S. SenGupta, Phys. Rev. Lett. **89**, 121101 (2002).
[35] B. Mukhopadhyaya, S. Sen, S. Sen, and S. SenGupta, Phys. Rev. D **70**, 066009 (2004).
[36] B. Mukhopadhyaya, S. Sen, and S. SenGupta, Phys. Rev. D **79**, 124029 (2009).
[37] H. R. Christiansen, M. S. Cunha, and M. O. Tahim, Phys. Rev. D **82**, 085023 (2010).
[38] R. R. Landim, G. Alencar, M. O. Tahim, M. A. M. Gomes, and R. N. C. Filho, arXiv:1010.1548.
[39] Y.-X. Liu, X.-H. Zhang, L.-D. Zhang, and Y.-S. Duan, J. High Energy Phys. 02 (2008) 067.
[40] X.-H. Zhang, Y.-X. Liu, and Y.-S. Duan, Mod. Phys. Lett. A **23**, 2093 (2008).
[41] D. Bazeia, F. A. Brito, and R. C. Fonseca, Eur. Phys. J. C **63**, 163 (2009); P. Koroteev and M. Libanov, Phys. Rev. D **79**, 045023 (2009); A. Flachi and M. Minamitsuji, Phys. Rev. D **79**, 104021 (2009); Z.-H. Zhao, Y.-X. Liu, and H.-T. Li, Classical Quantum Gravity **27**, 185001 (2010).
[42] A. E. R. Chumbes, A. E. O. Vasquez, and M. B. Hott, Phys. Rev. D **83**, 105010 (2011); L. B. Castro and L. A. Meza, arXiv:1011.5872.
[43] Z.-H. Zhao, Y.-X. Liu, H.-T. Li, and Y.-Q. Wang, Phys. Rev. D **82**, 084030 (2010).
[44] Y.-X. Liu, C.-E. Fu, H. Guo, S.-W. Wei, and Z.-H. Zhao, J. Cosmol. Astropart. Phys. 12 (2010) 031.
[45] Y. Kodama, K. Kokubu, and N. Sawado, Phys. Rev. D **79**, 065024 (2009); Y. Brihaye and T. Delsate, Phys. Rev. D **78**, 025014 (2008).
[46] Y.-X. Liu, C.-E. Fu, L. Zhao, and Y.-S. Duan, Phys. Rev. D **80**, 065020 (2009).

[47] Y.-X. Liu, H. Guo, C.-E. Fu, and J.-R. Ren, J. High Energy Phys. 02 (2010) 080.
[48] Y. Zhong, Y.-X. Liu, and K. Yang, Phys. Lett. B **699**, 398 (2011).
[49] T. Banks, M. R. Douglas, G. T. Horowitz, and E. Martinec, arXiv:hep-th/9808016.
[50] I. Bena, Phys. Rev. D **62**, 066007 (2000).
[51] Y.-X. Liu, H.-T. Li, Z.-H. Zhao, J.-X. Li, and J.-R. Ren, J. High Energy Phys. 10 (2009) 091.
[52] Y.-X. Liu, J. Yang, Z.-H. Zhao, C.-E. Fu, and Y.-S. Duan, Phys. Rev. D **80**, 065019 (2009).