

Classical and quantum correlations of scalar field in the inflationary universeYasusada Nambu^{*} and Yuji Ohsumi[†]*Department of Physics, Graduate School of Science, Nagoya University, Chikusa, Nagoya 464-8602, Japan*

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We investigate classical and quantum correlations of a quantum field in the inflationary universe using a particle detector model. By considering the entanglement and correlations between two comoving detectors interacting with a scalar field, we find that the entanglement between the detectors becomes zero after their physical separation exceeds the Hubble horizon. Furthermore, the quantum discord, which is defined as the quantum part of total correlation, approaches zero on the super horizon scale. These behaviors support the appearance of the classical nature of the quantum fluctuation generated during the inflationary era.

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I. INTRODUCTION

According to the inflationary scenario of cosmology, all structure in the Universe can be traced back to primordial quantum fluctuations during an accelerated expanding phase of the very early universe. Short wavelength quantum fluctuations generated during inflation are considered to lose quantum nature when their wavelengths exceed the Hubble horizon length. Then, the statistical property of generated fluctuations can be represented by classical distribution functions. This is the assumption of the quantum to classical transition of quantum fluctuations generated by the inflation. As the structure of the present Universe is classical objects, we must explain or understand how this transition occurred and how the quantum fluctuations changed to classical fluctuations.

We have investigated this problem from the viewpoint of entanglement [1,2]. For quantum fluctuations behaving in the classical way, quantum expectation values of any operators must be calculable using appropriate classical distribution functions. Restricting to two point correlation functions of quantum operators, this condition is equivalent to the separability of a bipartite state. For a separable state, the entanglement is zero and there exists a positive normalizable P function [3] and this function can play the role of classical distribution function. In our previous study, we defined two spatially separated regions in the inflationary universe and investigated the bipartite entanglement between these regions. We found that the entanglement between these two regions becomes zero after their physical separation exceeds the Hubble horizon. This behavior of the bipartite entanglement confirms our expectation that the long wavelength quantum fluctuations during inflation behave as classical fluctuations and can become seed fluctuations for the structure formation in the Universe. Our previous analysis concerning the entanglement of quantum fluctuations in the inflationary universe

relies on the separability criterion for continuous bipartite systems [4,5] of which dynamical variables are continuous. The applicability of this criterion is limited to systems with Gaussian states: the wave function or the density matrix of the system is represented in a form of Gaussian function. Thus, we cannot say anything about the entanglement for the system with non-Gaussian states such as excited states and thermal states. Furthermore, from a viewpoint of observation or measurement, information on quantum fluctuations can be extracted via interaction between quantum fields and measurement devices. Hence, it is more natural to consider a setup in which the entanglement of quantum fields is probed using detectors.

Following this direction, we consider particle detectors [6,7] with two internal energy levels interacting with a scalar field in this paper. By preparing two spatially separated equivalent detectors interacting with the scalar field, we can extract the information on entanglement of the scalar field by evaluating the entanglement between these two detectors. As a pair of such detectors is a two-qubit system, we have the necessity and sufficient condition for entanglement of this system [8,9]. Using this setup, B. Reznik *et al.* [10,11] studied the entanglement of the Minkowski vacuum. They showed that an initially non-entangled pair of detectors evolved to an entangled state through interaction with the scalar field. As the entanglement cannot be created by local operations, this implies that the entanglement of the quantum field is transferred to a pair of detectors. M. Cliche and A. Kempf [12] constructed the information-theoretic quantum channel using this setup and evaluated the classical and quantum channel capacities as a function of the spacetime separation. G. V. Steeg and N. C. Menicucci [13] investigated the entanglement between detectors in de Sitter spacetime and they concluded that the conformal vacuum state of the massless scalar field can be discriminated from the thermal state using the measurement of entanglement.

In this paper, we investigate the entanglement structure of the quantum field in the expanding universe using the particle detector model. We also consider correlations

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between detectors and explore the relation between classical and quantum parts of correlations. This paper is organized as follows. In Sec. II, we present our setup of a detector system. Then, in Sec. III, we review entanglement measure (negativity) and classical and quantum correlations for a two-qubit system. In Sec. IV, we calculate entanglement and correlations for quantum fields in de Sitter spacetime and discuss how the classical nature of quantum fluctuations appears. Section V is devoted to summary. We use units in which $c = \hbar = G = 1$ throughout the paper.

II. TWO DETECTOR SYSTEM

We consider a system with two equivalent detectors interacting with the massless scalar field in an expanding universe [11,13]. The detectors have two energy level states $|\uparrow\rangle, |\downarrow\rangle$ and their energy difference is given by Ω . The interaction Hamiltonian is assumed to be

$$V = g(t)(\sigma_A^+ + \sigma_A^-)\phi(\mathbf{x}_A(t)) + g(t)(\sigma_B^+ + \sigma_B^-)\phi(\mathbf{x}_B(t)), \quad (1)$$

where σ^+, σ^- are raising and lowering operators for the detector's state:

$$\sigma^+ = |\uparrow\rangle\langle\downarrow|, \quad \sigma^- = |\downarrow\rangle\langle\uparrow|.$$

Two detectors are placed at $\mathbf{x}_{A,B}(t)$ and $(t, \mathbf{x}_{A,B}(t))$ and represent their world lines. We assume that the detectors are comoving with respect to cosmic expansion. Strength of the coupling is controlled in accord with the following Gaussian window function

$$g(t) = g_0 \exp\left(-\frac{(t-t_0)^2}{2\sigma^2}\right). \quad (2)$$

This window function approximates the detector being “on” when $|t-t_0| \leq \sigma$ and “off” the rest of time. We assume that the detectors are both down state initially ($t \rightarrow -\infty$) and the scalar field is vacuum state $|0\rangle$. Thus, the initial state of the total system is $|\Psi_0\rangle = |\downarrow\downarrow\rangle|0\rangle$. Then, in the interaction representation, the final state ($t \rightarrow +\infty$) of the total system after interaction becomes

$$\begin{aligned} |\tilde{\Psi}\rangle &= \left[1 - i \int_{-\infty}^{\infty} dt_1 \tilde{V}_1 - \frac{1}{2} \int_{-\infty}^{\infty} dt_1 dt_2 T[\tilde{V}_1 \tilde{V}_2] + \cdots\right] |\tilde{\Psi}_0\rangle \\ &= \left(1 - \frac{1}{2} T[\Phi_A^- \Phi_A^+ + \Phi_B^- \Phi_B^+]\right) |\downarrow\downarrow\rangle|0\rangle \\ &\quad - i\Phi_A^+ |\uparrow\downarrow\rangle|0\rangle - i\Phi_B^+ |\downarrow\uparrow\rangle|0\rangle \\ &\quad - \frac{1}{2} T[\Phi_A^+ \Phi_B^+ + \Phi_B^+ \Phi_A^+] |\uparrow\uparrow\rangle|0\rangle + O(g^3), \end{aligned} \quad (3)$$

where T is time ordering and symbols with a tilde denote quantities in the interaction representation. We defined a field operator

$$\Phi_{A,B}^\pm = \int_{-\infty}^{\infty} dt_1 g(t_1) e^{\pm i\Omega t_1} \phi(t_1, \mathbf{x}_{A,B}(t_1)).$$

As we are interested in the detectors' state, by tracing out the degrees of freedom of the scalar field, the state for the detectors' degrees of freedom becomes

$$\rho^{AB} = \begin{pmatrix} X_4 & 0 & 0 & X \\ 0 & E & E_{AB} & 0 \\ 0 & E_{AB} & E & 0 \\ X^* & 0 & 0 & 1 - 2E - X_4 \end{pmatrix}, \quad (4)$$

where we use the basis $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$ for this matrix representation of the state. The matrix elements of the state (4) are

$$\begin{aligned} X &= -2 \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 g_1 g_2 e^{i\Omega(t_1+t_2)} \\ &\quad \times \langle\phi(t_1, \mathbf{x}_A)\phi(t_2, \mathbf{x}_B)\rangle, \\ E_{AB} &= \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 g_1 g_2 e^{-i\Omega(t_1-t_2)} \langle\phi(t_1, \mathbf{x}_A)\phi(t_2, \mathbf{x}_B)\rangle, \\ E &= E_{AB}(r=0), \\ X_4 &= 4 \int_{-\infty}^{\infty} dt_3 \int_{-\infty}^{t_3} dt_4 g_3 g_4 \int_{-\infty}^{\infty} dt_1 \\ &\quad \times \int_{-\infty}^{t_1} dt_2 g_1 g_2 e^{-i\Omega(t_3+t_4)} e^{i\Omega(t_1+t_2)} \\ &\quad \times \langle\phi(t_4, \mathbf{x}_B)\phi(t_3, \mathbf{x}_A)\phi(t_1, \mathbf{x}_A)\phi(t_2, \mathbf{x}_B)\rangle. \end{aligned}$$

In X_4 , the four point function can be written using two point functions

$$\begin{aligned} &\langle\phi(t_4, \mathbf{x}_B)\phi(t_3, \mathbf{x}_A)\phi(t_1, \mathbf{x}_A)\phi(t_2, \mathbf{x}_B)\rangle \\ &= \langle\phi(t_4, \mathbf{x}_B)\phi(t_1, \mathbf{x}_A)\rangle \langle\phi(t_3, \mathbf{x}_A)\phi(t_2, \mathbf{x}_B)\rangle \\ &\quad + \langle\phi(t_4, \mathbf{x}_B)\phi(t_3, \mathbf{x}_A)\rangle \langle\phi(t_1, \mathbf{x}_A)\phi(t_2, \mathbf{x}_B)\rangle \\ &\quad + \langle\phi(t_3, \mathbf{x}_A)\phi(t_1, \mathbf{x}_A)\rangle \langle\phi(t_4, \mathbf{x}_B)\phi(t_2, \mathbf{x}_B)\rangle. \end{aligned}$$

E, E_{AB} are $O(g^2)$ and X_4 is $O(g^4)$ quantities. For the purpose of obtaining entanglement only, it is not necessary to evaluate X_4 . We need X_4 to obtain quantum mutual information in which quantum parts of correlations are encoded. By changing integration variables to

$$\begin{aligned} x_1 &= \frac{t_1 + t_2}{2}, & y_1 &= \frac{t_1 - t_2}{2}, \\ x_2 &= \frac{t_3 + t_4}{2}, & y_2 &= \frac{t_3 - t_4}{2}, \end{aligned}$$

we have the following expressions for the matrix elements:

$$\begin{aligned}
 X &= -4g_0^2 e^{2i\Omega t_0} \int_{-\infty}^{\infty} dx e^{-x^2/\sigma^2 + 2i\Omega x} \int_0^{\infty} dy e^{-y^2/\sigma^2} D^+(x + t_0, y, r), \\
 E_{AB} &= 2g_0^2 \int_{-\infty}^{\infty} dx e^{-x^2/\sigma^2} \int_{-\infty}^{\infty} dy e^{-y^2/\sigma^2 - 2i\Omega y} D^+(x + t_0, y, r), \\
 X_4 &= 16g_0^4 \int_{-\infty}^{\infty} dx_1 e^{-x_1^2/\sigma^2 + 2i\Omega x_1} \int_0^{\infty} dy_1 e^{-y_1^2/\sigma^2} \int_{-\infty}^{\infty} dx_2 e^{-x_2^2/\sigma^2 - 2i\Omega x_2} \int_0^{\infty} dy_2 e^{-y_2^2/\sigma^2} \\
 &\quad \times \left[D^+\left(\frac{x_1 + y_1 + x_2 - y_2}{2} + t_0, \frac{x_2 - y_2 - x_1 - y_1}{2}, r\right) \times D^+\left(\frac{x_1 - y_1 + x_2 + y_2}{2} + t_0, \frac{x_2 + y_2 - x_1 + y_1}{2}, r\right) \right. \\
 &\quad + D^+(x_1 + t_0, y_1, r) D^+(x_2 + t_0, y_2, r) + D^+\left(\frac{x_1 + y_1 + x_2 + y_2}{2} + t_0, \frac{x_2 + y_2 - x_1 - y_1}{2}, 0\right) \\
 &\quad \left. \times D^+\left(\frac{x_1 - y_1 + x_2 - y_2}{2} + t_0, \frac{x_2 - y_2 - x_1 + y_1}{2}, 0\right) \right], \tag{5}
 \end{aligned}$$

where we have introduced the Wightman function for the scalar field

$$D^+(x_1, y_1, r) \equiv \langle \phi(t_1, \mathbf{x}_A) \phi(t_2, \mathbf{x}_B) \rangle, \quad r = |\mathbf{x}_A - \mathbf{x}_B|.$$

III. ENTANGLEMENT AND CORRELATIONS OF DETECTORS

As the response of detectors due to interaction with the scalar field is given by the state (4), we can extract information on entanglement and correlations of the scalar field indirectly by analyzing this state.

A. Entanglement of detectors

As a measure of the entanglement between two detectors, we consider the negativity [14] defined via a partially transposed operation to the density matrix (4) with respect to detector B's degrees of freedom. The eigenvalues of the partially transposed density matrix are

$$\lambda = E \pm |X| + O(g^4).$$

The negativity is defined using the eigenvalues of the partially transposed density matrix

$$\mathcal{N} = \sum_{\lambda_i < 0} |\lambda_i| = \max[0, |X| - E]. \tag{6}$$

From here on, we designate the following quantity as the negativity

$$\mathcal{N} = |X| - E. \tag{7}$$

The negativity gives the necessity and the sufficient condition of the entanglement for two-qubit systems [8,9]. Thus two detectors are entangled when $\mathcal{N} > 0$ and separable when $\mathcal{N} < 0$. For separable initial states of detectors $\mathcal{N} < 0$, $\mathcal{N} > 0$ after interaction with the scalar field implies the scalar field is entangled because entanglement cannot be generated by local operations.

B. Correlations of detectors

Using Bloch representation, the state (4) can be written as follows

$$\begin{aligned}
 \rho^{AB} &= \frac{1}{4} \left(I \otimes I + \mathbf{a} \cdot \boldsymbol{\sigma} \otimes I + I \otimes \mathbf{b} \cdot \boldsymbol{\sigma} + \sum_{\ell, m=1}^3 c_{\ell m} \sigma_{\ell} \otimes \sigma_m \right), \\
 \mathbf{a} = \mathbf{b} &= (0, 0, -1 + 2E + 2X_4), \\
 c_{\ell m} &= \begin{pmatrix} 2(E_{AB} + X_R) & -2X_I & 0 \\ -2X_I & 2(E_{AB} - X_R) & 0 \\ 0 & 0 & 1 - 4E \end{pmatrix}, \tag{8}
 \end{aligned}$$

where I is the identity operator, $\{\sigma_1, \sigma_2, \sigma_3\}$ are the Pauli spin matrices, $X_R = \text{Re}(X)$ and $X_I = \text{Im}(X)$. To quantify quantumness of quantum fluctuations in de Sitter space-time, we want to consider the ‘‘classical’’ and ‘‘quantum’’ part of the correlation between two detectors. The classical part of correlation is defined through a local measurement on each detector. By a measurement here we mean the von Neuman type; complete measurement consisting of orthogonal one-dimensional projectors.

1. Classical mutual information

To define the classical part of the correlation between two detectors (two qubits), we perform a local projective measurement of detector states. Of course, it is not possible to perform measurement of the scalar field in the inflationary era directly. We consider the following measurement procedure as a gedanken experiment to explore the nature of quantum fluctuation. The measurement operators for each detector are

$$\begin{aligned}
 M_{\pm}^A &= \frac{I \pm \mathbf{n}_A \cdot \boldsymbol{\sigma}}{2}, \quad M_{\pm}^B = \frac{I \pm \mathbf{n}_B \cdot \boldsymbol{\sigma}}{2}, \\
 |\mathbf{n}_A| &= |\mathbf{n}_B| = 1, \tag{9}
 \end{aligned}$$

where \pm denotes output of the measurement. \mathbf{n}_A , \mathbf{n}_B represent the internal direction of measurement. The joint

probability p_{jk} attaining measurement result j for detector A and k for detector B ($j, k = \pm 1$) is obtained as

$$\begin{aligned}
 p_{jk} &= \text{tr}(M_j^A \otimes M_k^B \rho^{AB}) \\
 &= \frac{1}{4} \left[(1 + (j)a_z n_z^A)(1 + (k)b_z n_z^B) \right. \\
 &\quad \left. + (jk) \sum_{\ell m} (c_{\ell m} - a_{\ell} b_m) n_{\ell}^A n_m^B \right], \\
 c_{\ell m} - a_{\ell} b_m &= \begin{pmatrix} 2(E_{AB} + X_R) & -2X_I & 0 \\ -2X_I & 2(E_{AB} - X_R) & 0 \\ 0 & 0 & -4E^2 + 4X_4 \end{pmatrix}.
 \end{aligned} \tag{10}$$

The probability p_j attaining a result j for detector A and p_k attaining a result k for detector B are

$$p_j = \sum_k p_{jk}, \quad p_k = \sum_j p_{jk}.$$

Using the joint probability (10) obtained by the measurement, the classical mutual information I_C is defined by [15]

$$\begin{aligned}
 I_C(p) &= H(p_j) + H(p_k) - H(p_{jk}), \\
 H(p_j) &= -\sum_j p_j \log_2 p_j, \\
 H(p_{jk}) &= -\sum_{jk} p_{jk} \log_2 p_{jk},
 \end{aligned}$$

where $H(p)$ is a Shannon entropy for a probability distribution p . The explicit form of $I_C(p)$ using p_{jk} is

$$I_C(p) = \sum_{jk} p_{jk} \log_2 \left(\frac{p_{jk}}{p_j p_k} \right). \tag{11}$$

The measure of classical correlation is defined via the maximization done over all possible projective measurements¹:

$$\mathcal{C}(p) = \sup_{\{\mathbf{n}_A, \mathbf{n}_B\}} I_C(p). \tag{12}$$

We evaluate the classical mutual information of p_{jk} separately for the cases where the vectors $\mathbf{n}_{A,B}$ are parallel to the z axes or not. For the case where \mathbf{n}_A and \mathbf{n}_B are both parallel to the z axes, we have

$$I_C(p) = \frac{1}{\ln 2} \left(E^2 - X_4 + X_4 \ln \left(\frac{X_4}{E^2} \right) \right) + O(g^6). \tag{13}$$

For the case where one of \mathbf{n}_A or \mathbf{n}_B is parallel to the z axes,

$$I_C(p) = O(g^6). \tag{14}$$

¹This definition of the classical correlation is based on two-sided measurements of correlations. We can also define the classical correlation using one-sided measurements of correlations.

For the case where both \mathbf{n}_A and \mathbf{n}_B are not parallel to the z axes,

$$\begin{aligned}
 I_C(p) &= \frac{1}{2 \ln 2} \frac{\left(\sum_{\ell m=x,y} \tilde{c}_{\ell m} (n_A)_{\ell} (n_B)_m \right)^2}{(1 - (n_{Az})^2)(1 - (n_{Bz})^2)} + O(g^6), \\
 \tilde{c}_{\ell m} &= \begin{pmatrix} 2(E_{AB} + X_R) & -2X_I \\ -2X_I & 2(E_{AB} - X_R) \end{pmatrix}.
 \end{aligned} \tag{15}$$

By introducing new direction vectors

$$\begin{aligned}
 \tilde{\mathbf{n}}_A &= \frac{1}{\sqrt{(n_{Ax})^2 + (n_{Ay})^2}} \begin{pmatrix} n_{Ax} \\ n_{Ay} \end{pmatrix}, \\
 \tilde{\mathbf{n}}_B &= \frac{1}{\sqrt{(n_{Bx})^2 + (n_{By})^2}} \begin{pmatrix} n_{Bx} \\ n_{By} \end{pmatrix},
 \end{aligned}$$

we obtain

$$I_C(p) = \frac{1}{2 \ln 2} \left(\sum_{\ell m} \tilde{c}_{\ell m} (\tilde{\mathbf{n}}_A)_{\ell} (\tilde{\mathbf{n}}_B)_m \right)^2. \tag{16}$$

As the eigenvalues of the matrix \tilde{c} is $2(E_{AB} \pm |X|)$, the maximum of classical mutual information for this case is given by

$$I_{C,\max}(p) = \frac{2}{\ln 2} (|E_{AB}| + |X|)^2. \tag{17}$$

Thus, the classical correlation of the state (8) obtained by the local projective measurement is

$$\mathcal{C} = \max \left[\frac{1}{\ln 2} \left(E^2 - X_4 + X_4 \ln \left(\frac{X_4}{E^2} \right) \right), \frac{2}{\ln 2} (|E_{AB}| + |X|)^2 \right]. \tag{18}$$

We comment on the relation between the classical mutual information and a correlation function of qubit variables. By the local projective measurement of the state (8), we obtain the following expectation values for qubit variables:

$$\begin{aligned}
 \langle \mathbf{n}_A \cdot \boldsymbol{\sigma} \rangle &= \mathbf{a} \cdot \mathbf{n}_A, \\
 \langle \mathbf{n}_B \cdot \boldsymbol{\sigma} \rangle &= \mathbf{b} \cdot \mathbf{n}_B, \quad \langle \mathbf{n}_A \cdot \boldsymbol{\sigma} \otimes \mathbf{n}_B \cdot \boldsymbol{\sigma} \rangle = \sum_{\ell, m} c_{\ell m} (n_A)_{\ell} (n_B)_m.
 \end{aligned}$$

By introducing a fluctuation part of qubit variables as $\Delta \sigma_n = \mathbf{n} \cdot \boldsymbol{\sigma} - \langle \mathbf{n} \cdot \boldsymbol{\sigma} \rangle$, the correlation function for fluctuations of qubit variables is

$$\begin{aligned}
 \langle \Delta \sigma_{nA} \Delta \sigma_{nB} \rangle &= \sum_{\ell, m} (c_{\ell m} - a_{\ell} b_m) (n_A)_{\ell} (n_B)_m \\
 &= \sum_{\ell, m} \tilde{c}_{\ell m} (\tilde{\mathbf{n}}_A)_{\ell} (\tilde{\mathbf{n}}_B)_m,
 \end{aligned} \tag{19}$$

where we have assumed $\mathbf{n}_A, \mathbf{n}_B$ are not parallel to the z axis at the last equality. Therefore, the classical mutual information (16) for joint probability p_{jk} corresponds to the square of the correlation function of the fluctuation part of the qubit variables.

2. Quantum mutual information and quantum discord

The quantum mutual information I_Q of the bipartite state ρ^{AB} is defined independently of measurement procedure:

$$I_Q(\rho^{AB}) = S(\rho^A) + S(\rho^B) - S(\rho^{AB}), \quad (20)$$

$$S(\rho) = -\text{tr}(\rho \log_2 \rho),$$

where $S(\rho)$ is the von Neumann entropy for the state ρ . This quantity represents the total correlations of the bipartite system including both the quantum and classical parts of correlations.

We evaluate the quantum mutual information for the state (8). The eigenvalues of the state (8) are $1 - 2E + |X|^2 - X_4$, $E \pm E_{AB}$, $X_4 - |X|^2$. The reduced density matrix for the subsystem A is

$$\rho^A = \text{tr}_B \rho^{AB} = \begin{pmatrix} E + X_4 & 0 \\ 0 & 1 - E - X_4 \end{pmatrix}.$$

Thus, the quantum mutual information for the two detector system is given by

$$I_Q(\rho) = \frac{1}{\ln 2} [-2E \ln E + (E + E_{AB}) \ln(E + E_{AB}) + (E - E_{AB}) \ln(E - E_{AB})] + \frac{1}{\ln 2} [E^2 - X_4 + |X|^2 - 2X_4 \ln E + (X_4 - |X|^2) \ln(X_4 - |X|^2)] + O(g^6). \quad (21)$$

The quantum discord [16–18] is introduced as the difference between the quantum mutual information and the classical mutual information

$$\mathcal{Q}(\rho) = I_Q(\rho) - \mathcal{C}(\rho). \quad (22)$$

For arbitrary local projective measurements, it can be shown that $\mathcal{Q} \geq 0$. Thus, the quantum mutual information can be decomposed into a positive classical mutual information and a positive quantum discord. The necessity and sufficient condition for zero quantum discord is expressed as the state has the following form [19]:

$$\rho^{AB} = \sum_{j,k} p_{jk} M_j^A M_k^B. \quad (23)$$

For this state, measurement (9) does not alter the form of state because M_j^A , M_k^B are projection operators. In this sense, the state with zero quantum discord can be termed a classical state and the quantum discord represents the quantum part of the total correlations. For pure state, we have $\mathcal{Q} = \mathcal{C}$ and the state with zero quantum discord has no classical correlations.

3. Bell-CHSH inequality

Related to classical correlations obtainable via local measurements, we consider the question whether correlations derived under the state (8) admit a local

hidden-variable (LHV) description; measured correlation functions can be mimicked by classical distribution functions. Let us consider the following operator (Bell operator):

$$\mathcal{B}_{\text{CHSH}} = \mathbf{a} \cdot \boldsymbol{\sigma} \otimes (\mathbf{b} + \mathbf{b}') \cdot \boldsymbol{\sigma} + \mathbf{a}' \cdot \boldsymbol{\sigma} \otimes (-\mathbf{b}') \cdot \boldsymbol{\sigma}, \quad (24)$$

where \mathbf{a} , \mathbf{a}' , \mathbf{b} , \mathbf{b}' are real unit vectors. Then, the Bell-Clauser-Horne-Shimony-Holt (CHSH) inequality [20] is

$$|\langle \mathcal{B}_{\text{CHSH}} \rangle| \leq 2. \quad (25)$$

If the state admits a LHV description of correlations, then this inequality holds. Violation of this inequality means existence of nonlocality. The two qubit state violates the Bell-CHSH inequality if and only if the following condition is satisfied [21]:

$$M(\rho) > 1,$$

$M(\rho) \equiv$ the sum of the two largest eigenvalues

$$\text{of the matrix } c c^\dagger, \quad (26)$$

where the matrix $c_{jk} = \text{tr}(\rho \sigma_j \otimes \sigma_k)$. For the state (8),

$$c_{jk} = \begin{pmatrix} c_{11} & c_{12} & 0 \\ c_{12} & c_{22} & 0 \\ 0 & 0 & c_{33} \end{pmatrix}, \quad c_{11} = 2(E_{AB} + X_R),$$

$$c_{22} = 2(E_{AB} - X_R), \quad c_{12} = -2X_I, \quad c_{33} = 1 - 4E.$$

As eigenvalues of c_{jk} are $1 - 4E$, $2(E_{AB} \pm |X|)$, the sum of the square of the two largest eigenvalues of c cannot exceed unity and the Bell-CHSH inequality holds. However, this does not mean the LHV description of correlations is possible; holding the Bell-CHSH inequality is only a necessary condition for the LHV description and does not guarantee existence of a LHV. Actually, by passing each detector through the filter

$$f_{A,B} = \begin{pmatrix} 1 & 0 \\ 0 & \eta \end{pmatrix},$$

there is a possibility revealing hidden nonlocality of the state [22]. After passing through the filter, the state is transformed according to $\rho \rightarrow \rho' = (f_A \otimes f_B) \rho (f_A \otimes f_B)$. Assuming that $\eta^2 = O(g^2)$, the transformed normalized state is

$$\frac{\rho'}{\text{tr} \rho'} = \frac{1}{X_4 + 2\eta^2 E + \eta^4} \times \begin{pmatrix} X_4 & 0 & 0 & \eta^2 X \\ 0 & \eta^2 E & \eta^2 E_{AB} & 0 \\ 0 & \eta^2 E_{AB} & \eta^2 E & 0 \\ \eta^2 X^* & 0 & 0 & \eta^4 \end{pmatrix}, \quad (27)$$

and the matrix c is transformed to

$$c' = \frac{2\eta^2}{X_4 + 2\eta^2 E + \eta^4} \times \begin{pmatrix} E_{AB} + X_R & -X_I & 0 \\ -X_I & E_{AB} - X_R & 0 \\ 0 & 0 & \frac{X_4 - 2\eta^2 E + \eta^4}{2\eta^2} \end{pmatrix}. \quad (28)$$

Assuming that the state is entangled $\mathcal{N} = |X| - E > 0$, eigenvalues of c' are

$$\pm \frac{2\eta^2 |X|}{X_4 + \eta^4}, \quad 1 - \frac{4E\eta^2}{X_4 + \eta^4},$$

and the condition for violation of the Bell-CHSH inequality $M(\rho') > 1$ is

$$\eta^4 - \frac{|X|^2}{2E} \eta^2 + X_4 < 0.$$

For existence of a real η satisfying this inequality, we need [10]

$$|X|^4 > 16X_4 E^2. \quad (29)$$

This provides a sufficient condition for violation of the Bell-CHSH inequality and existence of a hidden nonlocality.

IV. BEHAVIOR OF ENTANGLEMENT AND CORRELATIONS OF SCALAR FIELD

The matrix elements E_{AB} , X , X_4 of (5) can be evaluated using numerical integration after considering contributions of poles in integrands by contour integration on a complex plane [23]. However, numerical integrations of these functions are not so easy especially evaluating the fourfold integral of X_4 . Hence, in this paper, we consider asymptotic estimation of these functions to derive analytic approximate forms. We assume parameters $\Omega\sigma \gg 1$ with other dimensionless combinations of parameters such as $H\Omega\sigma^2$ contained in D^+ are kept in order of unity. That is, we are considering the asymptotic behavior of E_{AB} , X , X_4 in the range of parameters

$$\Omega\sigma \gg 1, \quad H\sigma \ll 1,$$

where H is the Hubble parameter. By rescaling the integration variables x and y ,

$$\begin{aligned} X &= -4g_0^2(\Omega\sigma^2)^2 e^{2i\Omega t_0} \int_{-\infty}^{\infty} dx e^{-(\Omega\sigma)^2(x^2 - 2ix)} \\ &\quad \times \int_0^{\infty} dy e^{-(\Omega\sigma)^2 y^2} D^+(t_0 + \Omega\sigma^2 x, \Omega\sigma^2 y, r), \\ E_{AB} &= 2g_0^2(\Omega\sigma^2)^2 \int_{-\infty}^{\infty} dx e^{-(\Omega\sigma)^2 x^2} \\ &\quad \times \int_{-\infty}^{\infty} dy e^{-(\Omega\sigma)^2(y^2 + 2iy)} D^+(t_0 + \Omega\sigma^2 x, \Omega\sigma^2 y, r). \end{aligned}$$

For $\Omega\sigma \gg 1$, x and y integrals in X can be evaluated approximately at the saddle point $x = i$ and $y = 0$ of the integrand. After performing Gaussian integrals about these saddle points, we have

$$\begin{aligned} X &\approx -4g_0^2(\Omega\sigma^2)^2 e^{2i\Omega t_0} \int_{-\infty}^{\infty} dx e^{-(\Omega\sigma)^2(x^2 + 1)} \\ &\quad \times \int_0^{\infty} dy e^{-(\Omega\sigma)^2 y^2} D^+(t_0 + i\Omega\sigma^2, 0, r) \\ &= -2\pi g_0^2 \sigma^2 e^{-(\Omega\sigma)^2} e^{2i\Omega t_0} D^+(t_0 + i\Omega\sigma^2, 0, r). \end{aligned}$$

In the same way, expression for E_{AB} is

$$\begin{aligned} E_{AB} &\approx 2g_0^2(\Omega\sigma^2)^2 \int_{-\infty}^{\infty} dx e^{-(\Omega\sigma)^2 x^2} \\ &\quad \times \int_{-\infty}^{\infty} dy e^{-(\Omega\sigma)^2(y^2 + 1)} D^+(t_0, -i\Omega\sigma^2, r) \\ &= 2\pi g_0^2 \sigma^2 e^{-(\Omega\sigma)^2} D^+(t_0, -i\Omega\sigma^2, r). \end{aligned}$$

Thus, the asymptotic forms of X , E_{AB} for $\Omega\sigma \gg 1$ are obtained as

$$\begin{aligned} X &\approx -2\pi g_0^2 \sigma^2 e^{-(\Omega\sigma)^2} D^+(t_0 + i\Omega\sigma^2, 0, r), \\ E_{AB} &\approx 2\pi g_0^2 \sigma^2 e^{-(\Omega\sigma)^2} D^+(t_0, -i\Omega\sigma^2, r), \\ X_4 &\approx 4\pi^2 g_0^4 \sigma^4 e^{-2(\Omega\sigma)^2} [D^+(t_0, -i\Omega\sigma^2, r)^2 \\ &\quad + |D^+(i\Omega\sigma^2 + t_0, 0, r)|^2 + D^+(t_0, -i\Omega\sigma^2, 0)^2] \\ &= E^2 + E_{AB}^2 + |X|^2. \end{aligned} \quad (30)$$

The negativity is

$$\begin{aligned} \mathcal{N} &\approx 2\pi g_0^2 \sigma^2 e^{-(\Omega\sigma)^2} [|D^+(t_0 + i\Omega\sigma^2, 0, r)| \\ &\quad - D^+(t_0, -i\Omega\sigma^2, 0)]. \end{aligned} \quad (31)$$

Using the asymptotic form of X_4 , we can reduce the condition of hidden nonlocality (29). For the entangled region $|X| \gg E$, using the asymptotic form of $X_4 \approx E^2 + |X|^2 + E_{AB}^2 \approx |X|^2$, the condition of hidden nonlocality (29) reduces to

$$|X| > 4E, \quad (32)$$

and provides a stronger condition than that for entanglement $|X| > E$.

Now, let us we consider entanglement and correlations of the scalar fields in Minkowski and de Sitter spacetime.

A. Minkowski vacuum

The Wightman function of the massless scalar with Minkowski vacuum state is

$$D^+ = -\frac{1}{4\pi^2} \frac{1}{4(y - i\epsilon)^2 - r^2}, \quad (33)$$

where a positive small constant $\epsilon > 0$ is introduced to regularize ultraviolet divergence in the Wightman function (see Appendix). Using (30),

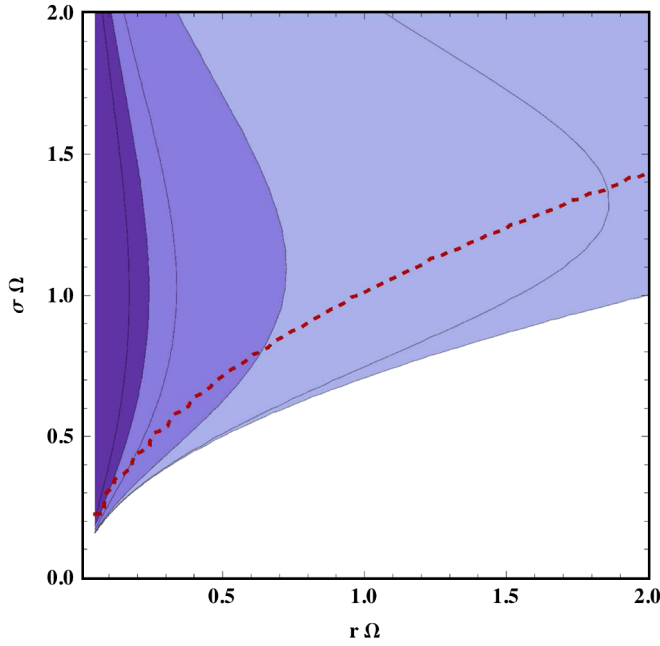


FIG. 1 (color online). Contours of negativity for the Minkowski vacuum. Two detectors are entangled with parameters in the shaded region. Darker areas correspond to larger negativities. In the region above the dotted line, the Bell-CHSH inequality is violated after operation of local filtering.

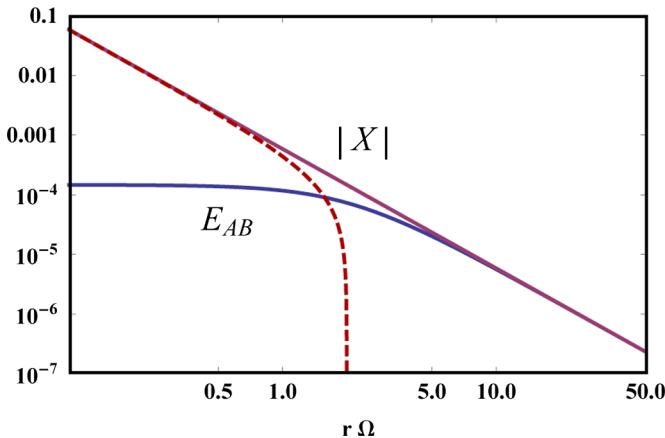
$$X = -(2\pi g_0^2) \frac{e^{-(\Omega\sigma)^2}}{4\pi^2} \left(\frac{\sigma}{r}\right)^2,$$

$$E_{AB} = (2\pi g_0^2) \frac{e^{-(\Omega\sigma)^2}}{4\pi^2} \frac{1}{(r/\sigma)^2 + 4(\Omega\sigma)^2}.$$

The negativity is

$$\mathcal{N} = \frac{g_0^2}{8\pi} e^{-\Omega^2\sigma^2} \left[4\left(\frac{\sigma}{r}\right)^2 - \frac{1}{(\Omega\sigma)^2} \right], \quad (34)$$

and the $\mathcal{N} = 0$ line is given by



$$\sigma\Omega = \frac{r}{2\sigma}. \quad (35)$$

For $r/\sigma < 2\Omega\sigma$, the negativity is positive and detectors are entangled (Fig. 1). As the prepared initial state of detectors is separable and entanglement cannot be created via local operations, this entanglement is due to the scalar field and it is swapped to a pair of detectors through interaction. For large separation $r/\sigma > 2\Omega\sigma$, detectors are separable. However, this does not mean the scalar field is separable for this separation; by increasing detector's parameter σ with fixed r , it is possible to make $\mathcal{N} > 0$. For any separation r , we can always choose an appropriate value of σ that makes detectors entangled. Thus, we can say that the Minkowski vacuum is always entangled.

Now, let us consider behavior of correlations. The left panel in Fig. 2 shows r dependence of $|X|$, E_{AB} . They are monotonically decreasing functions with respect to r . In the entangled region $r < 2\Omega\sigma^2$, E_{AB} is nearly constant $E_{AB} \approx E$ and in the separable region $r > 2\Omega\sigma^2$, $E_{AB} \approx |X| < E$. r dependence of the quantum mutual information I_Q and the classical correlation C is shown in the right panel in Fig. 2. These correlations also decrease monotonically with respect to r . In the entangled regions, the difference of these two correlations (quantum discord) increases as r increases until two detectors become separable. In the separable region, quantum discord remains constant.

As shown in Fig. 3, the ratio of classical correlation to total correlation C/I_Q is a good indicator of separability for this system. In the entangled region, the ratio decreases monotonically and in the separable region, the ratio approaches a constant value. We can derive this behavior using the following r dependence of X , E_{AB} :

$$\begin{aligned} r \ll r_c: E_{AB} \approx E \ll |X| (\text{entangled region}), \\ r \gg r_c: |X| \approx E_{AB} \ll E (\text{separable region}), \end{aligned} \quad (36)$$

where $r_c = 2\Omega\sigma^2$. For $r \ll r_c$,

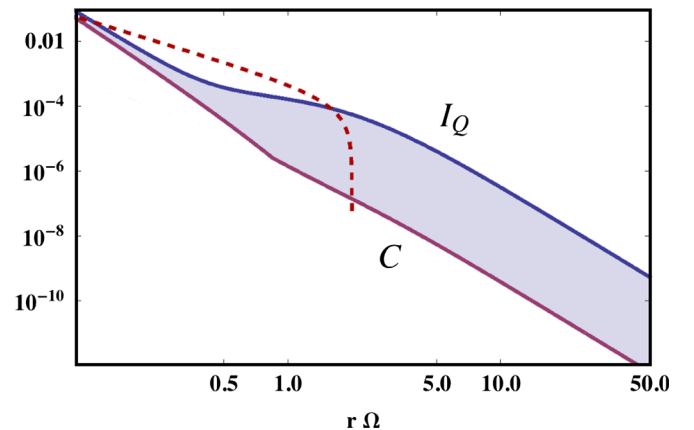


FIG. 2 (color online). r dependence of correlations ($\sigma\Omega = 1$, $g_0 = 0.1$). The right panel shows r dependence of I_Q and C . In both panels, dashed lines represent negativity.

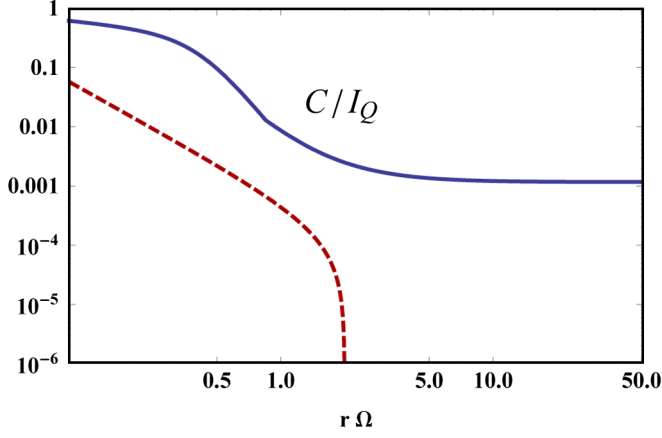


FIG. 3 (color online). r dependence of the ratio C/I_Q ($\sigma\Omega = 1$, $g_0 = 0.1$). The dashed line represents negativity.

$$C \approx \max\left\{\frac{2}{\ln 2}|X|^2 \ln \frac{|X|}{E}, \frac{2}{\ln 2}|X|^2\right\}, \quad (37)$$

$$I_Q \approx -\frac{2}{\ln 2}|X|^2 \ln E.$$

In this region, as $|X| \gg E$, the ratio is

$$\frac{C}{I_Q} \approx 1 - \frac{\ln|X|}{\ln E},$$

and this reproduces decreasing behavior of the ratio with respect to r . For $r \gg r_c$,

$$C \approx \frac{8}{\ln 2} E_{AB}^2, \quad I_Q \approx \frac{2}{\ln 2} \frac{E_{AB}^2}{E}, \quad (38)$$

and the ratio C/I_Q approaches a constant value independent of r and its value is given by

$$\frac{C}{I_Q} \approx 4E \sim g_0^2 \frac{e^{-(\Omega\sigma)^2}}{(\Omega\sigma)^2}. \quad (39)$$

The behavior of r dependence of the ratio C/I_Q changes at $|X| \approx E$ and this corresponds to the separable line $\mathcal{N} = 0$. The separability condition $\mathcal{N} < 0$ comes from the positive partially transposed criterion of the state [14]. Although the quantity C/I_Q is introduced independently of the positive partially transposed criterion, the change of its r dependence at $\mathcal{N} \approx 0$ arises and it corresponds to the separability condition.

B. Scalar field in de Sitter spacetime

In de Sitter spacetime with a spatially flat slice

$$ds^2 = -dt^2 + e^{2Ht} dx^2,$$

the Wightman function of a massless conformal scalar field with the conformal vacuum state [7] is

$$D_{\text{conf}}^+ = \frac{H^2}{16\pi^2} [-\sinh^2(H(y - i\epsilon)) + e^{2Hx}(Hr/2)^2]^{-1}. \quad (40)$$

The Wightman function of a massless minimal scalar field with the Bunch-Davis vacuum state is²

$$D_{\text{min}}^+ = D_{\text{conf}}^+ + D_2,$$

$$D_2 = -\frac{H^2}{8\pi^2} [\text{Ei}[i(Hr - 2e^{-Hx} \sinh(H(y - i\epsilon)))] + \text{Ei}[i(-Hr - 2e^{-Hx} \sinh(H(y - i\epsilon)))]]. \quad (41)$$

The massless minimal scalar field in de Sitter spacetime suffers from infrared divergence. Hence we have introduced the infrared cutoff $k_0 = H$ in D_2 which corresponds to a maximal comoving size of the inflating region; r is the comoving distance between two spatially separated points and satisfies $r < H^{-1}$. We have assumed that inflation starts at $t = 0$ and the comoving distance r must be smaller than the size of the inflationary universe H^{-1} at $t = 0$. The asymptotic form of X , E_{AB} are

$$X_{\text{conf}} = X_1, \quad E_{AB\text{conf}} = E_{AB1}, \quad (42)$$

$$X_{\text{min}} = X_1 + X_2, \quad E_{AB\text{min}} = E_{AB1} + E_{AB2},$$

where

$$X_1 = -(2\pi g_0^2) \frac{e^{-(\Omega\sigma)^2}}{4\pi^2} \frac{(H\sigma)^2}{(Hr_p)^2} e^{-2iH\Omega\sigma^2},$$

$$E_{AB1} = (2\pi g_0^2) \frac{e^{-(\Omega\sigma)^2}}{4\pi^2} \frac{(H\sigma)^2}{4\sin^2(H\Omega\sigma^2) + (Hr_p)^2},$$

$$X_2 = (2\pi g_0^2) \frac{e^{-(\Omega\sigma)^2}}{8\pi^2} (H\sigma)^2 \{\text{Ei}(-ie^{-Ht_0} Hr_p) + \text{Ei}(ie^{-Ht_0} Hr_p)\},$$

$$E_{AB2} = -(2\pi g_0^2) \frac{e^{-(\Omega\sigma)^2}}{8\pi^2} (H\sigma)^2 \{\text{Ei}(-ie^{-Ht_0} Hr_p) + 2e^{-Ht_0} \sin(\Omega H\sigma^2) + \text{Ei}(ie^{-Ht_0} Hr_p) + 2e^{-Ht_0} \sin(\Omega H\sigma^2)\}, \quad (43)$$

where Ht_0 is the e-folding time at which detectors are switched on. The physical distance between detectors at this instance is $r_p = e^{Ht_0} r_0$ with $r_0 < H^{-1}$. Unlike the Minkowski vacuum case, physical distance between two detectors increases in accord with cosmic expansion, and entanglement between detectors changes in time. For sufficiently large e-foldings $Ht_0 \gg 1$,

² $\text{Ei}(x) = -\int_x^\infty \frac{dk}{k} e^{-k} = \gamma + \ln x + \sum_{n=1}^\infty c_n x^n$.

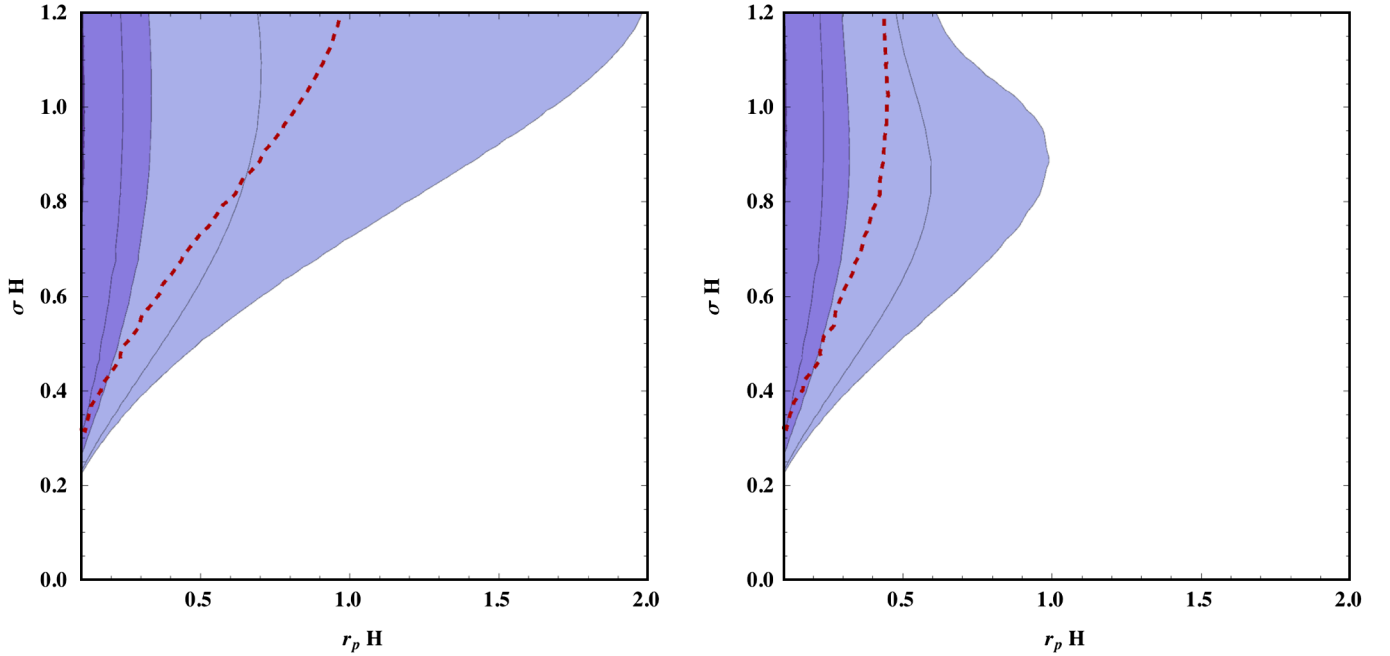


FIG. 4 (color online). Contours of negativity for the conformal invariant scalar field (left panel) and the minimal scalar field (right panel) ($\Omega/H = 1$, $r_0 = 0.1$). Two detectors are entangled with parameters in the shaded region. In the region above the dotted line, the Bell-CHSH inequality is violated after operation of local filtering.

$$X_{\min} \approx X_2 \approx -(2\pi g_0^2) \frac{e^{-(\Omega\sigma)^2}}{4\pi^2} (H\sigma)^2 [-\ln(Hr_p) + Ht_0],$$

$$E_{AB\min} \approx E_{AB2}$$

$$\approx (2\pi g_0^2) \frac{e^{-(\Omega\sigma)^2}}{4\pi^2} (H\sigma)^2 \left[\frac{1}{4\sin^2(H\Omega\sigma^2)} - \ln\sqrt{(Hr_p)^2 + 4\sin^2(H\Omega\sigma^2)} + Ht_0 \right],$$

and the negativity for the minimal scalar field is

$$\mathcal{N} = \frac{g_0^2}{2\pi} e^{-\Omega^2\sigma^2} (H\sigma)^2 \left(\ln \left[\frac{2\sin(H\Omega\sigma^2)}{Hr_p} \right] - \frac{1}{4\sin^2(H\Omega\sigma^2)} \right). \quad (44)$$

The separability condition $\mathcal{N} < 0$ yields

$$Hr_p \gtrsim 2\sin^2(H\Omega\sigma^2) \exp\left(-\frac{1}{4\sin^2 H\Omega\sigma^2}\right) \sim 1.0, \quad (45)$$

where the numerical value is obtained for $H\Omega\sigma^2 = 1$. Comparing this to the Minkowski vacuum case, we can find sufficiently large r_p at which detectors are separable for any value of a detector's parameters Ω , σ (see Fig. 4). As r_p grows in time t_0 , two entangled detectors $r_p < H^{-1}$ evolve to a separable state after their separation exceeds the Hubble horizon scale. This behavior is consistent with our previous analysis of entanglement using the lattice model and the coarse-grained model of the scalar field [1,2]: bipartite entanglement of the scalar field between

spatially separated regions in de Sitter spacetime disappears beyond the Hubble horizon scale. For the conformal invariant scalar field, the separability condition becomes

$$Hr_p \gtrsim 2\sin H\Omega\sigma^2 \sim 1.4. \quad (46)$$

The numerical value is for $H\Omega\sigma^2 = 1$. Contrary to the minimal scalar field, the entangled region extends to the super horizon scale. Although the size of the entangled region is larger than that of the minimal scalar field, the two detectors are separable when their separation reaches scale (46) and this behavior is the same as the minimal scalar field.

For the entangled state, we can check the violation of the Bell-CHSH inequality. The condition of violating the Bell-CHSH inequality after operation of local filtering is

$$Hr_p \lesssim 2\sin^2(H\Omega\sigma^2) \times \exp\left(-\frac{1}{\sin^2 H\Omega\sigma^2}\right) \sim 0.34 (\text{minimal scalar}),$$

$$Hr_p \lesssim \sin(H\Omega\sigma^2) \sim 0.84 \quad (\text{conformal scalar}). \quad (47)$$

Thus, for both scalar fields, the appearance of a hidden nonlocality is possible only on the subhorizon scale and violation of the Bell-CHSH inequality cannot be detected on the super horizon scale.

Behavior of correlations is completely different for these two types of scalar fields. Figure 5 shows r_p dependence of functions $|X|$, E_{AB} for the conformal invariant scalar field and the minimal scalar field.

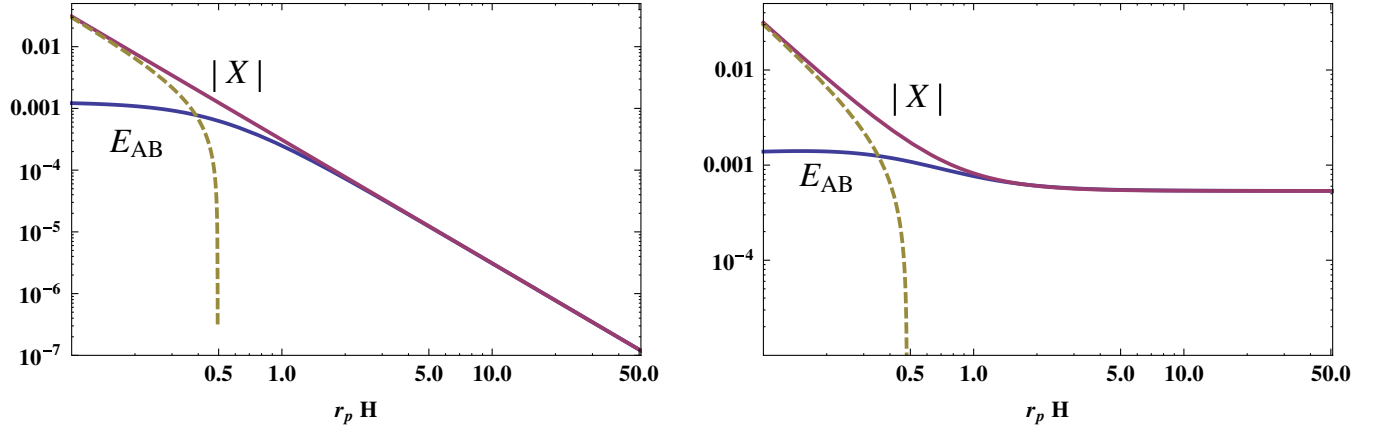


FIG. 5 (color online). r_p dependence of $|X|$ and E_{AB} (left panel: conformal invariant scalar field, right panel: minimal scalar field, $H\sigma = 0.5$, $\Omega/H = 1$, $r_0 = 0.1$, $g_0 = 0.1$). r_p is related to t_0 as $r_p = r_0 e^{Ht_0}$. The dashed line represents the negativity.

For the conformal invariant scalar field, these functions decay as $\sim r_p^{-2}$ on super horizon scale, which is the same behavior as the Minkowski vacuum case. On the other hand, for the minimal scalar field, these functions approach a constant value on the super horizon scale due to accumulation of long wavelength modes of quantum fluctuations. The different behavior of X , E_{AB} leads to different behavior of classical and quantum parts of correlations of these scalar fields.

In Fig. 6, the classical and total correlation \mathcal{C} , I_Q of the minimal scalar field also approach constant values on the super horizon scale while these correlations of the conformal invariant scalar field decay as $\sim r_p^{-4}$.

In Fig. 7, the ratio \mathcal{C}/I_Q for the conformal invariant scalar field approaches constant for $r_p H \gtrsim 1$ and this behavior is the same as the Minkowski vacuum case (see Fig. 3). However, for the minimal scalar field, value of this ratio has $\ln r_p$ dependence and increases as r_p increases:

$$\begin{aligned} \frac{\mathcal{C}}{I_Q} &\approx 4E \sim g_0^2 e^{-(\Omega\sigma)^2} (H\sigma)^2 (Ht_0) \\ &= g_0^2 e^{-(\Omega\sigma)^2} (H\sigma)^2 \ln\left(\frac{r_p}{r_0}\right). \end{aligned}$$

For a sufficiently large distance (large e-folding Ht_0) given by

$$\ln\left(\frac{r_p}{r_0}\right) \sim \frac{e^{(\Omega\sigma)^2}}{g_0^2 (H\sigma)^2} \gtrsim \frac{1}{g_0^2} \left(\frac{\Omega}{H}\right)^2, \quad (48)$$

$\mathcal{C} \approx I_Q$ and this means the quantum state of detectors approaches the zero quantum discord state and can be regarded as classical in the sense that measurement procedure does not alter the state. As discussed in Ref. [12], classical and quantum channel capacities of communication via detectors beyond the light cone identically vanishes and we presumably expect these capacities to be zero beyond the Hubble horizon. Thus the correlation with zero discord on the super horizon scale originates from quantum

fluctuations of the scalar field. Therefore, this behavior of correlations supports the long wavelength quantum fluctuations of the massless minimal scalar field in de Sitter spacetime being treated as classical fluctuations.

V. SUMMARY

We investigated quantum and classical correlations of the quantum field in de Sitter spacetime using the detector model. Entanglement of the scalar field is swapped to that of two detectors interacting with the scalar field and we can measure the entanglement of the quantum field by this setup of experiment. In de Sitter spacetime, the entanglement between detectors disappears on the super horizon scale and this behavior is consistent with our previous analysis using the lattice model and the coarse-grained model of the scalar field [1,2]. However, the behavior of correlations shows different behavior depending on the type of scalar fields. For the massless minimal scalar field, the ratio of classical correlation to the total correlation approaches unity for sufficiently large e-foldings. On the other hand, for the massless conformal scalar field, that ratio approaches a constant value smaller than unity and the condition for classicality is not achieved. These results support the long wavelength quantum fluctuation of the minimal scalar field being treated as classical fluctuations and becoming seed fluctuations for the structure in the our Universe.

As an application of our analysis presented in this paper, it is interesting to consider quantum effects in analogue curved spacetimes proposed using Bose-Einstein condensates or ion traps [24]. In these experiential setups of analogue models, we can directly measure entanglement and classical and quantum correlations of quantum fluctuation using detectors in the laboratory. We expect that investigation in this direction will increase understanding of the quantum and classical nature generated during the inflation.

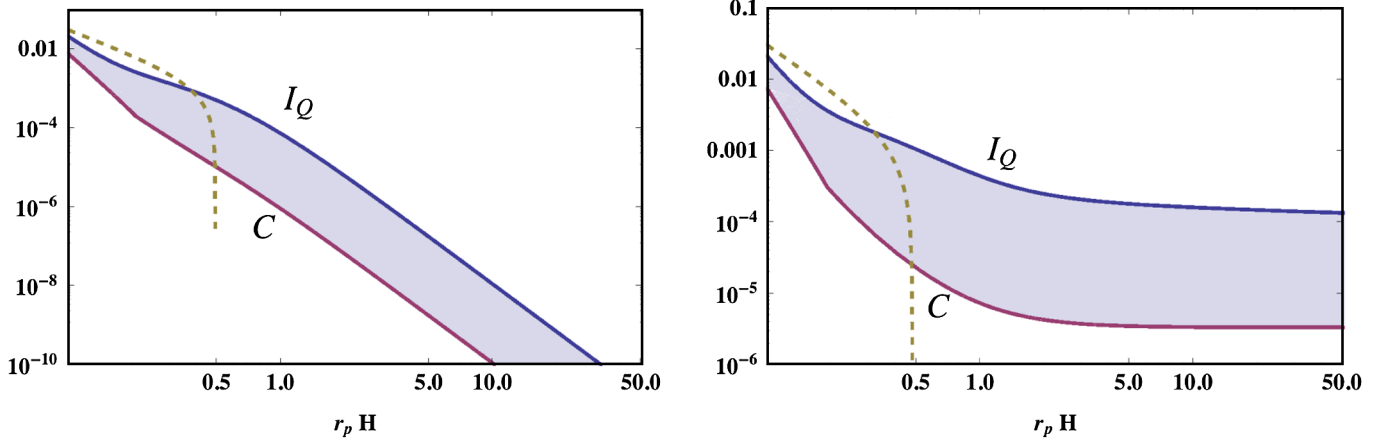


FIG. 6 (color online). r_p dependence of I_Q and C (left panel: conformal invariant scalar field, right panel: minimal scalar field, $H\sigma = 0.5$, $\Omega/H = 1$, $r_0 = 0.1$, $g_0 = 0.1$). The dashed line represents the negativity.

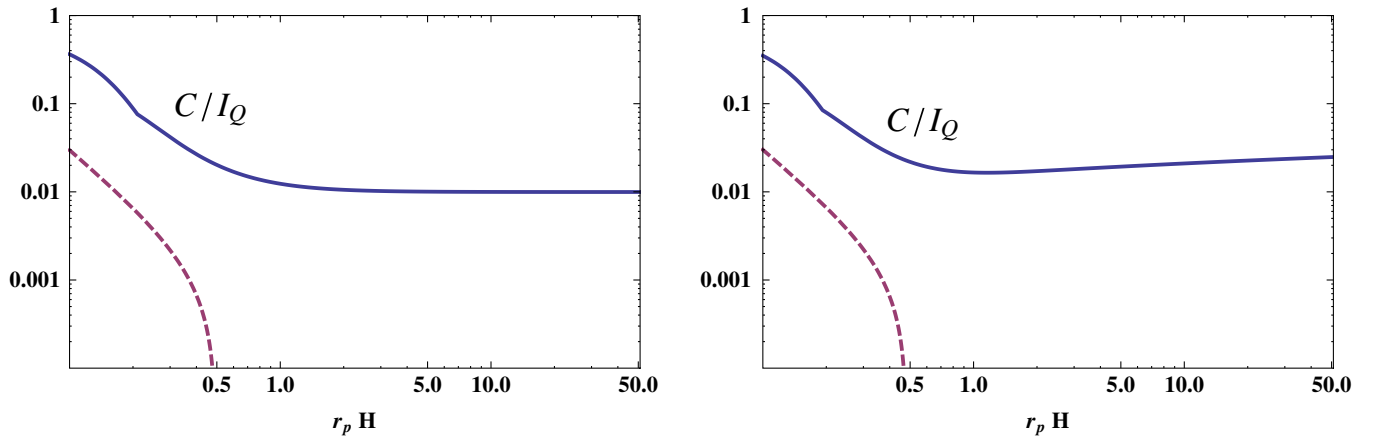


FIG. 7 (color online). r_p dependence of the ratio C/I_Q (left panel: conformal invariant scalar field, right panel: minimal scalar field, $H\sigma = 0.5$, $\Omega/H = 1$, $r_0 = 0.1$, $g_0 = 0.1$). The dashed line represents the negativity.

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APPENDIX A: WIGHTMAN FUNCTION OF MASSLESS MINIMAL SCALAR FIELD

In de Sitter spacetime with a spatially flat slice, the massless minimal scalar field obeys the following equation of motion

$$\ddot{\phi} + 3H\dot{\phi} - e^{-2Ht}\nabla^2\phi = 0. \quad (\text{A1})$$

The quantized field with the Bunch-Davis vacuum state is

$$\begin{aligned} \phi(t, \mathbf{x}) &= \int \frac{d^3k}{(2\pi)^{3/2}} (f_k(t)\hat{a}_k + f_k^*(t)\hat{a}_{-k}^\dagger) e^{ik\cdot\mathbf{x}}, \\ f_k &= \frac{-H}{\sqrt{2k}} \left(\eta - \frac{i}{k} \right) e^{-ik\eta}, \end{aligned} \quad (\text{A2})$$

where $\eta = -e^{-Ht}/H$ is conformal time. The Wightman function is

$$\begin{aligned} D^+(x_1, x_2) &= \langle \phi(x_1)\phi(x_2) \rangle \\ &= \frac{1}{2\pi^2} \int_0^\infty dk k^2 j_0(kr) f_k(\eta_1) f_k^*(\eta_2), \\ r &= |\mathbf{x}_1 - \mathbf{x}_2| \\ &= \frac{H^2 \eta_1 \eta_2}{4\pi^2 r} \int_0^\infty e^{-\epsilon k} \sin kr e^{-ik\Delta\eta} \\ &\quad + \frac{H^2}{4\pi^2 r} \int_0^\infty dk e^{-\epsilon k} \sin kr \left(-\frac{\partial}{\partial k} \right) \left(\frac{e^{-ik\Delta\eta}}{k} \right); \\ \Delta\eta &= \eta_1 - \eta_2, \end{aligned} \quad (\text{A3})$$

where we have introduced a damping factor $e^{-\epsilon k}$ with a small positive number ϵ to regularize ultraviolet divergence of the k integral. The first integral is

$$D_{\text{conf}}^+ = \frac{H^2}{4\pi^2} \frac{\eta_1 \eta_2}{-(\Delta\eta - i\epsilon)^2 + r^2} = \frac{H^2}{4\pi^2 \bar{y}}, \quad (\text{A4})$$

$$\bar{y} = \frac{-(\Delta\eta)^2 + r^2}{\eta_1 \eta_2}.$$

This term is the same as the Wightman function for the massless conformal invariant scalar field. The second integral diverges at $k = 0$, hence we introduce a lower bound k_0 of k integral:

$$D_2 = \frac{H^2}{8\pi^2} [-\text{Ei}(-k_0(ir - i\Delta\eta - \epsilon)) - \text{Ei}(-k_0(-ir - i\Delta\eta - \epsilon))]. \quad (\text{A5})$$

As the value of k_0 , we choose $k_0 = H$ which corresponds to the size of the universe (horizon scale) at the beginning of inflation.

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