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Non-Abelian gauge field inflation

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In [A. Maleknejad and M. M. Sheikh-Jabbari, arXiv:1102.1513.] we introduced an inflationary scenario, *non-Abelian gauge field inflation* or *gauge-flation* for short, in which slow-roll inflation is driven by non-Abelian gauge field minimally coupled to gravity. We present a more detailed analysis, both numerical and analytical, of the gauge-flation. By studying the phase diagrams of the theory, we show that getting enough number of e-folds during a slow-roll inflation is fairly robust to the choice of initial gauge field values. In addition, we present a detailed analysis of the cosmic perturbation theory in gauge-flation which has many special and interesting features compared the standard scalar-driven inflationary models. The specific gauge-flation model we study in this paper has two parameters, a cutoff scale Λ and the gauge coupling g. Fitting our results with the current cosmological data fixes $\Lambda \sim 10H \sim 10^{15}$ GeV (H is the Hubble parameter) and $g \sim 10^{-4}$, which are in the natural range of parameters in generic particle physics beyond standard models. Our model also predicts a tensor-to-scalar ratio $r > 0.05$, in the range detectable by the Planck satellite.

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I. INTRODUCTION

The idea of inflationary cosmology was originally proposed to provide a possible resolution to some of the theoretical problems of the big bang model for the early Universe cosmology [1]. However, with the advancement of the cosmological observations and most notably the cosmic microwave background (CMB) observations [1,2], the inflationary paradigm has received observational support and inflation is now considered an integral part of the standard model of cosmology with the following general picture. A patch of the early Universe which is a few Planck lengths in size under the gravitational effects of the matter present there undergoes a rapid (usually exponential) expansion, the inflationary period. The inflation ends while most of the energy content of the Universe is still concentrated in the field(s) driving inflation, the inflaton field(s). This energy should now be transferred to the other fields and particles, the (beyond) standard model particles, through the (p)reheating process. The rest of the picture is that of the standard hot big bang scenario, with radiation-dominated, matter-dominated and finally the dark-energy-dominated era that we live in.

In the absence of a direct observation for the primordial gravity waves, one of the main standing issues in inflation is what is the Hubble parameter during inflation H, or the energy density of the inflaton field(s). With the current observations, and within the slow-roll inflation scenario, the preferred scale is $H \lesssim 10^{-5} M_{\rm pl}$, where $M_{\rm pl} \equiv (8\pi G_N)^{-1/2} = 2.43 \times 10^{18}$ GeV is the reduced Planck mass. On the other hand, according to the lore in beyond standard particle physics models, the supersymmetric grand unified theories (SUSY GUTs) [3], the unification scale is around 10^{16} GeV, suggesting that inflationary model building should be sought for within various corners of such models. If so, the SUSY GUT setting will also provide a natural arena for building the (p)reheating models.

Almost all of inflationary models or model building ideas that appear in the literature use one or more scalar fields with a suitable potential to provide for the matter field inducing the inflationary expansion of the early Universe. The choice of scalar fields is made primarily because we work within the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) cosmology and that turning on spinor or gauge fields generically violates these symmetries. Another reason is that, from the model building viewpoint, turning on potential for the scalar fields is easier than for other fields, whose interactions are generically fixed by gauge symmetries or renormalizability conditions. Building inflationary models within the SUSY GUTs then amounts to exploring various corners of the theory/model in search of flat enough potential which supports successful slow-roll inflation, the flatness of which is respected by the loop and quantum corrections. Such models usually come under the D-term or F-term inflationary models [4].

Regardless of the details, non-Abelian gauge field theories are the widely accepted framework for building particle physics models, and, in particular, beyond standard

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models and GUTs. In view of the ubiquitous appearance of non-Abelian gauge fields, one may explore the idea of using gauge fields as inflaton fields, the fields which get nonzero background value during inflation and drive the inflationary dynamics. One of the main obstacles in this regard is the vector nature of the gauge fields and that turning them on in the background will spoil the rotation symmetry.

A related scenario in which this problem was pointed out and addressed is "vector inflation [5]." The idea in vector inflation, unlike ours, is to use vector fields and not gauge fields, as inflaton. In [5], two possible ways were proposed to overcome the broken rotational invariance caused by the vector inflaton fields: (1) introduce a large number of vectors each assuming a random direction in the 3D space, such that on the average we recover a rotational invariant background; or, (2) introduce three orthogonal vector fields of the same value which act as the triad of the spatial part of the spacetime, the "triad method" [6,7]. The other important obstacle in the way of driving inflation by vector fields is the exponential, $1/a(t)$ suppression of the massless vector fields in an inflationary background, causing inflation to end too fast. This problem has been overcome by adding nonminimal coupling to the gravity, usually a conformal mass type term [5,8]. To have a successful inflation, however, this is not enough and one should add quite nontrivial potentials for the vector field [5,6,8]. Dealing with vector fields, and not gauge fields, may bring instabilities in the theory: the longitudinal mode of the vector field which has a ghost type kinetic term and is not dynamical at tree level, in the absence of gauge invariance, can and will, become dynamical once quantum (loop) effects are taken into account. This latter will cause ghost instability, if we were studying the theory on a flat background. It has been argued that such instabilities can persist in the inflationary background too [9]; see, however, [10] for a counter argument. In any case the instability issue of vector inflationary models seems not to be settled yet.

In order not to face the above issue one should build a "gauge-invariant vector inflation." One can easily observe that it is not possible to get a successful inflation with some number of $U(1)$ gauge fields. The other option is to consider non-Abelian gauge theories. The "triad method" mentioned above is naturally realized within the non-Abelian gauge symmetry setting, irrespective of the gauge group in question. We then face the second obstacle, the $1/a(t)$ suppression. This may be achieved by changing the gravity theory, considering Yang-Mills action coupled to $F(R)$ modified gravity [11], or considering Einstein gravity coupled to a generic (not necessarily Yang-Mills) gauge theory action. This latter is the idea we will explore in this work. We should stress that, as will become clear, our approach and that of [11] are basically different. Using non-Abelian gauge fields has another advantage that, due to the presence of $[A_\mu, A_\nu]$ term in the gauge field strength $F_{\mu\nu}$, it naturally leads to a "potential" term for the gauge fields which, upon a suitable choice of the gauge theory action, can be used to overcome the $1/a(t)$ suppression problem mentioned above.

In this work, we present a detailed discussion and analysis of gauge-flation, inflation driven by non-Abelian gauge fields, which we introduced in [12]. In Sec. II, we show how the rotation symmetry breaking can be compensated by the $SU(2)$ (sub)group of the global part of non-Abelian gauge symmetry and how one can introduce a combination of the gauge field components which effectively behaves as spacetime scalar field (on the FRW background); and that there is a consistent truncation from the classical phase space of the non-Abelian gauge theory to the sector which only involves this scalar field.

Setting the stage, in Sec. III, we choose a specific action for the gauge theory that is Yang-Mills plus a specific F^4 term which can come from specific (one-) loop corrections to the gauge theory. In this work, however, we adopt a phenomenological viewpoint and choose this specific F^4 term primarily for the purpose of inflationary model building. The important point of providing field theoretical justifications for this F^4 term will be briefly discussed in the discussion section and dealt with in more detail in an upcoming publication. Our model has hence two parameters, the gauge coupling g and the ceofficient of this specific F^4 term κ. These two parameters will be determined only by focusing on the considerations coming from cosmological observations. In this section we present an analytic study of the inflationary dynamics of our gauge-flation model and show that the model allows for a successful slow-roll inflationary period which leads to enough number of e-folds. In Sec. IV, we present the diagrams and graphs for the numerical analysis of the gauge-flation model. Our numerical analysis reveals that the classical slow-roll inflationary trajectory is fairly robust to the choice of initial conditions.

Having studied the classical inflationary dynamics, in Sec. V, we turn to the question of cosmic perturbation theory in the gauge-flation. Because of the existence of other components of the gauge fields which has been turned off in the classical inflationary background, the situation here is considerably different than the standard cosmic perturbation theory developed in the literature. We hence first develop the cosmic perturbation theory for our model, discuss its subtleties and novelties; we discuss the scalar, vector and tensor perturbations, their power spectra and the spectral tilts. In Sec. VI, after completing the analysis of the model, we confront our model with the available cosmological and CMB data. We show that indeed it is possible to get a successful inflationary model with the gauge-flation setup. In the last section we summarize our results and make concluding remarks. In two appendices we have gathered some technical details of the cosmic perturbation theory.

II. THE SETUP

In this section we first demonstrate how the rotation symmetry is retained in the gauge-flation and then discuss truncation to the scalar sector. Here we will consider an $SU(2)$ gauge theory with gauge fields A^a_μ where $a = 1, 2, 3$ label the gauge algebra indices and $\mu = 0, 1, 2, 3$the space-time indices, the temporal components will be denoted by A^a_0, and the spatial components by A^a_i. Although we focus on the $SU(2)$ gauge theory, our analysis holds for any non-Abelian gauge group G, as any non-Abelian group always has an $SU(2)$ subgroup.

We will consider gauge- and Lorentz-invariant theories, where the gravity part is the usual Einstein-Hilbert action and the Lagrangian of the gauge theory part, which is *minimally coupled* to gravity, is of the form $\mathcal{L} = \mathcal{L}(F^a_{\mu\nu}; g_{\mu\nu})$, where $F^a_{\mu\nu}$ is the gauge field strength

$$F^a{}_{\mu\nu} = \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu - g\epsilon^a{}_{bc}A^b{}_\mu A^c{}_\nu. \qquad (2.1)$$

(For a generic gauge group, ϵ^{abc} should be replaced with the structure constant of that group.) Under the action of gauge transformation $U = \exp(-\lambda_a T^a)$, where T^a are generators of the $su(2)$ algebra,

$$[T^a, T^b] = i\epsilon^{ab}{}_c T^c \qquad (2.2)$$

$A^a{}_\mu$ transforms as

$$A_\mu \to UA_\mu U^\dagger - \frac{1}{g}U\partial_\mu U^\dagger. \qquad (2.3)$$

Therefore, out of 12 components of $A^a{}_\mu$, nine are physical and three are gauge freedoms, which may be removed by a suitable choice of gauge parameter λ_a. Since we are interested in isotropic and homogeneous FRW cosmology, the temporal gauge

$$A^a{}_0 = 0 \qquad (2.4)$$

appears to be a suitable gauge fixing. This fixes the gauge symmetry (2.3), up to the *global, time-independent* $SU(2)$ gauge transformations. This global $SU(2)$ is the key to restoring the rotation symmetry in the presence of the background gauge fields. We identify this $SU(2)$ with the three-dimensional rotations of the FRW background and, since the physical observables of the gauge fields are defined up to gauge transformations (or in other words, only gauge-invariant combinations are physical observables) the rotation symmetry may be preserved. This latter is done by turning on a specific gauge field configuration in which this identification can be made.

In order to see the above in a more technical language, consider the background FRW metric

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \qquad (2.5)$$

where $i, j = 1, 2, 3$ denote the indices along the spacelike three-dimensional hypersurface Σ, whose metric is chosen to be $a^2\delta_{ij}$. By choosing the (comoving cosmic) time direction, metric on Σ is then defined up to 3D foliation preserving diffeomorphisms. If we denote the metric on constant time hypersurfaces Σ by q_{ij}, we can introduce a set of three vector fields, $\{e^a_i(p)\}$, the triads, spanning the local Euclidian tangent space $T\Sigma_p$ to the Σ at the given point p. The triads satisfy the following orthonormality relations

$$q_{ij} = e^a{}_i e^b{}_j \delta_{ab}, \qquad \delta^{ab} = e^a{}_i e^b{}_j q^{ij}. \qquad (2.6)$$

The triads are then defined up to local 3D translations and rotations, which act on the "local indices" a, b. In particular the triads which are related to each other by local rotations $\Lambda^a{}_b \in SO(3)$

$$e^a{}_i \to \tilde{e}^a{}_i = \Lambda^a{}_b(p)e^b{}_i, \qquad (2.7)$$

at each point p, lead to the same metric q_{ij}. The elements of this local $SO(3)$ may be expressed in terms of $su(2)$ generators T^a as $\Lambda = \exp(-\lambda_a T^a)$. There are (infinitely) many possibilities for $e^a{}_i$ for the FRW metric (2.5), and one obvious choice is

$$e^a{}_i = a(t)\delta^a_i, \qquad (2.8)$$

which identifies the space coordinate indices $i, j, k, \cdots$, with the local frame indices $a, b, c, \cdots$.

We may now readily identify the remaining global $SU(2)$ gauge symmetry with the global part of the 3D rotation symmetry (2.7). This can be done through the following ansatz

$$A^a{}_i = \psi(t)e^a{}_i = a(t)\psi(t)\delta^a_i, \qquad (2.9)$$

where under both of 3D diffeomorphisms and gauge transformations, $\psi(t)$ acts as a genuine scaler field. Technically, the ansatz (2.9) identifies the combination of the gauge fields for which the rotation symmetry violation caused by turning on vector (gauge) fields in the background is compensated for (or undone by) the gauge transformations, leaving us with rotationally invariant background.

As a result of this identification, the energy-momentum tensor produced by the gauge field configuration (2.9) takes the form of a standard homogeneous, isotropic perfect fluid

$$T^\mu{}_\nu = \mathrm{diag}(-\rho, P, P, P). \qquad (2.10)$$

To see this, consider a general gauge- and Lorentz-invariant gauge field Lagrangian density $\mathcal{L} = \mathcal{L}(F^a_{\mu\nu}; g_{\mu\nu})$. The corresponding energy-momentum tensor is given by

$$T_{\mu\nu} \equiv \frac{-2}{\sqrt{-\mathfrak{g}}}\frac{\delta(\sqrt{-\mathfrak{g}}\mathcal{L})}{\delta g^{\mu\nu}} = 2\frac{\delta\mathcal{L}}{\delta F^a{}_\sigma{}^\mu}F^a{}_{\sigma\nu} + g_{\mu\nu}\mathcal{L}. \qquad (2.11)$$

To compute $T_{\mu\nu}$, we need to first calculate the field strength $F^a_{\mu\nu}$ for A^a_μ in the temporal gauge $A^a{}_0 = 0$, and for the field configuration of (2.9)

$$F^a{}_{0i} = \dot{\phi}\delta^a_i, \qquad F^a{}_{ij} = -g\phi^2\epsilon^a{}_{ij}, \qquad (2.12)$$

where dot denotes derivative with respect to the comoving time t and for the ease of notation we have introduced

$$\phi \equiv a(t)\psi(t). \tag{2.13}$$

(Note that ϕ, unlike ψ, is not a scalar.) It is now straightforward to calculate energy density ρ and pressure P, in terms of ϕ and its time derivatives. Plugging (2.12) into (2.11) yields

$$\rho = \frac{\partial \mathcal{L}_{\text{red.}}}{\partial \dot{\phi}}\dot{\phi} - \mathcal{L}_{\text{red.}}, \tag{2.14}$$

$$P = \frac{\partial(a^3\mathcal{L}_{\text{red}})}{\partial a^3}, \tag{2.15}$$

where $\mathcal{L}_{\text{red}}$ is the *reduced Lagrangian* density, which is obtained from calculating $\mathcal{L}(F^a_{\mu\nu}; g_{\mu\nu})$ for field strengths $F^a_{\mu\nu}$ given in (2.12) and FRW metric (2.5).

One can check that $\mathcal{L}_{\text{red}}$ is the true reduced Lagrangian for the reduced phase space of the field configurations in the ansatz (2.9) (and in the temporal gauge). In order to do this, one can show that the gauge field equations of motion

$$D_\mu \frac{\partial \mathcal{L}}{\partial F^a_{\mu\nu}} = 0, \tag{2.16}$$

where D_μ is the gauge covariant derivative, *(i)* allow for a solution of the form (2.9) and, *(ii)* once evaluated on the ansatz (2.9) become equivalent to the equation of motion obtained from the reduced Lagrangian $\mathcal{L}_{\text{red}}(\dot{\phi}, \phi; a(t))$

$$\frac{d}{a^3 dt}\left(a^3 \frac{\partial \mathcal{L}_{\text{red}}}{\partial \dot{\phi}}\right) - \frac{\partial \mathcal{L}_{\text{red}}}{\partial \phi} = 0. \tag{2.17}$$

In technical terms, there exists a consistent truncation of the gauge field theory to the sector specified by the scalar field ψ (or ϕ). In the next section we will study the cosmology of this reduced Lagrangian, with a specific choice for the gauge field theory action.

III. A SPECIFIC GAUGE-FLATION MODEL, ANALYTIC TREATMENT

In the previous section we showed how homogeneity and isotropy can be preserved in a specific sector of any non-Abelian gauge field theory. In this section we couple the gauge theory to gravity and search for gauge field theories which can lead to a successful inflationary background. The first obvious choice is Yang-Mills action minimally coupled to Einstein gravity. This will not lead to an inflating system because, as a result of scaling invariance of Yang-Mills action, one immediately obtains $P = \rho/3$ and $\rho \geq 0$, and in order to have inflation we should have $\rho + 3P < 0$. So, we need to consider modifications to Yang-Mills.

As will become clear momentarily, one such appropriate choice involving F^4 terms is

$$S = \int d^4x\sqrt{-g}\left[-\frac{R}{2} - \frac{1}{4}F^a{}_{\mu\nu}F_a{}^{\mu\nu} + \frac{\kappa}{384}(\epsilon^{\mu\nu\lambda\sigma}F^a{}_{\mu\nu}F^a{}_{\lambda\sigma})^2\right], \tag{3.1}$$

where we have set $8\pi G \equiv M_{\text{pl}}^{-2} = 1$ and $\epsilon^{\mu\nu\lambda\sigma}$ is the totally antisymmetric tensor. We stress that this specific F^4 term is chosen only for inflationary model building purposes and, since the contribution of this term to the energy-momentum tensor has the equation of state $P = -\rho$, it is perfect for driving inflationary dynamics. The justification of this term within a more rigorous quantum gauge field theory setting will be briefly discussed in Sec. VII. (To respect the weak energy condition for the F^4 term, we choose κ to be positive.)

The reduced (effective) Lagrangian is obtained from evaluating (3.1) for the ansatz (2.9)

$$\mathcal{L}_{\text{red}} = \frac{3}{2}\left(\frac{\dot{\phi}^2}{a^2} - \frac{g^2\phi^4}{a^4} + \kappa\frac{g^2\phi^4\dot{\phi}^2}{a^6}\right). \tag{3.2}$$

The energy density ρ and pressure P are

$$\rho = \frac{3}{2}\left(\frac{\dot{\phi}^2}{a^2} + \frac{g^2\phi^4}{a^4} + \kappa\frac{g^2\phi^4\dot{\phi}^2}{a^6}\right), \tag{3.3}$$

$$P = \frac{1}{2}\left(\frac{\dot{\phi}^2}{a^2} + \frac{g^2\phi^4}{a^4} - 3\kappa\frac{g^2\phi^4\dot{\phi}^2}{a^6}\right). \tag{3.4}$$

As we see, ρ and P have Yang-Mills parts and the F^4 parts, the κ terms. If we denote the Yang-Mills contribution to ρ by ρ_{YM} and the F^4 contribution by ρ_κ, i.e.,

$$\rho_{\text{YM}} = \frac{3}{2}\left(\frac{\dot{\phi}^2}{a^2} + \frac{g^2\phi^4}{a^4}\right), \qquad \rho_\kappa = \frac{3}{2}\frac{\kappa g^2\phi^4\dot{\phi}^2}{a^6}, \tag{3.5}$$

then

$$\rho = \rho_{\text{YM}} + \rho_\kappa, \qquad P = \frac{1}{3}\rho_{\text{YM}} - \rho_\kappa. \tag{3.6}$$

Field equations, the Friedmann equations and ϕ equation of motion, are then obtained as

$$H^2 = \frac{1}{2}\left(\frac{\dot{\phi}^2}{a^2} + \frac{g^2\phi^4}{a^4} + \kappa\frac{g^2\phi^4\dot{\phi}^2}{a^6}\right), \tag{3.7}$$

$$\dot{H} = -\left(\frac{\dot{\phi}^2}{a^2} + \frac{g^2\phi^4}{a^4}\right), \tag{3.8}$$

$$\left(1 + \kappa\frac{g^2\phi^4}{a^4}\right)\frac{\ddot{\phi}}{a} + \left(1 + \kappa\frac{\dot{\phi}^2}{a^2}\right)\frac{2g^2\phi^3}{a^3} + \left(1 - 3\kappa\frac{g^2\phi^4}{a^4}\right)\frac{H\dot{\phi}}{a} = 0. \tag{3.9}$$

We start our analysis by exploring the possibility of slow-roll dynamics. To this end it is useful to introduce slow-roll parameters[1]

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \qquad \eta \equiv -\frac{\ddot{H}}{2\dot{H}H}, \tag{3.10}$$

where ϵ is the standard slow-roll parameter and η is related to the time derivative of ϵ as

$$\eta = \epsilon - \frac{\dot{\epsilon}}{2H\epsilon}. \tag{3.11}$$

Therefore, to have a sensible slow-roll dynamics one should demand $\epsilon,\ \eta \ll 1$. Using the Friedmann Eqs. (3.7) and (3.8) and definitions (3.5) we have

$$\epsilon = \frac{2\rho_{\rm YM}}{\rho_{\rm YM} + \rho_\kappa}. \tag{3.12}$$

That is, to have slow-roll the κ-term contribution ρ_κ should dominate over the Yang-Mills contributions $\rho_{\rm YM}$, or $\rho_\kappa \gg \rho_{\rm YM}$. As we will see, the time evolution will then increase $\rho_{\rm YM}$ with respect to ρ_κ, and when $\rho_{\rm YM} \sim \rho_\kappa$, the slow-roll inflation ends. Noting that $\rho + 3P = 2(\rho_{\rm YM} - \rho_\kappa)$, inflation (accelerated expansion phase) will end when $\rho_{\rm YM} > \rho_\kappa$.

For having slow-roll inflation, however, it is *not* enough to make sure $\epsilon \ll 1$. For the latter, time-variations of ϵ and all the other physical dynamical variables of the problem, like η and the ψ field, must also remain small over a reasonably large period in time (to result in enough number of e-folds). To measure this latter we define

$$\delta \equiv -\frac{\dot{\psi}}{H\psi}, \tag{3.13}$$

in terms of which the Eqs. (3.7), (3.8), and (3.9) take the form

$$\epsilon = 2 - \kappa g^2\psi^6(1-\delta)^2, \tag{3.14}$$

$$\eta = \epsilon - (2-\epsilon)\left[\frac{\dot{\delta}}{H(1-\delta)\epsilon} + \frac{3\delta}{\epsilon}\right]. \tag{3.15}$$

Comparing (3.11) and (3.15) we learn that to have a successful slow roll, $\dot{\epsilon} \sim H\epsilon^2$ and $\eta \sim \epsilon$, we should demand that $\delta \sim \epsilon^2$. Explicitly, the equations of motion (3.7), (3.8), and (3.9) admit the solution[2]

$$\epsilon \simeq \psi^2(\gamma + 1), \tag{3.16}$$

$$\eta \simeq \psi^2 \Rightarrow \left(3 + \frac{\dot{\delta}}{H\delta}\right)\delta \simeq \frac{\gamma}{2(\gamma+1)}\epsilon^2, \tag{3.17}$$

$$\kappa \simeq \frac{(2-\epsilon)(\gamma+1)^3}{g^2\epsilon^3}, \tag{3.18}$$

where $\simeq$ means equality to first order in slow-roll parameter ϵ and[3]

$$\gamma = \frac{g^2\psi^2}{H^2}, \qquad \text{or equivalently } H^2 \simeq \frac{g^2\psi^4}{\epsilon - \psi^2} = \frac{g^2\epsilon}{\gamma(\gamma+1)}. \tag{3.19}$$

In the above, γ is a positive parameter which is slowly varying during slow-roll inflation.

Recalling (3.13) and that $\delta \sim \epsilon^2$, (3.19) implies that γH^2 remains almost a constant during the slow-roll inflation and hence [12]

$$\frac{\epsilon}{\epsilon_i} \simeq \frac{\gamma+1}{\gamma_i+1}, \qquad \frac{\gamma}{\gamma_i} \simeq \frac{H_i^2}{H^2}, \tag{3.20}$$

where ϵ_i, γ_i and H_i are the values of these parameters at the beginning of inflation. As discussed, the (slow-roll) inflation ends when $\epsilon = 1$, where

$$\gamma_f \simeq \frac{\gamma_i+1}{\epsilon_i}, \qquad \frac{H_f^2}{H_i^2} \simeq \frac{\gamma_i}{\gamma_i+1}\epsilon_i. \tag{3.21}$$

Using the above and (3.10) one can compute the number of e-folds N_e

$$N_e = \int_{t_i}^{t_f} H dt = -\int_{H_i}^{H_f}\frac{dH}{\epsilon H} \simeq \frac{\gamma_i+1}{2\epsilon_i}\ln\frac{\gamma_i+1}{\gamma_i}. \tag{3.22}$$

IV. NUMERICAL ANALYSIS

As pointed out, our gauge-flation model has two parameters, the gauge coupling g and the coefficient of the F^4 term κ. The degrees of freedom in the scalar sector of the model consists of the scalar field ψ and the scale factor $a(t)$ and hence our solutions are specified by four initial values for these parameters and their time derivatives. These were parameterized by H_i, ϵ_i and ψ_i and δ_i (or $\dot{\psi}_i$). The Friedmann equations, however, provide some relations between these parameters; assuming slow-roll dynamics these relations are (3.16), (3.17), and (3.18). As a result each inflationary trajectory may be specified by the values of four parameters, $(g, \kappa; \psi_i, \dot{\psi}_i)$. In what follows we present the results of the numerical analysis of the equations of motion (3.7), (3.8), and (3.9), for three sets of values for $(\psi_i, \dot{\psi}_i; g, \kappa)$.

[1]We note that our definition of slow-roll parameters ϵ, η for the standard single scalar inflationary theory $L = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$ reduces to [1] $\epsilon = \frac{M_{\rm pl}^2}{2}(\frac{V'}{V})^2$, $\eta = M_{\rm pl}^2\frac{V''}{V}$.

[2]Our numerical analysis reveals that even if we start with $\dot{\delta}/(H\delta) \sim \mathcal{O}(1)$, while $\psi_i^2 \sim \epsilon \ll 1$, after a short time it becomes very small and hence for almost all the inflationary period we may confidently use $\delta \simeq \frac{\gamma}{6(\gamma+1)}\epsilon^2$. See Sec. IV for a more detailed discussion.

[3]Note that all the dimensionful parameters, i.e.H, ψ and κ, are measured in units of $M_{\rm pl}$; H, ψ have dimension of energy while κ has dimension of one-over-energy density.

A. Discussion on diagrams in Fig. 1

The top left figure shows evolution of the effective inflaton field ψ as a function of $H_i t$. As we see, there is a period of slow roll, where ψ remains almost constant and ϵ is almost constant and very small. Toward the end of the slow roll ϵ grows and becomes one (the top right figure), (slow-roll) inflation ends and ψ suddenly falls off and starts oscillating. As we see from the top right figure, the slow-roll parameter ϵ has an upper limit which is equal to 2. This is understandable, recalling (3.12) and that ρ_κ is positive definite. At the end of slow-roll inflation, ρ_κ is negligible and the system is essentially governed by the Yang-Mills part $\rho_{\rm YM}$. In addition, the top left figure shows that amplitude of the ψ field in the oscillatory part is dropping like $t^{-1/2}$. (The minima of ϵ is also fit by a $t^{1/2}$ curve.) This behavior is of course expected, noting (3.5) and that in the oscillatory regime the dominant term is the $\rho_{\rm YM}$, that is, the system effectively behaves as a $g^2\psi^4$ chaotic inflation theory. And it is well-known that after the slow-roll phase $\psi(t)$ in the $g^2\psi^4$ theory oscillates as a Jacobi-cosine function whose amplitude drops like $t^{-1/2}$ [13]. In other words, the averaged value of ϵ and $a(t)$ behave like a radiation-dominated Universe (recall that for a radiation-dominated cosmology $\epsilon = -\dot{H}/H^2 = 2$).

The bottom left figure shows the phase diagram of the effective inflaton trajectory. Note that this diagram depicts $\dot{\phi}/a(t)$ vs $\phi/a(t)$ (rather than $\dot{\psi}$ vs ψ). The rightmost vertical line is where we have slow roll, because $\phi = a(t)\psi$ and during slow roll ψ is almost a constant. The curled up part is when inflation has ended, and when the system oscillates around a radiation-ominated phase. This latter may be seen in the figure noting that the amplitudes of oscillations of both $\dot{\phi}/a(t)$ and $\phi/a(t)$ drop by $t^{-1/2}$. The bottom right figure shows number of e-folds as a function of comoving time. As expected, the number of e-folds reaches its asymptotic value when $\epsilon \simeq 1$.

One can readily check that the behavior of ψ, ϵ, and the number of e-folds during slow-roll inflationary period has a perfect matching with our analytic results of previous section. We note that, as will be discussed in Sec. VI, the set of parameters H_i, γ_i, ψ_i, g corresponds to an inflationary model close to the range of values compatible with the current cosmological and CMB data.

B. Discussions on diagrams in Figs. 2 and 3

Figure 2 corresponds to a slow-roll trajectory which starts with a lower value of ϵ, but almost the same value for H, compared to the case of Fig. 1. For this case we hence get a larger number of e-folds. The qualitative shape of all four figures is essentially the same as those of Fig. 1, and both are compatible with our analytic slow-roll results of previous section. Our numeric analysis indicates that the behavior of the phase diagram for ψ field and ϵ do not change dramatically when the orders of magnitude of the initial parameters are within the range given in Figs. 1 or 2.

Figure 3 shows a trajectory with a relatively large $\dot{\psi}$. As we see, after a single fast falloff the field falls into usual slow-roll tracks, similar to what we see in Figs. 1 and 2. The oscillatory behavior after the inflationary phase, too, is the same as those of slow-roll inflation. The graph in the square in the bottom left figure shows, with a higher resolution, the upper part of the phase diagram which comes with under brace. This part corresponds to the dynamics of ψ field after inflation ends, and as expected has the same qualitative form as the phase diagrams in the slow-roll trajectories of Figs. 1 and 2. Our numeric analysis shows that getting a large enough number of e-folds and the generic after-inflation behavior of the fields is robust and does not crucially depend on the initial value of the fields, ψ_i or $\dot{\psi}_i$, but it is sensitive to the initial value of ϵ. More precisely, as long as ϵ remains small of order 0.01, regardless of the value of δ we can get arbitrarily large number of e-foldings. The examples of small and large δ values have been, respectively, given in Figs. 2 and 3.

It is also useful to work out the displacement of the ψ field during inflation. To this end, let us start from $\dot{\psi} = -\delta H\psi$ and use the value for δ given in (3.17). Our numerical analysis reveals that the dynamics of the system is such that even if we start with $\frac{\dot{\delta}}{H\delta}$ of order 1–10, but $\psi_i^2 \sim \epsilon_i$, after a short time $H_i t \sim 1$, $\dot{\delta}/(H\delta)$ becomes very small and hence in almost all the inflationary period, except for the first one or two e-folds, $\delta \simeq \frac{\gamma}{6(\gamma+1)}\epsilon^2$. Since variations of δ in this period happens very fast, and after that it remains almost zero, during this period ψ does not change much. Numerical analysis also shows that these arguments are generically true even if we start with a large value of δ, as in Fig. 3, provided that the other parameters are such that we get large number of e-folds (about $N_e \sim 60$ or larger). This latter can be easily arranged for. This is again confirming the robustness of the classical inflationary trajectories with respect to the choice of the initial conditions. Therefore, we may confidently compute roaming of ψ field using the equation

$$\frac{\dot{\psi}}{\psi^5} \simeq \frac{g^2}{6}\frac{\dot{H}}{H^3}.$$

Integrating the above equation one can compute the displacement of the scalar field ψ during inflation. If we denote the value of ψ in the beginning and end of inflation, respectively, by ψ_i and ψ_f we obtain

$$\psi_f^2 \simeq \frac{\sqrt{3}}{2}\psi_i^2. \tag{4.1}$$

Alternatively one could have used (3.18) $\kappa g^2\psi^6(1-\delta)^2 = 2-\epsilon$ to compute the change in ψ. If we are in slow-roll regime where we can drop δ-term, then the ratio of ψ_f (which is computed for $\epsilon_f = 1$) to ψ_i is obtained as $\psi_f^6 = \frac{1}{2}\psi_i^6$. And up to percent level, $\sqrt{3}/2$ and $2^{-1/3}$ are equal, a confirmation of the validity of the slow-roll

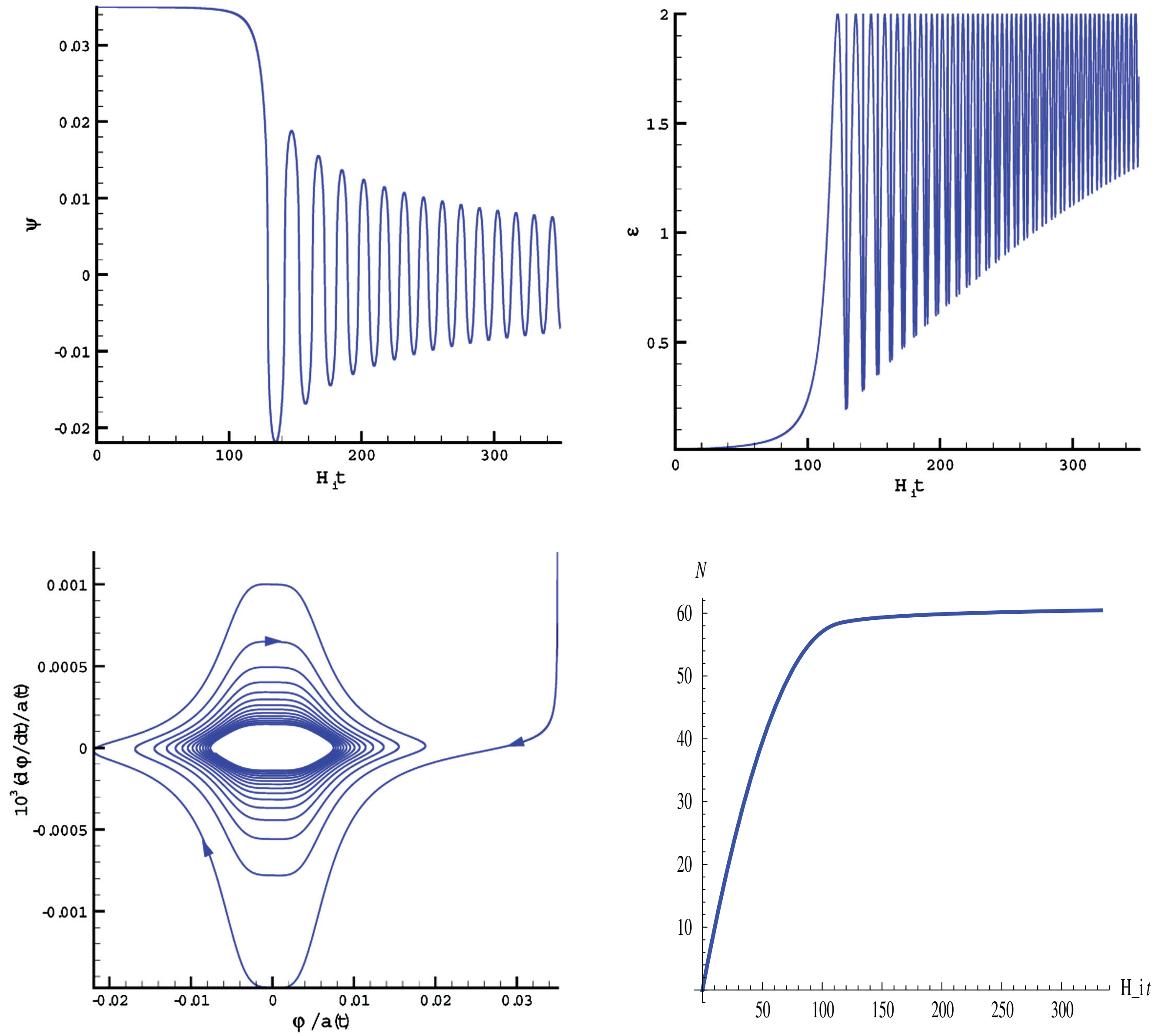

FIG. 1 (color online). The classical trajectory for $\psi_i = 0.035$, $\dot{\psi}_i = -10^{-10}$; $g = 2.5 \times 10^{-3}$, $\kappa = 1.733 \times 10^{14}$. These values correspond to a slow-roll trajectory with $H_i = 3.4 \times 10^{-5}$, $\gamma_i = 6.62$, $\epsilon_i = 9.3 \times 10^{-3}$, $\delta_i = 8.4 \times 10^{-5}$. These are the values very close to the range for which the gauge-flation is compatible with the current cosmological and CMB data (cf. discussions of Sec. VI). Note that κ, H_i and ψ_i are given in the units of $M_{\rm pl}$.

approximations we have used. Interestingly, at this level of approximation, the roaming of the ψ field is independent of the initial value of the γ (or the initial value of Hubble) parameter and ψ_i and ψ_f have the same orders of magnitude.

V. COSMIC PERTURBATION THEORY IN GAUGE-FLATION

So far we have shown that our gauge-flation model can produce a fairly standard slow-roll inflating Universe with enough number of e-folds. The main test of any inflationary model, however, appears in the imprints inflation has left on the CMB data, i.e., the power spectrum of curvature perturbations and primordial gravity waves, and the spectral tilt of these spectra. To this end, we should go beyond the homogeneous (x-independent) background fields and consider fluctuations around the background. This is what we will carry out in this section.

A. Gauge-invariant metric and gauge field perturbations

Although not turned on in the background, all of the components of metric and the gauge field in all gauge and spacetime directions will have quantum fluctuations and

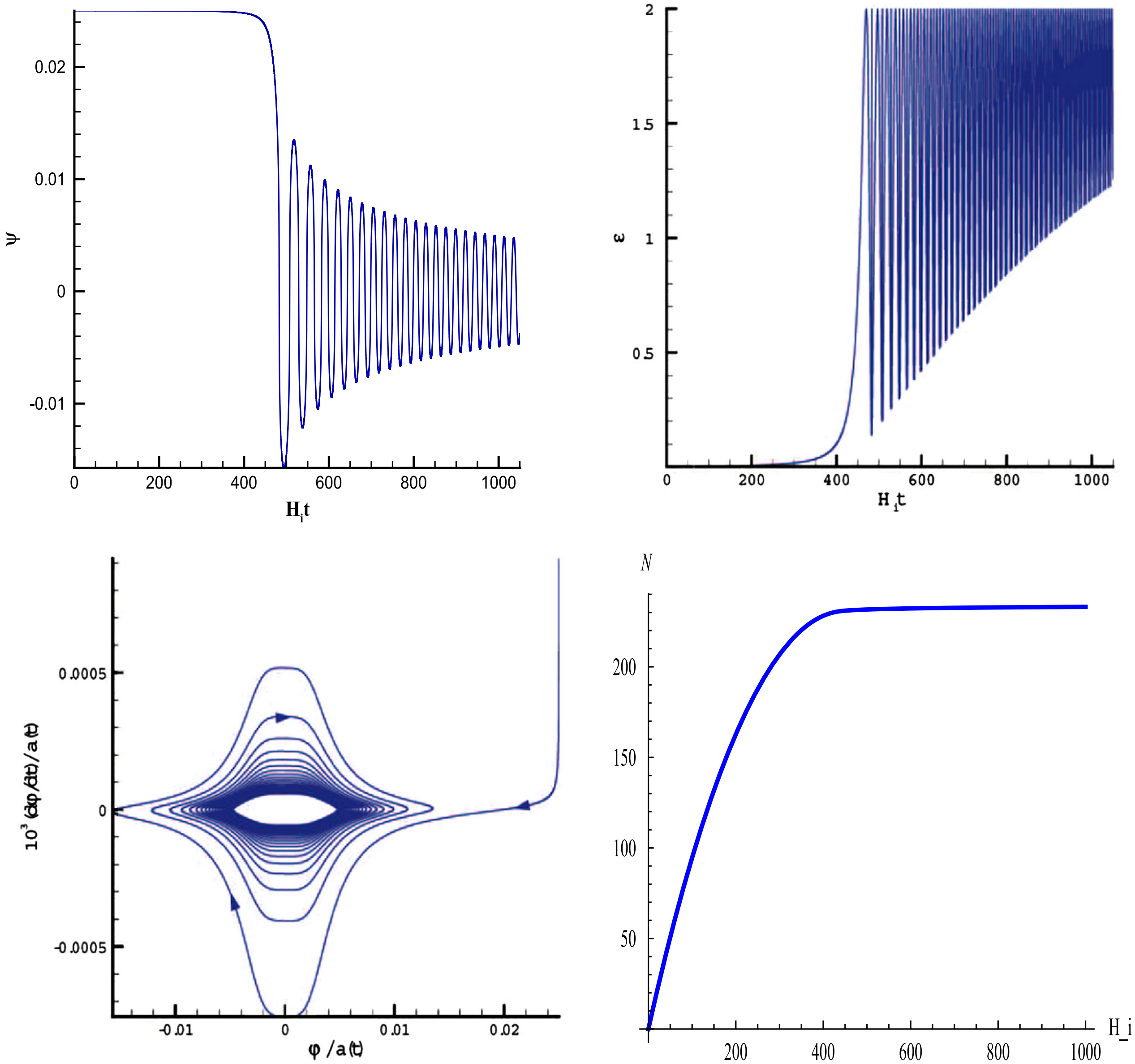

FIG. 2 (color online). The classical trajectory for $\psi_i = 0.025$, $\dot{\psi}_i = -10^{-10}$; $g = 2.507 \times 10^{-3}$, $\kappa = 1.3 \times 10^{15}$. These values correspond to a *slow-roll* trajectory with $H_i = 3.63 \times 10^{-5}$, $\gamma_i = 2.98$, $\epsilon_i = 2.5 \times 10^{-3}$, $\delta_i = 1.1 \times 10^{-4}$. These figures show that it is possible to get arbitrarily large numbers of e-folds within the *slow-roll* phase of our gauge-flation model.

should be considered. The metric perturbations may be parameterized in the standard form [1,2]

$$ds^2 = -(1+2A)dt^2 + 2a(\partial_i B - S_i)dx^i dt + a^2((1-2C)\delta_{ij} + 2\partial_{ij}E + 2\partial_{(i}W_{j)} + h_{ij})dx^i dx^j, \tag{5.1}$$

where ∂_i denotes partial derivative respect to x^i and A, B, C and E are scalar perturbations, S_i, W_i parameterize vector perturbations (these are divergence-free three-vectors) and h_{ij}, which is symmetric, traceless and divergence-free, is the tensor mode. The 12 components of the gauge field fluctuations may be parameterized as

$$\delta A^a{}_0 = \delta^k_a \partial_k \dot{Y} + \delta^j_a u_j, \tag{5.2}$$

$$\delta A^a{}_i = \delta^a_i Q + \delta^{ak}\partial_{ik} M + g\phi \epsilon^a{}_i{}^k \partial_k P + \delta^j_a \partial_i v_j + \epsilon^a{}_i{}^j w_j + \delta^{aj} t_{ij}, \tag{5.3}$$

where, as discussed in Sec. II, we have identified the gauge indices with the local Lorentz indices and the expansion is done around the background in $A^a_0 = 0$ temporal gauge. In the above, as has been made explicit, there are four scalar perturbations, Q, Y, M and P, three divergence-free three-vectors u_i, v_i and w_i, and a symmetric traceless divergence-free tensor t_{ij}, adding up to $4 + 3 \times 2 + 2 = 12$. Q is the perturbation of the background field ϕ, which

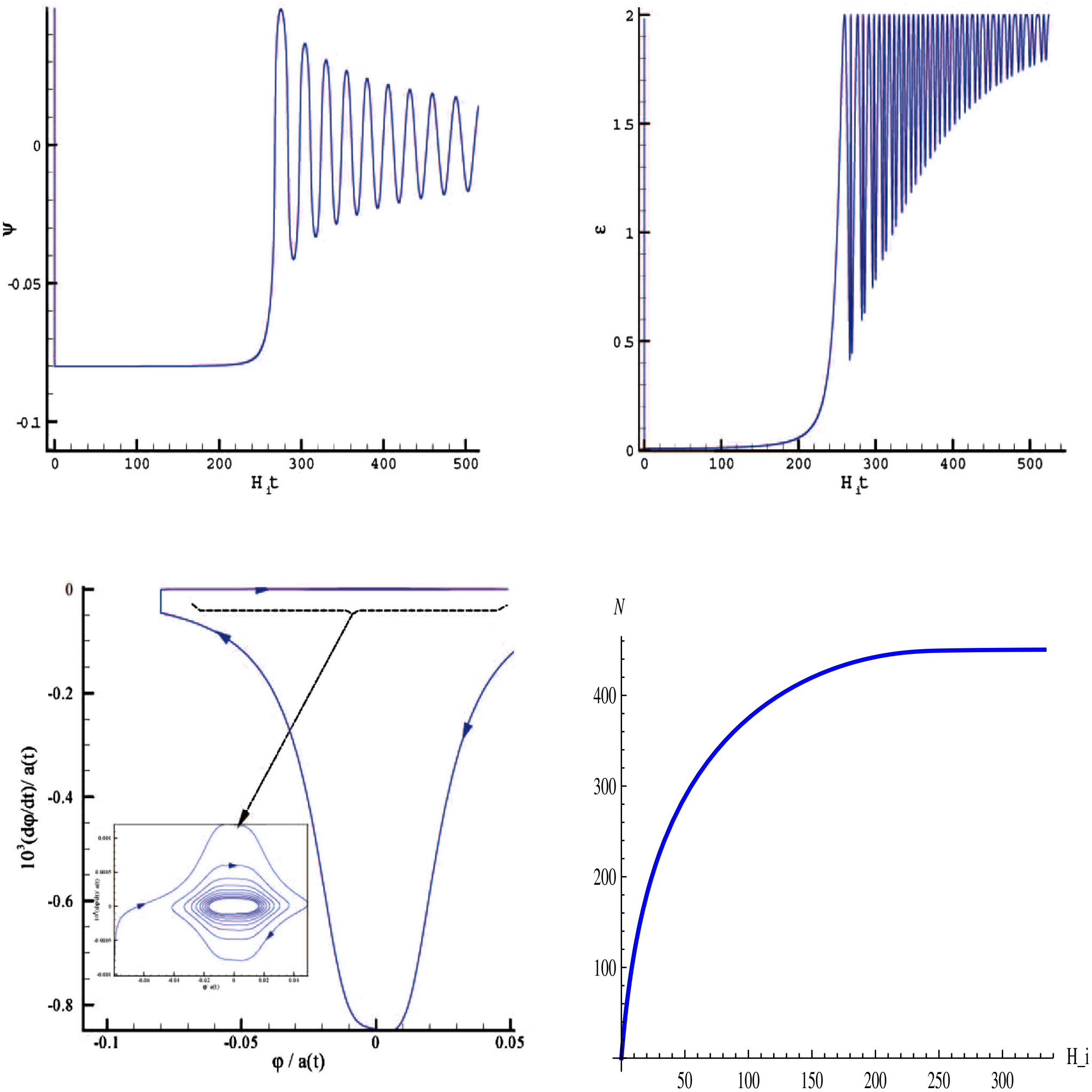

FIG. 3 (color online). The classical trajectory for $\psi_i = 8.0 \times 10^{-2}$, $\dot{\psi}_i = -10^{-4}$; $g = 4.004 \times 10^{-4}$, $\kappa = 4.73 \times 10^{13}$. These values correspond to a *non-slow-roll* trajectory with $\delta \sim 2$, $H_i = 6.25 \times 10^{-4}$, $\epsilon_i = 6.4 \times 10^{-3}$. We start far from the slow-roll regime for which $\delta \sim \epsilon^2 \ll 1$. This latter is also seen from the phase diagram (bottom left figure). Despite starting far from slow-roll regime, as we see from the top left figure, after an abrupt oscillation the field ψ loses its momentum and falls into the standard slow-roll trajectory. As shown in the bottom right figure, for this case we get a large number of e-folds. Getting a large enough number of e-folds seems to be a fairly robust result not depending much on the initial value of δ.

is the only scalar in the perturbed gauge field without spacial derivative. We are hence dealing with a situation similar to the multifield inflationary theories, where we have adiabatic and isocurvature perturbations. If the analogy held, Q would have then be like the adiabatic mode. However, as we will see this is not true and the curvature perturbations are dominated by other scalars and not Q. As another peculiar and specific feature of the gauge-flation cosmic perturbation theory, not shared by any other scalar-driven inflationary model, we note that the gauge field fluctuations contain a tensor mode t_{ij}. As we will show, the power spectrum of this mode is nonzero, but its effect is such that the power spectrum of the tensor modes ends up to be exactly those of usual scalar-driven inflationary models, with only the h_{ij} of the metric contributing to the tensor power spectrum.

Because of the presence of gauge symmetries not all 10 + 12 metric plus gauge field perturbations are physical. Altogether there are four diffeomorphisms and three local gauge symmetries, hence we have 15 physical degrees of freedom. The four diffeomorphisms remove two scalars and a divergence-free vector [1], and the three gauge transformations one scalar and one divergence-free vector. Therefore, we have *five* physical *scalar* perturbations, *three* physical divergence-free *vector*, and *two* physical *tensor* perturbations. (These amount to $5\times 1 + 3\times 2 + 2\times 2 = 15$ physical degrees of freedom.) The gauge degrees of freedom may be removed by gauge-fixing (working in a specific gauge) or working with gauge-invariant combinations of the perturbations. In what follows we work out the gauge- and diffeomorphism-invariant combinations of these modes.

1. Scalar modes

Let us first focus on the scalar perturbations A, B, C, E, Q, $\dot{Y}$, M and P. Under infinitesimal scalar coordinate transformations

$$t\to\tilde{t}=t+\delta t,\qquad x^i\to\tilde{x}^i=x^i+\delta^{ij}\partial_j\delta x, \tag{5.4}$$

where δt determines the time slicing and δx the spatial threading, the scalar fluctuations of the gauge field and metric transform as

$$\begin{aligned} &Q\to Q-\dot{\phi}\delta t, && \dot{Y}\to\dot{Y}-\phi\dot{\delta x},\\ &M\to M-\phi\delta x, && P\to P,\\ &A\to A-\dot{\delta t}, && C\to C+H\delta t,\\ &B\to B+\frac{\delta t}{a}-a\dot{\delta x}, && E\to E-\delta x. \end{aligned} \tag{5.5}$$

On the other hand, under an infinitesimal gauge transformation λ^a, fluctuations of the gauge field transform as

$$\delta A^a{}_\mu\to\delta A^a{}_\mu-\frac{1}{g}\partial_\mu\lambda^a-\epsilon^a{}_{bc}\lambda^bA^c{}_\mu. \tag{5.6}$$

The gauge parameters λ^a can be decomposed into a scalar and a divergence-free vector:

$$\lambda^a=\delta^{ai}\partial_i\lambda+\delta^a_i\lambda_V{}^i. \tag{5.7}$$

The scalar part of the gauge field perturbations under the action of the scalar gauge transformation λ transform as

$$\begin{aligned} &Q\to Q, && Y\to Y-\frac{1}{g}\lambda,\\ &M\to M-\frac{1}{g}\lambda, && P\to P+\frac{1}{g}\lambda. \end{aligned} \tag{5.8}$$

We note that Q is gauge-invariant and this is a result of identifying the gauge indices with the local Lorentz indices and that Q is a scalar.

Equipped with the above, one may construct five independent gauge-invariant combinations. One such choice is[4]

$$\Psi=C+a^2H\Big(\dot{E}-\frac{B}{a}\Big), \tag{5.9}$$

$$\Phi=A-\frac{d}{dt}\Big(a^2\Big(\dot{E}-\frac{B}{a}\Big)\Big), \tag{5.10}$$

$$\mathcal{Q}=Q-a^2\dot{\phi}\Big(\dot{E}-\frac{B}{a}\Big), \tag{5.11}$$

$$\mathcal{M}=M+P-\phi E, \tag{5.12}$$

$$\dot{\mathcal{Y}}=\dot{Y}+\dot{P}-\phi\dot{E}. \tag{5.13}$$

The first two, Φ and Ψ are the standard Bardeen potentials, while $\mathcal{Q}$, $\mathcal{M}$ and $\dot{\mathcal{Y}}$ are the three gauge- and diffeomorphism-invariant combinations coming from the gauge field fluctuations.

Finally, for the later use we also present the first-order perturbations of the gauge field strength sourced by the scalar perturbations

$$\delta F^a{}_{0i}=\delta^a_i\dot{Q}+\delta^{aj}\partial_{ij}(\dot{M}-\dot{Y})-g\epsilon^{aj}{}_i\partial_j((\phi\dot{P})+\phi\dot{Y}), \tag{5.14}$$

$$\begin{aligned}\delta F^a{}_{ij}=&\,2\delta^a_{[j}\partial_{i]}(Q+g^2\phi^2P)+2g\phi\epsilon^{ak}{}_{[i}\partial_{j]k}(P+M)\\ &-2g\epsilon^a{}_{ij}Q\phi.\end{aligned} \tag{5.15}$$

Note that $\delta F^a_{\mu\nu}$ are not gauge-invariant, as under gauge transformations $F^a_{\mu\nu}\to F^a_{\mu\nu}-g\epsilon^a{}_{bc}\lambda^bF^c_{\mu\nu}$.

2. Vector modes

Next, we consider the vector modes S_i, W_i, u_i, v_i and w_i. Under infinitesimal ''vector'' coordinate transformations

$$x^i\to\tilde{x}^i=x^i+\delta x^i_V, \tag{5.16}$$

where $\partial_i\delta x^V_i=0$,

$$\begin{aligned} &S_i\to S_i+a\delta\dot{x}^i_V,\quad W_i\to W_i-\delta x^i_V,\\ &u_i\to u_i-\phi\dot{\delta x}^i_V,\quad v_i\to v_i-\phi\delta x^i_V,\quad w_i\to w_i. \end{aligned} \tag{5.17}$$

On the other hand, under the vector part of infinitesimal gauge transformation (5.6),

[4]These choices are not unique and one can construct other gauge-invariant combinations too.

$$u_i \to u_i - \frac{1}{g}\dot{\lambda}^i_V,$$
$$v_i \to v_i - \frac{1}{g}\lambda^i_V, \tag{5.18}$$
$$w_i \to w_i + \phi\lambda^i_V,$$

and obviously S_i, W_i remain invariant.

The three gauge- and diffeomorphism-invariant divergence-free vector perturbations may be identified as

$$\mathcal{Z}_i = a\dot{W}_i + S_i, \tag{5.19}$$

$$\mathcal{U}_i = u_i - \dot{v}_i + \dot{\phi}W_i, \tag{5.20}$$

$$\mathcal{V}_i = w_i + g\phi(v_i - \phi W_i). \tag{5.21}$$

The contribution of vector perturbations to the first-order gauge field strength perturbations are

$$\delta F^a{}_{0i} = \delta^j_a\partial_i(\dot{v}_j - u_j) + \epsilon^a{}_i{}^j(\dot{w}_j + g\phi u_j),$$
$$\delta F^a{}_{ij} = 2\epsilon^a{}_{[j}{}^k\partial_{i]}(w_k + g\phi v_k) + 2g\phi\delta^a{}_{[i}w_{j]}. \tag{5.22}$$

3. Tensor modes

One can show that the tensor perturbations h_{ij} and t_{ij}, being symmetric, traceless, and divergence-free, are both gauge- and diffeomorphism-invariant. The contribution of t_{ij} to the first order perturbed $F^a_{\mu\nu}$ corresponding to t_{ij} is

$$\delta F^a_{0i} = \delta^{aj}\dot{t}_{ij}, \quad \delta F^a_{0i} = \delta^{ak}\partial_{[i}t_{j]k} - g\phi\epsilon^{ak}{}_{[j}t_{i]k}. \tag{5.23}$$

B. Field equations

Having worked out the gauge-invariant combinations of the field perturbations, we are now ready to study their dynamics. These first-order perturbations are governed by perturbed Einstein and gauge field equations

$$\delta G_{\mu\nu} = \delta T_{\mu\nu}, \qquad \delta\left(D_\mu \frac{\partial\mathcal{L}}{\partial F^a_{\mu\nu}}\right) = 0, \tag{5.24}$$

where by δ in the above we mean first order in field perturbations. Since we are dealing with an isotropic perfect fluid in the background, as it is customary in standard cosmology text books [1], it is useful to decompose energy-momentum perturbations as

$$\delta T_{ij} = P_0\delta g_{ij} + a^2(\delta_{ij}\delta P + \partial_{ij}\pi^s + \partial_i\pi^V_j + \partial_j\pi^V_i + \pi^T_{ij}), \tag{5.25a}$$

$$\delta T_{i0} = P_0\delta g_{i0} - (P_0 + \rho_0)(\partial_i\delta u + \delta u^V_i), \tag{5.25b}$$

$$\delta T_{00} = -\rho_0\delta g_{00} + \delta\rho, \tag{5.25c}$$

where subscript 0 denotes a background quantity and π^s, π^V_i, π^T_{ij} represent the anisotropic inertia and characterize departures from the perfect fluid form of the energy-momentum tensor; π^V_i and π^T_{ij} and the vorticity δu^V_i satisfy

$$\partial_i\delta u^V_i = \partial_i\pi^V_i = 0, \qquad \pi^T_{ii} = 0, \qquad \partial_i\pi^T_{ij} = 0. \tag{5.26}$$

Since being a perfect fluid or having irrotational flows are physical properties, their corresponding conditions are gauge-invariant.

1. Scalar modes

As is usually done in cosmic perturbation theory, it is useful to write down the equations of motion in a gauge-invariant form. In order this we note that $\delta T_{\mu\nu}$ has four gauge-invariant scalar parts $\delta\rho_g$, δP_g, δq_g,

$$\delta\rho_g = \delta\rho - \dot{\rho}_0 a^2\left(\dot{E} - \frac{B}{a}\right), \tag{5.27}$$

$$\delta P_g = \delta P - \dot{P}_0 a^2\left(\dot{E} - \frac{B}{a}\right), \tag{5.28}$$

$$\delta q_g = \delta q + (\rho_0 + P_0)a^2\left(\dot{E} - \frac{B}{a}\right), \tag{5.29}$$

and π^s [1], where $\delta q = (\rho_0 + P_0)\delta u$. Out of the 10 perturbed Einstein equations, there are four scalars, two (divergence-free) vectors, and one massless tensor mode (gravitons). Among the four scalar perturbed Einstein equations, one is dynamical and three are constraints. Therefore, they do not suffice to deal with five gauge-invariant scalar degrees of freedom and one equation is missing. This last equation is, of course, provided by the perturbed gauge field equations of motion. To this end, we use two observations:

(i) Using (5.14) and after a lengthy algebra (which was performed by MAPLE codes) one can show that the anisotropic inertia π^s is given by the following linear combination of the gauge-invariant quantities [5]

$$a^2\pi^s = 2\frac{\dot{\phi}}{a}(a\dot{\mathcal{Y}} - a\dot{\mathcal{M}}) + 2\frac{g^2\phi^3}{a^3}a\mathcal{M}. \tag{5.30}$$

We can write $\dot{\mathcal{Y}}$ in terms of $a^2\pi^s$ and the rest of variables.

(ii) Computing the second-order action for the gauge field perturbations, we observe that the momentum conjugate to the new variable $a^2\pi^s$, is identically zero and hence $a^2\pi^s$ is a nondynamical variable; it is a constant of motion. To see the latter, note that there is no $\ddot{\mathcal{Y}}$ in the $\delta F^a{}_{\mu\nu}$ (cf. (5.14)). In our analysis we can hence consistently choose the initial conditions so that π^s vanishes.

The above will then provide the last missing equation, and the perturbed Einstein equations are enough to solve for the scalar perturbations. The other perturbed gauge field

[5]Note that in the first-order perturbation theory the tensor and vector modes do not contribute to the scalar parts of the energy-momentum perturbations.

equations of motion do not lead to independent equations. Studying a generic constant $a^2\pi^s$ is an interesting question we postpone to future works.

With the above argument we have [1,2]

$$a^2\pi^s = 0 \Rightarrow \Phi = \Psi. \tag{5.31}$$

We hence remain with three equations for the three variables. Using (5.14), and after lengthy calculations (confirmed by MAPLE codes too), we obtain

$$\begin{aligned}\delta\rho_g = {}& 3\Big(1+\frac{\kappa g^2\phi^4}{a^4}\Big)\frac{\dot\phi\,\dot{\mathcal{Q}}}{a^2} - \frac{k^2}{a^2}\Big[\Big(1+\frac{\kappa g^2\phi^4}{a^4}\Big) \\ &+ 2\Big(1+\frac{\kappa\dot\phi^2}{a^2}\Big)\Big]\frac{g^2\phi^3}{a^3}a\mathcal{M} \\ &+ 6\Big(1+\frac{\kappa\dot\phi^2}{a^2}\Big)\frac{g^2\phi^4}{a^4}\Big(\Psi+\frac{\mathcal{Q}}{\phi}\Big),\end{aligned} \tag{5.32}$$

$$\begin{aligned}\delta P_g = {}& \Big(1-3\frac{\kappa g^2\phi^4}{a^4}\Big)\frac{\dot\phi\,\dot{\mathcal{Q}}}{a^2} - \frac{k^2}{3a^2}\Big[\Big(1-3\frac{\kappa g^2\phi^4}{a^4}\Big) \\ &+ 2\Big(1-3\frac{\kappa\dot\phi^2}{a^2}\Big)\Big]\frac{g^2\phi^3}{a^3}a\mathcal{M} \\ &+ 2\Big(1-3\frac{\kappa\dot\phi^2}{a^2}\Big)\frac{g^2\phi^4}{a^4}\Big(\Psi+\frac{\mathcal{Q}}{\phi}\Big),\end{aligned} \tag{5.33}$$

$$\delta q_g = 2\Big(-\frac{\dot\phi\,\mathcal{Q}}{a^2} - \frac{g^2\phi^3}{a^3}a\dot{\mathcal{M}} + \frac{g^4\phi^6}{a^5\dot\phi}a\mathcal{M}\Big). \tag{5.34}$$

Note that in the above expressions $\pi^s = 0$ condition and (5.31) have been employed.

We are now ready to write down the three perturbed Einstein equations, two of which are constraints and one is dynamical:

$$\delta q_g + 2(\dot\Psi + H\Psi) = 0, \tag{5.35}$$

$$\delta\rho_g - 3H\delta q_g + 2\frac{k^2}{a^2}\Psi = 0, \tag{5.36}$$

$$\delta P_g + \dot{\delta q}_g + 3H\delta q_g + (\rho_0 + P_0)\Psi = 0. \tag{5.37}$$

In the above we have already used $\Phi = \Psi$ relation. These relations provide enough number of equations for the gauge-invariant scalar perturbations to which we return in the next subsection.

2. Vector modes

To study the vector perturbations, we first work out vector parts of the perturbed energy-momentum tensor, δq_i^V and π_i^V, using (5.22):[6]

[6]Note that at first order in perturbation theory only the vector perturbations contribute to the vector part of energy-momentum tensor perturbations.

$$\begin{aligned}\delta q_i^V = {}& -2\frac{g\phi^2}{a^2}\Big(\dot w_i + \frac{g\phi}{a}\Big(au_i + \frac{\phi}{a}aS_i\Big)\Big) + 2\frac{g\phi\dot\phi}{a^2}w_i \\ &+ \frac{g\phi^2}{a^2}(\nabla\times(\dot{\vec v} - \vec u))_i - \frac{\dot\phi}{a^2}(\nabla\times(\vec w + g\phi\vec v))_i,\end{aligned} \tag{5.38}$$

$$\pi_i^V = \frac{g\phi^2}{a^2}\Big(\frac{w_i}{a} + \frac{g\phi}{a}v_i - \frac{g\phi^2}{a^2}W_i\Big) + \frac{\dot\phi}{a}\Big(u_i - \dot v_i + \frac{\dot\phi}{a}W_i\Big), \tag{5.39}$$

which as expected, are gauge-invariant and hence can be written in terms of physical gauge- and diffeomorphism-invariant variables as

$$\begin{aligned}\delta q_i^V = {}& -\frac{2g\phi}{a}\Big(\frac{\phi}{a}\dot{\mathcal{V}}_i - \frac{\dot\phi}{a}\mathcal{V}_i + \frac{g\phi^2}{a^2}a\mathcal{U}_i + \frac{g\phi^3}{a^3}a\mathcal{Z}_i\Big) \\ &- \Big(\nabla\times\Big(\frac{\dot\phi}{a}\frac{\vec{\mathcal{V}}}{a} + \frac{g\phi^2}{a^2}\vec{\mathcal{U}}\Big)\Big)_i,\end{aligned} \tag{5.40}$$

$$\pi_i^V = \frac{g\phi^2}{a^2}\frac{\mathcal{V}_i}{a} + \frac{\dot\phi}{a}\mathcal{U}_i. \tag{5.41}$$

The perturbed Einstein equations have two vector equations, one constraint and one dynamical equaiton. These equations are

$$\partial_i\Big(2a^2\pi_j^V - \frac{1}{a}(a^2\dot{\mathcal{Z}}_{\rm j})\Big) = 0, \tag{5.42}$$

$$2\delta q_i^V + \frac{k^2}{a^2}a\mathcal{Z}_i = 0. \tag{5.43}$$

To solve for the three gauge-invariant vector perturbations, the two Einstein equations are not enough and we should use information from the perturbed gauge field equations. As for the latter, we note that one can use (5.39) to write u_i in terms of $a^2\pi_i^V$ and the other variables. On the other hand, since there is no $\dot u_i$ in (5.22), once we write down the second-order action for the gauge field perturbations the momentum conjugate to π_i^V is vanishing and hence this variable is nondynamical; it is a constant of motion. We may then choose the initial conditions such that

$$a^2\pi_i^V = 0. \tag{5.44}$$

This completes the set of equations we need for solving vector perturbations. Equation (5.42) then implies that

$$\mathcal{Z}_i = \frac{c}{a^2}. \tag{5.45}$$

The above is the usual result of the scalar-driven inflationary models that the vector modes are diluted away by the (exponential) accelerated expansion of the Universe during inflation. In our model, despite having vector gauge fields as inflaton, we confirm the same result.[7]

3. Tensor modes

As discussed, there are two gauge- and diffeomorphism-invariant tensor modes h_{ij} and t_{ij}, while perturbed Einstein equations only lead to one equation for h_{ij}. This equation, which is sourced by the contribution of t_{ij} to the energy-momentum tensor, reads as

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 2\pi^T_{ij}. \tag{5.46}$$

The other equation of motion is provided with the perturbed gauge field equations of motion. After a tedious but straightforward calculation, which is also confirmed by the MAPLE codes, we obtain the following second-order action for the tensor modes

$$\begin{aligned}\delta S^{(2)}_T \simeq \frac{1}{2}\int d^3x dt a^3\Big[&\frac{1}{2}\Big(\frac{\dot\phi^2}{a^2} - \frac{g^2\phi^4}{a^4}\Big)h^2_{ij} \\ &+ \Big(-2\frac{\dot\phi}{a}\frac{\dot t_{ij}}{a} + 2\frac{g^2\phi^3}{a^3}\frac{t_{ij}}{a}\Big)h_{ij} \\ &+ \frac{1}{a^2}\Big(\dot t^2_{ij} - \frac{k^2}{a^2}t^2_{ij} - \epsilon\frac{\kappa g^2\phi^2}{a^2}\frac{\dot\phi^2}{a^2}t^2_{ij}\Big) \\ &+ \frac{1}{4}\Big(\dot h^2_{ij} - \frac{k^2}{a^2}h^2_{ij}\Big)\Big].\end{aligned} \tag{5.47}$$

Note that in the above we have already used the slow-roll approximation ($\dot\phi \simeq H\phi$). From the above second-order action one can readily compute π^T_{ij}

$$\pi^T{}_{ij} = \Big(\Big(\frac{\dot\phi^2}{a^2} - \frac{g^2\phi^4}{a^4}\Big)h_{ij} + 2\Big(-\frac{\dot\phi}{a}\frac{\dot t_{ij}}{a} + \frac{g^2\phi^3}{a^3}\frac{t_{ij}}{a}\Big)\Big). \tag{5.48}$$

Being traceless and divergence-free t_{ij} and h_{ij} each has 2 degrees of freedom which are usually decomposed into plus and cross ($+$ and $\times$) polarization states with the polarization tensors $e^{+,\times}_{ij}$. Since we have no parity-violating interaction terms in the action the equations for both of these polarization have the same time evolution and one may then introduce h and T variables instead

$$h^{+,\times}_{ij} = \frac{h}{a}e^{+,\times}_{ij}, \qquad t^{+,\times}_{ij} = T\psi e^{+,\times}_{ij}, \tag{5.49}$$

and in the computation of the power spectrum consider these variables, treating them as scalars, but multiplying by a factor of 2 to account for the two polarizations. The second-order action for h and T reads as

$$\begin{aligned}\delta S^{(2)}_T = \frac{1}{2}\int d^3x d\tau\Big[&\frac{1}{2}\psi^2\mathcal{H}^2(1-x)h^2 \\ &+ 2\psi^2\mathcal{H}^2\Big(-\frac{T'}{\mathcal{H}} + xT\Big)h \\ &+ \psi^2\Big(T'^2 - k^2T^2 - \mathcal{H}^2\epsilon\frac{\kappa g^2\phi^4}{a^2}T^2\Big) \\ &+ \frac{1}{4}(h'^2 + (2-\epsilon)\mathcal{H}^2h^2 - k^2h^2)\Big],\end{aligned} \tag{5.50}$$

where τ is the conformal time $dt = ad\tau$, $\mathcal{H} = \dot a$ and prime denotes a derivative with respect to the conformal time.

C. Primordial power spectra and the spectral indices

In the previous part, we provided the complete set of equations which govern the dynamics of scalar, vector and tensor modes. In this subsection we set about solving these equations, quantize their solutions, and compute the power spectra.

1. Scalar modes

In order to determine the power spectrum of the scalar perturbations we have to deal with two constraint (5.35) and (5.36) and one dynamical Eq. (5.37). In contrast to the case of single scalar field inflationary models which one of the constraints (a combination of δP and δq equations) reduces to the equation of motion of the background field, in our case both of them remain independent and should be considered. The constraint Eqs. (5.35) and (5.36) in the slow-roll regime take the form

$$\begin{aligned}&\mathcal{H}\Big(\Psi - \psi\frac{\mathcal{Q}}{a} + g^2\psi^3(\gamma+1)a\mathcal{M}\Big) \\ &\quad + (\Psi - g^2\psi^3(a\mathcal{M}))' = 0,\end{aligned} \tag{5.51}$$

$$\begin{aligned}&\mathcal{H}\Big(\frac{\mathcal{Q}}{\psi} + \Big(1 + \frac{\epsilon-\psi^2}{2}\Big)a\Psi\Big)' + \frac{k^2}{3}\Big(\Big(1 + \frac{\epsilon-\psi^2}{2}\Big)a\Psi \\ &\quad - \Big(1 + \frac{2}{\gamma} + \psi^2\Big(1 - \frac{1}{\gamma}\Big)\Big)g^2\psi^3\frac{a^2\mathcal{M}}{\psi^2}\Big) \\ &\quad + \mathcal{H}^2(2 + \psi^2(\gamma-1))\Big(a\Psi + \frac{\mathcal{Q}}{\psi}\Big) = 0.\end{aligned} \tag{5.52}$$

where $'$ denotes derivative with respect to the conformal time τ.

To analyze the dynamical Eq. (5.37), we note that for a general hydrodynamical fluid with pressure $P(\rho, S)$, the pressure perturbation δP can be decomposed as

[7] In contrast to standard scalar-driven inflationary models, in gauge-flation there is the possibility of a nonzero but constant $a^2\pi^V_i$ which leads to

$$Z_i = \frac{2a^2\pi^V_i}{aH} + \frac{c}{a^2}.$$

The above still shows a suppression by a $1/a$ factor (to be contrasted with standard $1/a^2$ suppression), and hence again the power spectrum of the vector modes are unimportant in inflationary cosmology.

$$\delta P = c_s^2 \delta\rho + \mathcal{T}\delta S, \tag{5.53}$$

where δS is the entropy perturbations, $c_s^2 = (\frac{\partial P}{\partial \rho})_S$ is the speed of sound, and $\mathcal{T} = (\frac{\partial P}{\partial S})_\rho$. With this decomposition, combining (5.36) and (5.37) yields

$$\Psi'' + 3\mathcal{H}(1+c_s^2)\Psi' + c_s^2 k^2\Psi + (2\mathcal{H}' + (1+3c_s^2)\mathcal{H}^2)\Psi = \frac{1}{2}a^2\mathcal{T}\delta S, \tag{5.54}$$

which upon introducing

$$\theta = \frac{1}{a}\sqrt{\frac{\rho_0}{\rho_0+P_0}}, \qquad \delta s = \frac{a^2\mathcal{T}\delta S}{2\sqrt{\rho_0+P_0}}, \qquad u = \frac{\Psi}{\sqrt{\rho_0+P_0}}, \tag{5.55}$$

where ρ_0 and P_0 are background energy density and pressure, simplifies to

$$u'' + c_s^2 k^2 u - \frac{\theta''}{\theta}u = \delta s, \tag{5.56}$$

where $c_s^2 = \frac{\dot P_0}{\dot\rho_0}$. Next, we need to compute δs explicitly. We will do this in two regions, the asymptotic past $k\tau \to -\infty$ and the superhorizon region $k\tau \to 0$.

The asymptotic past behavior of δs.—To study the effects of δs term in the asymptotic past $k\tau \to -\infty$ (Minkowski) limit, we note that in this limit constraints (5.51) and (5.52) take the following forms

$$\mathcal{Q} - \frac{1}{H}\left(\frac{\Psi}{\psi} - g^2\psi^2 a\mathcal{M}\right)' = 0, \tag{5.57}$$

$$\mathcal{Q}' - \frac{k^2}{3H}\left(1+\frac{2}{\gamma}\right)g^2\psi^2 a\mathcal{M} + \psi^2\frac{k^2}{3H}\left(\frac{\Psi}{\psi} - \left(1-\frac{1}{\gamma}\right)g^2\psi^2 a\mathcal{M}\right) = 0. \tag{5.58}$$

We solve (5.58) by considering the following *ansatz*[8]

$$\mathcal{Q}' - \frac{k^2}{3H}\left(1+\frac{2}{\gamma}\right)g^2\psi^2 a\mathcal{M} = \frac{\mathcal{W}}{2}\psi^2\mathcal{Q}', \tag{5.59}$$

with $\mathcal{W}$ being a constant number of order 1, to be determined momentarily. In this limit from (5.32), (5.33), and (5.53) one can read the value of δs

$$\delta s = -4\left(\frac{\gamma-1}{\gamma+2}\right)H\psi\frac{\mathcal{Q}'}{\sqrt{\rho_0+P_0}}. \tag{5.60}$$

From (5.59), the constraint Eq. (5.58) takes the form

$$\left(\mathcal{W} - 2\frac{\gamma-1}{\gamma+2}\right)H\psi\mathcal{Q}' + \frac{2k^2}{3}\Psi \simeq 0, \tag{5.61}$$

and hence

$$\delta s = \frac{8}{3}k^2\frac{\gamma-1}{(\gamma+2)\mathcal{W} - 2(\gamma-1)}u, \tag{5.62}$$

$$c_s^2 = \frac{(\gamma+2)\mathcal{W} + \frac{2}{3}(\gamma-1)}{2(\gamma-1) - (\gamma+2)\mathcal{W}}. \tag{5.63}$$

On the other hand, from (5.57), (5.59), and (5.61) we learn that

$$c_s^2 = \frac{2}{3}\frac{\gamma+2}{\mathcal{W}(\gamma+2)+2}.$$

Consequently, from equating the above two values for the speed of sound, we find a quadratic equation for $\mathcal{W}$ whose solutions denoted by $\mathcal{W}_\pm$ are

$$\mathcal{W}_+ = \frac{2}{3}\frac{\gamma-1}{\gamma+2} \Rightarrow c_{s+}^2 = 1, \tag{5.64}$$

$$\mathcal{W}_- = -2\frac{\gamma+1}{\gamma+2} \Rightarrow c_{s-}^2 = -\frac{1}{3}\frac{\gamma+2}{\gamma}. \tag{5.65}$$

As we see speed of sound squared for one of the solutions is 1 and for the other one, recalling that γ is positive by definition, is always negative. We also note that all the above analysis has been carried out in the slow-roll regime, and in the Minkowski limit $k\tau \to -\infty$.

The superhorizon behavior of δs.—We now turn to the question of large-scale superhorizon behavior of δs in $k\tau \to 0$. In this limit, from (5.32), (5.33), and (5.53) one can read the value of δs

$$\delta s = 2\mathcal{H}^2\psi^2\left(2\gamma\left(u + \frac{\mathcal{Q}/a}{\psi(\rho_0+P_0)^{1/2}}\right) + \frac{\mathcal{Q}'/a}{\mathcal{H}\psi(\rho_0+P_0)^{1/2}}\right). \tag{5.66}$$

On the other hand, constraint (5.52) in the $k\tau \to 0$ can be written as below

$$\left(\frac{\mathcal{Q}}{\psi} + a\Psi\right)' + 2\mathcal{H}\left(\frac{\mathcal{Q}}{\psi} + a\Psi\right) = O(\epsilon), \tag{5.67}$$

which implies that $\frac{\mathcal{Q}}{\psi} + a\Psi$ consists of a damping term proportional to a^{-2} and a part of the order ϵ. Hence, using (5.67), ignoring the damping terms and up to the first order of ϵ, δs takes the form

$$\delta s = -2\mathcal{H}^2\psi^2\left(\frac{u'}{\mathcal{H}} + u\right), \tag{5.68}$$

which is a small quantity of the order ϵ.

Upon using (5.64), (5.65), and (5.68), the equation for u (5.65) leads to two equations for each of the values of speed of sound $c_{s\pm}^2$:

$$u_+'' + (k^2 - \zeta\mathcal{H}^2)u_+ \simeq 0, \tag{5.69}$$

[8]The validity of the ansatz is proved in Appendix B.

$$u''_- - \left(\frac{1}{3}\frac{\gamma+2}{\gamma}k^2 + \zeta\mathcal{H}^2\right)u_- \simeq 0, \tag{5.70}$$

where, as before, $\simeq$ means first order in slow-roll parameters and

$$\zeta \equiv \frac{\theta''}{\theta} - 2\psi^2 \simeq \left(\frac{\gamma-1}{\gamma+1} + \frac{\dot\epsilon}{H\epsilon^2}\right)\epsilon = \frac{3\gamma-1}{\gamma+1}\epsilon. \tag{5.71}$$

In the first order in slow-roll parameter ϵ one may ignore the time-dependence of ϵ during slow-roll expansion, and

$$\tau \simeq -\frac{1}{(1-\epsilon)\mathcal{H}}, \tag{5.72}$$

and hence

$$\zeta\mathcal{H}^2 = \frac{\nu_u^2 - \frac{1}{4}}{\tau^2}, \qquad \nu_u \simeq \frac{1}{2} + \zeta. \tag{5.73}$$

To summarize, δs is of order 1 in the asymptotic past $k\tau \to -\infty$ limit and is of order ϵ in the superhorizon $k\tau \to 0$ limit, effecting the spectral tilt (cf. (5.71)).

The general solutions to (5.69) and (5.70) for $u_\pm$ can be expressed as a linear combination of Hankel $H_\nu^{(1)}$ and $H_\nu^{(2)}$, and modified Bessel functions I_ν and K_ν. Recalling (5.55), this leads to the following solutions for Ψ

$$\Psi_+(k,\tau) \simeq \frac{\sqrt{\pi|\tau|}}{2k}[b_1^+ H_{\nu_u}^{(1)}(k|\tau|) + b_2^+ H_{\nu_u}^{(2)}(k|\tau|)],$$
$$\Psi_-(k,\tau) \simeq \frac{\sqrt{|\tau|}}{\sqrt{\pi}k}\left[b_1^- I_{\nu_u}\left(\sqrt{\frac{1}{3}\frac{x+2}{x}}k|\tau|\right) + b_2^- K_{\nu_u}\left(\sqrt{\frac{1}{3}\frac{x+2}{x}}k|\tau|\right)\right]. \tag{5.74}$$

Before moving onto considering $\mathcal{Q}$ equations, some remarks are in order:

(i) It can immediately be seen that $\Psi_\pm$ given in (5.74), in the asymptotic past limit $k\tau \to -\infty$, indeed reproduce the solutions to

$$u'' + k^2 c_{s\pm}^2 u \simeq 0.$$

As such, we have found solutions which interpolate between the superhorizon regime $k\tau \to 0$ and deep subhorizon regime $k\tau \to -\infty$.

(ii) We stress that as we see δs in (5.62) and (5.68), in both $k\tau \to -\infty$ and $k\tau \to 0$ regimes, only depends on u, the variable which plays the role of adiabatic perturbations. Therefore, unlike the multifield inflationary models, δs does not represent an independent "entropy" mode. In other words, despite having two scalar modes $\mathcal{Q}$ and $\mathcal{M}$, we do not have entropy perturbations in our system. This is due to the fact that, unlike the usual two-field inflationary models, we have two independent constraints (5.51) and (5.52), rather than a single constraint in the usual two-field models [14], relating $\mathcal{Q}$ and $\mathcal{M}$. Therefore, we have a single adiabatic perturbation, as in the standard single field models.[9] Equation (5.74) is not exact in the sense that the constraint equations, the result of which have been plugged into the u Eq. (5.56), have only be solved in the $k\tau \to 0$ and $k\tau \to -\infty$ regimes.

(iii) Equation (5.69) implies that u_+ admits two physical regimes: oscillating regime in the asymptotic past $k\tau \to -\infty$, and the superhorizon limit $k\tau \to 0$,where u_+ freezes out. As given by (5.70), u_- has an exponentially damping regime in the asymptotic past Minkowski limit $k\tau \to -\infty$, while it freezes out, just like u_+, in the superhorizon $k\tau \to 0$ regime. The other two scalars, $\mathcal{Q}$ and $a\mathcal{M}$, have the same generic behavior.

(iv) Note also that the coefficients $b_i^\pm$, $i = 1, 2$ are not completely fixed from the above considerations and to determine them we need to know $\mathcal{Q}$, explicitly $\mathcal{Q}_+$, to which we will turn now.

Classical solutions for $\mathcal{Q}$.—To find the equations of motion for $\mathcal{Q}$, one may use the solutions (5.74), (5.57), and (5.58). Alternatively, one can work out the second-order action for the perturbations $\mathcal{Q}$, Ψ and $\mathcal{M}$ and insert the constraint Eqs. (5.57) and (5.58) into the action. The second-order action computation is indeed very tedious, lengthy and cumbersome, but that is necessary for quantization of the perturbations. This is because, for performing the canonical quantization of the modes, besides the equations of motion we need to have the canonical (conjugate) momentum too. In the Appendix A we have presented the explicit form of the second-order action, after imposing the gauge-fixing conditions and setting $\pi_s = 0$.

Appearance of negative c_s^2 modes may cause a concern about a possibility of ghost instability in our system. Theoretically, we do not expect finding ghosts in our theory because, i) we are dealing with a gauge-invariant action and we respect this gauge symmetry. (To be more precise, it is spontaneously broken by the choice of classical inflationary background. However, as is well-established, spontaneous gauge symmetry breaking does not lead to a breakdown of Slavnov-Taylor identity which reflects the gauge symmetry and its consequences about renormalizability and unitarity.) ii) Although we are dealing with a "higher derivative" action (3.1), the higher derivative term has a special form: it does not involve more than time-derivative squared terms. (This fact is also explicitly seen in (3.9) in that the ϕ equation of motion does not involve more than second time derivative.) As such, we expect not to see ghosts usually present in the higher derivative theories. Besides the above arguments, to make sure about the absence of ghosts, we have explicitly computed the second-order action. The expression for the

[9] Recall that the situation here is indeed very much like the single scalar case, where the entropy perturbations, like ours, is of order slow-roll parameter ϵ.

second-order action, after implementing the constraints (5.57) and (5.58), explicitly shows that neither $\mathcal{Q}$ nor $\mathcal{M}$ has negative kinetic terms and hence there is no ghost instability in our system. The explicit expression for the second-order action is presented in Appendix A and here we only present the simplified result for the canonical momenta and for the equations of motion for $\mathcal{Q}$.

The $\mathcal{Q}$ equations of motion in the slow-roll approximation, after using (5.59), (5.64), and (5.65), and some lengthy algebra is obtained to be

$$\begin{aligned}\mathcal{Q}_+'' + (k^2 - (2 + 3\zeta - 4\epsilon)\mathcal{H}^2)\mathcal{Q}_+ &\simeq 0,\\ \mathcal{Q}_-'' - \left(\frac{x+2}{3x}k^2 + (2+3\zeta-4\epsilon)\mathcal{H}^2\right)\mathcal{Q}_- &\simeq 0,\end{aligned} \tag{5.75}$$

where $\mathcal{Q}_\pm$ in an obvious notation is related to values of $\mathcal{W}_\pm$ and $c_{s\pm}^2$ given in (5.64) and (5.65). Defining ν_Q as

$$\nu_Q^2 - \frac{1}{4} = (2 + 3\zeta - 4\epsilon)\mathcal{H}^2\tau^2 \simeq 2 + 3\zeta, \tag{5.76}$$

leading to $\nu_Q \simeq \frac{3}{2} + \zeta$, (5.75) takes the form of a standard Bessel equation with the general solutions

$$\mathcal{Q}_+(k,\tau) \simeq \frac{\sqrt{\pi|\tau|}}{2} e^{i(1+2\nu_u)(\pi/4)}[d_1^+ H_{\nu_Q}^{(1)}(k|\tau|) + d_2^+ H_{\nu_Q}^{(2)}(k|\tau|)], \tag{5.77}$$

$$\mathcal{Q}_-(k,\tau) \simeq \frac{\sqrt{|\tau|}}{\sqrt{\pi}}\left[d_1^- I_{\nu_Q}\left(\sqrt{\frac{x+2}{3x}}k|\tau|\right) + d_2^- K_{\nu_Q}\left(\sqrt{\frac{x+2}{3x}}k|\tau|\right)\right]. \tag{5.78}$$

Quantization of $\mathcal{Q}$ modes.—

As in the standard textbook material in cosmic perturbation theory, the coefficients d_i^+ may be fixed using the canonical normalization of the modes in the Minkowski, deep subhorizon $k\tau \to -\infty$ regime. As discussed, in this limit $\mathcal{Q}_+$, which has an oscillatory behavior, is the only quantum field ($\mathcal{Q}_-$ has an exponentially damping behavior and is hence not a quantum mode). We should stress that, of course, not all coefficients $d_i^\pm$ and $b_i^\pm$ are fixed by the quantization normalization condition. To fix them, as we will do so below, we should impose the constraints (5.57) and (5.58) in both superhorizon and asymptotic past regimes. Note also that fulfilling these constraints is equivalent to maintaining the diffeomorphism and remainder of the gauge symmetry of the system; fluctuations both at classical and quantum levels must respect them.

From the second-order action given in Appendix A, the discussions alluded to above, and using the constraints (5.59) and (5.64), after some lengthy straightforward algebra the momentum conjugate to $\mathcal{Q}_+$ mode $P_{\mathcal{Q}_+}$ is obtained as

$$P_{\mathcal{Q}_+} = 2\mathcal{Q}_+' \tag{5.79}$$

in the $k\tau \to -\infty$ limit, and hence canonically normalized field is $\sqrt{2}\mathcal{Q}_+$. Imposing the usual Minkowski vacuum state for $\sqrt{2}\mathcal{Q}_+$

$$\mathcal{Q}_+ \simeq \frac{1}{2\sqrt{k}}e^{-ik\tau}$$

fixes the d_i^+ coefficients

$$d_1^+ = \frac{1}{\sqrt{2}}, \qquad d_2^+ = 0. \tag{5.80}$$

The constraint Eq. (5.61) can then be used to fix b_i^+ coefficients in (5.74):

$$b_1^+ = \sqrt{2}H\psi\frac{(\gamma-1)}{(\gamma+2)}, \qquad b_2^+ = 0. \tag{5.81}$$

We stress that with the above choice for b_i^+ we have only satisfied the constraint (5.61) in the asymptotic past $k\tau \to -\infty$ limit. As we will show momentarily, the constraint (5.61) in the superhorizon $k\tau \to 0$ limit, where $\mathcal{Q}_-$, Ψ_-, as well as $\mathcal{Q}_+$ and Ψ_+ assume constant values, can only be satisfied when the nonoscillatory $\mathcal{Q}_-$ and Ψ_- modes are also taken into account.

The coefficients d_i^- and b_i^-, although not fixed by quantization normalization, are related through the constraint Eq. (5.61). Recalling the exponential growth of the modified Bessel function $I_n(y)$ for large y [15], we learn that

$$b_1^- = d_1^- = 0.$$

Demanding the constraint Eq. (5.61) to be satisfied in the asymptotic past $k\tau \to -\infty$ then leads to

$$b_2^- \simeq \sqrt{\frac{12\gamma}{\gamma+2}}H\psi d_2^-. \tag{5.82}$$

To fix d_2^- and b_2^- we should now demand the constraint (5.61) to be satisfied in the superhorizon $k\tau \to 0$ regime too. This will, however, also involve the "oscillatory" $\mathcal{Q}_+$ and Ψ_+ modes. Putting these together and making use of the behavior of the Bessel functions $Z_n(y)$ (where Z_n is either H_n^1, H_n^2, or K_n) for small argument $y \to 0$ [15], we obtain

$$d_2^- \simeq -\frac{i}{\sqrt{2}}\left(\frac{\gamma+2}{3\gamma}\right)^{3/4}. \tag{5.83}$$

*The curvature power spectrum.—*Having fixed all the coefficients $b_i^\pm$ and $d_i^\pm$ we are now ready to compute the power spectrum of metric and curvature perturbations. The power spectrum for the metric perturbations is given by [1]

$$\mathcal{P}_\Psi = \frac{4\pi k^3}{(2\pi)^3}|\Psi|^2, \tag{5.84}$$

which on the large scales ($k \ll aH$) is

$$\mathcal{P}_{\Psi} \simeq \frac{\epsilon(2\gamma+1)^2}{2(\gamma+1)(\gamma+2)^2}\left(\frac{H}{\pi}\right)^2\left(\frac{|k\tau|}{2}\right)^{3-2\nu_Q}. \tag{5.85}$$

As is implicit, we have assumed slow-roll approximation and the value of ϵ and γ essentially remain constant during slow-roll period. The power spectrum of the comoving curvature perturbation $\mathcal{R}$, $\mathcal{R} = \Psi - \frac{H\delta q}{\rho_0+P_0}$, is hence

$$\mathcal{P}_{\mathcal{R}} \simeq \frac{(2\gamma+1)^2}{2\epsilon(\gamma+1)(\gamma+2)^2}\left(\frac{H}{\pi}\right)^2\bigg|_{k=aH}, \tag{5.86}$$

and becomes constant on super-Hubble scales.

The spectral index of the curvature perturbations, $n_{\mathcal{R}} - 1 = 3 - 2\nu_Q$, to the leading order in the slow-roll parameters is

$$n_{\mathcal{R}} - 1 \simeq -2\frac{3\gamma-1}{\gamma+1}\epsilon, \tag{5.87}$$

where (5.71) and (5.76) have been used. We note that the spectral tilt (5.87), depending on the value of γ, can be positive or negative; in order to have a red tilt we should consider $\gamma > 1/3$.

2. Tensor modes

To analyze the tensor modes h and T and the action (5.50), it proves useful to decompose T into h and a new variable w

$$T = Ah + w, \tag{5.88}$$

where A is a constant (to be determined). In terms of w the second-order action (5.50) takes the form

$$\begin{aligned}\delta S_T^{(2)} = \frac{1}{2}\int d^3x d\tau\bigg[&2A\psi^2\bigg(w'h' + \frac{1}{A}\mathcal{H}wh' - k^2wh \\ &+ (\gamma+1)\left(\frac{1-2A}{A}\right)\mathcal{H}^2wh\bigg) \\ &+ \frac{1}{4}(1+4A^2\psi^2)\left(h'^2 - k^2h^2 + \frac{z''}{z}h^2\right) \\ &+ \psi^2\left(w'^2 - k^2w^2 + \frac{\vartheta''}{\vartheta}w^2\right)\bigg],\end{aligned} \tag{5.89}$$

where

$$\begin{aligned}\frac{z''}{z} = \mathcal{H}^2[&2 + 2\psi^2(1 + 2A - A^2 + (4A-1)\gamma) \\ &- (1+8A^2)\epsilon],\end{aligned} \tag{5.90}$$

$$\frac{\vartheta''}{\vartheta} = -\mathcal{H}^2(2-\epsilon)(\gamma+1). \tag{5.91}$$

From the action (5.89) one can read the equations of motion for h and w

$$\begin{aligned} w'' + \left(k^2 - \frac{\vartheta''}{\vartheta}\right)w &= A\bigg(-h'' + \frac{1}{A}\mathcal{H}h' \\ &\qquad - k^2h + (\gamma+1)\left(\frac{1-2A}{A}\right)\mathcal{H}^2h\bigg), \\ h'' + \left(k^2 - \frac{z''}{z}\right)h &= -\frac{4A}{1+4A^2\psi^2}\bigg(w'' + \frac{1}{A}(\mathcal{H}w)' \\ &\qquad + k^2w - (\gamma+1)\left(\frac{1-2A}{A}\right)\mathcal{H}^2w\bigg). \end{aligned} \tag{5.92}$$

(To obtain these results, we have used the fact that ψ is almost a constant to first order in ϵ, *cf.* (3.13) and (3.17).) The above equations imply that h and w have both oscillatory behavior $e^{ik\tau}$ in the asymptotic past $k\tau \to -\infty$ region. However, since ϑ''/ϑ is negative while z''/z is positive, they behave differently in superhorizon $k\tau \to 0$ limit; $\frac{h}{a}$ freezes out and $\frac{w}{a}$ decays. Therefore, in this limit the leading contribution to the right-hand side of equation of motion for w, which is of order $(k\tau)^{-3}$, should vanish. That is,

$$\left(-2 + \frac{1}{A}\right)(2+\gamma)\mathcal{H}^2h \simeq 0, \tag{5.93}$$

which implies $A = \frac{1}{2}$. This choice for A has an interesting and natural geometric meaning, recalling the form of our ansatz for the background gauge field, $A^a{}_i = \psi e^a{}_i$, where ψ is a scalar (effective inflaton field) and e^a_i are the 3D triads, and that the triads are "square roots" of metric. Perturbing the ansatz and considering only the metric tensor perturbations h_{ij}, we have

$$\delta e^a{}_i = \frac{1}{2}h_{ij}\delta^{aj}. \tag{5.94}$$

Then, recalling (5.3) and the definition of t_{ij}, this implies that $A = \frac{1}{2}$ naturally removes the part of the gauge field tensor perturbations which is coming from the perturbation in the metric, and hence the "genuine" gauge field tensor perturbation is parameterized by w.

Inserting $T = \frac{h}{2} + w$ into the expression (5.48) and noting the exponential suppression of w in the superhorizon scales,

$$a^2\pi^T \simeq 0, \tag{5.95}$$

in the leading order in ϵ and in the $k\tau \to 0$ limit. Therefore, the equation of motion for h is

$$h'' + \left(k^2 - \frac{a''}{a}\right)h \simeq 0, \qquad \frac{a''}{a} = \mathcal{H}^2(2-\epsilon), \tag{5.96}$$

which is exactly the same as the standard single scalar field inflation result [2]. The w modes do not contribute to the tensor mode power spectrum.

In the slow-roll approximation (5.96) is a Bessel equation, whose solution after imposing the standard

Minkowski vacuum normalization on the $k\tau \to 0$ solutions [2], takes the form

$$h = \frac{\sqrt{\pi|\tau|}}{2} e^{i(1+2\nu_T)\pi/4} H^{(1)}_{\nu_T}(k|\tau|), \tag{5.97}$$

where $\nu_T \simeq \frac{3}{2} + \epsilon$. Thus, the gravitational waves' power spectrum to leading order in slow-roll and on super-Hubble scales is set by

$$\mathcal{P}_T \simeq 2\left(\frac{H}{\pi}\right)^2 \Big|_{k=aH}. \tag{5.98}$$

The spectral index of tensor perturbations, n_T is given by

$$n_T \simeq -2\epsilon. \tag{5.99}$$

To summarize, the gravitational power spectrum and the tensor spectral index of gauge-flation are just the $\mathcal{P}_T$ and n_T of the single scalar model and the tensor-to-scalar ratio r for our model is

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_\mathcal{R}} = \frac{4(\gamma+1)(\gamma+2)^2}{(2\gamma+1)^2}\epsilon. \tag{5.100}$$

This result, which can be written as $r = -\frac{2(\gamma+1)(\gamma+2)^2}{(2\gamma+1)^2} n_T$, may be contrasted with the usual single field consistency relation $r = -8n_T$ [16].

VI. FITTING GAUGE-FLATION RESULTS WITH THE COSMIC DATA

We are now ready to confront our model with the observational data. As discussed, our model for a specific range of its parameters allows for slow-roll inflation and for comparison with the observational data we use the results obtained in the slow-roll regime. First, we note that in order for inflation to solve the flatness and horizon problems it should have lasted for a minimum number of e-folds N_e. This amount, of course, depends on the scale of inflation and somewhat to the details of physics after inflation ends [1]. However, for a large inflationary scale, like $H \sim 10^{-4}$–$10^{-5} M_{\rm pl}$, it is usually demanded that $N_e \simeq 60$. As a standard benchmark we use $N_e = 60$.

As for the CMB data, current observations provide values for power spectrum of curvature perturbations $\mathcal{P}_\mathcal{R}$ and its spectral tilt $n_\mathcal{R}$ and impose an upper bound on the power spectrum of tensor modes $\mathcal{P}_T$, or equivalently an upper bound on tensor-to-scalar ratio r. These values vary (mildly) depending on the details of how the data analysis has been carried out. Here we use the best estimation of Komatsu *et al.* [17], which is based on WMAP seven years, combined with other cosmological data. These values are

$$\mathcal{P}_\mathcal{R} \simeq 2.5 \times 10^{-9}, \tag{6.1a}$$

$$n_\mathcal{R} = 0.968 \pm 0.012, \tag{6.1b}$$

$$r < 0.24. \tag{6.1c}$$

Our model has two parameter g and κ, and our results for physical observables depend also on other parameters, which are basically related to the initial values of the fields we have in our model. Out of these parameters we choose H, the value of Hubble, and ψ, the value of the effective inflaton field at the beginning of, or during, slow-roll inflation. The values of other parameters, ϵ, γ and δ (initial velocity of the ψ field (3.13)), are related to these two through (3.16), (3.17), (3.18), and (3.19). For convenience let us recollect all our results:

$$N_e = \frac{\gamma+1}{2\epsilon}\ln\frac{\gamma+1}{\gamma}, \tag{6.2}$$

$$n_\mathcal{R} - 1 \simeq -\frac{1}{N_e}(3\gamma-1)\ln\frac{\gamma+1}{\gamma}, \tag{6.3}$$

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_\mathcal{R}} = \frac{1}{2N_e}\left(\frac{2(\gamma+1)(\gamma+2)}{2\gamma+1}\right)^2 \ln\frac{\gamma+1}{\gamma}, \tag{6.4}$$

$$\mathcal{P}_\mathcal{R} \simeq \frac{1}{2\pi^2\epsilon}\left(\frac{H}{M_{\rm pl}}\right)^2 \frac{(2\gamma+1)^2}{(\gamma+2)^2(\gamma+1)} \simeq \frac{g^2}{2\pi^2}\frac{(2\gamma+1)^2}{\gamma(\gamma+1)^2(\gamma+2)^2}. \tag{6.5}$$

Interestingly, the spectral tilt and the number of e-folds only depend on γ and ϵ and hence their values may be fixed using these two, leading to

$$0.85 < \gamma < 6.35, \qquad \epsilon = (0.9\text{–}1.2) \times 10^{-2}. \tag{6.6}$$

Notice that lower value for ϵ corresponds to $\gamma = 6.35$. We may now use these values and the COBE normalization to read H and ψ

$$\psi = (3.5\text{–}8.0) \times 10^{-2} M_{\rm pl}, \qquad H \simeq 3.5 \times 10^{-5} M_{\rm pl}. \tag{6.7}$$

(We note that the variation in the value of H over the range (6.6) is about 0.3%.) Finally one may use definition of γ (3.19) to compute g and κ

$$\frac{g^2}{4\pi} = (0.13\text{–}5.0) \times 10^{-7}, \qquad \kappa = (4.6\text{–}17) \times 10^{13} M_{\rm pl}^{-4}. \tag{6.8}$$

One can easily check that with the above values for our parameters, our model *predicts* $\mathcal{P}_T = 2.45 \times 10^{-10}$ corresponding to $r \simeq 0.1$, which is well within the range to be observed by the Planck satellite.

As mentioned, however, the value of N_e is not exactly 60 and may be smaller or larger. This possibility has been explored in Fig. 4, considering the current observational data (6.1b) and (6.1c). The current observational value for $n_\mathcal{R} - 1$ and the bound on r, allows for $40 \lesssim N_e < 145$ and $0.05 < r < 0.24$. In other words, $r > 0.05$ is one of the predictions of our model, which can be tested by the Planck. As handy relations to remember, for $\gamma = 1$, $r = \frac{9.9}{N_e}$ and for $\gamma = 10$, $r = \frac{7.9}{N_e}$.

It is also instructive to compare our model with usual single field inflationary results. For the comparison it is more convenient to replace γ for η. Recalling (3.16) and (3.17) we learn that

$$\gamma \simeq \frac{\epsilon - \eta}{\eta}. \tag{6.9}$$

In our model by definition γ is a positive number and as is seen from Fig. 4 it is of order 1–10. In terms of ϵ and η

$$N_e \simeq \frac{1}{2\eta} \ln\frac{\epsilon}{\epsilon - \eta}, \qquad r \simeq \eta\left(\frac{2\epsilon(\epsilon + \eta)}{\eta(2\epsilon - \eta)}\right)^2, \\ \Delta\psi^2 = \psi_i^2 - \psi_f^2 \simeq \frac{\psi_i^2}{2^{1/3}} \simeq \frac{\eta}{2^{1/3}}. \tag{6.10}$$

As we see, the field ψ and its variation during inflation are both small and proportional to η while, as in the standard single field models, N_e is inversely proportional to η (or ϵ). This is to be contrasted with the usual results that leads to Lyth bound [18] where the *inverse* of slow-roll parameters ϵ or η are proportional to φ^2 and/or its variation during (slow-roll) inflation $(\Delta\varphi)^2$, with φ being the inflaton field. Therefore, our model can naturally produce large tensor-to-scalar ratio, $r \gtrsim 0.01$, which will be detectable by Planck satellite, without the need for super-Planckian field or field variations.

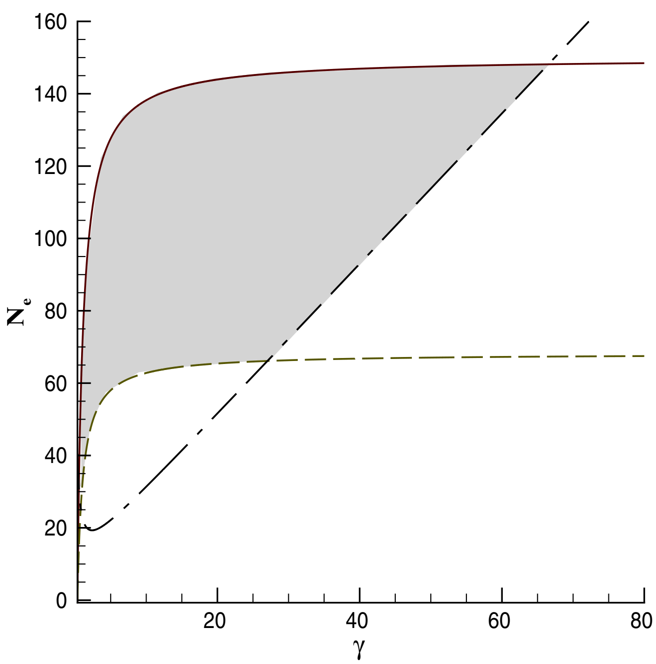

FIG. 4 (color online). The shaded region specifies the range for γ and N_e allowed by the current observations. The dot-dashed line corresponds to (6.4) with $r = 0.24$. The upper solid curve corresponds to (6.3) with $n_{\mathcal{R}} = 0.968 + 0.012$ and the lower dashed curve to $n_{\mathcal{R}} = 0.968 - 0.012$. As we see, the current observations allows for N_e as large as 145. One can show using (6.4) that for the values in the shaded region the tensor-to-scalar ratio is in the range $0.05 < r < 0.24$.

VII. SUMMARY, DISCUSSION AND OUTLOOK

In this work, we have presented a detailed analysis of the gauge-flation model which we introduced in [12]. We first showed that non-Abelian gauge field theory can provide the setting for constructing an isotropic and homogeneous inflationary background. We did so by using the global part of the gauge symmetry of the problem and identified the $SU(2)$ subgroup of that with the rotation group. We argued that this can be done for *any* non-Abelian gauge group, as any such group has an $SU(2)$ subgroup. Therefore, our discussions can open a new venue for building inflationary models, closer to particle physics high-energy models, where non-Abelian gauge theories have a ubiquitous appearance.

The Yang-Mills theory cannot serve the job of building inflationary models, and we have to consider more complicated gauge theory actions. Among the obvious choices, we have checked non-Abelian version of Born-Infeld action[10] (with the symmetric trace prescription [20]), which does not lead to a slow-roll dynamics within its space of parameters. We have checked F^4 terms which appear in one-loop level effective gauge theory action. If we parameterize such F^4 terms as $\mathrm{Tr}(\alpha F^4 + \beta(F^2)^2)$, our analysis shows that it is possible to get slow-roll inflationary background for specific range of α and β parameters. With the gauge group $SU(2)$, upon which we have mainly focused in this work, the $\mathrm{Tr}(F \wedge F)^2$ that we have considered here can be obtained from specific choices of α and β.

As discussed our motivation for considering a $\mathrm{Tr}(F \wedge F)^2$ term was primarily providing an explicit, simple realization of our gauge-flation scenario, which can lead to a satisfactory slow-roll inflation; in this work we were not concerned with explicit derivation or embedding of this term from particle physics models. At technical level, this happens because the dependence of this term on the background metric $g_{\mu\nu}$ appears only through $\det g$ and as a result the contribution of this term to the energy-momentum of the background will take the form of a perfect fluid with $P = -\rho$ equation of state, perfectly suited for driving an almost de Sitter expansion. It is, however, important to study appearance of this κ-term through a rigorous quantum gauge field theory analysis and in particle physics settings. From particle physics model building viewpoint, a $\mathrm{Tr}(F \wedge F)^2$ type term can be argued for, considering axions in a non-Abelian gauge theory [21] and recalling the axion-gauge field interaction term $\mathcal{L}_{\rm axion} \sim \frac{\varphi}{\Lambda}\mathrm{Tr}F \wedge F$. Then, integrating out the massive axion field φ leads to an action of the form we have considered. If we adopt this point of view, our κ parameter is then related to the cutoff scale Λ as $\frac{\kappa}{384} \sim \Lambda^{-4}$ [21], and hence leading to $\Lambda \sim 10^{-3} M_{\rm pl} \sim 10^{15}$ GeV. In order for this proposal to work, some points should be

[10] For an analysis of non-Abelian Born-Infeld theory within the FRW cosmology see [19].

checked: recalling that $H \lesssim 10^{14}$ GeV, $\Lambda \sim 10H$. For this one-loop effective action description to make sense, it is crucial that the cutoff Λ becomes larger than H, because only axion configurations with subhorizon momenta ($k \gtrsim H$) will contribute to (quantum) loop corrections. The superhorizon modes, as in any quantum field theory on (almost) de Sitter background, are frozen and have become classical, and hence do not contribute to quantum corrections. It is also crucial that we are in a perturbative regime of the gauge theory with $g \sim 10^{-3}$. Therefore, we need not worry about complications of dealing with a confining (non-Abelian) gauge theory. In our case, we are in a weakly coupled regime where the theory is in deconfined phase. We also remark that, as argued, during slow-roll inflation regime, the contribution of the κ-term to the energy density of the gauge field configuration should dominate over that of the Yang-Mills part. In order for the mechanism for generation of the κ-term sketched above to work, one should argue how the other possible higher-order terms, at F^4 level and higher loops (leading to higher powers of F in the effective action), are suppressed compared to the κ-term. These issues will be discussed in a later publication [22].

Another interesting feature of our gauge-flation model is its naturalness; that demanding to have a successful inflationary model compatible with the current data leads to parameters which are within their natural range: the Hubble during inflation H is of order 10^{14} GeV, and cutoff scale of the theory $\Lambda \sim 10^{15}$–10^{16} GeV which are natural within the (SUSY) GUT models. Moreover, as is required by the consistency of the theory H is less than cutoff Λ (by 1 order of magnitude). The other parameter of the theory, the gauge coupling $g \sim 10^{-3}$–10^{-4}, although a bit lower than the value expected for the coupling at the gauge unification scale, is also in a natural range. The field value ψ_i and its displacement during inflation $\psi_i - \psi_f$, are both of order $10^{-2} M_{\rm pl}$, well within the sub-Planckian regime. Therefore, as discussed, the arguments of standard single field inflationary models and the Lyth bound [18] do not apply to our model and we do not face the super-Planckian field problem, which is a generic feature of large-field inflation models, such as chaotic inflation, causing concerns about the validity of using classical Einstein gravity. We also note that the energy density during inflation $3H^2 M_{\rm pl}^2 \sim (2 \times 10^{16}\ \text{GeV})^4$, is the same order as the SUSY GUT scale.

Our other motivation for studying the gauge-flation scenario, which is at least in spirit close to beyond standard particle physics model settings, was to provide a setup to address cosmological questions after inflation. As we discussed and is also seen from the phase diagram in Fig. 1, after the slow-roll ends we enter a phase where the dynamics of the effective inflaton field, and gauge fields in general, is governed by the Yang-Mills term. The effective inflaton ψ starts an oscillatory phase and through standard (p)reheating arguments, e.g., see [13], it can lose its energy to the gauge fields. If we have an embedding of our gauge-flation scenario into beyond standard models, the energy of these gauge fields will then naturally be transferred to all the other standard model particles via standard gauge interactions. Therefore, our gauge-flation provides a natural setting for building (p)reheating models, to which we hope to return in future works.

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APPENDIX A: SECOND-ORDER ACTION

After a tedious but straightforward calculation, which is also confirmed by the MAPLE codes, the total action to second order in perturbations is

$$
\begin{aligned}
\delta_2 S = \int dx^4 a^3 \bigg\{ & \frac{3}{2}\Big(1 + \frac{\kappa g^2\phi^4}{a^4}\Big)\frac{\dot{\mathcal{Q}}^2}{a^2} + \Big(6\frac{\dot\phi}{a}\Big(-1 + \frac{\kappa g^2\phi^4}{a^4}\Big)\Psi - \frac{g^2\phi^3}{a^2\dot\phi}\Big(1 + \frac{4\kappa\dot\phi^2}{a^2} + \frac{\kappa g^2\phi^4}{a^4}\Big)\frac{k^2}{a^2}a\mathcal{M}\Big)\frac{\dot{\mathcal{Q}}}{a} \\
& + \Big(-\frac{k^2}{a^2} - \frac{9g^2\phi^2}{a^2}\Big(1 - \frac{\kappa\dot\phi^2}{a^2}\Big) - \frac{6\kappa g^2}{a}\Big(\frac{\phi^3\dot\phi}{a^3}\Big)\Big)\frac{\mathcal{Q}^2}{a^2} - \frac{2g^2\phi^3}{a^2\dot\phi}\frac{\kappa\dot\phi^2}{a^2}\frac{k^2}{a^2}a\dot{\mathcal{M}}\frac{\mathcal{Q}}{a} + \Big(\frac{12g^2\phi^3}{a^3}\Big(-1 + \frac{\kappa\dot\phi^2}{a^2}\Big)\Psi \\
& - \frac{2g^2\phi^2}{a^2}\Big(\frac{\kappa g^2\phi^4}{a^4} + \frac{3\kappa\dot\phi^2}{a^2} - 3\Big)\frac{k^2}{a^2}a\mathcal{M}\Big)\frac{\mathcal{Q}}{a} + \frac{g^2\phi^2}{a^2}k^2\dot{\mathcal{M}}^2 + \frac{g^2\phi^2}{a^2}\bigg(\Big(\frac{g^2\phi^3}{a^2\dot\phi}\Big)^2 + \frac{1}{a^3}\Big(\frac{g^4\phi^5}{a\dot\phi}\Big) \\
& + \frac{k^2}{a^2}\Big(-1 + \frac{2}{3}\Big(\frac{\kappa g^2\phi^4}{a^4} + \frac{\kappa\dot\phi^2}{a^2}\Big) + \frac{g^2\phi^4}{a^2\dot\phi^2}\Big(\frac{1}{6}\frac{\kappa g^2\phi^4}{a^4} + \frac{1}{2}\Big)\Big)\bigg)\frac{k^2}{a^2}a^2\mathcal{M}^2 - \frac{2g^2\phi^3}{a^3}\Big(\frac{2\kappa\dot\phi^2}{a^2} + \frac{\kappa g^2\phi^4}{a^4} - 3\Big)\frac{k^2}{a^2}a\mathcal{M}\Psi \\
& + 3\Big(\frac{\dot\phi^2}{a^2} - \frac{g^2\phi^4}{a^4} + 3\frac{\kappa g^2\phi^4}{a^4}\frac{\dot\phi^2}{a^2}\Big)\Psi^2 - 3\dot\Psi^2 + \frac{k^2}{a^2}\Psi^2 + 6\dot H\Psi^2 \bigg\}.
\end{aligned} \tag{A1}
$$

Note that the above action is computed fixing the Newtonian gauge $E = B = 0$ and imposing $\pi_s = 0$.

APPENDIX B: VALIDITY OF ANSATZ (5.59)

In this appendix we prove the validity of ansatz (5.59). In the asymptotic past $k\tau \to -\infty$ (Minkowski) limit and during the slow-roll inflation, Eq. (5.37) takes the following form

$$\left(\mathcal{Q}' - \frac{k^2}{3H}\left(1+\frac{2}{\gamma}\right)g^2\psi^2 a\mathcal{M}\right) \simeq -\frac{\psi^2}{3}\left(\mathcal{Q}' + \frac{g^2\psi^2}{H}(a\mathcal{M})''\right), \qquad \text{(B1)}$$

which can be rewritten as

$$\left(\mathcal{Q}' - \frac{k^2}{3H}\left(1+\frac{2}{\gamma}\right)g^2\psi^2 a\mathcal{M}\right) \simeq \frac{\mathcal{W}}{2}\psi^2\mathcal{Q}' - \frac{1}{3}\frac{g^2\psi^4}{H}(a\mathcal{M}_s)'', \qquad \text{(B2)}$$

where $\mathcal{W}$ is a constant. On the other hand, from (5.57) we learn that ψ should include a term $\psi_s \equiv g^2\psi^3 a\mathcal{M}_s$, which after combining with (5.58), yields

$$\left(1+\frac{2}{\gamma}+\frac{1}{\gamma}\psi^2\right)\mathcal{M}_s = 0, \qquad \text{(B3)}$$

and since $\gamma > 0$, it implies $\mathcal{M}_s = 0$, proving the validity of the ansatz (5.59).

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[1] V. Mukhanov, *Physical Foundations of Cosmology* (Cambridge University Press, Cambridge, England, 2005); S. Weinbege, *Cosmology* (Oxford University Press, New York, 2008); D. Lyth and A. Liddle, *Primordial Density Perturbations* (Cambridge University Press, Cambridge, England, 2009); M. Giovannini, *A Primer on the Physics of the Cosmic Microwave Background* (World Scientific Press, Singapore, 2008); A. D. Linde, Lect. Notes Phys. **738**, 1 (2008).
[2] B. A. Bassett, S. Tsujikawa, and D. Wands, Rev. Mod. Phys. **78**, 537 (2006).
[3] M. Dine, *Supersymmetry and String Theory: Beyond the Standard Model* (Cambridge Univ. Pr., Cambridge, UK, 2007), p. 515.
[4] R. Kallosh and A. D. Linde, J. Cosmol. Astropart. Phys. 10 (2003) 008.
[5] A. Golovnev, V. Mukhanov, and V. Vanchurin, J. Cosmol. Astropart. Phys. 06 (2008) 009; J. Cosmol. Astropart. Phys. 11 (2008) 018.
[6] G. Esposito-Farese, C. Pitrou, and J.-P. Uzan, Phys. Rev. D **81**, 063519 (2010); Y. Zhang, Y.-G. Gong, and Z -H. Zhu, Classical Quantum Gravity **27**, 135019 (2010).
[7] M. C. Bento, O. Bertolami, P. V. Moniz *et al.*, Classical Quantum Gravity **10**, 285 (1993).
[8] T. Chiba, J. Cosmol. Astropart. Phys. 08 (2008) 004; K. Dimopoulos and M. Karciauskas, J. High Energy Phys. 07 (2008) 119.
[9] B. Himmetoglu, C. R. Contaldi, and M. Peloso, Phys. Rev. Lett. **102**, 111301 (2009); Phys. Rev. D **80**, 123530 (2009); Phys. Rev. D **79**, 063517 (2009); B. Himmetoglu, C. R. Contaldi, and M. Peloso, Phys. Rev. D **79**, 063517 (2009); **80**, 123530 (2009).
[10] M. Karciauskas and D. H. Lyth, J. Cosmol. Astropart. Phys. 11 (2010) 023; K. Dimopoulos, M. Karciauskas, D. H. Lyth, and Y. Rodriguez, J. Cosmol. Astropart. Phys. 05 (2009) 013.
[11] K. Bamba, S. Nojiri, and S. D. Odintsov, Phys. Rev. D **77**, 123532 (2008); D. V. Gal'tsov and E. A. Davydov, Proc. Steklov Inst. Math. **272**, 119 (2011); D. V. Gal'tsov, arXiv:0901.0115.
[12] A. Maleknejad and M. M. Sheikh-Jabbari, arXiv:1102.1513.
[13] P. B. Greene, L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. D **56**, 6175 (1997).
[14] J. Garcia-Bellido and D. Wands, Phys. Rev. D **53**, 5437 (1996); C. Gordon, D. Wands, B. A. Bassett, and R. Maartens, Phys. Rev. D **63**, 023506 (2000).
[15] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic Press, New York, 2007), 7th ed..
[16] J. E. Lidsey, A. R. Liddle, E. W. Kolb *et al.*, Rev. Mod. Phys. **69**, 373 (1997).
[17] E. Komatsu *et al.*, Astrophys. J. Suppl. Ser. **192**, 18 (2011).
[18] D. H. Lyth, Phys. Rev. Lett. **78**, 1861 (1997).
[19] V. V. Dyadichev, D. V. Gal'tsov, A. G. Zorin, and M. Y. Zotov, Phys. Rev. D **65**, 084007 (2002).
[20] A. A. Tseytlin, Nucl. Phys. **B501**, 41 (1997).
[21] S. Weinberg, *The quantum theory of fields*, Vol. 2 Modern applications (Cambridge University Press, Cambridge, England, 1996).
[22] A. Maleknejad and M. M. Sheikh- Jabbari (work in progress).