Cross-correlation of cosmological birefringence with CMB temperature

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Theories for new particle and early-Universe physics abound with pseudo-Nambu-Goldstone fields that arise when global symmetries are spontaneously broken. The coupling of these fields to the Chern-Simons term of electromagnetism may give rise to cosmological birefringence (CB), a frequency-independent rotation of the linear polarization of photons as they propagate over cosmological distances. Inhomogeneities in the CB-inducing field may yield a rotation angle that varies across the sky. Here we note that such a spatially-varying birefringence may be correlated with the cosmic microwave background (CMB) temperature. We describe quintessence scenarios where this cross-correlation exists and other scenarios where the scalar field is simply a massless spectator field, in which case the cross-correlation does not exist. We discuss how the cross-correlation between CB-rotation angle and CMB temperature may be measured with CMB polarization. This measurement may improve the sensitivity to the CB signal, and it can help discriminate between different models of CB.

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I. INTRODUCTION

Much attention has focused recently on cosmological birefringence (CB), a frequency-independent rotation of the linear polarization of a photon that propagates over cosmological distances [1]. The rotation may arise if the pseudoscalar of electromagnetism $F^{\mu\nu}\tilde{F}_{\mu\nu}$ is coupled to a pseudo-Nambu-Goldstone field (PNGB field) that has variations on cosmological distances or timescales. This field may be identified with the quintessence field [2,3] introduced to account for cosmic acceleration [4–6]. In fact, the flatness required of the quintessence potential is naturally accommodated if quintessence is a PNGB field [7,8]. However, the scalar field may have nothing to do with quintessence—any PNGB field is expected to have such a coupling [9]. There may also be dark-matter mechanisms for CB [10].

Cosmological birefringence has been sought with polarized cosmological radio sources [1,11,12], but here we focus on cosmic-microwave-background (CMB) probes of CB [13]. If the CB-rotation angle α is uniform across the sky, as may result from the homogeneous evolution of quintessence, then there are parity-violating EB and TB correlations between the CMB temperature (T) and the curl-free (E) and curl (B) components of the CMB polarization [13]. Such a rotation has been sought for several years [14], and the tightest current limits on the rotation angle, $-1.41° < \alpha < 0.91°$ (95% C.L.), come from a combined analysis of the WMAP [15], BICEP [16,17], and QUaD [18,19] experiments [15]. It is worth noting, in the context of the current constraints, that the uniform-rotation angle is generally nonzero in quintessence models for CB, while the massless-scalar-field models have no homogeneous time evolution and thus predict no uniform rotation.

It has been pointed out that [9,10,20], more generally, the CB-rotation might be anisotropic, giving rise to $\alpha(\hat{\mathbf{n}})$ as a function of direction $\hat{\mathbf{n}}$ in the sky. Refs. [20–22] showed how measurement of the characteristic non-Gaussianities in the CMB polarization induced by a spatially-varying CB can be used to reconstruct $\alpha(\hat{\mathbf{n}})$ from the CMB. The current best constraint to the root-variance of α, $\langle(\Delta\alpha)^2\rangle^{1/2} \lesssim 4°$, comes from observations of active galactic nuclei [12].

In this paper we explore the possibility that CB may be correlated with primordial density perturbations and thus also with temperature fluctuations in the CMB. Such correlations are to be expected, for example, if the CB-inducing field is a quintessence field with adiabatic primordial perturbations seeded during inflation. On the other hand, correlations between the CB angle and primordial perturbations may be absent if, for example, the CB-inducing field is a massless scalar [9].

We first work out the predictions for spatially-varying $\alpha(\hat{\mathbf{n}})$ for a massless-scalar-field model in which there is no uniform rotation. In this case, the rotation-angle pattern $\alpha(\hat{\mathbf{n}})$ is completely uncorrelated with the CMB-temperature pattern $T(\hat{\mathbf{n}})$, and so we calculate only the CB-angle autocorrelation power spectrum $C_L^{\alpha\alpha}$. We then move on to quintessence models in which the αT cross-correlation exists and calculate this cross-correlation power spectrum $C_L^{\alpha T}$.

We derive the minimum-variance estimators for the αT cross correlation and estimate the detectability of CB-angle fluctuations and CB-angle-temperature correlations with current and forthcoming CMB experiments. We find that the cross-correlation can help improve the sensitivity of experiments to a signal in some cases where the signal would otherwise be only marginally detectable. We show that experiments like SPIDER [23] and Planck [24] may be able to detect a cross-correlation if the CB signal is near its current upper bounds, while the cross-correlation may be detectable with a future experiment, like CMBPol/EPIC [25], even if the CB power spectrum is several orders of magnitude smaller than the current upper limit.

This paper is organized as follows: In Sec. II we introduce the two anisotropic-CB scenarios and their parameters and calculate the corresponding $\alpha\alpha$ and αT power spectra. In Sec. III we discuss how α can be reconstructed from a CMB temperature/polarization map and then how the cross-correlations can be measured. Here, we present the expressions for the minimum-variance estimators of the autocorrelation and cross-correlation power spectra, and expressions for their variances. We then evaluate those variances for SPIDER, Planck, and CMBPol/EPIC and estimate detectability thresholds for these three experiments. A summary and concluding remarks are presented in Sec. IV. Appendix A details the evolution of the scalar-field perturbations, and Appendix B provides the full expression for the variance of the αT cross-correlation.

II. SCENARIOS FOR ANISOTROPIC ROTATION

We consider theories of a cosmic scalar $\phi(x^\mu)$ coupled to the Chern-Simons term of electromagnetism via the Lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\beta\phi}{2M}F^{\mu\nu}\tilde{F}_{\mu\nu}, \tag{1}$$

where $\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}/2$ is the dual of the electromagnetic tensor, $\epsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita tensor (totally antisymmetric), and M is a parameter with dimensions of mass. If ϕ is a PNGB field, then M is the vacuum expectation value for the broken global symmetry and β is a coupling [7,8]. Such a parity-violating term in the Lagrangian introduces a modification of Maxwell's equations that results in different dispersion relations for left- and right-circularly polarized photons. Consequently, linearly-polarized electromagnetic waves that propagate over cosmological distances undergo CB, a frequency-independent rotation of the plane of polarization by an angle α, where [1]

$$\alpha = \frac{\beta}{M}\int d\tau\left(\frac{\partial}{\partial\tau} - \hat{\mathbf{n}}\cdot\vec{\nabla}\right)\phi = \frac{\beta}{M}\Delta\phi, \tag{2}$$

where $\Delta\phi$ is the change in ϕ over the photon trajectory, and τ is the conformal time. For the CMB, the polarization rotation is determined by the change in ϕ since recombination, when the CMB polarization pattern was largely established.

Allowing for spatial fluctuations $\delta\phi$ in the cosmic scalar field, the anisotropy in the CB-rotation angle is then $\Delta\alpha(\hat{\mathbf{n}}) = (\beta/M)\delta\phi(\hat{\mathbf{n}})$, evaluated at recombination.

Below we consider two scenarios for the scalar field. In the first, the scalar is massless, with no homogeneous time evolution, while in the second, the scalar is quintessence. In both cases, CB-angle fluctuations arise from scalar-field fluctuations at the surface of last scatter (LSS).

A. Massless scalar field

In the first scenario, we suppose that the ϕ field is simply a massless scalar with a potential that vanishes, $V = 0$. In this case, the value of the field is completely uncorrelated with primordial density perturbations[1] [9]. If ϕ is effectively massless during inflation there will be a scale-invariant power spectrum of perturbations to ϕ, $P_{\delta\phi}(k) = H_I^2/2k^3$, with an amplitude fixed by the Hubble parameter H_I evaluated during inflation.[2] If we split the field into a smooth background component and a perturbation on top of it, the evolution of the homogeneous component is given by the following equation of motion

$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + a^2 V' = 0, \tag{3}$$

where $\mathcal{H} = \dot{a}/a$, a is the scale factor, and dots denote derivatives with respect to conformal time. For a vanishing potential, this equation has only a decaying and a constant solution; thus, the value of the field is fixed in time in each causally disconnected region of the early Universe. This precludes the scalar-field perturbations from having any correlation with perturbations in the matter/radiation density. This is manifest in the absence of any source term in the perturbed equation of motion for the scalar field [compare to the Fourier transform of the full equation, Eq. (A2), after taking $(d\phi/d\tau) = 0$, and $V = 0$],

$$\delta\ddot{\phi} + 2\mathcal{H}\delta\dot{\phi} - k^2\delta\phi = 0. \tag{4}$$

A solution to Eq. (4) is a transfer function $T_k(\tau) \propto j_1(k\tau)/(k\tau)$, which describes the conformal-time evolution of a given Fourier mode of wave number k during matter domination.

The angular power spectrum $C_L^{\alpha\alpha}$ for the rotation angle is then

$$\begin{aligned} C_L^{\alpha\alpha} &= 4\pi\left(\frac{\beta}{M}\right)^2 \int \frac{k^2 dk}{2\pi^2} P_\phi(k)[j_L(k\Delta\tau)T_k(\tau_{\rm lss})]^2 \\ &= \frac{1}{\pi}\left(\frac{\beta H_I}{M}\right)^2 \int \frac{dk}{k}[j_L(k\Delta\tau)T_k(\tau_{lss})]^2. \end{aligned} \tag{5}$$

Here $\Delta\tau$ is the conformal-time difference between last scattering and today, and $\tau_{\rm lss}$ is the conformal time at the LSS. For large angular scales, $L \lesssim 100$, the transfer function evaluates to $T_k(\tau_{\rm lss}) \simeq 1$, in which case

$$C_L^{\alpha\alpha} \simeq \frac{(\beta H_I/M)^2}{2\pi L(L+1)}, \quad \text{for } L \lesssim 100. \tag{6}$$

[1]We imagine that some mechanism has nullified the quantum-gravity effects that generically break global symmetries [26].

[2]It is also imaginable that a white-noise spectrum of ϕ fluctuations is imprinted by some post-inflation phase transition, but we will not consider that scenario here.

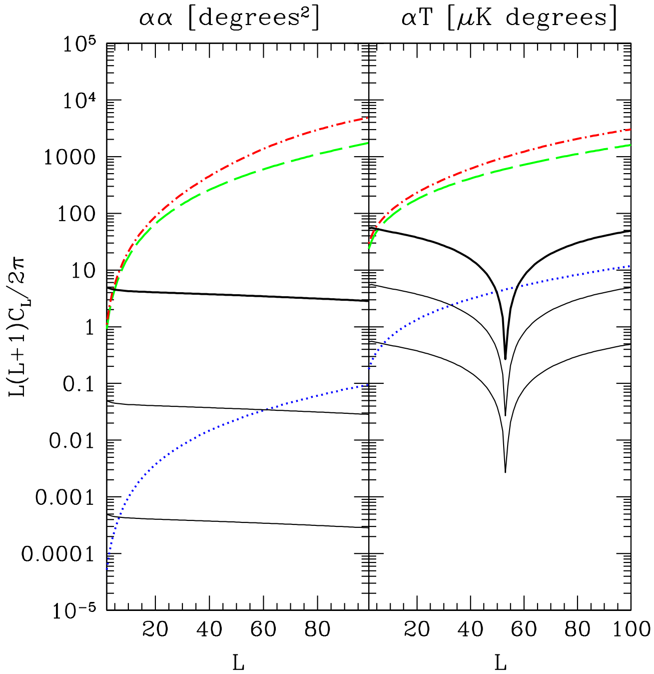

FIG. 1 (color online). Shown are the power spectra for the cosmological-birefringence rotation angle $C_L^{\alpha\alpha}$ and its cross-correlation with the CMB temperature $C_L^{\alpha T}$ (logarithm of the absolute value) for a quintessence model in which the CB-angle fluctuations are due to scalar-field fluctuations at the LSS. The black solid curves are the theoretical prediction for (from top to bottom) $\alpha_4 = 1$, 0.1, and 0.01, where α_4 is the fluctuation amplitude for the CB angle in units of the maximum currently allowed amplitude [12]. We also show the noise power spectra anticipated for SPIDER (red, dot-dashed line), Planck (green, dashed line), and CMBPol (blue, dotted line).

The left-hand panel of Fig. 1 shows the result of a numerical calculation of $C_L^{\alpha\alpha}$ for this scale-invariant power spectrum.

The mean-square rotation amplitude measured by a probe with angular resolution of $\sim 1°$ is

$$\langle(\Delta\alpha)^2\rangle = \sum_{L=2}^{\infty} \frac{2L+1}{4\pi} C_L^{\alpha\alpha}[W_L(\theta)]^2 \simeq 332\left(\frac{\beta H_I}{M}\right)^2 \mathrm{deg}^2. \quad (7)$$

Here, $W_L(\theta) \equiv \exp[-L^2\theta^2/(16\ln 2)]$ is a Gaussian window function of full-width half-maximum θ (in radians). The best current constraint to the variance of the rotation angle, $\langle(\Delta\alpha)^2\rangle \lesssim (4°)^2$, from AGN data [12], places a bound $\beta H_I/M \lesssim 0.2$ to the combination of parameters that control the rotation-angle amplitude in this scenario. We may therefore write the rotation-angle power spectrum as

$$C_L^{\alpha\alpha} = 0.015\alpha_4^2 \int \frac{dk}{k}[j_L(k\Delta\tau)T_k(\tau_{ls})]^2 \simeq \frac{7.7\times 10^{-3}\alpha_4^2}{L(L+1)}, \quad \text{for } L \lesssim 100. \quad (8)$$

Here, $\alpha_4 \equiv \langle(\Delta\alpha)^2\rangle^{1/2}/4°$ parametrizes the amplitude of the rotation-angle power spectrum in units of the maximum value currently allowed.

The CMB temperature power spectrum is given by

$$C_L^{TT} = \frac{2}{\pi}\int k^2 dk[\Delta_{T,L}(k)]^2 P_\Psi(k), \quad (9)$$

where $P_\Psi(k)$ is the primordial power spectrum for the gravitational potential, and $\Delta_{T,L}(k)$ is the transfer function that quantifies the contribution of a density mode of wave number k to C_L^{TT}, and may be obtained from numerical Boltzmann codes [27].

As discussed above, scalar-field fluctuations are not sourced by the gravitational potentials for this $V = 0$ model; similarly, energy-density fluctuations in the scalar field have only second-order corrections due to $\delta\phi$, and so their effect on gravitational potentials is also small. In this case, the αT cross-correlation power spectrum vanishes, $C_L^{\alpha T} = 0$.

B. Quintessence

In the second scenario, we suppose that ϕ is a quintessence field with a nonzero potential and homogeneous component that undergoes time evolution. In this case, gravitational-potential perturbations directly source (and are also sourced by) scalar-field fluctuations, see Eq. (A2). A cross-correlation between CB-angle and CMB-temperature fluctuations is therefore inevitable, although its amplitude and detailed features depend on the specific potential V.

Since every CMB photon that comes from a given direction $\hat{\mathbf{n}}$ last scattered at the spacetime point in the direction $\hat{\mathbf{n}}$, when the Universe had some fixed temperature, the CB-rotation angle $\alpha(\hat{\mathbf{n}})$ is determined by the value of ϕ at that point of spacetime. In other words, the CB-angle anisotropies are determined by the scalar-field perturbations on surfaces of constant CMB temperature, or equivalently, on surfaces of constant synchronous-gauge time.

We suppose that the initial value of ϕ is set by some post-inflationary physics so that the primordial perturbation to ϕ is adiabatic. In this case, the synchronous-gauge scalar-field perturbation $(\delta\phi)_{\mathrm{syn}}$ is initially zero. However, the scalar-field perturbation is sourced by the gravitational potentials, as described by Eqs. (A1) or (A2). The synchronous-gauge scalar-field perturbation at the LSS is then approximately (see Appendix A),

$$(\delta\phi)_{\mathrm{syn,lss}} = -\frac{2}{9}\left(\frac{3\Omega_\phi(1+w_\phi)}{8\pi}\right)^{1/2} M_{\mathrm{Pl}}\Psi, \quad (10)$$

where the equation-of-state parameter w_ϕ and the energy-density parameter Ω_ϕ are evaluated at recombination. The primordial power spectrum for the gravitational potential, for large scales (small k) is given by

$$P_\Psi = \frac{9}{25}\frac{2\pi^2}{k^3}\Delta_{\mathcal{R}}^2, \quad (11)$$

where we have taken a scalar spectral index to be $n_s = 1$ for simplicity, and the curvature-perturbation amplitude is

$\Delta^2_{\mathcal{R}}(k_0) = 2.43(\pm 0.11) \times 10^{-9}$ [28]. To evolve the power spectrum from primordial to the LSS, we need to multiply it by transfer functions, which are a suppression factor for small scales (large k's). The angular power spectrum for the CB-rotation angle in the quintessence model is then

$$C_L^{\alpha\alpha} = \frac{2}{27}\Omega_\phi(1+w_\phi)\left(\frac{\beta M_{\rm Pl}}{M}\right)^2 \int \frac{k^2 dk}{2\pi^2} P_\Psi(k) \times [j_L(k\Delta\tau)T_k(\tau_{\rm lss})]^2. \tag{12}$$

For large scales, $L \lesssim 100$, we can approximate $T_k(\tau_{\rm lss}) \simeq 1$, in which case we can again write the CB-rotation-angle power spectrum as in Eq. (8), but now with

$$\alpha_4 \simeq 6.7 \times 10^{-5}\sqrt{\Omega_\phi(1+w_\phi)}(\beta M_{\rm Pl}/M). \tag{13}$$

In other words, the $\alpha\alpha$ power spectrum for the quintessence scenario will be similar to that for the massless-scalar-field scenario in the small-L limit where $T_k(\tau_{\rm lss})$ can be approximated as a constant.

However, in the quintessence model, there will also be a cross-correlation with the CMB temperature, since the CMB temperature is determined largely by the potential Ψ at the LSS. From Eqs. (9) and (12), we get

$$C_L^{\alpha T} = -\frac{4\pi}{3}\sqrt{\frac{\Omega_\phi(1+w_\phi)}{6\pi}}\frac{\beta M_{\rm Pl}}{M} \times \int \frac{k^2 dk}{2\pi^2} P_\Psi(k)\Delta_{T,L}(k) j_L(k\Delta\tau)T_k(\tau_{\rm lss}). \tag{14}$$

The absolute value of this cross-correlation is also shown in Fig. 1. The passage through zero at $L \sim 50$ arises because of the relative contributions of the monopole and dipole contributions of the photon distribution function to $\Delta_{T,L}(k)$.

III. PROSPECTS FOR DETECTION

In this Section, we first review the procedure presented in Refs. [20–22] for constructing the minimum-variance estimator for the CB-rotation angle from a CMB temperature/polarization map. We then extend this work to show how the cross-correlation with the temperature can be reconstructed. At the end, we evaluate the detectability of the CB rotation with both the autocorrelation and the cross-correlation for the CB scenarios discussed in Sec. II.

A. Measuring the rotation angle

References [20,21] show how the CB-rotation-angle spherical-harmonic coefficients α_{LM} can be reconstructed from a full-sky CMB temperature/polarization map. While these coefficients can be obtained from EE, TE, TB, and EB cross-correlations, the best sensitivity will ultimately come from the EB cross-correlation. We therefore restrict our attention to reconstruction of $\alpha(\hat{\mathbf{n}})$ from the EB power spectra.

To begin, the E/B spherical-harmonic coefficients, $E_{lm}^{\rm map}$ and $B_{lm}^{\rm map}$, are constructed from the full-sky map of the Stokes parameters, $Q(\hat{\mathbf{n}})$ and $U(\hat{\mathbf{n}})$, in the usual way [29,30]. Following Refs. [20,21], the minimum-variance estimator for the rotation-angle spherical-harmonic coefficient is

$$\hat{\alpha}_{LM} = C_L^{\alpha\alpha,\rm noise} \sum_{mm',l'\geq l} \xi^{LM}_{lml'm'}[V^L_{ll'}E^{\rm map}_{l'm'}B^{\rm map}_{lm} + V^L_{l'l}E^{\rm map}_{lm}B^{\rm map}_{l'm'}], \tag{15}$$

where

$$V^L_{ll'} \equiv \frac{F^{L,BE}_{ll'}}{(1+\delta_{ll'})C_l^{BB,\rm map}C_{l'}^{EE,\rm map}}, \tag{16}$$

$$F^{L,\rm BE}_{ll'} \equiv 2C^{\rm EE}_{l'}\begin{pmatrix} l & L & l' \\ 2 & 0 & -2\end{pmatrix} W_l W_{l'}, \quad F^{L,\rm EB}_{ll'} \equiv F^{L,\rm BE}_{l'l}, \tag{17}$$

and

$$\xi^{LM}_{lml'm'} \equiv (-1)^m\sqrt{\frac{(2l+1)(2l'+1)(2L+1)}{4\pi}}\begin{pmatrix} l & L & l' \\ -m & M & m'\end{pmatrix}. \tag{18}$$

Here, the objects in parentheses are Wigner-3j symbols, and W_l is the window function defined in Sec. II. $C_l^{EE,\rm map}$ and $C_l^{BB,\rm map}$ are, respectively, power spectra for the E and B modes from the map (including instrumental noise); i.e.,

$$C_l^{XX',\rm map} \equiv C_l^{XX'}|W_l|^2 + C_l^{XX',\rm noise}, \tag{19}$$

where $XX' \in \{TT, EE, BB, ET, EB, TB\}$. The noise power spectra are

$$\begin{aligned} C_l^{TT,\rm noise} &\equiv \frac{4\pi f^0_{\rm sky}({\rm NET})^2}{t_{\rm obs}}, \\ C_l^{EE,\rm noise} &= C_l^{BB,\rm noise} \equiv 2C_l^{TT,\rm noise}, \\ C_l^{EB,\rm noise} &= C_l^{TB,\rm noise} \equiv 0, \end{aligned} \tag{20}$$

where $t_{\rm obs}$ is the total observation time, $f^0_{\rm sky}$ is the fraction of the sky surveyed (taken to be different from 1 only for SPIDER, where $f^0_{\rm sky} = 0.5$), and NET is the noise-equivalent temperature. We assume no cross-correlation between the noises in polarization and temperature and apply the null assumption (no B modes in the signal), so there are no TB and EB correlations. The power spectrum $C_l^{BB,\rm map}$ thus contains only the contribution from instrumental noise.

We note here that weak gravitational lensing induces a contribution to the B mode. However, the power spectrum for this B-mode contribution is smaller than that of the noise, even for CMBPol [31], and so our sensitivity estimates should be unaffected by neglecting it. While weak gravitational lensing also induces off diagonal EB correlations, the EB correlations from weak lensing can be distinguished geometrically from those due to CB, see Appendix B in Ref. [21]. Moreover, weak lensing affects the temperature map, while CB does not; this provides an additional avenue to distinguish their relative contributions.

Under the null hypothesis of no rotation, the expectation value of the estimator in Eq. (15) is zero, and its variance is the $\alpha\alpha$ noise power spectrum as given in Ref. [21],

$$C_L^{\alpha\alpha,\mathrm{noise}} \equiv \langle|\hat{\alpha}_{LM}|^2\rangle = \left[\sum_{ll'} \frac{(2l+1)(2l'+1)(F_{ll'}^{L,BE})^2}{4\pi C_l^{BB,\mathrm{map}} C_{l'}^{EE,\mathrm{map}}}\right]^{-1}. \tag{21}$$

If the polarization pattern at the LSS is a realization of a statistically isotropic field, then there are $2L+1$ statistically independent M modes for each L in $\hat{\alpha}_{LM}$. In this case, each M mode provides an independent estimator of the rotation power spectrum, $C_L^{\alpha\alpha}$. The minimum-variance estimator is then

$$\hat{C}_L^{\alpha\alpha} = \frac{1}{2L+1}\sum_{M=-L}^{L} |\hat{\alpha}_{LM}|^2. \tag{22}$$

Each $\hat{\alpha}_{LM}$ is a sum of products of Gaussian random variables and is thus not a Gaussian random variable. However, if the number of terms in the sum is large, the central-limit theorem holds and $\hat{\alpha}_{LM}$ can be approximated as Gaussian. In this case, the expression for the variance of $\hat{C}_L^{\alpha\alpha}$ takes on the usual form,

$$(\Delta\hat{C}_L^{\alpha\alpha})^2 \simeq \frac{2}{f_{\mathrm{sky}}(2L+1)}(C_L^{\alpha\alpha,\mathrm{noise}})^2, \tag{23}$$

where f_{sky} is the sky cut used in the analysis, taken to be 0.8 for Planck and CMBPol and 0.5 for SPIDER.

B. Measurement of the rotation-temperature cross-correlation

In analogy with the derivation in Ref. [29] of the estimator for C_l^{TE}, the estimator for $C_L^{\alpha T}$ is

$$\hat{C}_L^{\alpha T} = \frac{1}{2L+1}\sum_{M=-L}^{L} \hat{\alpha}_{LM}(T_{LM}^{\mathrm{map}})^* W_L^{-1}, \tag{24}$$

where T_{LM}^{map} is the temperature spherical-harmonic coefficient obtained from the map. Under the null hypothesis, $\hat{T}_{LM}$ has no correlation with any B_{LM}'s, and it is correlated with E_{LM} with the same L and M but uncorrelated with any other E_{LM}. The estimator $\hat{\alpha}_{LM}$ depends on a large number of E_{lm}'s but does not include $\{lm\} = \{LM\}$. There is therefore no correlation (under the null hypothesis) of $\hat{\alpha}_{LM}$ and $\hat{T}_{LM}$; i.e., there is no noise contribution to $C_L^{\alpha T}$. Again, if $\hat{\alpha}_{LM}$ is approximately Gaussian, then the variance with which $C_L^{\alpha T}$ can be measured is approximately that obtained assuming $\hat{\alpha}_{LM}$ is Gaussian. To check the validity of this assumption for the purpose of calculating the sensitivity of future CMB experiments to the CB signal (see Sec. III C), we evaluate the full expression for this variance [without assuming Gaussianity of $\hat{\alpha}_{LM}$, see Eq. (B1)] and confirm that the numerical results agree up to a level of a few percent. Thus, for simplicity and without any loss in accuracy, we can invoke analogy with the variance of C_l^{TE} (see, e.g., Ref. [29]) to get

$$(\Delta\hat{C}_L^{\alpha T})^2 \simeq \frac{1}{f_{\mathrm{sky}}(2L+1)} C_L^{\alpha\alpha,\mathrm{noise}} C_L^{TT,\mathrm{map}} W_L^{-2}. \tag{25}$$

C. Sensitivity to detection: αT vs $\alpha\alpha$

We now return to our two models for CB which predict that the rotation α is a realization of a random field with the power spectra $C_L^{\alpha\alpha}$ and $C_L^{\alpha T}$ presented in Fig. 1. Our aim here is to evaluate the smallest signal amplitude detectable by measurement of the rotation alone, as well as the smallest amplitude detectable by measurement of the rotation-temperature cross-correlation.

We write the power spectra as $C_L^{\alpha\alpha} \equiv \alpha_4^2 C_L^{\alpha\alpha,\mathrm{fiducial}}$ and $C_L^{\alpha T} \equiv \alpha_4 C_L^{\alpha T,\mathrm{fiducial}}$, where the fiducial model ($\alpha_4 = 1$) is the quintessence model in Fig. 1 with the largest amplitude allowed by current rotation-angle constraints. The inverse variance with which the amplitude α_4^2 of the $\alpha\alpha$ power spectrum can be obtained from the rotation-angle autocorrelation is [32]

$$\frac{1}{[\Delta(\alpha_4^2)]^2} = \sum_L \left(\frac{\partial C_L^{\alpha\alpha}}{\partial(\alpha_4^2)}\right)^2 \frac{1}{(\Delta\hat{C}_L^{\alpha\alpha})^2} = \sum_L \left(\frac{C_L^{\alpha\alpha,\mathrm{fiducial}}}{\Delta\hat{C}_L^{\alpha\alpha}}\right)^2. \tag{26}$$

Similarly, the inverse-variance with which the amplitude α_4 of the αT power spectrum can be obtained from the cross-correlation of the rotation with the temperature is

$$\frac{1}{(\Delta\alpha_4)^2} = \sum_L \left(\frac{\partial C_L^{\alpha T}}{\partial\alpha_4}\right)^2 \frac{1}{(\Delta\hat{C}_L^{\alpha T})^2} = \sum_L \left(\frac{C^{\alpha T,\mathrm{fiducial}}}{\Delta\hat{C}_L^{\alpha T}}\right)^2. \tag{27}$$

From these relations, we can estimate the signal-to-noise ratio for measurement of α_4^2 from the $\alpha\alpha$ autocorrelation to be $(S/N)_{\alpha\alpha} = \alpha_4^2/[\Delta(\alpha_4^2)]$ and a signal-to-noise for measurement of α_4 from the αT cross-correlation to be $(S/N)_{\alpha\mathrm{T}} = \alpha_4/(\Delta\alpha_4)$. We evaluate these expressions for our fiducial model ($\alpha_4 = 1$), for different instrumental parameters in Sec. III D. The smallest α_4 detectable at the 2σ level from the cross-correlation and autocorrelation are then $2\Delta\alpha_4$ and $[2\Delta(\alpha_4^2)]^{1/2}$, respectively.

D. Numerical results

We now present numerical results for the $\alpha\alpha$ and αT noise power spectra and evaluate the largest possible signal-to-noise and the smallest detectable amplitude α_4 for three CMB polarization experiments: (i) SPIDER's 150 GHz channel [23], (ii) Planck's 143 GHz channel [24], and (iii) CMBPol's (EPIC-2m) 150 GHz channel [25]. We obtain the CMB temperature-polarization power

TABLE I. Instrumental parameters from Refs. [23–25] for the three experiments considered in this work: beamwidth θ (in arcminutes), noise-equivalent temperature (in $\mu\mathrm{K}\,\mathrm{sec}^{1/2}$), and observation time t_{obs} (in years). The last two columns list signal-to-noise ratios (S/N) for the CB-angle autocorrelation and its cross-correlation with the CMB temperature for our fiducial quintessence model ($\alpha_4 = 1$) shown in Fig. 1. Note that the signal-to-noise scales with the signal amplitude α_4 as $(S/N)_{\alpha\alpha} \propto \alpha_4^2$ and $(S/N)_{\alpha\mathrm{T}} \propto \alpha_4$.

Instrument	θ	NET	t_{obs}	$(S/N)_{\alpha\alpha}$	$(S/N)_{\alpha T}$
SPIDER	60	3.1	0.016	9	7
Planck	7.1	62	1.2	11	9
CMBPol/EPIC	5	2.8	4	2×10^5	1200

spectra from CMBFAST [27] using WMAP-7 cosmological parameters [15]. The instrumental parameters we use are listed in Table I. Figure 1 shows the noise power spectra[3] $C_L^{\alpha\alpha,\mathrm{noise}}$ and $C_L^{\alpha T,\mathrm{noise}}$. For $C_L^{\alpha T}$, strictly speaking, there is no instrumental-noise contribution, only the effective noise, arising from cosmic variance. Table I lists signal-to-noise ratios, assuming $\alpha_4 = 1$, for the auto- and cross-correlations, for these three experiments. We find that SPIDER and Planck may already have the sensitivity to detect not only the signal but also its cross-correlation with the temperature in the best-case scenario of $\alpha_4 \simeq 1$, where the signal is just below the current detection limit.[4] In both cases, the sensitivity to the signal may be improved if the auto- and cross-correlations are measured in tandem. CMBPol should have sensitivity to a signal as small as $\alpha_4 \sim 10^{-5}$, and a detection of the cross-correlation of very high signal-to-noise may be obtained with CMBPol if $\alpha_4 \simeq 1$.

IV. SUMMARY AND DISCUSSION

If a quintessence field gives rise to cosmological birefringence, then a correlation between CB-rotation-angle fluctuations and CMB-temperature fluctuations is inevitable. We calculated that cross-correlation assuming the initial quintessence perturbations are adiabatic. We also discussed, by way of contrast, a scenario in which the CB-inducing field is just a massless scalar field that has no correlation with primordial perturbations.

We derived the minimum-variance estimator for the αT power spectrum that can be obtained from a CMB temperature-polarization map. We find that measurement of this cross-correlation may improve sensitivity to the CB signal in some cases where the signal would otherwise be only marginally detectable. We further show that a high signal-to-noise measurement of this cross-correlation is conceivable with forthcoming and future CMB experiments if the rotation-angle power-spectrum amplitude is near its current upper limit. Measurement of this cross-correlation may thus provide another empirical handle with which to discover new physics indicated by cosmological birefringence.

We have restricted our attention to the EB estimator for the rotation angle, as it is expected to provide the best sensitivity. However, there may be some improvement, though probably small, with the inclusion of the TE, TB, and EE estimators for the rotation. We leave this calculation for future work. Likewise, we have left more careful investigation of the impacts of partial-sky analysis, foregrounds, uneven noise, and the effect of CB on cosmological parameter extraction [33] for future work.

We have refrained from discussing details of the quintessence model here, as the angular dependence of the CB power spectra at superhorizon scales at the time of recombination, $L \lesssim 100$, is insensitive to these details. The dependence of the amplitudes of the $\alpha\alpha$ and αT power spectra is given in terms of the quintessence parameters Ω_ϕ and w at the LSS by Eqs. (12) and (14). However, if the quintessence field couples to the pseudoscalar of electromagnetism, it is natural to expect it to be a pseudo-Nambu-Goldstone field, and if so, then its potential should be $V(\phi) \propto [1 - \cos(\phi/f)]$. In this case, the quintessence field ϕ is frozen at early times leading to spatial variations in α that are unobservably small. In this case, though, additional fluctuations in α may be produced during the epoch of reionization [34].

For the massless scalar field, the uniform CB-rotation angle is expected to be zero, and so a search for the fluctuations is essential to detect the signal. For quintessence, however, the uniform rotation is expected to be nonzero and generically quite a bit larger than the fluctuations, which, given the current best constraint, may imply a relatively small amplitude of the fluctuations power spectrum. However, CMBPol may be sensitive to a fluctuation amplitude as small as $\sim 10^{-5}$ of the current upper limit to the uniform rotation, which, if detected, would help distinguish between different CB scenarios. Moreover, the fluctuation amplitude in the quintessence scenario could be larger than a measured uniform-rotation angle. This could occur if, for example, the uniform-rotation angle (which can only be recovered modulo π) happens to be close to an integer multiple of π. It will be interesting, with forthcoming precise CMB maps, to address these questions empirically rather than through theoretical speculation.

[3] Note that there is a difference in normalization between the *noise* and the *variance*: $C_L^{XX',\mathrm{noise}} \equiv \sqrt{(2L+1)/2}\Delta\hat{C}_L^{XX'}$, where $XX' = \{\alpha\alpha, \alpha T\}$. It is customary to plot the noise power spectra, even though the variance enters the expressions for signal-to-noise.

[4] Here we have assumed that the errors to the rotation-angle estimators are approximately Gaussian. However, if the signal is just detectable (for example, for experiments like SPIDER if $\alpha_4 \simeq 1$), then this assumption may break down, and if so, the precise quantitative forecasts for the signal-to-noise may differ slightly.

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APPENDIX A: QUINTESSENCE PERTURBATIONS

As discussed in the paper, the CB-angle fluctuation is determined by the synchronous-gauge scalar-field fluctuation $(\delta\phi)_{\rm syn}$ at the LSS. To obtain this fluctuation, we start from adiabatic initial conditions and then evolve the scalar-field-perturbation equation of motion forward in time, from the early radiation-dominated epoch to the LSS. The equation of motion is

$$\delta\ddot{\phi} + 2\mathcal{H}\delta\dot{\phi} + a^2V''\delta\phi - \nabla^2\delta\phi = -\frac{1}{2}\dot{h}\,\dot{\phi}, \quad \text{(A1)}$$

in the synchronous gauge, and

$$\delta\ddot{\phi} + 2\mathcal{H}\delta\dot{\phi} + a^2V''\delta\phi - \nabla^2\delta\phi = \dot{\phi}(3\dot{\Phi} + \dot{\Psi}) - 2a^2V'\Psi, \quad \text{(A2)}$$

in the conformal-Newtonian/longitudinal gauge. (See Ref. [35] for definitions of the metric variables Φ, Ψ, η, and h.) Adiabatic initial conditions require that the perturbations of the scalar field vanish at early times. However, the subsequent evolution of the scalar field is not adiabatic, meaning that $(\delta\phi)_{\rm syn,lss}$ does not necessarily vanish at the LSS even though all the matter and radiation perturbations do. All of our numerical integrations that give the power spectra presented in the Figure are done in the synchronous gauge, using a modified version of CMBFast [27].

For the numerical work, we assume a quintessence potential of the PNGB form $V(\phi) = m^4(1 - \cos\phi/f)$, as expected if ϕ is an axion-like field. We take an initial value ϕ, m, and f so that $\Omega_\phi = 0.7$ today and the density-weighted average equation-of-state parameter $\langle w\rangle \simeq -0.95$, which gives $\Delta\phi = 0.045M_{\rm Pl}$ for the change in the scalar field between decoupling and today. However, the numerical results presented in Fig. 1 will be similar for any quintessence potential that has $w_\phi \to -1$ at early times.

The numerical results can be largely reproduced with the analytic approximation for $(\delta\phi)_{\rm syn,lss}$, given in Eq. (10), which we now derive. We now work in the conformal-Newtonian/longitudinal gauge, and we make the approximation that decoupling takes place well into matter domination; we assume that most of the growth in perturbations happens during this epoch. For $w_\phi \to -1$, the V'' term in Eq. (A2) is negligible. Additionally, in the superhorizon limit, valid for multipoles $L \lesssim 100$, we can neglect the spatial-gradient term. The simplified equation of motion is then

$$\delta\ddot{\phi} + 2\mathcal{H}\delta\dot{\phi} \simeq -2a^2V'\Psi. \quad \text{(A3)}$$

Aside from the homogeneous solutions that are either constant or decaying, it also has an inhomogeneous solution that grows as

$$(\delta\phi)_{\rm con} \simeq -a^2\tau^2V'\Psi/27 \quad \text{(A4)}$$

during matter domination. The potential derivative V' can be expressed, using the quintessence slow-roll approximation, from

$$a^2V' \simeq -3\mathcal{H}\dot{\phi}. \quad \text{(A5)}$$

Also,

$$\dot{\phi}^2 = a^2\rho_\phi(1 + w_\phi), \quad \text{(A6)}$$

with $\rho_\phi = \Omega_\phi\rho_c$ and $\rho_c = 3H^2M_{\rm Pl}^2/(8\pi)$. We then find

$$(\delta\phi)_{\rm con} = \frac{4}{9}\left[\frac{3}{8\pi}\Omega_\phi(1 + w_\phi)\right]^{1/2}M_{\rm Pl}\Psi. \quad \text{(A7)}$$

The result in Eq. (10) is then obtained by going back to the synchronous gauge, using the gauge-transformation equations [35]

$$(\delta\phi)_{\rm syn} = (\delta\phi)_{\rm con} - \alpha\dot{\phi} \quad \text{(A8)}$$

$$(\delta\dot{\phi})_{\rm syn} = (\delta\dot{\phi})_{\rm con} - \alpha\ddot{\phi}, \quad \text{(A9)}$$

after noting that $\alpha \simeq (2/3)\Psi/\mathcal{H}$ during matter domination.

We derive the initial conditions in the conformal-Newtonian/longitudinal gauge, for the sake of completeness, which can then be used to evolve Eq. (A2). To obtain the initial conditions in this gauge, we use Eq. (A9), where $\alpha = (1/2)\Psi/\mathcal{H}$ during radiation domination. At early times, deep in the radiation era, we can set the fractional energy-density perturbation in the radiation field to $\delta_r = -2\Psi$ [35] and assume that the equation-of-state parameter $w_\phi \to -1$ and changes slowly with time. Furthermore, the pressure and energy density of the scalar are given as

$$p_\phi = \frac{1}{2a^2}\dot{\phi}^2 - V(\phi), \quad \rho_\phi = \frac{1}{2a^2}\dot{\phi}^2 + V(\phi),$$
$$p_\phi + \rho_\phi = \frac{1}{a^2}\dot{\phi}^2, \quad \text{(A10)}$$

and the perturbation in the energy density is

$$(\delta\rho_\phi)_{\rm con} = \frac{1}{a^2}\dot{\phi}\,\dot{\delta}\,\phi + V'(\phi)\delta\phi - \frac{1}{a^2}\dot{\phi}^2\Phi, \quad \text{(A11)}$$

while the adiabatic initial conditions require that the entropy density perturbation vanishes at early times, so that

$$S \equiv \frac{\delta\rho_\phi}{\rho_\phi + p_\phi} - \frac{\delta\rho_r}{\rho_r + p_r}. \quad \text{(A12)}$$

Combining these assumptions into the gauge-transformation equations, along with the observation that

$\delta\phi$ vanishes in the synchronous gauge, we get the initial conditions for the scalar-field perturbations in the conformal-Newtonian/longitudinal gauge,

$$(\delta\phi)_{\rm con} = \frac{1}{2}\frac{\dot{\phi}}{\mathcal{H}}\Psi, \tag{A13}$$

$$(\delta\dot{\phi})_{\rm con} = \dot{\phi}\Phi - \frac{3}{2}\dot{\phi}\Psi - \frac{1}{2}\frac{a^2V'}{\mathcal{H}}\Psi. \tag{A14}$$

These initial conditions can also be derived by requiring that S and $\dot{S}$ vanish at early times.

APPENDIX B: FULL EXPRESSION FOR THE VARIANCE OF αT CROSS-CORRELATION

If we do not assume $\hat{\alpha}_{LM}$ is a Gaussian, then the full expression for the variance of its cross-correlation with the CMB temperature becomes a 6-point correlation function. After applying Wick's theorem and taking into account the properties of the Wigner 3j symbols to simplify the terms, the full expression becomes

$$\begin{aligned}(\Delta\hat{C}_L^{\alpha T})^2 = \frac{(C_L^{\alpha\alpha,\rm noise})^2 C_L^{BB,\rm noise}}{4\pi W_L^2}\Bigg[& 2(V_{LL}^L)^2(C_L^{TE,\rm map})^2 \\ & + \sum_l \bigg[2\frac{(2l+1)^2}{(2L+1)^2}(V_{ll}^L)^2 C_l^{EE,\rm map} C_L^{TT,\rm map} \\ & + (1+\delta_{lL})(V_{lL}^L)^2 (C_L^{TE,\rm map})^2 \bigg] \\ & + \sum_{ll'}(1+\delta_{ll'})\frac{(2l+1)(2l'+1)}{(2L+1)} \\ & \times (V_{ll'}^L)^2 C_L^{TT,\rm map} C_{l'}^{EE,\rm map}\Bigg]. \end{aligned} \tag{B1}$$

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