Gluon polarization in the nucleon demystified

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Recently, X. Chen *et al.* proposed a new approach to the gauge invariant decomposition of the nucleon spin into the helicity and orbital angular momentum of quarks and gluons. The key ingredient in their construction is the separation of the gauge field into physical and "pure gauge" parts. We suggest a simple separation scheme and show that the resulting gluon helicity coincides with the first moment of the conventional polarized gluon distribution measurable in high energy experiments.

Despite its intuitive clarity, the decomposition of the nucleon spin into the helicity and orbital angular momentum of quarks and gluons has remained one of the most elusive problems in QCD spin physics [1]. The current unsatisfactory situation may be epitomized by the following dilemma: On one hand, continuous efforts have been made both at experimental facilities and by the theorists to assess the gluon helicity contribution ΔG [2] defined as the first moment of the polarized gluon distribution. On the other hand, since the seminal work of Ji [3], it has been widely recognized by the community that the gluonic angular momentum cannot be decomposed into helicity and orbital parts in a gauge invariant way. This implies that ΔG extracted from measurements and global QCD analyses has unfortunately no natural counterpart in Ji's framework, making one wonder what exactly the physical meaning of ΔG is. The case of the gluon orbital angular momentum is even murkier since there is no known way of directly measuring it, nor is its operator representation available.

Recently, however, the situation took an interesting turn when Chen *et al.* proposed a new, complete decomposition of the nucleon spin [4,5]. The key ingredient in their construction is the separation of the total gauge field into "physical" and "pure gauge" parts

$$A^{\mu} = A^{\mu}_{\text{phys}} + A^{\mu}_{\text{pure}},$$
$$F^{\mu\nu}_{\text{pure}} = \partial^{\mu}A^{\nu}_{\text{pure}} - \partial^{\nu}A^{\mu}_{\text{pure}} + ig[A^{\mu}_{\text{pure}}, A^{\nu}_{\text{pure}}] = 0, \tag{1}$$

which transform differently under gauge transformations

$$A^{\mu}_{\text{phys}} \rightarrow U^{\dagger}A^{\mu}_{\text{phys}}U, \qquad A^{\mu}_{\text{pure}} \rightarrow U^{\dagger}A^{\mu}_{\text{pure}}U - \frac{i}{g}U^{\dagger}\partial^{\mu}U. \tag{2}$$

The separation (1) is not unique, and accordingly, the gluon helicity contribution is scheme dependent. Chen *et al.* imposed a subsidiary condition that can be used to construct A^{μ}_{phys} perturbatively, and found that the corresponding gluon helicity indeed differs from ΔG [6]. [See, also, [7].] While their scheme has some attractive physical features, how to test their predictions in experiment is currently unknown. The experimental observability is a

prerequisite for a good definition of the gluon helicity. One then asks the question: "Is there a separation scheme (1) in which the gluon helicity coincides with ΔG?" In this paper we answer this question positively in the hope that the conflicting opinions about the nature of the gluon helicity are reconciled with each other.

The original proposal by Chen *et al.* [4,5] achieves a complete decomposition of the QCD angular momentum operator $\vec{J}$ into quarks' and gluons' helicity and orbital angular momentum. This was further elaborated by Wakamatsu [8,9], where the covariant generalization of the decomposition was derived. The result for the QCD angular momentum tensor $M^{\mu\nu\lambda}$ is [9]

$$M^{\mu\nu\lambda}_{\text{quark-spin}} = \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\bar{\psi}\gamma_5\gamma_{\sigma}\psi, \tag{3}$$

$$M^{\mu\nu\lambda}_{\text{quark-orbit}} = \bar{\psi}\gamma^{\mu}(x^{\nu}iD^{\lambda} - x^{\lambda}iD^{\nu})\psi, \tag{4}$$

$$M^{\mu\nu\lambda}_{\text{gluon-spin}} = F^{\mu\lambda}_a A^{\nu a}_{\text{phys}} - F^{\mu\nu}_a A^{\lambda a}_{\text{phys}}, \tag{5}$$

$$M^{\mu\nu\lambda}_{\text{gluon-orbit}} = F^{\mu\alpha}_a(x^{\nu}(D^{\lambda}_{\text{pure}}A^{\text{phys}}_{\alpha})_a - x^{\lambda}(D^{\nu}_{\text{pure}}A^{\text{phys}}_{\alpha})_a)$$
$$+ (D_{\alpha}F^{\alpha\mu})_a(x^{\nu}A^{\lambda a}_{\text{phys}} - x^{\lambda}A^{\nu a}_{\text{phys}}), \tag{6}$$

where $D^{\mu}_{\text{pure}} \equiv \partial^{\mu} + igA^{\mu}_{\text{pure}}$ and $a, b = 1, 2, \cdots, 8$ are the color indices. We use the convention $\epsilon_{0123} = +1$. The second term on the right hand side of (6) is gauge–invariant on its own, and Chen *et al.* included it in the quark–orbital part. (This would result in the change $D^{\nu} \rightarrow D^{\nu}_{\text{pure}}$ in (4).) Following Wakamatsu [8], we have relocated it to the gluon–orbital part. With this modification the quark part agrees with Ji's decomposition and can be extracted from GPD analyses [3]. The decomposition of the gluon spin into the helicity (5) and orbital (6) parts has been made possible at the cost of introducing nonlocality: In general, A^{μ}_{phys} and A^{μ}_{pure} are nonlocally related to the total A^{μ}.

Let us focus on the gluon helicity operator (5). We go to the infinite momentum frame and use the light–cone coordinates $x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$. In the framework of Chen *et al.*, the gluon helicity contribution is given by the nucleon

matrix element of the $\mu\nu\lambda = +12$ tensor component of (5)

$$\frac{-1}{2P^+}\langle PS|F_a^{+\mu}(0)\epsilon_{\mu\nu}^{+-}A_{\text{phys}}^{\nu a}(0)|PS\rangle. \tag{7}$$

On the other hand, the conventional and experimentally accessible gluon helicity is given by the first moment of the polarized gluon distribution (see, e.g., [10,11])

$$\Delta G = \int_0^1 dx_B \Delta g(x_B)$$
$$= \frac{1}{4P^+}\int_{-\infty}^{\infty} dy^- \epsilon(y^-)\langle PS|F_a^{+\mu}(0)$$
$$\times \mathcal{P}\exp\left(-ig\int_{y^-}^0 A^+(y'^-)dy'^-\right)_{ab}$$
$$\times \epsilon^{+-}{}_{\mu\nu}F_b^{+\nu}(y^-)|PS\rangle, \tag{8}$$

where x_B is the usual Bjorken variable and the Wilson line is in the adjoint representation. ($\mathcal{P}$ denotes path–ordering.)

If we insist that the two definitions (7) and (8) are equivalent, we must have that, using the notation $x^\mu = (x^-, \vec{x})$ with $\vec{x} = (x^+, x^1, x^2)$,

$$A_{\text{phys}}^{\mu a}(x) \overset{??}{=} -\frac{1}{2}\int_{-\infty}^{\infty} dy^- \epsilon(y^- - x^-)$$
$$\times \mathcal{P}\exp\left(-ig\int_{y^-}^{x^-} A^+(y'^-,\vec{x})dy'^-\right)_{ab} F_b^{+\mu}(y^-,\vec{x}). \tag{9}$$

Does this identification make sense? The right hand side obeys the gauge transformation law (2) as expected for A_{phys}^μ, but it is far from obvious that the difference $A_{\text{pure}}^\mu = A^\mu - A_{\text{phys}}^\mu$ is pure gauge. Remarkably, however, there exists a special, but very simple scheme of separation (1) in which (9) becomes an identity rather than a definition [12].

In order to find such a scheme, we first observe that (9) immediately implies that

$$A_{\text{phys}}^+ = 0. \tag{10}$$

This motivates us to write, denoting fields as matrices in the adjoint representation,

$$A^+ = A_{\text{pure}}^+ = -\frac{i}{g}VW\partial^+(VW)^\dagger = \frac{i}{g}\partial^+(VW)(VW)^\dagger, \tag{11}$$

where

$$V(x) = \mathcal{P}\exp\left(-ig\int_{\pm\infty}^{x^-} A^+(x'^-,\vec{x})dx'^-\right),$$
$$W(\vec{x}) = \mathcal{P}\exp\left(-ig\int_{\infty\vec{n}}^{\vec{x}} \vec{A}(\pm\infty,\vec{x}')\cdot d\vec{x}'\right). \tag{12}$$

Note that W is evaluated at $x^- = \pm\infty$ where the plus (minus) sign corresponds to the choice $x^- = +\infty$

($x^- = -\infty$) in the lower limit of the integration in V. The path to spatial infinity (denoted as '$\vec{x} = \infty\vec{n}$' with $\vec{n}$ being a constant vector) is arbitrary assuming that the field strength vanishes at $x^- = \pm\infty$.

Promoting (11) to a four–dimensional relation, we define

$$A_{\text{pure}}^\mu \equiv -\frac{i}{g}VW\partial^\mu(VW)^\dagger, \tag{13}$$

which guarantees that $F_{\text{pure}}^{\mu\nu} = 0$, and

$$A_{\text{phys}}^\mu \equiv A^\mu - A_{\text{pure}}^\mu. \tag{14}$$

In order for A_{pure}^μ to transform according to (2) under gauge transformation, we require that

$$\lim_{\substack{x^- = \pm\infty \\ \vec{x}\to\infty\vec{n}}} \partial_\mu U(x^-, \vec{x}) = 0, \tag{15}$$

that is, we allow only for global gauge rotations as $(x^-, \vec{x}) \to (\pm\infty, \infty\vec{n})$. A_{pure}^μ should vanish (by a gauge choice) in the same limit and this is already implied by (13). Except for this minor qualification, our separation scheme is independent of the gauge choice.

Still, it will be very convenient in the following to consider the light–cone gauge which has a special status in our scheme and which can be accessed by setting $U = VW$ (consistently with (15)). Denoting fields in the light–cone gauge with a tilde, we find

$$\tilde{A}_{\text{phys}}^\mu = (VW)^\dagger A_{\text{phys}}^\mu VW, \qquad \tilde{A}_{\text{pure}}^\mu = 0, \tag{16}$$

so that $\tilde{A}^\mu = \tilde{A}_{\text{phys}}^\mu$ in this gauge. The residual (x^-–independent) gauge symmetry in the light–cone gauge is essentially contained in $W(\vec{x})$. It may seem more natural to let $\tilde{A}_{\text{pure}}^\mu$ carry these degrees of freedom. However, we have absorbed them in $\tilde{A}_{\text{phys}}^\mu$ for our purpose. The point is that, by using these degrees of freedom, one can fix the boundary condition for $\tilde{A}^\mu = \tilde{A}_{\text{phys}}^\mu$ as $x^- \to \pm\infty$.

We are now ready to prove (9). The last factor can be written as, suppressing color indices,

$$\mathcal{P}\exp\left(-ig\int_{y^-}^{x^-} A^+(y'^-,\vec{x})dy'^-\right)F^{+\mu}(y^-,\vec{x})$$
$$= VW(x)(VW)^\dagger(y^-,\vec{x})F^{+\mu}(y^-,\vec{x})$$
$$= VW(x)\tilde{F}_{\text{phys}}^{+\mu}(y^-,\vec{x})$$
$$= VW(x)\frac{\partial}{\partial y^-}\tilde{A}_{\text{phys}}^\mu(y^-,\vec{x}), \tag{17}$$

where in the second equality we have used the fact that $\tilde{F}^{\mu\nu} = \tilde{F}_{\text{phys}}^{\mu\nu}$ in the light–cone gauge. [Remember that matrices are in the adjoint representation so that, for instance, the following identity holds: $(VW)_{ab}^\dagger F_b^{+\mu} = [(VW)^\dagger F^{+\mu}(VW)]_a = \tilde{F}_a^{+\mu}$.] The right hand side of (9) then becomes

$$-\frac{1}{2}\int dy^-\,\epsilon(y^--x^-)VW(x)\frac{\partial}{\partial y^-}\tilde{A}^\mu_{\text{phys}}(y^-,\vec{x})$$

$$=VW(x)\tilde{A}^\mu_{\text{phys}}(x)-\frac{1}{2}VW(x)(\tilde{A}^\mu_{\text{phys}}(\infty,\vec{x})+\tilde{A}^\mu_{\text{phys}}(-\infty,\vec{x}))$$

$$=A^\mu_{\text{phys}}(x)-\frac{1}{2}VW(x)(\tilde{A}^\mu_{\text{phys}}(\infty,\vec{x})+\tilde{A}^\mu_{\text{phys}}(-\infty,\vec{x})),\quad (18)$$

where we integrated by parts. Equation (18) differs from A^μ_{phys} by the surface terms at $x^-=\pm\infty$. However, these surface terms can be consistently eliminated. To see this, suppose that (9) is valid. Then

$$A^\mu_{\text{phys}}(\infty,\vec{x})=\frac{1}{2}\int dy^-\,\mathcal{P}\exp\left(-ig\int_{y^-}^{\infty}A^+(y'^-,\vec{x})dy'^-\right)$$
$$\times F^{+\mu}(y^-,\vec{x}),$$

$$A^\mu_{\text{phys}}(-\infty,\vec{x})=-\frac{1}{2}\int dy^-\,\mathcal{P}\exp\left(-ig\int_{y^-}^{-\infty}A^+(y'^-,\vec{x})dy'^-\right)$$
$$\times F^{+\mu}(y^-,\vec{x}).\quad (19)$$

Going to the light–cone gauge, we get

$$\tilde{A}^\mu_{\text{phys}}(\infty,\vec{x})=(VW)^\dagger(\infty,\vec{x})A^\mu_{\text{phys}}(\infty,\vec{x})$$
$$=\frac{1}{2}W^\dagger(\vec{x})\int dy^-\,\mathcal{P}\exp\left(-ig\int_{y^-}^{-\infty}A^+(y'^-,\vec{x})dy'^-\right)F^{+\mu}(y^-,\vec{x}),$$
$$=-W^\dagger(\vec{x})A^\mu_{\text{phys}}(-\infty,\vec{x})$$
$$=-\tilde{A}^\mu_{\text{phys}}(-\infty,\vec{x}),\quad (20)$$

where, for definiteness, we have chosen $x^-=-\infty$ as the lower limit of the integration in (12). (The other case $x^-=\infty$ is a trivial modification.) Therefore, the surface terms in (18) cancel and this completes the proof of (9).

Note that the cancellation we have just observed is nothing but the well–known antisymmetric boundary condition of the gauge field in the light–cone gauge. Thus, in the above proof we have implicitly chosen this boundary condition by adjusting $W(\vec{x})$. This conforms to the sign function $\epsilon(y^-)$ in (8), or equivalently, the principal value prescription for the $1/x_B$ pole in $\Delta g(x_B)$

$$\int_{-\infty}^{\infty}dx_B\,\text{p.v.}\left(\frac{1}{x_B}\right)e^{iy^-P^+x_B}=i\pi\epsilon(y^-).\quad (21)$$

Different prescriptions for the $1/x_B$ pole lead to different boundary conditions for $\tilde{A}^\mu=\tilde{A}^\mu_{\text{phys}}$ at $x^-\to\pm\infty$, just like the prescription for the $1/k^+$ pole of the gluon propagator in the light–cone gauge [13]. It does not matter which prescription one uses, since the difference is proportional to $\delta(x_B)$ and vanishes under the assumption that the x_B–integral converges as $x_B\to0$. However, it does change the appearance of ΔG. Had we chosen a different prescription, say, $1/(x_B-i\epsilon)$, we would have obtained a formula for ΔG similar to (8), but with the step function $2\theta(y^-)$ in place of the sign function $\epsilon(y^-)$. In the light–cone gauge, this corresponds to the advanced boundary condition $\tilde{A}^\mu(\infty,\vec{x})=0$. For each different prescription, the surface terms will be different. But they always vanish under the corresponding boundary condition.

In conclusion, the gauge–invariant decomposition of the gluonic contribution to the nucleon spin into helicity and orbital parts is not possible if one restricts to *local* operators [3]. Once one allows for *nonlocal* operators, it becomes possible [4]. We have shown that the traditional definition of the gluon helicity, ΔG, can be nicely accommodated in this latter approach, thereby dispelling any concerns about the physical meaning of ΔG. After all, ΔG is measurable, gauge invariant, and meets the criterion by Chen *et al.* for a proper definition of the gluon helicity in QCD.

By using the explicit relation between A^μ_{phys} and A^μ, one can write down the all–order expression for the gluon orbital angular momentum (6) as well. While it is measurable as the difference between the total gluon contribution (from the GPD) and ΔG, more direct access to the orbital component would of course be desirable.

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