

Gravitational corrections to fermion masses in grand unified theories

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We reconsider quantum gravitational threshold corrections to the unification of fermion masses in grand unified theories. We show that the running of the Planck mass can have a sizable effect on these thresholds that are thus much more important than naively expected. These corrections make any extrapolation from low energy measurements challenging.

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There are several hints that the strong and electroweak forces unify at some very large energy typically assumed to be at around 10^{16} GeV. The quantum fields of the standard model fit nicely into simple representations of a grand unified theory [1] such as e.g. $SU(5)$ or $SO(10)$. The idea of unification is extremely attractive for several reasons. For example, a grand unification drastically reduces the number of independent coupling constants. Furthermore, when extrapolated using renormalization group equations, the value of the strong and electroweak interactions measured at low energy seem to converge amazingly to some common value at around 10^{16} GeV [2–4] if the standard model is replaced by the minimal supersymmetric standard model at around a TeV. An important feature of grand unified theories is that they predict the existence of many, potentially heavy, new particles. This is due to the very nature of grand unified theories that need to be based on groups large enough to incorporate the standard model $SU(3) \times SU(2) \times U(1)$ groups. Besides having to be large as such, unified theories often incorporate multiplets with a large number of fields to obtain viable phenomenology. When the unified theory is supersymmetric, the number of fundamental fields is even larger. It has been argued that the LHC data could be used to reconstruct, using renormalization group techniques, the fundamental grand unified theory, see e.g. [5], or differentiate between different supersymmetry breaking patterns [5]. In [6,7] it was shown that there are potentially sizable quantum gravitational corrections to the unification conditions for the gauge couplings of the standard model. The thresholds have been known for a while [8–11], but it had not been realized that they could potentially be larger than the two-loop corrections [6]. The aim of this work is to show that this quantum gravity blur has a similar effect on the unification conditions for the masses of the fermions in a grand unified framework.

An important consequence of the large number of fundamental fields mentioned above, which can easily reach 1000, is that the scale at which quantum gravitational effects are expected to become large is not necessarily as

expected some 10^{19} GeV but is given by the renormalized Planck mass:

$$M(\mu)^2 = M(0)^2 - \frac{\mu^2}{12\pi}(N_0 + N_{1/2} - 4N_1), \quad (1)$$

with $M(0)$ being the Planck mass at low energy, i.e. Newton's constant is given by $G = M(0)^{-2}$, and N_0 , $N_{1/2}$, and N_1 are, respectively, the numbers of real scalar fields, Weyl spinors, and spin-one vector bosons.

If the strength of gravitational interactions is scale dependent, the scale μ_* at which quantum gravity effects are large is the one at which

$$M(\mu_*) \sim \mu_*. \quad (2)$$

It was shown in [12] that the presence of a large number of fields can dramatically impact the value μ_* . In many grand unified models, the large number of fields can cause the true scale μ_* of quantum gravity to be significantly lower than the naive value $M_{\text{Pl}} \sim 10^{19}$ GeV. In fact, from the above equations, one finds

$$\mu_* = M_{\text{Pl}}/\eta, \quad (3)$$

where, for a theory with $N \equiv N_0 + N_{1/2} - 4N_1$,

$$\eta = \sqrt{1 + \frac{N}{12\pi}}. \quad (4)$$

In [6], quantum gravity effects have been shown to affect the unification of gauge couplings (see [8–11,13–18], for a nonexhaustive list of papers). The lowest order effective operators induced by a quantum theory of gravity are of dimension five, such as [8,9]

$$\frac{c}{\hat{\mu}_*} \text{Tr}(G_{\mu\nu}G^{\mu\nu}H), \quad (5)$$

where $G_{\mu\nu}$ is the grand unified theory field strength and H is a scalar multiplet. This operator is expected to be induced by strong nonperturbative effects at the scale of quantum gravity, so has coefficient $c \sim \mathcal{O}(1)$ and is suppressed by the reduced true Planck scale $\hat{\mu}_* = \mu_*/\sqrt{8\pi} = \hat{M}_{\text{Pl}}/\eta$ with $\hat{M}_{\text{Pl}} = 2.43 \times 10^{18}$ GeV.

The importance of gravitational effects were illustrated in [6] using the example of SUSY- $SU(5)$. Operators

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similar to (5) are present in all grand unified theory models and an equivalent analysis applies.

In $SU(5)$ the multiplet H in the adjoint representation acquires, upon symmetry breaking at the unification scale M_X , a vacuum expectation value $\langle H \rangle = M_X(2, 2, 2, -3, -3)/\sqrt{50\pi\alpha_G}$, where α_G is the value of the $SU(5)$ gauge coupling at M_X . Inserted into the operator (5), this modifies the gauge kinetic terms of $SU(3) \times SU(2) \times U(1)$ below the scale M_X to

$$-\frac{1}{4}(1 + \epsilon_1)F_{\mu\nu}F_{U(1)}^{\mu\nu} - \frac{1}{2}(1 + \epsilon_2)\text{Tr}(F_{\mu\nu}F_{SU(2)}^{\mu\nu}) - \frac{1}{2}(1 + \epsilon_3)\text{Tr}(F_{\mu\nu}F_{SU(3)}^{\mu\nu}) \quad (6)$$

with

$$\epsilon_1 = \frac{\epsilon_2}{3} = -\frac{\epsilon_3}{2} = \frac{\sqrt{2}}{5\sqrt{\pi}} \frac{c\eta}{\sqrt{\alpha_G}} \frac{M_X}{\hat{M}_{\text{Pl}}}. \quad (7)$$

After a finite field redefinition $A_\mu^i \rightarrow (1 + \epsilon_i)^{1/2} A_\mu^i$ the kinetic terms have familiar form, and it is then the corresponding redefined coupling constants $g_i \rightarrow (1 + \epsilon_i)^{-1/2} g_i$ that are observed at low energies and that obey the usual renormalization-group equations below M_X , whereas it is the *original* coupling constants that need to meet at M_X in order for unification to happen. In terms of the observable rescaled couplings, the unification condition therefore reads

$$\alpha_G = (1 + \epsilon_1)\alpha_1(M_X) = (1 + \epsilon_2)\alpha_2(M_X) = (1 + \epsilon_3)\alpha_3(M_X). \quad (8)$$

It was shown in [6] that the effects can be larger than the two loop effects considered in e.g. [4] and that it could either invalidate claims of a perfect unification SUSY-standard model or on the contrary help to unify models which apparently would not unify their gauge couplings.

In this work we point out that the same physical effect can have important implications for fermion masses. Again we will be using a simple $SU(5)$ model to make our point more explicit, but our results can be trivially generalized to any grand unified theory. One of the most interesting predictions of a grand unified theory, besides the unification of the gauge couplings at the unification scale, is the unification of some of the fermion masses at the unification scale. Fermion masses are generated by the Yukawa interactions. For example, in the simple $SU(5)$ grand unification model with a Higgs boson in the **5** representation, one has

$$\mathcal{L} = \{G_d \bar{\Psi}_{jR}^c \Psi_{kL}^j H^k(5) + G_u \varepsilon_{ijklmn} \bar{\Psi}_L^{cjk} \Psi_L^{lm} H^n(5)\} + \text{H.c.} \quad (9)$$

$$= -\frac{2M_w}{\sqrt{2}g_2} [G_d(\bar{d}d + \bar{e}e) + G_u 8(\bar{u}u)], \quad (10)$$

and one obtains

$$m_d(M_X) = m_e(M_X) = -\frac{2M_w}{\sqrt{2}g_2} G_d, \quad (11)$$

where M_w is the W -boson mass, g_2 the $SU(2)$ gauge coupling, and G_i are Yukawa couplings. This is one of the most exciting results of grand unified theories, namely, at the unification scale M_X the masses of the down-type quarks are equal to the masses of the charged leptons, while the mass of the u -type quarks are not related to other parameters of the model. The up-type quark masses are given by $m_u(X) = -\frac{16M_w}{\sqrt{2}g_2} G_u$ at the unification scale.

In analogy to (5), there are also dimension five operators which can affect the fermions masses. They were considered a while ago by Ellis and Gaillard [19] (see also [20])

$$\frac{c}{\hat{\mu}_*} \bar{\Psi} \phi \Psi H + \text{H.c.}, \quad (12)$$

where Ψ are fermion fields, ϕ and H some scalar bosons multiplets chosen in appropriate representations. In a simple $SU(5)$ toy model with scalar fields in the **24** and **5** representations, one gets

$$\mathcal{O}_5 = \frac{a_1}{\hat{\mu}_*} \{\phi_{mn} \bar{f}^{mk} H_k^l \Psi_l^n\} + \frac{a_2}{\hat{\mu}_*} \{\phi_{mn} H^{mk} \bar{f}_k^l \Psi_l^n\} + \frac{a_3}{\hat{\mu}_*} \varepsilon^{mnpql} \{\Psi_{mn} \Psi_{pq} H_k \phi_l^k\}, \quad (13)$$

where Ψ and f are fermion fields in **10** and **5**, respectively. These operators have been studied extensively; see e.g. [21] and references therein for more recent works in that direction. However, the renormalization group improvement considered here has not been previously studied. In $SU(5)$, the values of the expectations value of $\phi(24)$ and $H(5)$ are fixed by the requirement that the grand unified theory be broken at some 10^{16} GeV, i.e. $\langle \phi(24) \rangle \sim 10^{16}$ GeV and that the spontaneous symmetry breaking of the electroweak interactions takes place at the weak scale, i.e. $\langle H(5) \rangle = 246$ GeV.

These operators lead to a modification of the unification condition for the down-type quarks and their respective charged leptons. One finds

$$m_d(M_X)[1 + \frac{3}{2}\zeta_1 - \zeta_2] = m_e(M_X)[1 + \frac{3}{2}\zeta_1 + \frac{3}{2}\zeta_2], \quad (14)$$

with

$$\zeta_i = \frac{-2\sqrt{2}}{5G_d g_u} \frac{M_X}{\hat{M}_{\text{Pl}}} a_i \eta, \quad (15)$$

where g_u is the unified coupling constant. We note that u -type quark masses do receive a correction due to one of these operators:

$$m_u(M_X)(1 - \frac{3}{2}\zeta_3). \quad (16)$$

Clearly since the scale $\hat{\mu}_*$, i.e., the effective reduced Planck mass, is very poorly known and depends of the number of fields in the unified theory, it is very difficult to

argue that these quantum gravitational effects can be neglected. While in this simple $SU(5)$ model η is only equal to 0.74 as shown in [7] η can easily be as large as 8 in $SO(10)$ models. The running of the Planck mass has thus potentially a large impact on the splitting at the unification scale of the down-type quarks and down-type leptons. It is easy to evaluate the magnitude of the effect. One finds $\zeta_i \sim 10^{-2} a_i / G_e \eta$, where we used $\alpha_u \sim 1/40$ and $M_X / \bar{M}_{\text{Pl}} \sim 10^{-2}$. Even if the a_i are as tiny as the corresponding Yukawa couplings, one can get a 10% effect for grand unified theories with a large matter content and thus large η . Once again we see that renormalization effects of the Planck mass can have sizable effects on the unification conditions of grand unified theories.

It is easy to generalize our results to other grand unified theories. For example, in supersymmetric $SU(5)$ with Higgs fields in the **24**, **5**, and $\bar{\mathbf{5}}$ representations, the Yukawa couplings including dimension five operators suppressed by the effective reduced Planck mass are given by

$$W = \frac{1}{4} \epsilon_{abcde} \left(Y_1^{ij} \mathbf{10}_i^{ab} \mathbf{10}_j^{cd} H^e + \frac{1}{\hat{\mu}_*} f_1^{ij} \mathbf{10}_i^{ab} \mathbf{10}_j^{cd} \phi_f^e H^f \right. \\ \left. + \frac{1}{\hat{\mu}_*} f_2^{ij} \mathbf{10}_i^{ab} \mathbf{10}_j^{cf} H^d \phi_f^e \right) + \sqrt{2} \left(Y_2^{ij} \bar{H}_a \mathbf{10}_i^{ab} \mathbf{5}_{jb}^* \right. \\ \left. + \frac{1}{\hat{\mu}_*} h_1^{ij} \bar{H}^a \phi_b^a \mathbf{10}_i^{bc} \mathbf{5}_{jc}^* + \frac{1}{\hat{\mu}_*} h_2^{ij} \bar{H}^a \mathbf{10}_i^{ab} \phi_c^b \mathbf{5}_{jc}^* \right), \quad (17)$$

where $\mathbf{10} = (Q, u^c, e^c)$ and $\mathbf{5}^* = (d^c, L)$ are the chiral matter multiplets. Using the results of [22], we find

$$Y_d - Y_e = \frac{5}{\sqrt{50} \pi \alpha_G} \frac{M_X}{\hat{\mu}_*} h_2 \quad (18)$$

for the new unification condition. Note that in SUSY $SU(5)$, the value of η is typically between 2.3 and 4.4 depending on the choice of representations for the Higgs bosons [7].

$SO(10)$ grand unified theories offer even more options. If one considers, for example, a SUSY $SO(10)$ model with Higgs bosons in the **10**, $\bar{\mathbf{126}}$, and **210** representations as in [23], one has to consider terms of the type $\psi \psi H \Phi$ where ψ stands for the fermions in the **16** representation of

$SO(10)$, H for the Higgs bosons in the **120** of $SO(10)$, and Φ for the Higgs bosons in the **210** of $SO(10)$. These terms lead to corrections to the unification condition of the masses of the type [23]

$$\frac{1}{\hat{\mu}_*} \psi \psi \langle H \rangle \langle \Phi \rangle = \frac{M_X}{\hat{\mu}_*} \mathbf{v}_t (\bar{u}_R u_L + \bar{\nu}_R \nu_L) \\ + \mathbf{v}_b \frac{M_X}{\hat{\mu}_*} (\bar{d}_R d_L + \bar{e}_R e_L), \quad (19)$$

with $\mathbf{v}_t = \mathbf{v}_1 + i\mathbf{v}_2$ and $\mathbf{v}_b = -\mathbf{v}_1 - i\mathbf{v}_2$ with $\mathbf{v}_1 = \langle H_9 \rangle$ and $\mathbf{v}_2 = \langle H_0 \rangle$. For comparison, the renormalizable Yukawa mass term yields

$$\psi \psi \langle H \rangle = \mathbf{v}_t (\bar{u}_R u_L + \bar{\nu}_R \nu_L) - \mathbf{v}_b (\bar{d}_R d_L + \bar{e}_R e_L). \quad (20)$$

Clearly, the nonrenormalizable operators can have a sizable impact on the unification condition. In $SO(10)$, the value of η varies between 3.5 and 8.1, again, depending on the representations introduced in the model [7].

There are several implications of these results. Without a precise knowledge of the quantum gravitational corrections, i.e. of the full theory of quantum gravity, it is very difficult to extrapolate from low energy measurements to check whether fermion masses unify or not. This casts some doubts concerning the feasibility of reconstructing the parameters of a grand unified theory by using low energy measurements performed at the large hadron collider. On the other hand, these threshold effects can help to explain the low energy pattern of fermion masses and can revive models that naively would predict the incorrect pattern in the low energy regime.

As a summary, we have reconsidered quantum gravitational threshold effects studied a long time ago by Ellis and Gaillard. We have shown that the running of the Planck mass can have a sizable effect and that these threshold corrections are much more important than naively expected. This result is in line with our previous observations concerning the quantum gravitational threshold corrections to the unification of the coupling constants of the standard model.

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