Relating quarks and leptons without grand unification

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In combination with supersymmetry, flavor symmetry may relate quarks with leptons, even in the absence of a grand-unification group. We propose an $SU(3) \times SU(2) \times U(1)$ model where both supersymmetry and the assumed A_4 flavor symmetries are softly broken, reproducing well the observed fermion mass hierarchies and predicting: (i) a relation between down-type quarks and charged lepton masses, and (ii) a correlation between the Cabibbo angle in the quark sector and the reactor angle θ_{13} characterizing CP violation in neutrino oscillations.

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I. INTRODUCTION

Understanding the observed pattern of quark and lepton masses and mixing [1,2] constitutes one of the deepest challenges in particle physics. Flavor symmetries provide a very useful approach toward reducing the number of free parameters describing the fermion sector [3]. It has long been advocated that grand unification offers a suitable framework to describe flavor. In what follows we will adopt the alternative approach, assuming that flavor is implemented directly at the $SU(3) \times SU(2) \times U(1)$ level. Typically this requires several $SU(2)$ doublet scalars in order to break spontaneously the flavor symmetry so as to obtain an acceptable structure for the masses and mixing matrices. (One may alternatively introduce ‘‘flavons’’ instead of additional Higgs doublets, but in this case one would have to give up renormalizability).

In order to construct a ‘‘realistic’’ extension of the standard model (SM) with flavor symmetry one needs a suitable alignment of the scalar vacuum expectation values (VEVs) in the theory [4–7]. There are several multidoublet extensions of the SM with flavor in the market, but renormalizable supersymmetric extensions of the SM with a flavor symmetry are only a few [8], usually because the existence of additional Higgs doublets spoils the unification of the coupling constants.

Here we choose to renounce to this theoretical argument, noting that gauge coupling unification may happen in multidoublet schemes due to other effects. What we now present is a supersymmetric extension of the SM based on the A_4 group where all the matter fields as well as the Higgs doublets belong to the same A_4 representation, namely, the triplet. This leads us to two theoretical predictions. The first a mass relation

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_d m_s}}, \tag{1}$$

involving down-type quarks and charged lepton mass ratios. Such a relation can be obtained by a suitable combination of the three Georgi-Jarlskog (GJ) mass relations [9],

$$m_b = m_\tau, \qquad m_s = 1/3 m_\mu, \qquad m_d = 3 m_e, \tag{2}$$

which arise within a particular ansatz for the $SU(5)$ model and hold at the unification scale. In contrast to Eq. (2), our relation requires no unification group and holds at the electroweak scale. It would, in any case, be rather robust against renormalization effects as it involves only mass ratios.

The second prediction obtained in our flavor model is a correlation between the Cabibbo angle for the quarks and the so-called ‘‘reactor angle’’ θ_{13} characterizing the strength of CP violation in neutrino oscillations [10,11]. Within a reasonable approximation we find

$$\lambda_C \approx \frac{1}{\sqrt{2}} \frac{m_\mu m_b}{m_\tau m_s} \sqrt{\sin^2 2\theta_{13}} - \sqrt{\frac{m_u}{m_c}}, \tag{3}$$

which arises mainly from the down-type quark sector [12] with a correction coming from the up isospin diagonalization matrix. This is a very interesting relation, discussed below in more detail.

II. THE MODEL

Here we propose a supersymmetric model based on an A_4 flavor symmetry realized in an $SU(3) \times SU(2) \times U(1)$ gauge framework. The field representation content is given in Table I. Note that all quarks and leptons transform as A_4 triplets. Similarly the Higgs superfields with opposite hypercharge characteristic of the minimal supersymmetric standard model (MSSM) are now upgraded into two sets, also transforming as A_4 triplets. Note that since all matter fields transform in the same way under the flavor symmetry one may in principle embed the model into a grand-unified

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TABLE I. Basic multiplet assignments of the model.

Fields	$\hat{L}$	$\hat{E}^c$	$\hat{Q}$	$\hat{U}^c$	$\hat{D}^c$	$\hat{H}^u$	$\hat{H}^d$
$SU(2)_L$	2	1	2	1	1	2	2
A_4	3	3	3	3	3	3	3

scheme. However, given the large number of scalar doublets, gauge coupling unification must proceed differently; see, for example, Ref. [13].

The most general renormalizable Yukawa Lagrangian for the charged fermions in the model is [14]

$$L_{\rm Yuk} = y^l_{ijk}\hat{L}_i\hat{H}^d_j\hat{E}^c_k + y^d_{ijk}\hat{Q}_i\hat{H}^d_j\hat{D}^c_k + y^u_{ijk}\hat{Q}_i\hat{H}^u_j\hat{U}^c_k, \quad (4)$$

where $y^{u,d,l}_{ijk}$ are A_4 tensors, assumed real at this stage.

The Higgs scalar potential invariant under A_4 is

$$\begin{aligned} V = {} & (|\mu|^2 + m^2_{H_u})(|H^u_1|^2 + |H^u_2|^2 + |H^u_3|^2) \\ & + (|\mu|^2 + m^2_{H_d})(|H^d_1|^2 + |H^d_2|^2 + |H^d_3|^2) \\ & - [b(H^u_1H^d_1 + H^u_2H^d_2 + H^u_3H^d_3) + \text{c.c.}] \\ & + \frac{1}{8}(g^2 + g'^2)(|H^u_1|^2 + |H^u_2|^2 + |H^u_3|^2 \\ & + -|H^d_1|^2 - |H^d_2|^2 - |H^d_3|^2)^2. \end{aligned} \quad (5)$$

Assuming that the Higgs doublet scalars take real VEVs $\langle H^{u,d}_i\rangle = v^{u,d}_i$ one can show that the minimization of the potential V gives as possible local minima the alignments $\langle H^{0u,d}\rangle \sim (1,0,0)$ and $(1,1,1)$. Only the first is viable and we verify that minimization leads to this solution within a wide region of parameters. By adding A_4 soft breaking terms to the A_4-invariant scalar potential in Eq. (5)

$$V_{\rm soft} = \sum_{ij}(\mu^u_{ij}H^{u*}_iH^u_j + \mu^d_{ij}H^{d*}_iH^d_j) + \sum_{ij}b_{ij}H^d_iH^u_j,$$

one finds that

$$\langle H^u\rangle = (v^u, \varepsilon^u_1, \varepsilon^u_2), \qquad \langle H^d\rangle = (v^d, \varepsilon^d_1, \varepsilon^d_2), \quad (6)$$

where $\varepsilon^u_{1,2} \ll v^u$ and $\varepsilon^d_{1,2} \ll v^d$.

A. Charged fermions

By using A_4 product rules it is straightforward to show that the charged fermion mass matrix takes the following universal structure [14]:

$$M_f = \begin{pmatrix} 0 & y^f_1\langle H^f_3\rangle & y^f_2\langle H^f_2\rangle \\ y^f_2\langle H^f_3\rangle & 0 & y^f_1\langle H^f_1\rangle \\ y^f_1\langle H^f_2\rangle & y^f_2\langle H^f_1\rangle & 0 \end{pmatrix}, \quad (7)$$

where f denotes any charged lepton, up- or down-type quarks. Note that, in addition to the "texture" zeros in the diagonal, one has additional relations among the parameters. This may be seen explicitly by rewriting Eq. (7) as

$$M_f = \begin{pmatrix} 0 & a^f\alpha^f & b^f \\ b^f\alpha^f & 0 & a^fr^f \\ a^f & b^fr^f & 0 \end{pmatrix}, \quad (8)$$

where $a^f = y^f_1\varepsilon^f_1$, $b^f = y^f_2\varepsilon^f_1$, with $y^f_{1,2}$ denoting the only two couplings arising from the A_4 tensor in Eq. (4), $r^f = v^f/\varepsilon^f_1$, and $\alpha^f = \varepsilon^f_2/\varepsilon^f_1$. Thanks to the fact that the same Higgs doublet H^d couples to the lepton and to the down-type quarks one has, in addition, the following relations:

$$r^l = r^d, \qquad \alpha^l = \alpha^d, \quad (9)$$

involving down-type quarks and charged leptons.

It is straightforward to obtain analytical expressions for a^f, b^f, and r^f from Eq. (8) in terms of the charged fermion masses and α^f,

$$\frac{r^f}{\sqrt{\alpha^f}} \approx \frac{m^f_3}{\sqrt{m^f_1m^f_2}}, \quad a^f \approx \frac{m^f_2}{m^f_3}\frac{\sqrt{m^f_1m^f_2}}{\sqrt{\alpha^f}}, \quad b^f \approx \frac{\sqrt{m^f_1m^f_2}}{\sqrt{\alpha^f}}. \quad (10)$$

From Eqs. (9) and (10) it follows that

$$\frac{m_\tau}{\sqrt{m_em_\mu}} \approx \frac{m_b}{\sqrt{m_dm_s}},$$

a formula relating quark and lepton mass ratios (to a very good approximation this formula also holds for complex Yukawa couplings). This relation is a strict prediction of our model, and appears in a way similar to the celebrated $SU(5)$ mass relation, despite the fact that we have not assumed any unified group, but just the $SU(3)\times SU(2)\times U(1)$ gauge structure. It allows us to compute the down-quark mass in terms of the charged fermion masses and the s and b quarks, as

$$m_d \approx m_e\frac{m_\mu}{m_s}\left(\frac{m_b}{m_\tau}\right)^2. \quad (11)$$

This mass formula predicts the down-quark mass at the scale of the Z boson mass, to lie in the region

$$\begin{aligned} 1.71\ \text{MeV} < m^{\rm th}_d < 3.35\ \text{MeV} \\ 1.71\ \text{MeV} < m_d < 4.14\ \text{MeV}, \end{aligned} \quad (12)$$

at 1σ [15]. This is illustrated in Fig. 1 where, to guide the eye, we have also included the 1σ experimental ranges from Ref. [15], as well as the best fit point and the GJ prediction.

Note also that, thanks to supersymmetry, we obtain a relation only among the charged lepton and down-type quark mass ratios, avoiding the unwanted relation found by Wilczek and Zee in Ref. [16].

B. Neutrinos

To the renormalizable model we have so far we now add an effective dimension-five A_4-preserving lepton-number violating operator

$$\mathcal{L}_{5d} = \frac{f_{ijlm}}{\Lambda} \hat{L}_i \hat{L}_j \hat{H}^u_l \hat{H}^u_m, \tag{13}$$

where the A_4 tensor f_{ijlm} takes into account all the possible contractions of the product of four A_4 triplets.[1]

Neutrino masses are induced after electroweak symmetry breaking from the operator in Eq. (13). In order to determine the flavor structure of the resulting mass matrix we take the limit where the VEV hierarchy $\langle H^u_1 \rangle \gg \langle H^u_2 \rangle, \langle H^u_3 \rangle$ holds, leading to [14]

$$M_\nu = \begin{pmatrix} x r^{u2} & \kappa r^u & \kappa r^u \alpha^u \\ \kappa r^u & y r^{u2} & 0 \\ \kappa r^u \alpha^u & 0 & z r^{u2} \end{pmatrix}, \tag{14}$$

where x, y, z, and κ are coupling constants, while r^u and α^u already have been introduced above in the up-quark sector.

The best fit of neutrino oscillation data [2] yields maximally mixed μ and τ neutrinos. This is possible, in the basis where the charged lepton is diagonal, if and only if the light-neutrino mass matrix is approximately μ-τ invariant. In turn this holds true if $y \approx z$ and $\alpha^u \approx 1$ [14].[2] When $\alpha^u < 1$ the "atmospheric angle" deviates from the maximality. We have verified that for $\alpha^u \gtrsim 0.5$ the atmospheric angle is within its 3σ allowed range.

III. RELATING THE CABIBBO ANGLE TO θ_{13}

In the CP conserving limit we have taken so far, we have in total 14 free parameters to describe the fermion sector: six a^f and b^f parameters (three for each charged fermion type), plus four r^f and α^f [here only down-type are counted, in view of Eq. (9)], plus four parameters describing the neutrino mass induced by the dimension-5 operator: x, y, z, κ. These parameters describe 18 observables, which may be taken as the nine charged fermion masses, the two neutrino squared mass differences describing neutrino oscillations, the three neutrino mixing angles, the neutrinoless double beta decay effective mass parameter, the Cabibbo angle, in addition to V_{ub} and V_{cb}. Hence we have four relations.

The first of these we have already seen, namely, the mass relation in Eq. (1) and Fig. 1. The second is a quark-lepton mixing angle relation concerning the Cabibbo angle λ_C and the reactor angle θ_{13} describing neutrino oscillations. To derive it note first that the matrix in Eq. (8) is diagonalized on the left by a rotation in the 12 plane, namely

$$\sin\theta^f_{12} \approx \sqrt{\frac{m^f_1}{m^f_2}} \frac{1}{\sqrt{\alpha^f}}. \tag{15}$$

[1] Specific realizations of $\mathcal{L}_{5d}$ within various seesaw schemes [17] can, of course, be envisaged.

[2] The charged lepton mass matrix is mainly diagonalized by a rotation in the 12 plane.

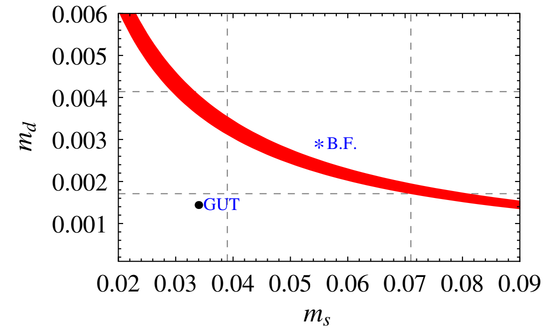

FIG. 1 (color online). The shaded band gives our prediction for the down-strange quark masses at the M_z scale, Eq. (11). Vertical and horizontal lines are the 1σ experimental ranges from Ref. [15].

In order to give an analytical expression for the relation between Cabibbo and reactor angles, we neglect mixing of the third family of quarks and go in the limit where our neutrino mass matrix Eq. (15) is μ-τ invariant, that is $\alpha^u = 1$ and $y = z$. In this approximation, the reactor mixing angle is given by

$$\sin\theta_{13} = \frac{1}{\sqrt{2}} \sin\theta^l_{12} = \frac{1}{\sqrt{2}} \sqrt{\frac{m_e}{m_\mu}} \frac{1}{\sqrt{\alpha^l}}. \tag{16}$$

Using our mass relation in Eq. (1) one finds that the Cabibbo angle may be written as

$$\lambda_C = \frac{m_b}{m_s} \frac{\sqrt{m_e m_\mu}}{m_\tau} \frac{1}{\sqrt{\alpha^d}} - \sqrt{\frac{m_u}{m_c}}. \tag{17}$$

Comparing Eq. (16) with Eq. (17) leads immediately to Eq. (3). In order to display this prediction graphically we take the quark masses at 1σ, obtaining the curved band shown in Fig. 2. The narrow horizontal band indicates current determination of the Cabibbo angle, while the two vertical dashed lines represent the expected sensitivities of the Double Chooz [18] and Daya-Bay [19]

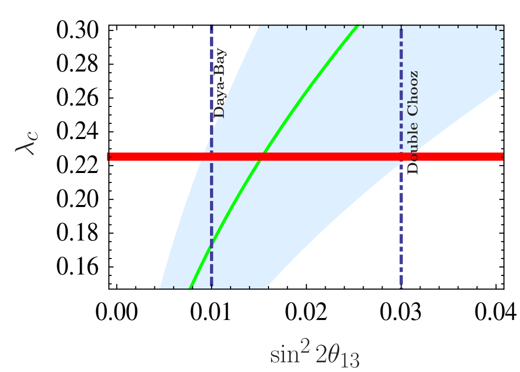

FIG. 2 (color online). The shaded band gives our predicted 1σ correlation between the Cabibbo angle and the reactor angle, as above. Vertical lines give the expected sensitivities on θ_{13} [18,19].

experiments on the reactor mixing angle θ_{13}. The curved line corresponds to the analytical approximation for the best-fit value of the quark masses in Eq. (3). Clearly the width of the curved band characterizing our prediction is dominated by quark mass determination uncertainties.

Finally note that mixing parameters of the third family of quarks $U^q_{13} \approx \frac{m^q_2}{m^q_3} \frac{\sqrt{m^q_1 m^q_2}}{m^q_3} \frac{1}{\sqrt{\alpha^q}}$ and $U^q_{23} \approx \frac{m^q_1 (m^q_2)^2}{(m^q_3)^3} \frac{1}{\alpha^q}$ $(q = u, d)$ are negligible, and cannot account for the measured values of V_{ub} and V_{cb}. The predicted values obtained for these are too small so that in its simplest presentation described above our model cannot describe the CP violation found in the decays of neutral kaons. However there is a simple solution which maintains the good predictions described above, namely, adding colored vectorlike $SU(2)_L$ singlet states. In this case acceptable values for V_{ub} and V_{cb}, leading to adequate CP violation can arise solely from nonunitarity effects of the quark mixing matrix.

IV. OUTLOOK

We proposed a supersymmetric extension of the standard model with an A_4 flavor symmetry, where all matter fields in the model transform as triplets of the flavor group. Charged fermion masses arise from renormalizable Yukawa couplings while neutrino masses are treated in an effective way. The scheme illustrates how, in combination with supersymmetry, flavor symmetry may relate quarks with leptons, even in the absence of a grand-unification group. Two good predictions emerge: (i) a relation between down-type quarks and charged lepton masses, and (ii) a correlation between the Cabibbo angle in the quark sector and the reactor angle θ_{13} characterizing CP violation in neutrino oscillations, which lies within the sensitivities of upcoming experiments.

Although the predicted values for the other mixing parameters V_{uc} and V_{cb} of the Cabibbo-Kobayashi-Maskawa matrix are too small, we mentioned a simple way to circumvent this, making the scheme fully realistic.

Finally note that, with few exceptions such as those in Refs. [20,21], grand-unified flavor models are not more predictive than the novel idea proposed here and illustrated through this simple scheme. As it stands the model fits well with the idea that gauge coupling unification may be an effect of the presence of extra dimensions rather than of grand-unified interactions [13]. Notwithstanding, we wish to stress that our model is manifestly embeddable into a standard grand-unified scenario, which would result in further predictive power. A detailed study of this particular model lies outside the scope of this paper and will be taken up elsewhere.

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