Discrete R symmetries and F-term supersymmetry breaking

Pritibhajan Byakti

Saha Institute of Nuclear Physics, 1/AF Bidhan-Nagar, Kolkata 700064, India

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We have shown that in a large number of generic and renormalizable Wess-Zumino models, existence of a Z_n R-symmetry is sufficient to break supersymmetry spontaneously. This implies that the existence of a Z_n R-symmetry is a necessary condition for supersymmetry breaking in generic and renormalizable Wess-Zumino models.

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I. INTRODUCTION

In discussions of F-term $N = 1$ supersymmetry (SUSY) breaking, R-symmetry plays a crucial role. The importance of a U(1) R-symmetry in spontaneous SUSY breaking was clearly addressed by Nelson and Seiberg [1]. It was shown that, "a continuous R-symmetry is a necessary condition for spontaneous supersymmetry breaking and a spontaneously broken R-symmetry is a sufficient condition, in models where the gauge dynamics can be integrated out and in which the effective superpotential is a generic function consistent with the symmetries of the theory." It was also shown [1] that if the requirement of genericity is relaxed, one can find models of SUSY breaking without any R-symmetry. In this paper, we address the question of the necessary condition for SUSY breaking in Wess-Zumino (WZ) models which are both generic and renormalizable.

We show that there exist a large number of generic and renormalizable WZ models where existence of a Z_n R-symmetry is sufficient to break SUSY spontaneously. It is well known that if there is no R-symmetry, continuous or discrete, in generic WZ models with canonical Kähler potentials, then the global minima preserve SUSY [1,2]. With the help of the above two results, we can conclude that existence of a Z_n R-symmetry is a necessary condition for spontaneous SUSY breaking into a global minimum in generic and renormalizable WZ models.

Models of F-term SUSY breaking with a Z_n R-symmetry can be obtained most simply from the models of F-term SUSY breaking with U(1) R-symmetry by adding a completely different and decoupled sector to break U(1) R-symmetry explicitly down to a Z_n R-symmetry. In Sec. II, we illustrate this idea by adding some more terms to the famous O'Raifeartaigh [3] (O'R) model. However, these models are trivial. In Sec. III, we explicitly discuss a nontrivial model where the superpotential cannot be broken into such two noninteracting parts. We have also shown that this model has a vacuum where SUSY as well as discrete R-symmetry is spontaneously broken for large regions of parameter space. We then give some variations of this model by adding more fields. In Sec. IV, we identify the form of these models, and then three series of models are given where each series contains a large number of such models.

A note about notations. For U(1) R-symmetry we use $R(\phi)$ to denote the U(1) R-charge of any chiral scalar superfield ϕ in the normalization where the R-charge of θ is 1. We use $R_d(\theta)$ to denote Z_n R-charge of the superspace coordinate θ. Discrete R-charge of pseudomoduli superfield X is $2R_d(\theta)\,\mathrm{mod}\,n$ and that of any other superfield is just the subscript of that field.

II. SOME TRIVIAL EXAMPLES

Adding a completely different and decoupled sector to any one of F-term SUSY breaking models with U(1) R-symmetry (we will call it as old sector), we can break U(1) R-symmetry to a Z_n R-symmetry explicitly. Value of n is controlled by the new sector. If we find a new sector (or in other words, a Z_n R-symmetry) for which no new term to the old sector is allowed even though R-symmetry becomes weaker, then SUSY breaking conditions coming from the old sector will not alter. In this way, we get models of F-term SUSY breaking with a discrete R-symmetry in generic theories.

To illustrate our idea, we consider the famous O'R model as an example of the old sector:

$$W = fX + a\phi_0\phi_2 + \frac{1}{2}b\phi_0^2 X. \tag{1}$$

The above superpotential is generic with a U(1) R-symmetry where $R(X) = R(\phi_2) = 2$, $R(\phi_0) = 0$ and a Z_2 internal symmetry under which X transforms trivially whereas ϕ's transform nontrivially. Let us now add five new fields, ϕ'_0, ϕ'_1, ϕ'_2, ϕ'_3, ϕ'_4, to the old sector and get the following superpotential:

$$\begin{aligned} W = fX + a\phi_0\phi_2 + \frac{1}{2}b\phi_0^2X + \frac{1}{2}\lambda'_{002}\phi_0'^2\phi'_2 \\ + \lambda'_{034}\phi'_0\phi'_3\phi'_4 + \frac{1}{2}\lambda'_{011}\phi'_0\phi_1'^2 + \frac{1}{6}\lambda'_{444}\phi_4'^3 \\ + \lambda'_{124}\phi'_1\phi'_2\phi'_4 + \frac{1}{2}\lambda'_{133}\phi'_1\phi_3'^2 + \frac{1}{2}\lambda'_{223}\phi_2'^2\phi'_3. \end{aligned} \tag{2}$$

This superpotential does not have a U(1) R-symmetry, as can be easily checked. However, spontaneous SUSY breaking still occurs because the F-terms, F_X and F_{ϕ_2}, are not changed due to the inclusion of the new terms. The above superpotential is generic with the following three symmetries.

(1) A Z_5 R-symmetry with $R_d(\theta) = 1$.

(2) A Z_2 internal symmetry under which all the ϕ_i's transform nontrivially, whereas X and ϕ'_α's transform trivially.

(3) A Z_3 internal symmetry under which all the ϕ'_α's transform as $\phi'_\alpha \to \omega\phi'_\alpha$ where $\omega \neq 1$ is a cube root of 1 and the remaining fields are invariant.

We can get different variations of the above model easily, for a Z_n R-symmetry with $n \geq 5$ and $R_d(\theta) = 1$ as follows:

$$W = fX + a\phi_0\phi_2 + \frac{1}{2}b\phi_0^2 X + \frac{1}{6}\lambda'_{\alpha\beta\gamma}\phi'_\alpha\phi'_\beta\phi'_\gamma \tag{3}$$

where $\lambda'_{\alpha\beta\gamma} \neq 0$ when $\alpha + \beta + \gamma = 2\,\mathrm{mod}\,n$. Thus, we have proved that there exists a large number of generic (trivial) models where existence of a Z_n R-symmetry is sufficient to break SUSY via F-terms. One can also use this technique to other SUSY breaking models [4,5] with $U(1)$ R-symmetry.

One side comment about these models. Old and new sectors of any one of these models can communicate to each other through gauge interactions if we gauge some of the internal symmetries. For example, we can easily promote the fields ϕ and ϕ' to transform under adjoint representation and promote the field X to remain invariant under a gauge group, without forbidding any term of the old and new sectors. However, these models will then no longer be WZ models.

Now, we can ask whether it is possible to construct generic F-term SUSY breaking models with the following characteristics: (a) there is no U(1) R-symmetry in the superpotential; (b) the superpotential cannot be subdivided into two disjoint sectors/parts. In the rest of the paper, we show examples of models with all these characteristics.

III. A NONTRIVIAL EXAMPLE WITH SOME VARIATIONS

We consider a renormalizable WZ model with Z_{26} R-symmetry and $R_d(\theta) = 25$. We also consider that other than X, there are ϕ_3, ϕ_6, ϕ_8, ϕ_9, ϕ_{11}, ϕ_{12}, and ϕ_{13} fields in the theory. So we will have the following generic superpotential:

$$\begin{aligned} W_1 = {} & fX + M_{11,13}\phi_{11}\phi_{13} + \frac{1}{2}M_{12,12}\phi_{12}^2 + \frac{1}{2}N_{13,13}X\phi_{13}^2 \\ & + \lambda_{3,8,13}\phi_3\phi_8\phi_{13} + \lambda_{3,9,12}\phi_3\phi_9\phi_{12} + \frac{1}{2}\lambda_{6,6,12}\phi_6^2\phi_{12} \\ & + \frac{1}{2}\lambda_{6,9,9}\phi_6\phi_9^2 + \frac{1}{6}\lambda_{8,8,8}\phi_8^3 \end{aligned} \tag{4}$$

where, without loss of generality we can take all the parameters, except $\lambda_{8,8,8}$, to be real and positive. The above superpotential does not have a U(1) R-symmetry. With $\lambda_{3,9,12} = 0$, there is a U(1) R-symmetry with the following R-charge assignments.

$$\begin{array}{cccccccc} X & \phi_3 & \phi_6 & \phi_8 & \phi_9 & \phi_{11} & \phi_{12} & \phi_{13} \\ 2 & \frac{4}{3} & \frac{1}{2} & \frac{2}{3} & \frac{3}{4} & 2 & 1 & 0 \end{array} \tag{5}$$

For $\lambda_{3,9,12} \neq 0$, this U(1) R-symmetry is explicitly broken down to Z_{26} R-symmetry. Smaller symmetry means lesser restriction on the superpotential, and so, the superpotential contain more terms. For this reason, one cannot say that necessity of discrete R-symmetry for SUSY breaking is a trivial consequence of that of U(1) R-symmetry.

There is spontaneous SUSY breaking. This can be easily realized by observing the following F-terms:

$$-F_X^* = f + \frac{1}{2}N_{13,13}\phi_{13}^2 \tag{6}$$

$$-F_{\phi_{11}}^* = M_{11,13}\phi_{13}. \tag{7}$$

Notice that vacuum expectation values (VEVs) of F_X and $F_{\phi_{11}}$ terms cannot be simultaneously zero.

We can vanish all other F-terms for any value of $\phi_{13}^{(0)}$ (VEV of ϕ_{13}) by choosing appropriate VEVs of other fields. Now minimum of scalar potential depends on $\phi_{13}^{(0)}$. Like the O'Raifeartaigh model [3] we have two cases, (a) for $y = \frac{fN_{13,13}}{M_{11,13}^2} < 1$, minimum is at $\phi_{13}^{(0)} = 0$, whereas (b) for $y > 1$, minimum is at $\phi_{13}^{(0)} = \pm i\frac{M_{11,13}}{N_{13,13}}\sqrt{2(y-1)}$. Hence we have a vacuum where supersymmetry as well as discrete R-symmetry get spontaneously broken [6].

Tree level scalar potentials of SUSY breaking often have flat directions [7,8]. For example, for the case of $y < 1$, minimum of tree level potential is at $\phi_6^{(0)} = \phi_8^{(0)} = \phi_9^{(0)} = \phi_{11}^{(0)} = \phi_{13}^{(0)} = \phi_{13}^{(0)} = 0$ with arbitrary $X^{(0)}$ and $\phi_3^{(0)}$. So, it is necessary to calculate 1-loop correction to check whether these flat directions are lifted or not. One loop correction is given by the Coleman-Weinberg (CW) [9,10] potential,

$$V_{\mathrm{CW}} = \frac{1}{64\pi^2}\left(\mathrm{tr}\left(M_B^4\log\frac{M_B^2}{\Lambda_{\mathrm{cutoff}}^2}\right) - \mathrm{tr}\left(M_F^4\log\frac{M_F^2}{\Lambda_{\mathrm{cutoff}}^2}\right)\right), \tag{8}$$

where M_B and M_F are mass matrices for scalar and fermion fields. Nonzero eigenvalues λ^F and λ^B of M_F^2 and M_B^2 respectively for $y < 1$ are as follows:

$$\lambda^F_{1,\eta} = \lambda^F_{2,\eta} = \frac{1}{2}(M^2_{12,12} + 2\lambda_{3,9,12}|\phi^{(0)}_3|^2 + \eta M_{12,12}\sqrt{M^2_{12,12} + 4\lambda^2_{3,9,12}|\phi^{(0)}_3|^2})$$

$$\lambda^F_{3,\eta} = \lambda^F_{4,\eta} = \frac{1}{2}(2M^2_{11,13} + N^2_{13,13}|X^{(0)}|^2 + 2\lambda^2_{3,8,13}|\phi^{(0)}_3|^2 + \eta N_{13,13}|X^{(0)}|\sqrt{4M^2_{11,13} + N^2_{13,13}|X^{(0)}|^2 + 4\lambda^2_{3,8,13}|\phi^{(0)}_3|^2})$$

$$\lambda^B_{1,\eta} = \lambda^B_{2,\eta} = \lambda^F_{1,\eta}$$

$$\lambda^B_{3,\eta_1,\eta_2} = \frac{1}{2}(\eta_2 f N_{13,13} + 2M^2_{11,13} + N^2_{13,13}|X^{(0)}|^2 + 2\lambda^2_{3,8,13}|\phi^{(0)}_3|^2 + \eta_1 N_{13,13}\sqrt{f^2 + |X^{(0)}|^2(2\eta_2 f N_{13,13} + 4M^2_{11,13} + N^2_{13,13}|X^{(0)}|^2 + 4\lambda^2_{3,8,13}|\phi^{(0)}_3|^2)}),$$

where η, η_1 and η_2 denote ± 1. Putting these eigenvalues to Eq. (8) and expanding $V_{\rm CW}$ about $X^{(0)} = \phi^{(0)}_3 = 0$, we find

$$V_{\rm CW} = \text{const.} + m^2_{X^{(0)}}|X^{(0)}|^2 + m^2_{\phi^{(0)}_3}|\phi^{(0)}_3|^2 + \cdots \quad (9)$$

where ellipses denote terms higher order in $X^{(0)}$ and $\phi^{(0)}_3$, and constants

$$m^2_{X^{(0)}} = \frac{M^2_{11,13}N^2_{13,13}}{32\pi^2} y^{-1}((1+y)^2\log(1+y) - (1-y)^2\log(1-y) - 2y)$$

$$m^2_{\phi^{(0)}_3} = \frac{\lambda^2_{3,8,13}M^2_{11,13}}{64\pi^2}((1+y)\log(1+y) + (1-y)\log(1-y)), \quad (10)$$

are positive. In Ref. [11], it is shown that the CW potential for the O'R model is a monotonically increasing function of $X^{(0)}$ with minimum at $X^{(0)} = 0$. The eigenvalues $\lambda^F_{3,\eta}$, $\lambda^F_{4,\eta}$ and $\lambda^B_{3,\eta_1,\eta_2}$ have similarity with those of the famous O'R model. For fixed $\phi^{(0)}_3$, we can treat $\tilde{M}^2_{11,13} = M^2_{11,13} + \lambda^2_{3,8,13}|\phi^{(0)}_3|^2$ as a parameter. And the eigenvalues $\lambda^F_{3,\eta}$, $\lambda^F_{4,\eta}$, and $\lambda^B_{3,\eta_1,\eta_2}$ are the eigenvalues of the O'R model with mass term $\tilde{M}_{11,13}\phi_{11}\phi_{13}$. Hence, the CW potential of our model is also a monotonically increasing function of $X^{(0)}$ with minimum at $X^{(0)} = 0$ for fixed $\phi^{(0)}_3$. Now, for $X^{(0)} = 0$, the CW potential can be written as

$$V_{\rm CW} = \frac{f^2N^2_{13,13}}{64\pi^2}\Big(\tilde{y}^{-2}\log(1-\tilde{y}^2) + 2\tilde{y}^{-1}\log\frac{1+\tilde{y}}{1-\tilde{y}} + \log\frac{1-\tilde{y}^2}{\tilde{y}^2} + \log\frac{f^2N^2_{13,13}}{\Lambda^4_{\rm cutoff}}\Big), \quad (11)$$

where $\tilde{y} = \frac{fN_{13,13}}{M^2_{11,13} + \lambda^2_{3,8,13}|\phi^{(0)}_3|^2}$. This is a monotonically increasing function of $\phi^{(0)}_3$. Hence, after the addition of the 1-loop correction, total scalar potential has a global minimum at $X^{(0)} = \phi^{(0)}_i = 0$.

So, we have seen that there exists a nontrivial generic and renormalizable WZ model where existence of U(1) R-symmetry is not necessary for spontaneous SUSY breaking. However, the above model cannot be considered as a counter example of the demand of Ref. [1] because in that paper no assumption about renormalizability was considered. If we add higher-dimensional terms to the superpotential consistent with the discrete R-symmetry, then SUSY gets restored. However, we then have a metastable vacuum at $X^{(0)} = \phi^{(0)}_i = 0$ due to the CW potential because contribution of higher-dimensional terms to the scalar potential will dominate only when fields get sufficiently large VEV. We can also obtain metastable SUSY breaking by using the noncanonical Kähler potential [12], and for this case SUSY breaking vacuum need not necessarily be at the origin of field space. Now, if our superpotential is a generic polynomial of degree d, then can we find any model like the above model? We will address this question in the Appendix.

We can get variations of the above model by adding more ϕ fields in the theory. For example, we can add any number of fields from the list $\{\phi_{16}, \phi_{19}, \phi_{21}, \phi_{22}, \phi_{25}\}$. In this way, we get 31 more models. If we add all the fields from the list, then the superpotential takes the following form:

$$W = W_1 + M_{3,21}\phi_3\phi_{21} + M_{8,16}\phi_8\phi_{16} + \frac{1}{2}M_{25,25}\phi^2_{25} + \lambda_{3,22,25}\phi_3\phi_{22}\phi_{25} + \lambda_{6,19,25}\phi_6\phi_{19}\phi_{25} + \frac{1}{2}\lambda_{6,22,22}\phi_6\phi^2_{22} + \frac{1}{2}\lambda_{8,21,21}\phi_8\phi^2_{21} + \lambda_{9,16,25}\phi_9\phi_{16}\phi_{25} + \lambda_{9,19,22}\phi_9\phi_{19}\phi_{22} + \lambda_{12,13,25}\phi_{12}\phi_{13}\phi_{25} + \lambda_{12,16,22}\phi_{12}\phi_{16}\phi_{22} + \frac{1}{2}\lambda_{12,19,19}\phi_{12}\phi^2_{19} + \lambda_{13,16,25}\phi_{13}\phi_{16}\phi_{25}. \quad (12)$$

Note that the addition of these fields does not change F_X and $F_{\phi_{13}}$, and hence there is SUSY breaking.

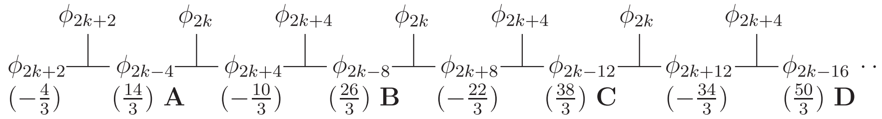

FIG. 1. Diagram representing some cubic terms in superpotentials for series I. Values inside parentheses represent U(1) R-charges. For any field ϕ_{2k+4i}, U(1) R-charge is $\frac{-12i+2}{3}$ where i is an integer. The chain will be truncated at **A**, **B**, **C**, ... for $k = 4, 8, 12, \ldots$

IV. THREE SERIES OF MODELS

In this section, we are going to show that there exists a large number of nontrivial models where existence of a discrete R-symmetry is sufficient to break supersymmetry. We consider the superpotentials are of the following form with a Z_{6k+2q} R-symmetry and $R_d(\theta) = 6k + q$:

$$W = X\Big(f + \frac{1}{2}N_{3k+q,3k+q}\phi_{3k+q}^2\Big) + M_{3k-q,3k+q}\phi_{3k-q}\phi_{3k+q} + m(\phi) + \lambda(\phi), \quad (13)$$

where k, q are natural numbers, $\lambda(\phi)$ contains cubic terms which are independent of ϕ_{3k-q}, and $m(\phi)$ denotes quadratic terms for ϕ fields other than $\phi_{3k\pm q}$ fields. Note that the superpotentials of the previous section are of the above form with $k = 4$ and $q = 1$. Because of this form of superpotentials, SUSY gets spontaneously broken, as can be checked by observing the F-terms of X and ϕ_{3k-q}.

A. Series I

In this series of models, k is a multiple of four and $q = 1$, i.e. superpotentials have a Z_{6k+2} R-symmetry with $R_d(\theta) = 6k + 1$. Field content of this series for any k is given below:

$$\Big\{X, \phi_k, \phi_{2k-1}, \phi_{2k}, \phi_{2k+2}, \phi_{3k-1}, \phi_{3k}, \phi_{3k+1}, \phi_{2k\pm 4i}\Big(i = 1, 2, \ldots, \frac{k}{4} - 1\Big)\Big\}. \quad (14)$$

Note that the model with $k = 4$ has the same R-symmetry and $R_d(\theta)$ as the models given in the previous section. But this model is different from those models because it has different field content.

To show that there is no U(1) R-symmetry in this series of models, we use method of contradiction. If there were a U(1) R-symmetry in any model, the existence of terms $X\phi_{3k+1}^2$, ϕ_{3k}^2, and ϕ_{2k}^3 would imply that $R(\phi_{3k+1}) = 0$, $R(\phi_{3k}) = 1$, and $R(\phi_{2k}) = \frac{2}{3}$. From the terms $\phi_k\phi_{2k}\phi_{3k}$, $\phi_k\phi_{2k-1}\phi_{3k+1}$, and $\phi_{2k-1}^2\phi_{2k+2}$, we could conclude $R(\phi_k) = \frac{1}{3}$, $R(\phi_{2k-1}) = \frac{5}{3}$ and $R(\phi_{2k+2}) = -\frac{4}{3}$. Similarly, we could construct a R-charge assignment chain for other fields as shown in Fig. 1. Now, for $k = 4i$ we have $2k - 4i = k$. But according to the chain, $R(\phi_{2k-4i}) = \frac{12i+2}{3} \neq R(\phi_k)$. Hence the superpotential do not have a U(1) R-symmetry for any k.

We are now going to prove that all the generic and renormalizable superpotentials of this series of models are of the form as given in Eq. (13).

There is no term quadratic in X in the superpotentials because field content for any k does not contain the field ϕ_2. Also, the cubic term in X is not allowed by discrete R-symmetries. Similarly, one can show that the terms $\phi_{3k-1}^2\phi_2$ and ϕ_{3k-1}^3 are also not allowed.

For a cubic term of the form $\phi_i\phi_j\phi_{3k-1}$ to exist, we need $i + j = 3k + 1$. Without loss of generality, we can take $i \leq j$. From field content given in Eq. (14), we find $i \geq k$ and hence $j \leq 2k + 1$. Also, one of them must be odd since their sum is odd. The only field having odd discrete R-charge in this range is ϕ_{2k-1}. But field content of any model does not contain the field ϕ_{k+2}, and so λ-terms are independent of the field ϕ_{3k-1}. There is only one cubic term containing X, $\frac{1}{2}N_{3k+1,3k+1}X\phi_{3k+1}^2$, because discrete R-charges of the ϕ fields lie between k and $3k + 1$. Thus, superpotentials of this series are of form as given in Eq. (13) and hence there is F-term SUSY breaking.

Let us now give the superpotential for $k = 4$:

$$W = fX + M_{11,13}\phi_{11}\phi_{13} + \frac{1}{2}M_{12,12}\phi_{12}^2 + \frac{1}{2}N_{13,13}X\phi_{13}^2 + \lambda_{4,7,13}\phi_4\phi_7\phi_{13} + \lambda_{4,8,12}\phi_4\phi_8\phi_{12} + \frac{1}{2}\lambda_{4,10,10}\phi_4\phi_{10}^2 + \frac{1}{2}\lambda_{7,7,10}\phi_7^2\phi_{10} + \frac{1}{6}\lambda_{8,8,8}\phi_8^3. \quad (15)$$

One can explicitly verify that the above superpotential does not have a U(1) R-symmetry yet there is F-term SUSY breaking.

B. Series II

Superpotentials of this series have Z_{6k+4} R-symmetry where k is a multiple of 6 with starting value 12. Discrete R-charge of superspace coordinate θ is $6k + 2$ or $q = 2$. Field content for any k is given below:

$$\Big\{X, \phi_k, \phi_{2k-6}, \phi_{2k-2}, \phi_{2k-1}, \phi_{2k}, \phi_{2k+1}, \phi_{2k+3}, \phi_{2k+6}, \phi_{3k-2}, \phi_{3k}, \phi_{3k+2}, \phi_{2k\pm 6i}\Big(i = 2, 3, \ldots, \frac{k}{6} - 1\Big)\Big\}. \quad (16)$$

To show that there is no U(1) R-symmetry, we have taken same the strategy as of the earlier case. Let us first tabulate U(1) R-charges of first 12 fields from the above list.

$$\begin{array}{cccccccccccc} X & \phi_k & \phi_{2k-6} & \phi_{2k-2} & \phi_{2k-1} & \phi_{2k} & \phi_{2k+1} & \phi_{2k+3} & \phi_{2k+6} & \phi_{3k-2} & \phi_{3k} & \phi_{3k+2} \\ 2 & \frac{1}{3} & \frac{11}{3} & \frac{5}{3} & \frac{7}{6} & \frac{2}{3} & \frac{1}{6} & -\frac{5}{6} & -\frac{7}{3} & 2 & 1 & 0 \end{array} \quad (17)$$

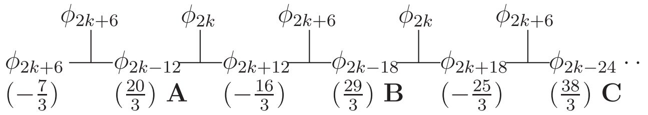

FIG. 2. Diagram representing some cubic terms in superpotentials for series II. Values inside parentheses represent U(1) R-charges. For any field ϕ_{2k+6i}, discrete R-charge is $\frac{-9i+2}{3}$ where i is an integer. The chain will truncate at **A**, **B**, **C**, ... for $k = 12, 18, 24, \ldots$.

These fields are common to all models of this series. U(1) R-charges for the fields ϕ_{3k+2}, ϕ_{3k}, ϕ_{3k-2}, ϕ_{2k-2}, ϕ_{2k} and ϕ_k can be derived easily. We obtained R-charges for $\phi_{2k+1}, \phi_{2k-1}, \phi_{2k+3}, \phi_{2k-6}, \phi_{2k+6}$ from the terms $\phi_{2k-2}\times \phi^2_{2k+1}$, $\phi_{2k+1}\phi_{2k}\phi_{2k-1}$, $\phi_{2k-2}\phi_{2k-1}\phi_{2k+3}$, $\phi^2_{2k+3}\phi_{2k-6}$, and $\phi_{2k-6}\phi_{2k}\phi_{2k+6}$, respectively. U(1) R-charge assignment chain for rest of the fields is given in the Fig. 2. From this given information, one can easily show that there is no U(1) R-symmetry in any model of this series.

There will be a $\lambda_{i,j,3k-2}$-term only if $i + j = 3k + 2$. Without loss of generality, we can take $i \leq j$. Minimum value for i is k. As there is no field with R-charge $2k + 2$, i cannot be equals to k. Next, the higher value of the discrete R-charge is $k + 6$ and hence $k + 6 \leq i \leq j < 2k - 2$. In this range, discrete R-charges of the fields are multiples of 6. As $3k + 2$ is not a multiple of 6, there cannot be a λ-term for ϕ_{3k-2}. So, superpotentials of this series are also of the form as given in Eq. (13) and in turn guarantee spontaneous breakdown of SUSY.

Let us give the first model of this series.

$$W = fX + M_{34,38}\phi_{34}\phi_{38} + \frac{1}{2}M_{36,36}\phi^2_{36} + \frac{1}{2}XN_{38,38}\phi^2_{38} + \lambda_{12,22,38}\phi_{12}\phi_{22}\phi_{38} + \lambda_{12,24,36}\phi_{12}\phi_{24}\phi_{36}$$
$$+ \frac{1}{2}\lambda_{12,30,30}\phi_{12}\phi^2_{30} + \lambda_{22,23,27}\phi_{22}\phi_{23}\phi_{27} + \frac{1}{2}\lambda_{22,25,25}\phi_{22}\phi^2_{25} + \lambda_{23,24,25}\phi_{23}\phi_{24}\phi_{25} + \frac{1}{6}\lambda_{24,24,24}\phi^3_{24}$$
$$+ \lambda_{18,24,30}\phi_{18}\phi_{24}\phi_{30} + \frac{1}{2}\lambda_{18,27,27}\phi_{18}\phi^2_{27} + \frac{1}{2}\lambda_{18,18,36}\phi^2_{18}\phi_{36} \qquad (18)$$

One can explicitly verify that there is no U(1) R-symmetry and SUSY is spontaneously broken.

C. Series III

Superpotentials of this series have Z_{6k+6} R-symmetry with $R_d(\theta) = 6k + 3$ and $k = 8, 10, 12, 14, \ldots$. Field content for any k is given below:

$$\left\{X, \phi_k, \phi_{2k-4}, \phi_{2k}, \phi_{2k+2}, \phi_{2k+8}, \phi_{3k-3}, \phi_{3k}, \phi_{3k+3}, \phi_{4k+2}, \phi_{6k+2}, \phi_{6k+3}, \phi_{6k+4}, \phi_{6k+5}, \phi_{2k\pm 2i}\left(i = 5, 6, \ldots, \frac{k}{2} - 1\right)\right\}. \qquad (19)$$

If we demand that the superpotentials have an U(1) R-symmetry, then we will have a table (Eq. (20)) and a chain (Fig. (3)) of R-charge assignments. From these inputs, one can conclude that there is no U(1) R-symmetry in any model of this series.

X	ϕ_k	ϕ_{2k}	ϕ_{2k+2}	ϕ_{3k-3}	ϕ_{3k}	ϕ_{3k+3}	ϕ_{4k+2}	ϕ_{6k+2}	ϕ_{6k+3}	ϕ_{6k+4}	ϕ_{6k+5}
2	$\frac{1}{3}$	$\frac{2}{3}$	0	2	1	0	$\frac{2}{3}$	$\frac{4}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$

(20)

There will be a λ-term for ϕ_{3k-3} only if $i + j = 3k + 3 \bmod (6k + 6)$ where i, j are discrete R-charges of fields coupled to it. Thus,

$$i + j = 3k + 3 \quad \text{or} \quad i + j = 9k + 9. \qquad (21)$$

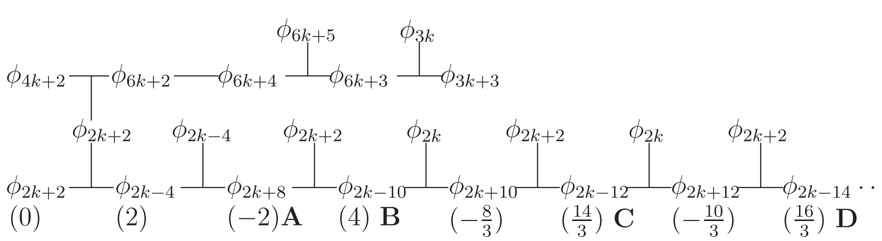

FIG. 3. Diagram representing some cubic terms in superpotentials for $n = 6k + 6$. Values inside parentheses represent U(1) R-charges. For any field ϕ_{2k+2i}, U(1) R-charge is $\frac{-2i+2}{3}$ where i is an integer. Most of the fields from Eq. (20) are also added to the chain so that one can easily determine the U(1) R-charge easily.

For the first case, $k \leq i$, $j \leq 2k + 3$. As there is no field with odd discrete R-charge in this range, this possibility is ruled out. One can check that second possibility is also ruled out. Hence, superpotentials of this series are also of the form given in Eq. (13) and there are F-term SUSY breaking.

Let us give the first model of this series, i.e. for $k = 8$, so that one can verify nonexistence of a U(1) R-symmetry and spontaneous breakdown of SUSY:

$$\begin{aligned} W = {} & fX + M_{21,27}\phi_{21}\phi_{27} + \frac{1}{2}M_{24,24}\phi_{24}^2 + M_{50,52}\phi_{50}\phi_{52} + \frac{1}{2}M_{51,51}\phi_{51}^2 + \frac{1}{2}XN_{27,27}\phi_{27}^2 + \lambda_{8,16,24}\phi_8\phi_{16}\phi_{24} \\ & + \frac{1}{2}\lambda_{12,12,24}\phi_{12}^2\phi_{24} + \frac{1}{2}\lambda_{12,18,18}\phi_{12}\phi_{18}^2 + \frac{1}{6}\lambda_{16,16,16}\phi_{16}^3 + \lambda_{16,34,52}\phi_{16}\phi_{34}\phi_{52} + \lambda_{18,34,50}\phi_{18}\phi_{34}\phi_{50} \\ & + \lambda_{24,27,51}\phi_{24}\phi_{27}\phi_{51} + \frac{1}{6}\lambda_{34,34,34}\phi_{34}^3 + \frac{1}{2}\lambda_{50,53,53}\phi_{50}\phi_{53}^2 + \lambda_{51,52,53}\phi_{51}\phi_{52}\phi_{53} + \frac{1}{6}\lambda_{52,52,52}\phi_{52}^3. \end{aligned} \tag{22}$$

In the above, we have given only three series of models. However, one can construct many series of such models.

V. CONCLUSIONS

We have shown that there exists a large number of generic and renormalizable Wess-Zumino models where existence of a Z_n R-symmetry is sufficient to break SUSY spontaneously. And it is well known that if there is no R-symmetry in generic WZ models with canonical Kähler potential, then global minima always preserve SUSY. So, existence of a Z_n R-symmetry in a generic and renormalizable Wess-Zumino model is a necessary condition for F-term SUSY breaking. However, for even n with $R_d(\theta) = \frac{n}{2}$, one cannot have models of SUSY breaking because for these cases, superpotentials as a whole transform trivially and terms which are allowed or forbidden by these R-symmetries can always be reproduced by some internal symmetries.

Our results do not go against the Nelson-Seibergs necessary condition for SUSY breaking because they have not assumed renormalizability of the superpotentials. And if we add all higher-dimensional terms consistent with discrete R-symmetry, SUSY gets restored in our case. But we then obtain metastable SUSY breaking. We have also given some models of SUSY breaking where superpotential is a generic polynomial of some degree $d \geq 4$.

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APPENDIX: NONRENORMALIZABLE SUPERPOTENTIALS

Here we will try to show that if the superpotential is a generic polynomial of some degree $d \geq 4$, then we can find models where existence of a Z_n R-symmetry is sufficient to break SUSY.

As an example, we consider a superpotential which is a generic polynomial of degree 4 and have Z_{39} R-symmetry with $R_d(\theta) = 38$. Other than X, this model contains the fields ϕ_8, ϕ_{11}, ϕ_{13}, ϕ_{19}, ϕ_{24}, and ϕ_{36}. So the superpotential is

$$\begin{aligned} W = {} & fX + gX\phi_{13}^3 + h_1\phi_{13}\phi_{24} + h_2\phi_{11}\phi_{13}^2 + h_3\phi_{19}^4 \\ & + h_4\phi_{19}\phi_{24}\phi_{36}^2 + h_5\phi_8\phi_{13}\phi_{19}\phi_{36} + h_6\phi_8^3\phi_{13} \\ & + h_7\phi_8^2\phi_{24}\phi_{36}. \end{aligned} \tag{23}$$

One can easily verify that the above superpotential does not have a U(1) R-symmetry and observing the F-terms of X and ϕ_{11}, it can be concluded that there is SUSY breaking. Though we have not found a general proof, we believe that one can find such models for any finite d. We are giving two more models with $d = 5$, 6 in support to our belief.

d	n	$R_d(\theta)$	Field content
5	92	91	X, ϕ_{13}, ϕ_{18}, ϕ_{21}, ϕ_{23}, ϕ_{26}, ϕ_{31}, ϕ_{59}
6	185	184	X, ϕ_{22}, ϕ_{23}, ϕ_{37}, ϕ_{72}, ϕ_{73}, ϕ_{109}, ϕ_{123}

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