

# Big-bang nucleosynthesis with a long-lived charged massive particle including ${}^4\text{He}$ spallation processes

Toshifumi Jitto,<sup>1</sup> Kazunori Kohri,<sup>2,3</sup> Masafumi Koike,<sup>1</sup> Joe Sato,<sup>1</sup> Kenichi Sugai,<sup>1</sup>  
Masato Yamanaka,<sup>2,4</sup> and Koichi Yazaki<sup>5,6</sup>

<sup>1</sup>*Department of Physics, Saitama University, Shimo-okubo, Sakura-ku, Saitama, 338-8570, Japan*

<sup>2</sup>*Theory Center, Institute of Particle and Nuclear Studies, KEK (High Energy Accelerator Research Organization),  
1-1 Oho, Tsukuba 305-0801, Japan*

<sup>3</sup>*Department of Physics, Tohoku University, Sendai 980-8578, Japan*

<sup>4</sup>*Maskawa Institute for Science and Culture, Kyoto Sangyo University, Kyoto 603-8555, Japan*

<sup>5</sup>*Hashimoto Mathematical Physics Laboratory, Nishina Accelerator Research Center, RIKEN, Wako, Saitama 351-0198, Japan*

<sup>6</sup>*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

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We propose helium-4 spallation processes induced by long-lived stau in supersymmetric standard models, and investigate an impact of the processes on light elements abundances. We show that, as long as the phase space of helium-4 spallation processes is open, they are more important than stau-catalyzed fusion and hence constrain the stau property.

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## I. INTRODUCTION

Quests for the physics beyond the standard model will reach a new stage at the TeV scale. Among the expected interesting signals of the new physics are those provided by exotic charged particles (charged massive particles; CHAMPs) with a long lifetime. The presence of such particles is predicted in many notable models beyond the standard model, although its identity depends on the models one assumes. CHAMP hunting is indeed one of the major issues of the high energy experiments, and its collider phenomenology is enthusiastically studied [1–14]; it also motivates other researches including neutrino telescope observations [15–17] and cosmology [18–21].

Long-lived CHAMPs will play interesting roles in the big-bang nucleosynthesis (BBN) as well. The light nuclei will interact not only with the CHAMPs during the BBN processes [22–40], but also with the decay products of the CHAMPs in the post-BBN era [41–48]. The standard scenario of the BBN will thus be altered, and so is the abundance of the light elements at the present time. One can thus constrain the models beyond the standard model by evaluating their prediction on the light elements abundance and comparing it with the current observations. We can then give stringent predictions for the forthcoming experiments and observations according to these constraints.

The standard model extended with supersymmetry (SUSY) is one of the models that can accommodate such long-lived CHAMPs. With the  $R$ -parity conservation, the lightest SUSY particle (LSP) is stable and becomes a cold dark matter. Interestingly, it can offer a long-lived CHAMPs if the LSP is the bino-like neutralino  $\tilde{\chi}_1^0$ . A coannihilation mechanism is required to account for the dark matter abundance in this case [49], where the LSP and the next-lightest SUSY particle are almost degenerate in mass. Staus, denoted by  $\tilde{\tau}$  and a possible candidate of the

next-lightest SUSY particle, can acquire a long lifetime when the mass difference with the LSP is less than the mass of tau leptons. This is due to the phase space suppression of the final state that necessarily consists of three particles or more. Note that the presence of intergenerational mixing allows  $\tilde{\tau}$  to decay into  $\tilde{\chi}_1^0$  and electron (or muon). In this case, the longevity of  $\tilde{\tau}$  is guaranteed as long as the mixing is tiny, and the peculiar analysis is needed to precisely predict the information of  $\tilde{\tau}$ . Since the main subject here is the exotic nucleosynthesis with a long-lived charged massive particle, we investigate it in the absence of intergenerational mixing.

Such long-lived staus will be copious during the BBN [50,51], we have shown in [29,33,38] that their presence indeed alters the prediction of the standard BBN and possibly solves the discrepancy of the lithium abundance in the Universe through the internal conversion reactions.

In this article, we improve our analyses by including new reactions of

$$(\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + t + n, \quad (1a)$$

$$(\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + d + n + n, \quad (1b)$$

$$(\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + p + n + n + n, \quad (1c)$$

in which  $(\tilde{\tau}^4\text{He})$  represents a bound state of a stau and the  ${}^4\text{He}$  nucleus. Reaction (1) is essentially a spallation of the  ${}^4\text{He}$  nucleus, producing a triton  $t$ , a deuteron  $d$ , and neutrons  $n$ . Presence of such spallation processes has been ignored so far due to the naïve expectation that the rate of the stau-catalyzed fusion [23]

$$(\tilde{\tau}^4\text{He}) + d \rightarrow \tilde{\tau} + {}^6\text{Li}, \quad (2)$$

is larger than the reaction (1). Indeed, the cross section of Eq. (2) is much larger than that of  ${}^4\text{He} + d \rightarrow {}^6\text{Li} + \gamma$  by (6–7) orders of magnitude [52].

We point out that this expectation is indeed naïve; the reaction Eq. (1) is more effective than Eq. (2) as long as the spallation processes are kinematically allowed. The former reaction rapidly occurs due to the large overlap of their wave functions in a bound state. On the other hand, the latter proceeds slowly since it requires an external deuteron which is sparse at the BBN era. The overproduction of  $t$  and  $d$  is more problematic than that of  ${}^6\text{Li}$ . This puts new constraints on the parameters of the minimal supersymmetric standard model. Note that there is no reaction corresponding to Eq. (1) in the gravitino LSP scenario [53].

The purpose of this work is to understand the impact of  ${}^4\text{He}$  spallation processes on light element abundances. In Sec. II, we analytically calculate its reaction rates, and compare its time scale with that of the reaction Eq. (2). In Sec. III, we calculate all of light element abundances including exotic reactions, i.e., the  ${}^4\text{He}$  spallation processes, the stau-catalyzed fusion, and the internal conversion processes. We show the minimal supersymmetric standard model parameter space in which we can reproduce the observed abundances of both dark matter and light elements including  ${}^7\text{Li}$  and  ${}^6\text{Li}$ . Section IV is devoted to a summary.

## II. SPALLATION OF HELIUM 4

Two types of reactions are possible for the bound state of a stau and a  ${}^4\text{He}$  nucleus: (1) the stau-catalyzed fusion and (2) the spallation of the  ${}^4\text{He}$  nucleus. The property of stau is stringently constrained in order to evade the overproduction of the various light elements due to these processes.

In this section, we calculate the rate of the spallation of the  ${}^4\text{He}$  nucleus. We compare the result with the rate of the stau-catalyzed fusion to show that the former is larger than the latter in a large part of the parameter space, and thereby show that the spallation plays a significant role in the BBN.

The  ${}^4\text{He}$  spallation processes of Eq. (1) are described by the Lagrangian

$$\mathcal{L} = \tilde{\tau}^* \bar{\chi}_1^0 (g_L P_L + g_R P_R) \tau + \sqrt{2} G_F \nu_\tau \gamma^\mu P_L \tau J_\mu + \text{H.c.}, \quad (3)$$

where  $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$  is the Fermi coupling constant,  $P_{L(R)}$  represents the chiral projection operator, and  $J_\mu$  is the weak current. The effective coupling constants  $g_L$  and  $g_R$  are given by

$$g_L = \frac{g}{\sqrt{2} \cos \theta_W} \sin \theta_W \cos \theta_\tau, \quad g_R = \frac{\sqrt{2} g}{\cos \theta_W} \sin \theta_W \sin \theta_\tau e^{i\gamma_\tau}, \quad (4)$$

where  $g$  is the  $SU(2)_L$  gauge coupling constant and  $\theta_W$  is the Weinberg angle. The mass eigenstate of staus is given by the linear combination of  $\tilde{\tau}_L$  and  $\tilde{\tau}_R$ , the superpartners of left-handed and right-handed tau leptons, as

$$\tilde{\tau} = \cos \theta_\tau \tilde{\tau}_L + \sin \theta_\tau e^{-i\gamma_\tau} \tilde{\tau}_R. \quad (5)$$

Here  $\theta_\tau$  is the left-right mixing angle of staus and  $\gamma_\tau$  is the  $CP$  violating phase.

$$\mathbf{A.} \quad (\tilde{\tau}^4 \text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + t + n$$

First we consider the process of Eq. (1a). The rate of this process is expressed as

$$\Gamma_{\text{in}} = |\psi|^2 \cdot \sigma \nu_{\text{in}}, \quad (6)$$

where  $|\psi|^2$  stands for the overlap of the wave functions of the stau and the  ${}^4\text{He}$  nucleus. We estimate the overlap by

$$|\psi|^2 = \frac{(Z\alpha m_{\text{He}})^3}{\pi}, \quad (7)$$

where  $Z$  and  $m_{\text{He}}$  represent the atomic number and the mass of  ${}^4\text{He}$ , respectively, and  $\alpha$  is the fine structure constant. We assumed that the stau is pointlike particle and is much heavier than the  ${}^4\text{He}$  nucleus so that the reduced mass of the bound state is equal to the mass of the  ${}^4\text{He}$  nucleus itself. The cross section of the elementary process for this reaction is denoted by  $\sigma \nu_{\text{in}}$  and calculated as

$$\begin{aligned} \sigma \nu_{\text{in}} &\equiv \sigma \nu((\tilde{\tau}^4 \text{He}) \rightarrow \tilde{\chi}_1^0 \nu_\tau t n) \\ &= \frac{1}{2E_{\tilde{\tau}}} \int \frac{d^3 \mathbf{p}_\nu}{(2\pi)^3} \frac{d^3 \mathbf{p}_{\tilde{\chi}}}{2E_{\tilde{\chi}}} \frac{d^3 \mathbf{q}_n}{(2\pi)^3} \frac{d^3 \mathbf{q}_t}{(2\pi)^3} \\ &\quad \times |\mathcal{M}((\tilde{\tau}^4 \text{He}) \rightarrow \tilde{\chi}_1^0 \nu_\tau t n)|^2 (2\pi)^4 \\ &\quad \times \delta^{(4)}(p_{\tilde{\tau}} + p_{\text{He}} - p_\nu - q_t - q_n). \end{aligned} \quad (8)$$

Here  $p_i$  and  $E_i$  are the momentum and the energy of the particle species  $i$ , respectively.

We briefly show the calculation of the amplitude of this process, leaving the full calculation in the Appendix. The amplitude is deconstructed as

$$\begin{aligned} \mathcal{M}((\tilde{\tau}^4 \text{He}) \rightarrow \tilde{\chi}_1^0 \nu_\tau t n) &= \langle t n \tilde{\chi}_1^0 \nu_\tau | \mathcal{L}_{\text{int}} | {}^4\text{He} \tilde{\tau} \rangle \\ &= \langle t n | J^\mu | {}^4\text{He} \rangle \langle \tilde{\chi}_1^0 \nu_\tau | j_\mu | \tilde{\tau} \rangle. \end{aligned} \quad (9)$$

Here we omitted the delta function for the momentum conservation and the spatial integral. The weak current  $J_\mu$  consists of a vector current  $V_\mu$  and an axial vector current  $A_\mu$  as  $J_\mu = V_\mu + g_A A_\mu$ , where  $g_A$  is the axial coupling constant. The relevant components of the currents in this reaction are  $V^0$  and  $A^i$  ( $i = 1, 2, 3$ ). We take these operators as a sum of a single-nucleon operators as

$$V^0 = \sum_{a=1}^4 \tau_a^- e^{i\mathbf{q}\cdot\mathbf{r}_a}, \quad A^i = \sum_{a=1}^4 \tau_a^- \sigma_a^i e^{i\mathbf{q}\cdot\mathbf{r}_a}, \quad (10)$$

where  $\mathbf{q}$  is the momentum carried by the current,  $\mathbf{r}_a$  is the spatial coordinate of the  $a$ th nucleon ( $a \in \{1, 2, 3, 4\}$ ), and  $\tau_a^-$  and  $\sigma_a^i$  denote the isospin ladder operator and the spin operator of the  $a$ th nucleon, respectively. Each component leads to a part of the hadronic matrix element:

$$\begin{aligned} \langle t n | V^0 | {}^4\text{He} \rangle &= \sqrt{2} \mathcal{M}_{\text{in}}, \\ \langle t n | g_A A^+ | {}^4\text{He} \rangle &= \sqrt{2} g_A \mathcal{M}_{\text{in}}, \\ \langle t n | g_A A^- | {}^4\text{He} \rangle &= -\sqrt{2} g_A \mathcal{M}_{\text{in}}, \\ \langle t n | g_A A^3 | {}^4\text{He} \rangle &= -\sqrt{2} g_A \mathcal{M}_{\text{in}}, \end{aligned} \quad (11)$$

TABLE I. Input values of the matter radius  $R_{\text{mat}}$  for  $d$ ,  $t$ , and  ${}^4\text{He}$ , the magnetic radius  $R_{\text{mag}}$  for  $p$  and  $n$ , nucleus mass  $m_X$ , excess energy  $\Delta_X$  for the nucleus  $X$ , and each reference.

nucleus	$R_{\text{mat(mag)}} [\text{fm}]/[\text{GeV}^{-1}]$	$m_X [\text{GeV}]$	$\Delta_X [\text{GeV}]$
$p$	0.876/4.439 [54]	0.9383 [55]	$6.778 \times 10^{-3}$ [56]
$n$	0.873/4.424 [57]	0.9396 [55]	$8.071 \times 10^{-3}$ [56]
$d$	1.966/9.962 [58]	1.876 [56]	$1.314 \times 10^{-2}$ [56]
$t$	1.928/9.770 [59]	2.809 [56]	$1.495 \times 10^{-2}$ [56]
${}^4\text{He}$	1.49/7.55 [60]	3.728 [56]	$2.425 \times 10^{-3}$ [56]

where  $A^\pm = (A^1 \pm iA^2)/\sqrt{2}$ . Given the relevant wave functions of a  ${}^4\text{He}$  nucleus, a triton, and a neutron in the Appendix, we obtain the hadronic matrix element as

$$\mathcal{M}_{in} = \left( \frac{128\pi}{3} \frac{a_{\text{He}} a_t^2}{(a_{\text{He}} + a_t)^4} \right)^{3/4} \left\{ \exp\left[ -\frac{\mathbf{q}_t^2}{3a_{\text{He}}} \right] - \exp\left[ -\frac{\mathbf{q}_n^2}{3a_{\text{He}}} - \frac{(\mathbf{q}_t + \mathbf{q}_n)^2}{6(a_{\text{He}} + a_t)} \right] \right\}. \quad (12)$$

Here  $\mathbf{q}_t$  and  $\mathbf{q}_n$  are three-momenta of the triton and the neutron, respectively, and  $a_{\text{He}}$  and  $a_t$  are related to the mean square matter radius  $R_{\text{mat}}$  by

$$a_{\text{He}} = \frac{9}{16} \frac{1}{(R_{\text{mat}}^2)_{\text{He}}}, \quad a_t = \frac{1}{2} \frac{1}{(R_{\text{mat}}^2)_t}. \quad (13)$$

We list in Table I input values of the matter radius for the numerical calculation in this article.

The remaining part is straightforwardly calculated to be

$$|\langle \tilde{\chi}_1^0 \nu_\tau | j_0 | \tilde{\tau} \rangle|^2 = |\langle \tilde{\chi}_1^0 \nu_\tau | j_z | \tilde{\tau} \rangle|^2 = 4G_F^2 |g_R|^2 \frac{m_{\tilde{\chi}_1^0} E_\nu}{m_\tau^2}, \quad (14)$$

$$|\langle \tilde{\chi}_1^0 \nu_\tau | j_\pm | \tilde{\tau} \rangle|^2 = 4G_F^2 |g_R|^2 \frac{m_{\tilde{\chi}_1^0} E_\nu}{m_\tau^2} \left( 1 \mp \frac{p_\nu^z}{E_\nu} \right),$$

where  $E_\nu$  and  $p_\nu^z$  are the energy and the  $z$  component of the momentum of the tau neutrino, respectively. We assumed that the stau and the neutralino are nonrelativistic. This equation includes not only all the couplings such as  $G_F$ ,  $g_L$ , and  $g_R$ , but also the effect of the virtual tau propagation in Fig. 1. Note here that the  $g_L$  coupling does not contribute. This is because the virtual tau ought to be left-handed at the weak current, and it flips its chirality during the propagation since the transferred momentum is much less than its mass.

Combining the hadronic part with the other part, we obtain the squared amplitude as

$$|\mathcal{M}(\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 \nu_\tau t n|^2 = \frac{8m_{\tilde{\chi}_1^0} G_F^2 |g_R|^2}{m_\tau^2} (1 + 3g_A^2) \mathcal{M}_{in}^2 E_\nu. \quad (15)$$

Integrating on the phase space of the final states, we obtain the cross section as

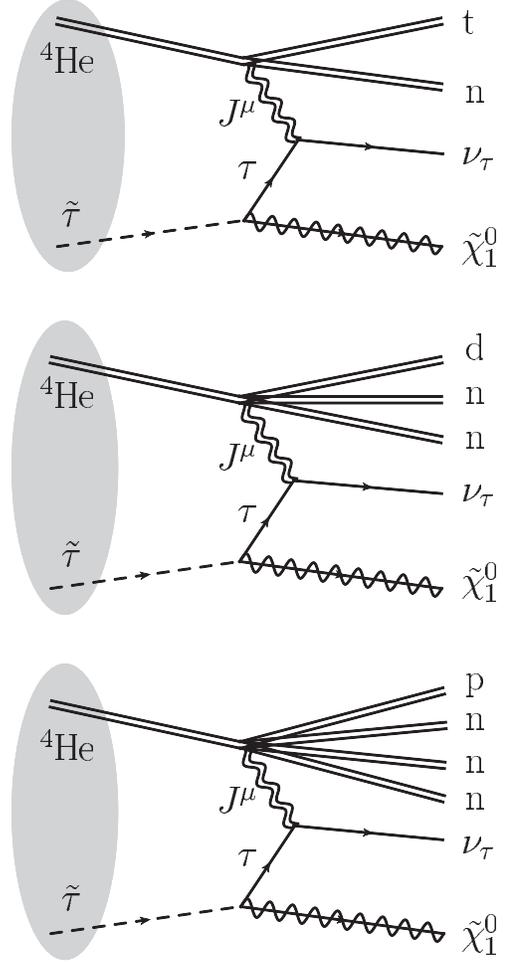


FIG. 1.  ${}^4\text{He}$  spallation processes.

$$\sigma \nu_{in} = \frac{8}{\pi^2} \left( \frac{32}{3\pi} \right)^{3/2} g^2 \tan^2 \theta_W \sin^2 \theta_\tau (1 + 3g_A^2) G_F^2 \Delta_{in}^4 \times \frac{m_t m_n}{m_\tau m_\tau^2} \frac{a_{\text{He}}^{3/2} a_t^3}{(a_{\text{He}} + a_t)^5} I_{in}, \quad (16)$$

$$I_{in} = 12 \int_0^1 ds \int_0^{\sqrt{1-s^2}} dt (1-s^2-t^2)^2 st \left\{ \frac{1}{6} \frac{k_t k_n}{a_{\text{He}} + a_t} \times st \exp\left[ -\frac{2}{3} \frac{k_t^2}{a_{\text{He}}} t^2 \right] + \frac{1}{4} \exp\left[ -\frac{2}{3} \frac{k_n^2}{a_{\text{He}}} s^2 \right] - \frac{1}{3} \frac{k_n^2 s^2 + k_t^2 t^2}{a_{\text{He}} + a_t} \right\} \sinh\left[ \frac{2}{3} \frac{k_t k_n}{a_{\text{He}} + a_t} st \right] - \exp\left[ -\frac{1}{3} \frac{k_n^2 s^2 + k_t^2 t^2}{a_{\text{He}}} - \frac{1}{6} \frac{k_n^2 s^2 + k_t^2 t^2}{a_{\text{He}} + a_t} \right] \times \sinh\left[ \frac{1}{3} \frac{k_t k_n}{a_{\text{He}} + a_t} st \right]. \quad (17)$$

Here  $\Delta_{in}$ ,  $k_t$ , and  $k_n$  are defined as

$$\Delta_{tn} \equiv \delta m + \Delta_{\text{He}} - \Delta_t - \Delta_n - E_b, \quad k_t \equiv \sqrt{2m_t \Delta_{tn}}, \quad k_n \equiv \sqrt{2m_n \Delta_{tn}}, \quad (18)$$

where  $\Delta_X$  is the excess energy of the nucleus  $X$ , and  $E_b$  is the binding energy of ( $\tilde{\tau}^4\text{He}$ ) system.

### B. ( $\tilde{\tau}^4\text{He}$ ) $\rightarrow \tilde{\chi}_1^0 + \nu_\tau + d + n + n$

The rate of another spallation process of Eq. (1b) is similarly calculated. The cross section is calculated to be

$$\sigma \nu_{dnn} = \frac{192}{\pi^4} g^2 \tan^2 \theta_W \sin^2 \theta_\tau G_F^2 \Delta_{dnn}^4 \frac{m_n m_d}{m_\tau m_\tau^2} \left( \frac{2a_d}{a_{\text{He}}(a_d + a_{\text{He}})} \right)^{3/2} I_{dnn}, \quad (19)$$

where

$$\begin{aligned} I_{dnn} = & \int_0^1 ds \int_0^{\sqrt{1-s^2}} dt \int_0^{\sqrt{1-s^2-t^2}} du (1-s^2-t^2-u^2)^2 \left\{ (1+3g_A^2) a_{\text{He}} k_n^3 s t^2 u \exp\left[-\frac{3k_d^2 s^2 + 4k_n^2 u^2}{4a_{\text{He}}}\right] \sinh\left[\frac{k_n k_d}{a_{\text{He}}} s u\right] \right. \\ & - \sqrt{2}(1+g_A^2) a_{\text{He}} k_n^3 s t u^2 \exp\left[-\frac{3k_d^2 s^2 + 2k_n^2 t^2 + 2k_n^2 u^2}{4a_{\text{He}}}\right] \sinh\left[\frac{1}{\sqrt{2}} \frac{k_n k_d}{a_{\text{He}}} s t\right] \\ & + 2\sqrt{2} g_A^2 (a_{\text{He}} + a_d) k_n^3 s t u^2 \exp\left[-\frac{k_n^2 (2t^2 + u^2)}{2a_{\text{He}}} - \frac{k_d^2 s^2 + 2k_n^2 t^2}{4(a_{\text{He}} + a_d)}\right] \sinh\left[\frac{1}{\sqrt{2}} \frac{k_d k_n}{a_{\text{He}} + a_d} s t\right] \\ & \left. - 16\sqrt{2} g_A^2 \frac{a_{\text{He}}^2}{a_{\text{He}} + a_d} \sqrt{5a_{\text{He}}^2 + 6a_{\text{He}} a_d + 2a_d^2} k_n s u \exp(-A_1 k_d^2 s^2 - A_2 k_n^2 t^2 - A_3 k_n^2 u^2) \sinh(A_4 k_d k_n s t) \sinh(A_5 k_n^2 t u) \right\}. \end{aligned} \quad (20)$$

Here  $\Delta_{dnn}$ ,  $k_d$ ,  $k_n$  and  $A_i$  ( $i = 1-5$ ) are defined as follows:

$$\begin{aligned} \Delta_{dnn} & \equiv \delta m + \Delta_{\text{He}} - \Delta_d - 2\Delta_n - E_b, & k_n & \equiv \sqrt{2m_n \Delta_{dnn}}, & k_d & \equiv \sqrt{2m_d \Delta_{dnn}}, & A_1 & \equiv \frac{4a_{\text{He}} + 3a_d}{8a_{\text{He}}(a_{\text{He}} + a_d)}, \\ A_2 & \equiv \frac{22a_{\text{He}}^3 + 44a_{\text{He}}^2 a_d + 30a_{\text{He}} a_d^2 + 7a_d^3}{4a_{\text{He}}(a_{\text{He}} + a_d)(5a_{\text{He}}^2 + 6a_{\text{He}} a_d + 2a_d^2)}, & A_3 & \equiv \frac{8a_{\text{He}}^2 + 9a_{\text{He}} a_d + 3a_d^2}{4a_{\text{He}}(5a_{\text{He}}^2 + 6a_{\text{He}} a_d + 2a_d^2)}, \\ A_4 & \equiv \frac{1}{4a_{\text{He}}(a_{\text{He}} + a_d)} \sqrt{10a_{\text{He}}^2 + 12a_{\text{He}} a_d + 4a_d^2}, & A_5 & \equiv \frac{(a_{\text{He}} + a_d)^2}{2a_{\text{He}}(5a_{\text{He}}^2 + 6a_{\text{He}} a_d + 2a_d^2)}. \end{aligned} \quad (21)$$

The rate is then obtained in the same manner as Eq. (6).

### C. ( $\tilde{\tau}^4\text{He}$ ) $\rightarrow \tilde{\chi}_1^0 + \nu_\tau + p + n + n + n$

The cross section of the spallation process of Eq. (1c) is calculated to be

$$\sigma \nu_{pnnn} = \frac{8}{\pi^9} \left( \frac{32\pi^3}{a_{\text{He}}^3} \right)^{3/2} g^2 \tan^2 \theta_W \sin^2 \theta_\tau (1+3g_A^2) G_F^2 a_{\text{He}} \Delta_{pnnn}^7 \frac{m_N^5}{m_\tau m_\tau^2} I_{pnnn}, \quad (22)$$

where

$$\begin{aligned} I_{pnnn} = & \int_0^1 ds \int_0^{\sqrt{1-s^2}} dt \int_0^{\sqrt{1-s^2-t^2}} du \int_0^{\sqrt{1-s^2-t^2-u^2}} dv (1-s^2-t^2-u^2-v^2)^2 s t^2 u v^2 \left\{ \frac{1}{\sqrt{2}} \exp\left[-\frac{k_N^2}{2a_{\text{He}}}(3s^2 + t^2 + 2u^2)\right] \right. \\ & \left. \times \sinh\left[\frac{\sqrt{2}k_N^2}{a_{\text{He}}} s u\right] - \exp\left[-\frac{k_N^2}{2a_{\text{He}}}(3s^2 + t^2 + u^2 + v^2)\right] \sinh\left[\frac{k_N^2}{a_{\text{He}}} s u\right] \right\}, \end{aligned} \quad (23)$$

where  $\Delta_{pnnn}$  and  $k_N$  are defined as follows:

$$\Delta_{pnnn} \equiv \delta m + \Delta_{\text{He}} - \Delta_p - 3\Delta_n - E_b, \quad k_N \equiv \sqrt{2m_N \Delta_{pnnn}}. \quad (24)$$

In this calculation, we assumed the proton and neutron have an identical kinetic energy, and then the factor  $k_p$  and  $k_n$ , which are introduced to factorize their kinetic energies, are also identical.  $k_N$  is the identical factor, and here we took  $m_N = m_n$ .

The reaction rate is obtained in the same manner as Eq. (6).

### D. Comparing the rate of spallation reaction with that of stau-catalyzed fusion

We compare the rate of the spallation and that of the stau-catalyzed fusion. We first note that the rate of stau-catalyzed fusion strongly depends on the temperature [52], and we fix the reference temperature to be 30 keV. Staus begin to form a bound state with  ${}^4\text{He}$  at this temperature, which corresponds to a cosmic time of  $10^3$  s. Thus the bound state is formed when the lifetime of staus is longer than  $10^3$  s.

Figure 2 shows the time scale of the spallation processes as a function of  $\delta m$ . The lifetime of a free stau is plotted by a solid line. We took the reference values of  $m_{\tilde{\tau}} = 350$  GeV,  $\sin\theta_{\tilde{\tau}} = 0.8$ , and  $\gamma_{\tilde{\tau}} = 0$ . The inverted rate of the stau-catalyzed fusion at the temperature of 30 keV is also shown by the horizontal dashed line. Once a bound state is formed, as long as the phase space of spallation processes are open sufficiently that is  $\delta m \gtrsim 0.026$  GeV, those processes dominate over other processes. There  $\tilde{\tau}$  property is constrained to evade the overproduction of  $d$  and/or  $t$ . For  $\delta m \lesssim 0.026$  GeV, the dominant process of ( $\tilde{\tau}{}^4\text{He}$ ) is stau-catalyzed fusion, since the free  $\tilde{\tau}$  lifetime is longer than the time scale of stau-catalyzed fusion. Thus the light gray region is forbidden due to the overproduction of  ${}^6\text{Li}$ .

This interpretation of Fig. 2 is not much altered by varying the parameters relevant with  $\tilde{\tau}$ . First cross sections of spallation processes are inversely proportional to  $m_{\tilde{\tau}}$ , and then the time scale of each process linearly increases as  $m_{\tilde{\tau}}$  increases. Thus, even when  $m_{\tilde{\tau}}$  is larger than  $m_{\tilde{\tau}} = 350$  GeV by up to a factor of 10, the region of  ${}^6\text{Li}$  overproduction scarcely changes. Next we point out that our results depend only mildly on the left-right mixing of the stau. Indeed, the cross section of the  ${}^4\text{He}$  spallation is proportional to  $\sin^2\theta_{\tilde{\tau}}$ . Its order of magnitude will not change as long as the right-handed component is significant.

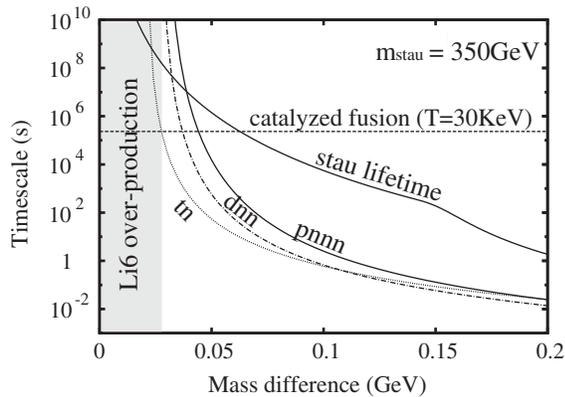


FIG. 2. Time scale of spallation processes as a function of  $\delta m$  and the stau-catalyzed fusion at the universe temperature  $T = 30$  keV [52]. The lifetime of free  $\tilde{\tau}$  (solid line) is also depicted. Here we took  $m_{\tilde{\tau}} = 350$  GeV,  $\sin\theta_{\tilde{\tau}} = 0.8$ , and  $\gamma_{\tilde{\tau}} = 0$ .

### III. LIGHT ELEMENTS ABUNDANCES AND ALLOWED PARAMETER SPACE

We numerically calculate the primordial abundances of light elements including  ${}^4\text{He}$  spallation processes and  $\tilde{\tau}$  catalyzed nuclear fusion. Then we can search for allowed regions of the parameter space to fit observational light element abundances.

So far it has been reported that there is a discrepancy between the theoretical value of the  ${}^7\text{Li}$  abundance predicted in the standard BBN and the observational one. This is called the  ${}^7\text{Li}$  problem. Standard BBN predicts the  ${}^7\text{Li}$  to H ratio to be  $\text{Log}_{10}({}^7\text{Li}/\text{H}) = -9.35 \pm 0.06$  when we adopt a recent value of baryon to photon ratio  $\eta = (6.225 \pm 0.170) \times 10^{-10}$  (68% C.L.) reported by the WMAP satellite [61], and experimental data of the rate for the  ${}^7\text{Li}$  or  ${}^7\text{Be}$  production through  ${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma$  [62] ( ${}^7\text{Li}$  is produced from  ${}^7\text{Be}$  by its electron capture,  ${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e$  at a later epoch). On the other hand, the primordial  ${}^7\text{Li}$  abundance is observed in metal-poor halo stars as absorption lines [63]. Recent observationally-inferred value of the primordial  ${}^7\text{Li}$  to hydrogen ratio is  $\text{Log}_{10}({}^7\text{Li}/\text{H}) = -9.63 \pm 0.06$  [64] for a high value, and  $\text{Log}_{10}({}^7\text{Li}/\text{H}) = -9.90 \pm 0.09$  [65] for a low value. (See also Refs. [66–68] for other values.) Therefore there is a discrepancy at more than three sigma between theoretical and observational values even when we adopt the high value of [64]. This discrepancy can hardly be attributed to the correction of the cross section of nuclear reaction [69,70]. Even if we consider nonstandard astrophysical models such as those including diffusion effects [71,72], it might be difficult to fit all of the data consistently [73].

In Figs. 3 and 4, we plot the allowed parameter regions which are obtained by comparing the theoretical values to

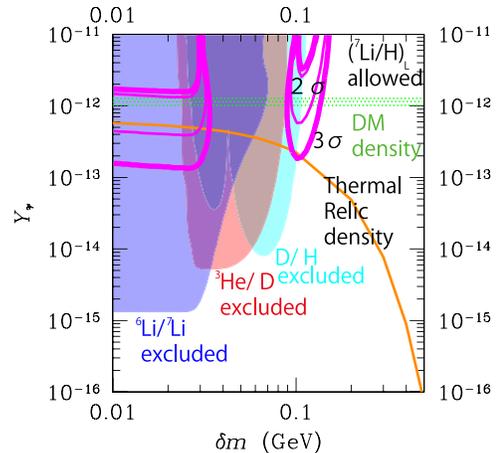


FIG. 3 (color online). Allowed regions from observational light element abundances at  $2\sigma$ . Here we have adopted the higher value of the observational  ${}^7\text{Li}/\text{H}$  in [64] denoted by  $({}^7\text{Li}/\text{H})_{\text{H}}$ , and have plotted both the  $2\sigma$  (thin line) and  $3\sigma$  (thick line) only for  ${}^7\text{Li}/\text{H}$ . The horizontal band means the observationally-allowed dark matter density. We have adopted  $m_{\tilde{\tau}} = 350$  GeV,  $\sin\theta_{\tilde{\tau}} = 0.8$ , and  $\gamma_{\tilde{\tau}} = 0$ , respectively.

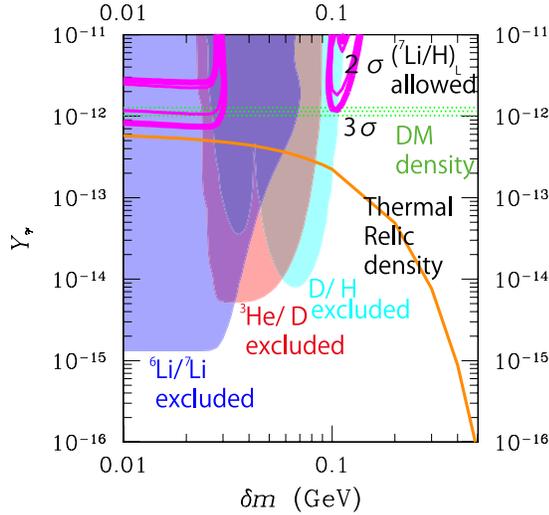


FIG. 4 (color online). Same as Fig. 3, but for the lower value of observational  ${}^7\text{Li}/\text{H}$  reported in [65], which is denoted by  $({}^7\text{Li}/\text{H})_L$ .

observational ones for the high and low  ${}^7\text{Li}/\text{H}$ , respectively. The vertical axis is the yield value of  $\tilde{\tau}$  at the time of the formation of the bound states with nuclei,  $Y_{\tilde{\tau}} = n_{\tilde{\tau}}/s$  ( $s$  is the entropy density), and the horizontal axis is the mass difference of  $\tilde{\tau}$  and  $\tilde{\chi}_1^0$ . We have adopted following other observational constraints on the light element abundances: an upper bound on the  ${}^6\text{Li}$  to  ${}^7\text{Li}$  ratio,  ${}^6\text{Li}/{}^7\text{Li} < 0.046 + 0.022$  [67], the deuteron to hydrogen ratio,  $\text{D}/\text{H} = (2.80 \pm 0.20) \times 10^{-5}$  [74], and an upper bound on the  ${}^3\text{He}$  to deuteron ratio,  ${}^3\text{He}/\text{D} < 0.87 + 0.27$  [75].

The solid line (orange line) denotes a theoretical value of the thermal relic abundance for staus [38] while keeping observationally-allowed dark matter density  $\Omega_{\text{DM}} h^2 = 0.11 \pm 0.01$  ( $2\sigma$ ) [61] as total  $\tilde{\chi}_1^0 + \tilde{\tau}$  abundance. For reference, we also plot the observationally-allowed dark matter density in the figures by a horizontal band.

At around  $\delta m \sim 0.1$  GeV, we find that  ${}^7\text{Li}/\text{H}$  can be fitted to the observational value without conflicting with the other light element abundances.<sup>1</sup> As shown in Fig. 3, it should be impressive that the relic density is consistent with the allowed region at  $Y_{\tilde{\tau}} = 2 \times 10^{-13}$  at  $3\sigma$  in case of the high value of  ${}^7\text{Li}/\text{H}$  in [64].

#### IV. SUMMARY

We calculated primordial abundances of all of the light elements involving the helium-4 spallation processes, the catalyzed fusion, and the internal conversion processes. Newly included in the present work is the spallation of the  ${}^4\text{He}$  in the stau- ${}^4\text{He}$  bound state given in Eq. (1). This process is only present in the model which predicts the

long-lived charged particles due to the phase space suppression with the weakly interacting daughter particle.

We calculated the rate of the helium-4 spallation processes analytically, and compared it with that of catalyzed fusion. We found that the spallation of  ${}^4\text{He}$  nuclei dominate over the catalyzed fusion as long as the phase space of the spallation processes are open and hence the property of the long-lived stau is constrained from avoiding the overproduction of a deuteron and/or a triton. In spite of these new constraints, we found that the lithium discrepancy and the dark matter abundance can be simultaneously solved in the parameter regions presented in Fig. 3.

#### ACKNOWLEDGMENTS

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#### APPENDIX A: AMPLITUDE OF THE SPALLATION REACTION OF ${}^4\text{He}$

The interaction relevant to our scenario is

$$\mathcal{L}_{\text{int}} = \tilde{\tau}^* \tilde{\chi}_1^0 (g_L P_L + g_R P_R) \tau + \sqrt{2} G_F (\nu_\tau \gamma^\mu P_L \tau) J_\mu + \text{H.c.}, \quad (\text{A1})$$

where  $J_\mu$  is the hadronic current. This interaction allows the spallation of the nuclei such as

$$(\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + t + n, \quad (\text{A2a})$$

$$(\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + d + n + n, \quad (\text{A2b})$$

$$(\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + p + n + n + n, \quad (\text{A2c})$$

to take place. In this section, we illustrate how we calculate the amplitude of the processes of Eqs. (A2).

##### 1. ${}^4\text{He} \rightarrow tn$

The amplitude of the process (A2a) is given by

$$\mathcal{M}((\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 \nu_\tau tn) = \langle tn | J_\mu | {}^4\text{He} \rangle \langle \tilde{\chi}_1^0 \nu_\tau | j_\mu | \tilde{\tau} \rangle, \quad (\text{A3})$$

in which we define the leptonic matrix element by

$$\begin{aligned} \langle \tilde{\chi}_1^0 \nu_\tau | j_\mu | \tilde{\tau} \rangle &= \sqrt{2} G_F \langle \tilde{\chi}_1^0 \nu_\tau | [\nu_\tau \gamma^\mu P_L \tau] \\ &\quad \times [\tilde{\tau} (g_L P_R + g_R^* P_L) \tilde{\chi}_1^0 \tilde{\tau}] | \tilde{\tau} \rangle \end{aligned} \quad (\text{A4})$$

<sup>1</sup>See also [27,40,43,48,76] for other mechanisms to reduce  ${}^7\text{Li}/\text{H}$ .

and the hadronic matrix element by  $\langle tn|J^\mu|^4\text{He}\rangle$ . We separately calculate these two matrix elements.

### a. Leptonic matrix element

The leptonic matrix element is directly calculated under the following simplifications:

- (i) The neutralino is treated as a nonrelativistic particle since its mass  $m_{\tilde{\chi}_1^0}$  is much larger than the mass of the nuclei.
- (ii) The momentum of a virtual tau is negligibly smaller than its mass due to the assumption that the stau and the neutralino are nearly degenerate with the mass difference of  $O(10\text{--}100)$  MeV.

A straightforward calculation leads to

$$|\langle \tilde{\chi}_1^0 \nu_\tau | j_0 | \tilde{\tau} \rangle|^2 = |\langle \tilde{\chi}_1^0 \nu_\tau | j_3 | \tilde{\tau} \rangle|^2 = 4G_F^2 |g_R|^2 \frac{m_{\tilde{\chi}_1^0} E_\nu}{m_\tau^2},$$

$$|\langle \tilde{\chi}_1^0 \nu_\tau | j_\pm | \tilde{\tau} \rangle|^2 = 4G_F^2 |g_R|^2 \frac{m_{\tilde{\chi}_1^0} E_\nu}{m_\tau^2} \left( 1 \mp \frac{p_\nu^z}{E_\nu} \right), \quad (\text{A5})$$

$$|^4\text{He}\rangle = \frac{1}{2\sqrt{6}} [ |pnpn\rangle (|\uparrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle - |\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\downarrow\rangle) + |pnnp\rangle (-|\uparrow\uparrow\downarrow\rangle - |\downarrow\downarrow\uparrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle) + |nppn\rangle (-|\uparrow\uparrow\downarrow\rangle - |\downarrow\downarrow\uparrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle) + |pnnp\rangle (|\uparrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle - |\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\downarrow\rangle) + |ppnn\rangle (-|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle - |\downarrow\downarrow\uparrow\rangle) + |nppp\rangle (-|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle - |\downarrow\downarrow\uparrow\rangle) ]. \quad (\text{A6})$$

The above wave function is antisymmetric under the exchange of two particles. Thus, the spatial part ought to be symmetric, and is constructed as

$$\psi_{\text{He}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = \left( 2 \frac{a_{\text{He}}^3}{\pi^3} \right)^{3/4} \exp \left\{ -a_{\text{He}} \left[ r_1^2 + r_2^2 + r_3^2 + r_4^2 - \frac{1}{4} (\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4)^2 \right] \right\}, \quad (\text{A7})$$

where  $a_{\text{He}}$  parametrizes the nuclear radius and is related to mean square matter radius  $R_{\text{mat}}$  by

$$a_{\text{He}} = \frac{9}{16} \frac{1}{(R_{\text{mat}})_{\text{He}}^2}. \quad (\text{A8})$$

We applied the nonrelativistic normalization for the nuclear wave function, which is

$$\int d^3\mathbf{r}_1 d^3\mathbf{r}_2 d^3\mathbf{r}_3 d^3\mathbf{r}_4 |\psi_{\text{He}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)|^2 \times \delta^3 \left( \frac{1}{4} (\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4) \right) = 1. \quad (\text{A9})$$

The wave function (A7) is independent of the coordinate of the center of mass since the initial  $^4\text{He}$  is taken to be stationary. The complete wave function of the  $^4\text{He}$  nucleus is a direct product of Eqs. (A6) and (A7).

where  $E_i$ ,  $p_i$ , and  $m_i$  individually stand for energy, four-momentum and mass of particles.

### b. Hadronic matrix element

Calculation of the hadronic matrix element requires the explicit form of the hadronic current and the wave functions of the nuclei.

We need the wave functions of initial helium, final triton, and nucleon. Building up these wave functions requires special attention to the symmetry. The wave function consists of spatial, spin, and isospin parts, and should be antisymmetric under the exchange of the two nucleons. The spin and isospin of the nucleus dictates the spin and isospin parts of the wave function. We then arrange the spatial part so that the total wave function will be antisymmetric under the permutation of the nucleons. We model the spatial wave functions by Gaussian functions in terms of Jacobi coordinates.

Let us make a wave function of  $^4\text{He}$  by this prescription. The spin and isospin parts of the wave function are constructed according to  $S = 0$  and  $I = 0$ , and turn out to be

The wave functions of other nuclei are similarly obtained. For the triton, the spin and isospin parts of the wave function is formed according to  $S = 1/2$ ,  $I = 1/2$ , and  $I_z = -1/2$ ; it is given by

$$|t \uparrow\rangle = \frac{1}{\sqrt{6}} [ |pnn\rangle (|\uparrow\downarrow\rangle - |\uparrow\uparrow\rangle) - |nnp\rangle (|\uparrow\downarrow\rangle - |\uparrow\uparrow\rangle) + |nnp\rangle (|\uparrow\uparrow\rangle - |\uparrow\downarrow\rangle) ], \quad (\text{A10a})$$

$$|t \downarrow\rangle = \frac{1}{\sqrt{6}} [ |pnn\rangle (|\uparrow\downarrow\rangle - |\uparrow\uparrow\rangle) - |nnp\rangle (|\uparrow\downarrow\rangle - |\uparrow\uparrow\rangle) + |nnp\rangle (|\uparrow\uparrow\rangle - |\uparrow\downarrow\rangle) ]. \quad (\text{A10b})$$

Its spatial wave function is

$$\psi_t(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \left( \frac{4 a_t^2}{3 \pi^2} \right)^{3/4} \exp \left( i \mathbf{q}_t \cdot \frac{1}{3} (\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3) \right) \times \exp \left\{ -a_t \left[ r_1^2 + r_2^2 + r_3^2 - \frac{1}{3} (\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3)^2 \right] \right\}, \quad (\text{A11})$$

where  $\mathbf{q}_t$  is the center-of-mass momentum of the triton and  $a_t$  is related to the mean square matter radius as

$$a_t = \frac{1}{2} \frac{1}{(R_{\text{mat}})_t^2}. \quad (\text{A12})$$

The wave function of the final neutron is simply taken to be

$$\psi_n(\mathbf{r}) = e^{i\mathbf{q}_n \cdot \mathbf{r}} \quad (\text{A13})$$

with trivial spin and isospin parts.

Using these wave functions, we can calculate the hadronic matrix elements of Eq. (A3) as

$$\begin{aligned} \langle tn|J^0|^4\text{He}\rangle &= \sqrt{2}\mathcal{M}_{in}, & \langle tn|J^+|^4\text{He}\rangle &= \sqrt{2}g_A\mathcal{M}_{in}, \\ \langle tn|J^-|^4\text{He}\rangle &= -\sqrt{2}g_A\mathcal{M}_{in}, & \langle tn|J^3|^4\text{He}\rangle &= -\sqrt{2}g_A\mathcal{M}_{in}, \end{aligned} \quad (\text{A14})$$

$$\begin{aligned} |\mathcal{M}((\bar{\tau}^4\text{He}) \rightarrow \bar{\chi}_1^0\nu_\tau tn)|^2 &= |\langle tn|J^0|^4\text{He}\rangle|^2 |\langle \bar{\chi}_1^0\nu_\tau|j_0|\bar{\tau}\rangle|^2 + |\langle tn|J^-|^4\text{He}\rangle|^2 |\langle \bar{\chi}_1^0\nu_\tau|j_+|\bar{\tau}\rangle|^2 + |\langle tn|J^+|^4\text{He}\rangle|^2 |\langle \bar{\chi}_1^0\nu_\tau|j_-|\bar{\tau}\rangle|^2 \\ &+ |\langle tn|J^3|^4\text{He}\rangle|^2 |\langle \bar{\chi}_1^0\nu_\tau|j_3|\bar{\tau}\rangle|^2 = \frac{8m_{\bar{\chi}_1^0}G_F^2|g_R|^2}{m_\tau^2} (1 + 3g_A^2)\mathcal{M}_{in}^2 E_\nu. \end{aligned} \quad (\text{A16})$$

## 2. $^4\text{He} \rightarrow dnn$

The calculation presented in the previous section is applicable to another spallation process of Eq. (A2b). The amplitude we need is

$$\mathcal{M}(^4\text{He} \rightarrow dnn) = \langle \bar{\chi}_1^0\nu_\tau|j_\mu|\bar{\tau}\rangle \langle dnn|J^\mu|^4\text{He}\rangle. \quad (\text{A17})$$

The leptonic matrix element is the same as in Eq. (A3) and is already calculated in Eq. (A5), while the hadronic matrix element requires a new calculation.

### a. Hadronic matrix element

We need the wave function of the final states, which include a deuteron and two neutrons. The spin and isospin of a deuteron is  $S = 1$  and  $I = 0$ , and the corresponding wave functions are

$$|d, +1\rangle = \frac{1}{\sqrt{2}}(|p \uparrow\rangle|n \uparrow\rangle - |n \uparrow\rangle|p \uparrow\rangle), \quad (\text{A18})$$

$$|d, 0\rangle = \frac{1}{2}(|p \uparrow\rangle|n \downarrow\rangle - |n \uparrow\rangle|p \downarrow\rangle + |p \downarrow\rangle|n \uparrow\rangle - |n \downarrow\rangle|p \uparrow\rangle), \quad (\text{A19})$$

$$|d, -1\rangle = \frac{1}{\sqrt{2}}(|p \downarrow\rangle|n \downarrow\rangle - |n \downarrow\rangle|p \downarrow\rangle). \quad (\text{A20})$$

The spatial part of the wave function is given by

$$\psi_d(\mathbf{r}_1, \mathbf{r}_2) = \left(\frac{a_d}{\pi}\right)^{3/4} \exp\left(i\mathbf{q}_d \cdot \frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right) \exp\left[-\frac{1}{2}a_d(\mathbf{r}_1 - \mathbf{r}_2)^2\right], \quad (\text{A21})$$

where  $\mathbf{q}_d$  is the center-of-mass momentum and  $a_d$  is related to the mean square matter radius as

where  $\mathcal{M}_{in}$  is defined as

$$\begin{aligned} \mathcal{M}_{in} &= \left(\frac{128\pi}{3} \frac{a_{\text{He}}a_t^2}{(a_{\text{He}} + a_t)^4}\right)^{3/4} \left\{ \exp\left[-\frac{\mathbf{q}_t^2}{3a_{\text{He}}}\right] \right. \\ &\quad \left. - \exp\left[-\frac{\mathbf{q}_n^2}{3a_{\text{He}}} - \frac{(\mathbf{q}_t + \mathbf{q}_n)^2}{6(a_{\text{He}} + a_t)}\right] \right\}. \end{aligned} \quad (\text{A15})$$

### c. Amplitude

Combining the hadronic matrix elements with the leptonic matrix elements, we obtain the square amplitude of Eq. (A3) as

$$a_d = \frac{3}{8} \frac{1}{(R_{\text{mat}})_d^2}. \quad (\text{A22})$$

The spin of two neutrons can be  $S = 0$  and  $S = 1$ . For each case, spin and isospin parts of the wave functions are

$$|n_0\rangle = \frac{1}{\sqrt{2}}(|n \uparrow\rangle|n \downarrow\rangle - |n \downarrow\rangle|n \uparrow\rangle), \quad (\text{A23})$$

$$|n_1, +1\rangle = |n \uparrow\rangle|n \uparrow\rangle, \quad (\text{A24})$$

$$|n_1, 0\rangle = \frac{1}{\sqrt{2}}(|n \uparrow\rangle|n \downarrow\rangle + |n \downarrow\rangle|n \uparrow\rangle), \quad (\text{A25})$$

$$|n_1, -1\rangle = |n \downarrow\rangle|n \downarrow\rangle, \quad (\text{A26})$$

where  $|n_i\rangle$  expresses the spin and isospin parts of the wave function of  $S = i$ . Since the spin and isospin parts of  $S = 1$  are symmetric under the exchange of two particles, the spatial part of the wave function ought to be antisymmetric. The spin and isospin parts of  $S = 0$ , on the other hand, is antisymmetric under the exchange of two particles, then the spatial part of the wave function ought to be symmetric. Therefore the spatial parts of each wave function are given by

$$\begin{aligned} \psi_{n_0}(\mathbf{r}_1, \mathbf{r}_2) &= \frac{1}{\sqrt{2}} \{ \exp[i(\mathbf{q}_{n_1} \cdot \mathbf{r}_1 + \mathbf{q}_{n_2} \cdot \mathbf{r}_2)] \\ &\quad + \exp[i(\mathbf{q}_{n_2} \cdot \mathbf{r}_1 + \mathbf{q}_{n_1} \cdot \mathbf{r}_2)] \}, \end{aligned} \quad (\text{A27})$$

$$\begin{aligned} \psi_{n_1}(\mathbf{r}_1, \mathbf{r}_2) &= \frac{1}{\sqrt{2}} \{ \exp[i(\mathbf{q}_{n_1} \cdot \mathbf{r}_1 + \mathbf{q}_{n_2} \cdot \mathbf{r}_2)] \\ &\quad - \exp[i(\mathbf{q}_{n_2} \cdot \mathbf{r}_1 + \mathbf{q}_{n_1} \cdot \mathbf{r}_2)] \}. \end{aligned} \quad (\text{A28})$$

Using these wave functions, we can calculate the Hadronic matrix elements of Eq. (A3) as

$$\begin{aligned}
\langle dn_1|J^0|^4\text{He}\rangle &= \sqrt{3}\mathcal{M}_{dnn}, & \langle dn_1|J^+|^4\text{He}\rangle &= -\sqrt{2}g_A\mathcal{M}_{dnn}, & \langle dn_1|J^-|^4\text{He}\rangle &= \sqrt{2}g_A\mathcal{M}_{dnn}, \\
\langle dn_1|J^3|^4\text{He}\rangle &= \sqrt{2}g_A\mathcal{M}'_{dnn}, & \langle dn_0|J^+|^4\text{He}\rangle &= \sqrt{2}g_A\mathcal{M}'_{dnn}, & \langle dn_0|J^-|^4\text{He}\rangle &= -\sqrt{2}g_A\mathcal{M}'_{dnn}, \\
\langle dn_0|J^3|^4\text{He}\rangle &= -\sqrt{2}g_A\mathcal{M}'_{dnn},
\end{aligned} \tag{A29}$$

where  $\mathcal{M}_{dnn}$  and  $\mathcal{M}'_{dnn}$  are defined as

$$\begin{aligned}
\mathcal{M}_{dnn} &= \left(\frac{32\pi^2 a_d}{a_{\text{He}}(a_{\text{He}} + a_d)^2}\right)^{3/4} \left[ \exp\left(-\frac{4q_{n2}^2 + 4q_{n2} \cdot q_d + 3q_d^2}{8a_{\text{He}}}\right) - \exp\left(-\frac{4q_{n1}^2 + 4q_{n1} \cdot q_d + 3q_d^2}{8a_{\text{He}}}\right) \right], \\
\mathcal{M}'_{dnn} &= \frac{1}{\sqrt{2}} \left(\frac{32\pi^2 a_d}{a_{\text{He}}(a_{\text{He}} + a_d)^2}\right)^{3/4} \left[ \exp\left(-\frac{4q_{n2}^2 + 4q_{n2} \cdot q_d + 3q_d^2}{8a_{\text{He}}}\right) + \exp\left(-\frac{4q_{n1}^2 + 4q_{n1} \cdot q_d + 3q_d^2}{8a_{\text{He}}}\right) \right. \\
&\quad \left. - 2 \exp\left(-\frac{3q_{n1}^2 + 2q_{n1} \cdot q_{n2} + 3q_{n2}^2}{8a_{\text{He}}} - \frac{1}{8} \frac{(q_d + q_{n1} + q_{n2})^2}{a_{\text{He}} + a_d}\right) \right].
\end{aligned} \tag{A30}$$

### b. Amplitude

Combining the hadronic matrix elements with the leptonic matrix elements, we obtain the square amplitude of Eq. (A17) as

$$\begin{aligned}
|\mathcal{M}((\bar{\tau}^4\text{He}) \rightarrow \bar{\chi}_1^0 \nu_\tau dnn)|^2 &= |\langle dnn|J^0|^4\text{He}\rangle|^2 |\langle \bar{\chi}_1^0 \nu_\tau |j_0|\bar{\tau}\rangle|^2 + |\langle dnn|J^-|^4\text{He}\rangle|^2 |\langle \bar{\chi}_1^0 \nu_\tau |j_+|\bar{\tau}\rangle|^2 \\
&\quad + |\langle dnn|J^+|^4\text{He}\rangle|^2 |\langle \bar{\chi}_1^0 \nu_\tau |j_-|\bar{\tau}\rangle|^2 + |\langle dnn|J^3|^4\text{He}\rangle|^2 |\langle \bar{\chi}_1^0 \nu_\tau |j_3|\bar{\tau}\rangle|^2 \\
&= \frac{12m_{\bar{\chi}_1^0} G_F^2 |g_R|^2}{m_\tau^2} ((1 + 2g_A^2)\mathcal{M}_{dnn}^2 + 2g_A^2\mathcal{M}_{dnn}'^2) E_\nu.
\end{aligned} \tag{A31}$$

### 3. $^4\text{He} \rightarrow pnnn$

The matrix element for the  $^4\text{He} \rightarrow pnnn$  process is

$$\mathcal{M}(^4\text{He} \rightarrow pnnn) = \langle \bar{\chi}_1^0 \nu_\tau |j_\mu|\bar{\tau}\rangle \langle pnnn|J^\mu|^4\text{He}\rangle. \tag{A32}$$

The calculation is also performed with an identical step as that of other  $^4\text{He}$  spallation processes.

#### a. Hadronic matrix element

The final state of the process is a system composed of a proton and three neutrons, and two types of the systems could be brought; (1)  $S = 0$  and  $S_z = 0$  via vector current (2)  $S = 1$  and  $S_z = \{-1, 0, +1\}$  via axial vector current.

The spin and isospin parts of the system for  $S = 0$  and  $S_z = 0$  is given by

$$\begin{aligned}
|pnnn(S=0, S_z=0)\rangle &= \frac{1}{4\sqrt{3}} \epsilon_{ijkl} [ |n\uparrow\rangle_i |n\uparrow\rangle_j |n\downarrow\rangle_k |p\downarrow\rangle_l \\
&\quad + |n\downarrow\rangle_i |n\downarrow\rangle_j |n\uparrow\rangle_k |p\uparrow\rangle_l ],
\end{aligned} \tag{A33}$$

where we set  $\epsilon_{ijkl} = +1$ . Here indices  $i, j, k, l$  are corresponding to spatial coordinates, i.e., the spatial part of the wave function for  $|n\uparrow\rangle_i |n\uparrow\rangle_j |n\downarrow\rangle_k |p\downarrow\rangle_l$  is

$$\psi(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k, \mathbf{r}_l) \equiv \exp[i(\mathbf{q}_{n1} \cdot \mathbf{r}_i + \mathbf{q}_{n2} \cdot \mathbf{r}_j + \mathbf{q}_{n3} \cdot \mathbf{r}_k + \mathbf{q}_p \cdot \mathbf{r}_l)]. \tag{A34}$$

Here each spatial part of proton and neutron are simply taken to be a plane wave form. Similarly, the spin and

isospin parts of the system for  $S = 1$  and  $S_z = \{-1, 0, +1\}$  are given by

$$|pnnn(S=1, S_z=+1)\rangle = \frac{1}{2\sqrt{6}} \epsilon_{ijkl} |n\uparrow\rangle_i |n\uparrow\rangle_j |n\downarrow\rangle_k |p\uparrow\rangle_l, \tag{A35}$$

$$\begin{aligned}
|pnnn(S=1, S_z=0)\rangle &= \frac{1}{4\sqrt{3}} \epsilon_{ijkl} [ |n\uparrow\rangle_i |n\uparrow\rangle_j |n\downarrow\rangle_k |p\downarrow\rangle_l \\
&\quad - |n\downarrow\rangle_i |n\downarrow\rangle_j |n\uparrow\rangle_k |p\uparrow\rangle_l ],
\end{aligned} \tag{A36}$$

$$|pnnn(S=1, S_z=-1)\rangle = \frac{1}{2\sqrt{6}} \epsilon_{ijkl} |n\downarrow\rangle_i |n\downarrow\rangle_j |n\uparrow\rangle_k |p\downarrow\rangle_l. \tag{A37}$$

Spatial parts of them are the same as the system for  $S = 0$  and  $S_z = 0$ .

The hadronic matrix element in (A32) is calculated with built wave functions and explicit forms of each current as follows,

$$\begin{aligned}
\langle pnnn|J^0|^4\text{He}\rangle &= \sqrt{2}\mathcal{M}_{pnnn}, \\
\langle pnnn|J^+|^4\text{He}\rangle &= -\sqrt{2}g_A\mathcal{M}_{pnnn}, \\
\langle pnnn|J^-|^4\text{He}\rangle &= \sqrt{2}g_A\mathcal{M}_{pnnn}, \\
\langle pnnn|J^3|^4\text{He}\rangle &= \sqrt{2}g_A\mathcal{M}_{pnnn},
\end{aligned} \tag{A38}$$

where  $\mathcal{M}_{pnnn}$  is define as follows:

$$\mathcal{M}_{pnnn} = \left(\frac{32\pi^3}{a_{\text{He}}^3}\right)^{3/4} \left\{ \exp\left[-\frac{1}{2a_{\text{He}}}(\mathbf{q}_{n2}^2 + \mathbf{q}_{n3}^2 + \mathbf{q}_p^2 + \mathbf{q}_{n2} \cdot \mathbf{q}_{n3} + \mathbf{q}_{n2} \cdot \mathbf{q}_p + \mathbf{q}_{n3} \cdot \mathbf{q}_p)\right] - \exp\left[-\frac{1}{2a_{\text{He}}}(\mathbf{q}_{n1}^2 + \mathbf{q}_{n3}^2 + \mathbf{q}_p^2 + \mathbf{q}_{n1} \cdot \mathbf{q}_{n3} + \mathbf{q}_{n1} \cdot \mathbf{q}_p + \mathbf{q}_{n3} \cdot \mathbf{q}_p)\right] \right\}. \quad (\text{A39})$$

### b. Amplitude

Combining the hadronic matrix elements with the leptonic matrix elements, we obtain the square amplitude of Eq. (A32) as

$$\begin{aligned} |\mathcal{M}(\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 \nu_\tau pnnn|^2 &= |\langle pnnn|J^0|^4\text{He}\rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau |j_0|\tilde{\tau}\rangle|^2 + |\langle pnnn|J^-|^4\text{He}\rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau |j_+|\tilde{\tau}\rangle|^2 \\ &\quad + |\langle pnnn|J^+|^4\text{He}\rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau |j_-|\tilde{\tau}\rangle|^2 + |\langle pnnn|J^3|^4\text{He}\rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau |j_3|\tilde{\tau}\rangle|^2 \\ &= \frac{8m_{\tilde{\chi}_1^0} G_F^2 |g_R|^2}{m_\tau^2} (1 + 3g_A^2) \mathcal{M}_{pnnn}^2 E_\nu. \end{aligned} \quad (\text{A40})$$

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