PHYSICAL REVIEW D **84,** 034046 (2011)

Exploring the structure of the proton through polarization observables in $lp \to$ jet X

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(Received 23 June 2011; published 31 August 2011)

We present results for a complete set of polarization observables for jet production in lepton-proton collision, where the final-state lepton is not observed. The calculations are carried out in collinear factorization at the level of Born diagrams. For all the observables we also provide numerical estimates for typical kinematics of a potential future electron-ion collider. On the basis of this numerical study, the prospects for the transverse single target spin asymmetry are particularly promising. This observable is given by a certain quark-gluon correlation function, which has a direct relation to the transverse momentum dependent Sivers parton distribution.

DOI: 10.1103/PhysRevD.84.034046 PACS numbers: 13.88.+e, 13.87.Ce

I. INTRODUCTION

In lepton-nucleon scattering one normally detects the scattered lepton in order to determine the virtuality Q^2 of the exchanged gauge boson. Provided that Q^2 is sufficiently large, one can use, for a number of final states, the machinery of QCD factorization (see Ref. [1] for an overview) in order to separate the short-distance physics from the nonperturbative long-distance physics encoded in different parton correlation functions. In data analyses for lepton-nucleon scattering, QCD factorization not only has been used to get a handle on objects like ordinary forward parton distributions, but also to address generalized parton distributions and transverse momentum dependent parton distributions.

In the present work we study inclusive jet production in lepton-proton scattering with the scattered lepton going unobserved, i.e., $lp \to \mathrm{jet}X$. In this case, the transverse momentum of the jet can serve as the large scale, which is needed for justifying a calculation in perturbative QCD. The kinematics of this process is rather simple—in particular, simpler than the kinematics of semi-inclusive deep-inelastic scattering (DIS)—and in essence coincides with the one of, e.g., single jet production in hadronic collisions. We focus here on jet production, as opposed to hadron production, because this process, in principle, can provide more direct information about parton correlation functions of the proton as no uncertainties from parton fragmentation are involved.

We consider all possible polarization observables for the process $lp \to \mathrm{jet}X$ at the level of Born diagrams. Neglecting parity violating effects as well as transverse polarization of the initial-state lepton, one can identify three spin-dependent cross sections: σ_{LL}, σ_{UT}, and σ_{LT}, where the first index refers to the lepton polarization and the second one to the proton polarization. While σ_{LL}, like the unpolarized cross section σ_{UU}, is a twist-2 observable, the latter two are twist-3 effects.

In order to compute σ_{UT} and σ_{LT} we make use of collinear twist-3 factorization, which was pioneered in the early 1980s [2,3] and applied for the first time to single spin asymmetries (SSAs) in [2]. These early treatments of twist-3 SSAs were later revisited and improved [4,5]. In the meantime many papers on SSAs and related observables in the collinear twist-3 formalism exist; see, e.g., Refs. [6–26]. In particular, various works are dealing with the QCD evolution of the relevant twist-3 correlators (see [27–31] and references therein). Moreover, the relation between the collinear twist-3 factorization and a factorization in terms of transverse momentum dependent parton correlators [32–34] has been studied in detail for semi-inclusive deep-inelastic scattering and the Drell-Yan process [35–38].

Based on our numerical estimate for a typical kinematics of the currently discussed/planned Electron Ion Collider (see, e.g., Refs. [39,40]), the transverse SSA $A_{UT} = \sigma_{UT}/\sigma_{UU}$ appears to be rather promising. The QCD description of A_{UT} contains a specific twist-3 quark-gluon-quark correlator—the so-called ETQS (Efremov-Teryaev-Qiu-Sterman) matrix element [2,4]. As pointed out in [41,42], the ETQS matrix element is related to the transverse momentum dependent Sivers function [43]. (For experimental studies of the Sivers effect in semi-inclusive DIS we refer to [44,45], while extractions of the Sivers function from data were discussed in [46–51]). Thus, measuring A_{UT} in $lp \to \mathrm{jet}X$ would give a direct (complementary) handle on the Sivers effect. Our prediction for the longitudinal double spin asymmetry $A_{LL} = \sigma_{LL}/\sigma_{UU}$ is at the percent level, while for A_{LT}, computed in a Wandzura-Wilczek-type approximation, we obtain only a tiny effect.

Note also that a detailed study of A_{UT} in $lp \to \mathrm{jet}X$, based on factorization in terms of transverse momentum dependent correlators, can be found in Ref. [52]. That work represents an extension and update of a related earlier investigation of A_{UT} for $lp \to \pi X$ [53]. Moreover, A_{UT}

for pion production was also considered in the collinear twist-3 approach in two conference proceedings [54].

II. FACTORIZATION AND PREDICTIVE POWER

In this section, we present a QCD factorization formalism for the inclusive high transverse momentum jet production in lepton-hadron collision and provide brief arguments why this factorization formalism should be valid. We also provide the prescription for calculating the short-distance hard parts of the formalism, and discuss the predictive power of perturbative calculations.

A. Factorization formulas

With a large momentum transfer, the transverse momentum $P_{JT} \equiv |\vec{P}_{JT}|$ of the inclusive jet in high-energy collisions, the short-distance dynamics takes place at a time scale of $1/P_{JT}$, which is much shorter than the typical time scale of hadronic physics, $\mathcal{O}$(fm). The quantum interference taking place between these two very different time scales is likely suppressed by the ratio of these two scales. Perturbative QCD factorization of hadronic cross sections effectively neglects the power suppressed quantum interference and factorizes the cross section into a product or a convolution of two probabilities: one for finding active parton(s) inside the identified hadron(s), and the other is the short-distance part of partonic cross section(s). For example, the leading power contribution to single inclusive jet production at large transverse momentum in hadron-hadron collisions, $h(P) + h'(P') \to \text{jet}(P_J) + X$, can be factorized as [1],

$$\frac{d\sigma^{hh'\to \text{jet}(P_J)X}}{dP_{JT}dy} \approx \sum_{ab} \int dx f_1^{a/h}(x,\mu) \int dx' f_1^{b/h'}(x',\mu) \times \frac{d\hat{\sigma}^{ab\to \text{jet}(P_J)X}}{dP_{JT}dy}(x, x', P_{JT}, y, \mu), \qquad (1)$$

where $\sum_{ab}$ runs over all parton flavors, $f_1^{a/h}(x,\mu)$ is the (unpolarized) parton distribution function (PDF) of flavor a and momentum fraction x of hadron h, and μ is the factorization scale. Since P_{JT} is the only large observed momentum, Eq. (1) is a collinear factorization formalism [1]. The $\frac{d\hat{\sigma}^{ab\to \text{jet}(P_J)X}}{dP_{JT}dy}(x, x', P_{JT}, y, \mu)$ in Eq. (1) is the perturbatively calculable short-distance hard part, which is effectively equal to the partonic jet cross section for the collision between two partons a and b with all collinear sensitive contribution removed. The predictive power of Eq. (1) relies on our ability to calculate the partonic hard parts systematically in perturbative QCD order-by-order in α_s, and the universality of PDFs. The factorization formalism in Eq. (1) works extremely well for describing single inclusive jet data at the Tevatron for over 10 orders of magnitude in the production rate [55].

Our ability to calculate the short-distance partonic hard parts in Eq. (1) relies on the fact that the factorization of short-distance dynamics is not sensitive to the long-distance details of the colliding hadron(s). That is, the factorization formalism in Eq. (1), which is valid for two colliding hadrons, should also be valid for the collision of two asymptotic partons. By applying Eq. (1) to the collision of two partons of various flavors, we can derive all short-distance hard parts order-by-order in powers of α_s [56]. Since the factorization formalism in Eq. (1) is not sensitive to the details of the colliding particles, we expect that the same factorization formalism in Eq. (1) is also valid for single inclusive jet production at high P_{JT} in lepton-hadron collision, $l(l) + h(p) \to \text{jet}(P_J) + X$, as

$$\frac{d\sigma^{lh\to \text{jet}(P_J)X}}{dP_{JT}dy} \approx \sum_{ab} \int dx f_1^{a/l}(x,\mu) \int dx' f_1^{b/h}(x',\mu) \times \frac{d\hat{\sigma}^{ab\to \text{Jet}(P_J)X}}{dP_{JT}dy}(x, x', P_{JT}, y, \mu), \qquad (2)$$

where $\sum_a$ runs over the lepton, the photon, and all parton flavors, while $\sum_b$ runs over all parton flavors, $f_1^{a/l}(x,\mu)$ is the nonperturbative distribution to find a lepton, photon or parton inside the colliding lepton with the momentum fraction x. As in the case of hadronic collisions, the short-distance hard part $\frac{d\hat{\sigma}^{ab\to \text{Jet}(P_J)X}}{dP_{JT}dy}(x, x', P_{JT}, y, \mu)$ can be perturbatively calculated order-by-order in the coupling constant by applying the factorized formalism in Eq. (2) to the collision between various lepton, photon or parton states. For the leading contribution, it might be reasonable to keep the cross section at the lowest power in α_{em} while including radiative corrections from the strong interaction in powers of α_s.

Like all perturbative QCD factorization approaches, the predictive power of Eq. (2) relies on the infrared safety of the hard parts and the universality of the long-distance distributions. Unlike the hadronic case in Eq. (1), the jet production in lepton-hadron collisions requires a set of new nonperturbative distributions, $f_1^{a/l}(x,\mu)$ with $a = l, \gamma, q, \bar{q}, g$. The operator definition of the lepton PDFs should be the same as the proton PDFs' except that the proton state is replaced by the state of the lepton [57]. The operator definition of the lepton distribution inside a lepton is very similar to that of a quark distribution,

$$f_1^{l'/l}(x,\mu) = \int \frac{dy^-}{2\pi} e^{ixP^+\xi^-} \langle l, \vec{s}_l | \bar{\psi}^{l'}(0) \times \frac{\gamma^+}{2} \mathcal{W}_\gamma(0,\xi^-) \psi^{l'}(\xi^-) | l, \vec{s}_l \rangle, \qquad (3)$$

where $\mathcal{W}_\gamma(0,\xi^-) = \exp[-ie\int_0^{\xi^-} dy^- A_\gamma^+(y^-)]$ is the gauge link in an Abelian gauge for the lepton moving in the "$+z$" direction. If we neglect the role of the strong interaction, we can calculate the lepton distribution perturbatively in QED. At the lowest order, $f_1^{l'/l}(x,\mu) = \delta^{l'l}\delta(1-x)$. The operator definition of the photon distribution inside a lepton is the same as the operator definition for the gluon distribution inside a proton with the color dependence removed and the proton state replaced by the lepton state,

$$f_1^{\gamma/l}(x,\mu) = \frac{1}{xP^+}\int\frac{d\xi^-}{2\pi}e^{ixP^+\xi^-} \times \langle l,\vec{s}_l|F_{\rm em}^{+\alpha}(0)F_{\rm em}^{+\beta}(\xi^-)|l,\vec{s}_l\rangle(-g_{\alpha\beta}), \tag{4}$$

where $F_{\rm em}^{\mu\nu}$ is the electromagnetic field strength tensor, which can be expressed in terms of gauge invariant electric and magnetic fields. The photon distribution of the lepton, $f_1^{\gamma/l}(x,\mu)$, could be calculated perturbatively in QED if we neglect the strong interaction. But, in general, it is a nonperturbative distribution.

Based on the same argument that the factorization of the short-distance dynamics is not sensitive to the details of the colliding particles, and the fact that the single inclusive jet cross section at high transverse momentum has only one observed large momentum transfer P_{JT}, we expect that the twist-3 collinear factorization formalism for calculating the transverse-spin-dependent cross section in hadronic collisions [2,4,5] can be applied to the single inclusive jet cross section in lepton-hadron collisions. Therefore,

$$\frac{d\Delta\sigma^{lh\to{\rm jet}(P_J)X}(S_T)}{dP_{JT}dy} \approx \sum_{ab}\int dx f_1^{a/l}(x,\mu) \times \int dx' T_F^{b/h}(x',x',\mu,S_T)H^{ab\to{\rm Jet}(P_J)X}(x,x',P_{JT},y,\mu), \tag{5}$$

where $\Delta\sigma(S_T) \equiv [\sigma(S_T) - \sigma(-S_T)]/2$ is the transverse-spin-dependent cross section, with the spin vector $S_T \equiv |\vec{S}_T|$ of the transversely polarized colliding hadron. The $T_F^{b/h}(x,x,\mu,\vec{S}_T)$ with $b = q,\bar{q},g$ in Eq. (5) is the universal twist-3 parton correlation function relevant for the SSAs [4,27], and the $H^{ab\to{\rm Jet}(P_J)X}(x,x',P_{JT},y,\mu)$ with $a = l,\gamma,q,\bar{q},g$ and $b = q,\bar{q},g$ are the process-dependent short-distance hard parts whose leading order contributions are derived below.

B. Partonic hard parts

In this subsection, we provide the prescription for calculating the partonic hard parts of the factorization formulas in Eqs. (2) and (5).

To calculate the short-distance hard parts, $\frac{d\hat{\sigma}^{ab\to{\rm Jet}(P_J)X}}{dP_{JT}dy} \times (x,x',P_{JT},y,\mu)$ in Eq. (2), we apply the factorization formalism to the collision between all possible combinations of two asymptotic incoming lepton, photon, or parton states. For the leading order (LO) contribution, the jet cross section is given by the lowest order lepton-quark scattering, as sketched in Fig. 1(a), and the jet is effectively given by the final-state quark: ${\rm jet}(P_J) \to q(P_J)$ at the lowest order. The corresponding hard part can be uniquely derived by applying Eq. (2) to the collision of the lepton on a quark state: $l \to l$ and $h \to q$,

$$\sigma_{(2,0)}^{lq\to q(P_J)\to{\rm jet}(P_J)} = f_{1(0)}^{l/l}\otimes f_{1(0)}^{q/q}\otimes\hat{\sigma}_{(2,0)}^{lq\to q(P_J)\to{\rm jet}(P_J)} = \hat{\sigma}_{(2,0)}^{lq\to q(P_J)\to{\rm jet}(P_J)}, \tag{6}$$

where the subscript "(2,0)" indicates two powers of α_{em} and zeroth order in α_s, and $\otimes$ represents the convolution over the lepton or parton momentum fraction as shown in Eq. (2). At the lowest order, the short-distance hard part for the jet cross section is effectively the same as the lepton-quark scattering cross section for producing the quark $q(P_J)$ and is perturbatively finite.

If we apply the factorization formalism in Eq. (2) to photon-quark collision by letting $l\to\gamma$ and $h\to q$, as sketched in Fig. 2(a), we can have two additional tree-level contributions to the jet cross section,

$$\begin{aligned}\sigma_{(1,1)}^{\gamma q\to q(P_J)\to{\rm jet}(P_J)} &= f_{1(0)}^{\gamma/\gamma}\otimes f_{1(0)}^{q/q}\otimes\hat{\sigma}_{(1,1)}^{\gamma q\to q(P_J)\to{\rm jet}(P_J)} = \hat{\sigma}_{(1,1)}^{\gamma q\to q(P_J)\to{\rm jet}(P_J)},\\ \sigma_{(1,1)}^{\gamma q\to g(P_J)\to{\rm jet}(P_J)} &= f_{1(0)}^{\gamma/\gamma}\otimes f_{1(0)}^{q/q}\otimes\hat{\sigma}_{(1,1)}^{\gamma q\to g(P_J)\to{\rm jet}(P_J)} = \hat{\sigma}_{(1,1)}^{\gamma q\to g(P_J)\to{\rm jet}(P_J)}.\end{aligned} \tag{7}$$

The difference of these two contributions is whether the jet is generated by an energetic quark or a gluon. Similarly, if

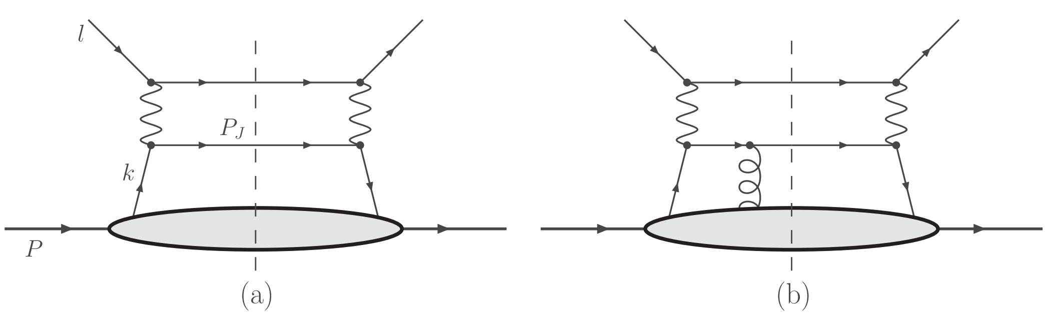

FIG. 1. Diagram (a): parton model representation for $lp \to {\rm jet}X$. The jet is produced by the struck quark, and the final-state lepton goes unobserved. Diagram (b): contribution from quark-gluon-quark correlation. This diagram, together with its Hermitian conjugate which is not displayed, needs to be taken into account when computing twist-3 observables.

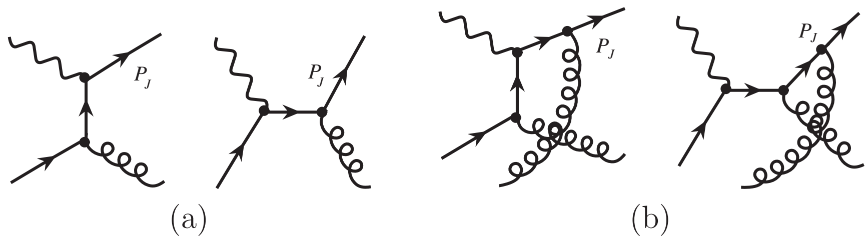

FIG. 2. (a) Leading order tree diagrams for photon scattering on an asymptotic quark state. (b) Leading order tree diagrams for photon scattering on an asymptotic quark-gluon composite state. Note that only the gluon interaction with the observed final-state parton is nonzero, while the interaction with the unobserved parton cancels.

we apply Eq. (2) to photon-gluon collision, we obtain two more tree-level contributions to the jet cross section: $\hat{\sigma}_{(1,1)}^{\gamma g\to q(P_J)\to \text{jet}(P_J)}$ and $\hat{\sigma}_{(1,1)}^{\gamma g\to \bar{q}(P_J)\to \text{jet}(P_J)}$. Since the photon distribution of the lepton $f_1^{\gamma/l}(x,\mu)$ carries at least one power of α_{em} higher than the leading term of $f_1^{l/l}(x,\mu)$, these photon-parton contributions could be formally considered as an order α_s correction to the LO term in Eq. (6). However, since the photon distribution of the lepton $f_1^{\gamma/l}(x,\mu)$ has a large QED logarithm perturbatively, for certain kinematics these terms could be more important than typical higher order corrections.

If we apply the factorization formalism in Eq. (2) to parton-parton collision by letting $l\to a$ and $h\to b$ with a, $b = q, \bar{q}, g$, the partonic hard parts are to be the same as those in the hadronic collisions [4,9]. Since the parton distribution functions of the lepton $f_1^{a/l}(x,\mu)$ have at least two powers of α_{em} perturbatively, even the leading Born contribution here should be considered as higher order corrections in powers of α_s.

Higher order corrections to the short-distance hard parts of the factorization formalism can be derived in the same way by applying Eq. (2) to the scattering of various partonic states at higher orders in α_s. For example, we can calculate the next-to-leading order (NLO) contribution $\hat{\sigma}_{(2,1)}^{lq\to \text{jet}(P_J)}$ by applying Eq. (2) to the lepton-quark collision at order α_s,

$$\begin{aligned}\sigma_{(2,1)}^{lq\to \text{jet}(P_J)} &= f_{1(0)}^{l/l}\otimes f_{1(0)}^{q/q}\otimes \hat{\sigma}_{(2,1)}^{lq\to \text{jet}(P_J)}\\ &+ f_{1(0)}^{l/l}\otimes f_{1(1)}^{q/q}\otimes \hat{\sigma}_{(2,0)}^{lq\to \text{jet}(P_J)}\\ &+ f_{1(1)}^{\gamma/l}\otimes f_{1(0)}^{q/q}\otimes [\hat{\sigma}_{(1,1)}^{\gamma q\to q(P_J)} + \hat{\sigma}_{(1,1)}^{\gamma q\to g(P_J)}],\end{aligned} \quad (8)$$

which can be written as

$$\begin{aligned}\hat{\sigma}_{(2,1)}^{lq\to \text{jet}(P_J)} &= \sigma_{(2,1)}^{lq\to \text{jet}(P_J)} - f_{1(1)}^{q/q}\otimes \hat{\sigma}_{(2,0)}^{lq\to \text{jet}(P_J)}\\ &- f_{1(1)}^{\gamma/l}\otimes [\hat{\sigma}_{(1,1)}^{\gamma q\to q(P_J)} + \hat{\sigma}_{(1,1)}^{\gamma q\to g(P_J)}],\end{aligned} \quad (9)$$

where the first term on the right-hand side (RHS) is the partonic cross section given by the real Feynman diagrams sketched in Fig. 3(a) plus the virtual diagrams from one-loop corrections to the tree-diagram in Fig. 1(a). The real diagrams in Fig. 3(a) have three potential collinear logarithmic divergences. One comes from the final-state gluon radiation when the gluon is parallel to the parent quark. Such final-state collinear divergence is taken care of by the jet definition and its finite cone size. The other two come from the initial state: when the gluon line is almost parallel to the incoming quark or when the photon is about parallel to the incoming lepton. As a result of QCD factorization, these two divergences are systematically removed by the second and the third terms in Eq. (9), respectively. The hard $2\to 2$ scattering cross section of the second term, $\hat{\sigma}_{(2,0)}^{lq\to \text{jet}(P_J)}$ is derived in Eq. (6), while $\hat{\sigma}_{(1,1)}^{\gamma q\to q(P_J)}$ and $\hat{\sigma}_{(1,1)}^{\gamma q\to g(P_J)}$ of the third term are given in Eq. (7).

Similarly, we can derive another short-distance contribution at the same order, $\hat{\sigma}_{(2,1)}^{lg\to \text{jet}(P_J)}$, by applying Eq. (2) to the collision between a lepton and a gluon,

$$\begin{aligned}\hat{\sigma}_{(2,1)}^{lg\to \text{jet}(P_J)} &= \sigma_{(2,1)}^{lg\to \text{jet}(P_J)} - f_{1(1)}^{q/g}\otimes \hat{\sigma}_{(2,0)}^{lq\to \text{jet}(P_J)}\\ &- f_{1(1)}^{\gamma/l}\otimes [\hat{\sigma}_{(1,1)}^{\gamma g\to q(P_J)} + \hat{\sigma}_{(1,1)}^{\gamma g\to \bar{q}(P_J)}],\end{aligned} \quad (10)$$

where the second and the third terms on the RHS again remove the collinear divergence of the partonic scattering

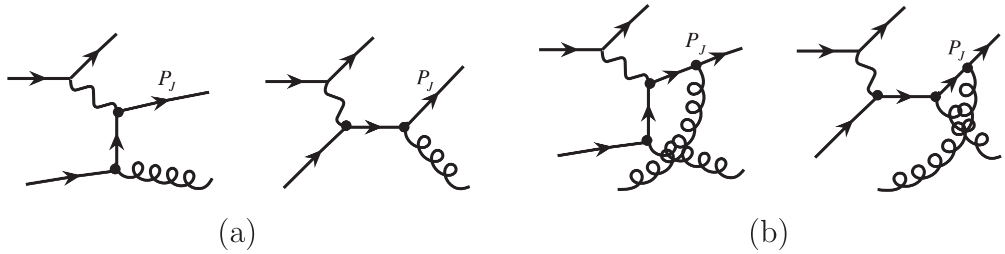

FIG. 3. (a) Next-to-leading order tree diagrams for lepton scattering on an asymptotic quark state. (b) Next-to-leading order tree diagrams for lepton scattering on an asymptotic quark-gluon composite state.

cross section. In general, the perturbatively calculated hard parts are effectively equal to the partonic cross sections with all collinear divergences removed.

In the collinear factorization approach, the spin-dependent cross section to the SSAs comes from the interference of the real part of the scattering amplitude with one active parton and the imaginary part of the scattering amplitude with two active partons [2,4,5,9]. For the LO contribution, the partonic hard part $H^{lq\to q(P_J)X}(x, x', P_{JT}, y)$ in Eq. (5) is given by the diagram in Fig. 1(b), and will be calculated in the following section.

For the higher order corrections, the partonic hard part $H^{ab\to \mathrm{Jet}(P_J)X}(x, x', P_{JT}, y, \mu)$ in Eq. (5) can be calculated in the same way by applying the formalism to the various partonic states. For example, the LO photon-parton scattering contribution to the SSA comes from the interference of Feynman diagrams in Fig. 2(a) and 2(b). Similarly, the NLO lepton-quark scattering contribution to the SSA comes from the interference of Feynman diagrams in Fig. 3(a) and 3(b). In the rest of this paper, we will present our derivation and results of the LO contribution to various spin asymmetries. We will leave the explicit treatment of higher order corrections to future work.

III. KINEMATICS AND ANALYTICAL RESULTS

In this section we present some details of the kinematics for the process $l(l) + p(P) \to \mathrm{jet}(P_J) + X$, as well as the tree-level formulas for the various observables. We use the momenta of the particles to fix a coordinate system according to $\hat{e}_z = \vec{P}/|\vec{P}| = -\vec{l}/|\vec{l}|$, $\hat{e}_x = \vec{P}_{JT}/|\vec{P}_{JT}|$, and $\hat{e}_y = \hat{e}_z \times \hat{e}_x$. Mandelstam variables are defined by

$$s = (l+P)^2, \qquad t = (P-P_J)^2, \qquad u = (l-P_J)^2, \tag{11}$$

while on the partonic level one has

$$\hat{s} = (l+k)^2 = xs, \qquad \hat{t} = (k-P_J)^2 = xt, \qquad \hat{u} = (l-P_J)^2 = u, \tag{12}$$

with k denoting the momentum of the active quark in the proton; see also Fig. 1(a). The momentum fraction x specifies the plus momentum of the quark through $k^+ = xP^+$.[1] Using $\hat{s} + \hat{t} + \hat{u} = 0$ one finds that $x = -u/(s+t)$. In other words, the longitudinal momentum of the struck quark is fixed by the external kinematics of the process, like it is in fully inclusive DIS. Of course, this no longer applies once higher order corrections are taken into account. For the numerical estimates we will use P_{JT}, and the Feynman variable x_F (defined in the lepton-proton *cm*-frame) for which one has

$$x_F = \frac{2P_{Jz}}{\sqrt{s}} = \frac{t-u}{s}. \tag{13}$$

Next, we turn to the polarization observables for $lp \to \mathrm{jet}X$, which we compute in the collinear factorization framework. We restrict ourselves to one-photon exchange between the leptonic and the hadronic part of the process. Allowing for longitudinal polarization of the initial-state lepton, as well as longitudinal and transverse polarization of the proton target, one finds the following expression for the cross section[2]:

$$\begin{aligned} P_J^0\frac{d^3\sigma}{d^3P_J} = \frac{\alpha_{em}^2}{s}\sum_a \frac{e_a^2}{(s+t)x}\Big\{ & f_1^a(x)H_{UU} + \lambda_l\lambda_p g_1^a(x)H_{LL} \\ & + 2\pi M\epsilon_T^{ij}S_T^i P_{JT}^j\Big[T_F^a(x,x) - x\frac{d}{dx}T_F^a(x,x)\Big]\frac{\hat{s}}{\hat{t}\hat{u}}H_{UU} \\ & + \lambda_l 2M\vec{S}_T\cdot\vec{P}_{JT}\Big[(\tilde{g}^a(x) - x\frac{d}{dx}\tilde{g}^a(x))\frac{\hat{s}}{\hat{t}\hat{u}}H_{LL} \\ & + xg_T^a(x)\frac{2}{\hat{t}}\Big]\Big\}. \end{aligned} \tag{14}$$

In Eq. (14), which is the main analytical result of our work, λ_l and λ_p represent the helicity of the lepton and the proton, respectively. One can project out the four independent components of the cross section in (14) according to

$$\sigma_{UU} = \frac{1}{4}(\sigma(+,+) + \sigma(-,+) + \sigma(+,-) + \sigma(-,-)), \tag{15}$$

$$\sigma_{LL} = \frac{1}{4}([\sigma(+,+) - \sigma(-,+)] - [\sigma(+,-) - \sigma(-,-)]), \tag{16}$$

$$\sigma_{UT} = \frac{1}{4}([\sigma(+,\uparrow_y) + \sigma(-,\uparrow_y)] - [\sigma(+,\downarrow_y) + \sigma(-,\downarrow_y)]), \tag{17}$$

$$\sigma_{LT} = \frac{1}{4}([\sigma(+,\uparrow_x) - \sigma(-,\uparrow_x)] - [\sigma(+,\downarrow_x) - \sigma(-,\downarrow_x)]). \tag{18}$$

In these formulas, '+' and '−' indicate particle helicities, whereas '$\uparrow_{x/y}$' ('$\downarrow_{x/y}$') denotes transverse polarization of the proton along $\hat{e}_{x/y}(-\hat{e}_{x/y})$.

As already mentioned, both σ_{UU} and σ_{LL} are twist-2 observables. We computed them on the basis of diagram (a) in Fig. 1 by applying the collinear approximation to the momentum k of the active quark. In the case of σ_{UU} the result contains the unpolarized quark distribution f_1^a, while for σ_{LL} the quark helicity distribution g_1^a shows up. The hard scattering coefficients for these two terms in (14),

[1] For a generic four-vector v, we define light-cone coordinates according to $v^\pm = (v^0 \pm v^3)/\sqrt{2}$ and $\vec{v}_T = (v^1, v^2)$.

[2] Polarization degrees are suppressed in the cross section formula (14).

expressed through the partonic Mandelstam variables in (12), read

$$H_{UU} = \frac{2(\hat{s}^2 + \hat{u}^2)}{\hat{t}^2}, \qquad H_{LL} = \frac{2(\hat{s}^2 - \hat{u}^2)}{\hat{t}^2}. \tag{19}$$

The cross sections σ_{UT} and σ_{LT} (3rd and 4th term on the RHS of (14), respectively) represent twist-3 observables. (Note that M is the proton mass, and $\varepsilon_T^{ij} \equiv \varepsilon^{-+ij}$ with $\varepsilon^{0123} = 1$.) The transverse SSA $A_{UT} = \sigma_{UT}/\sigma_{UU}$ is analogous to the SSA A_N which has been extensively studied in one-particle inclusive production for hadron-hadron collisions; see also Ref. [52], and [58–60] for experimental results from RHIC. A similar observable was proposed in [61,62] for semi-inclusive DIS, but in this case the final-state lepton still needs to be observed.

Calculational details for such twist-3 observables in collinear factorization can be found in various papers; see, e.g., Refs. [4,5,9,22]. We merely mention that one has to expand the hard scattering contributions around vanishing transverse parton momenta. While for twist-2 effects only the leading term of that expansion matters, in the case of twist-3 the second term is also relevant. In addition, the contribution from quark-gluon-quark correlations, as displayed in diagram (b) in Fig. 1, needs to be taken into consideration. The sum of all the terms can be written in a color gauge invariant form, which provides a consistency check of the calculation.

The quark-gluon-quark correlator showing up in σ_{UT} is the aforementioned ETQS matrix element $T_F^a(x,x)$ [2,4]. The peculiar feature of this object is the vanishing gluon momentum—that's why it is also called a "soft gluon pole matrix element". If the gluon momentum becomes soft one can hit the pole of a quark propagator in the partonic scattering process, providing an imaginary part (nontrivial phase) which, quite generally, can lead to single spin effects [2,4]. Note also that in our lowest order calculation of σ_{UT} no so-called soft fermion pole contribution (see [20] and references therein) emerges. For σ_{LT} another quark-gluon-quark matrix element—denoted as $\tilde{g}^a$; see, in particular, Refs. [8,22,28]—appears, together with the familiar twist-3 quark-quark correlator g_T^a.

We use the common definitions for g_1 and g_T. The quark-gluon-quark correlators T_F and $\tilde{g}$ are specified according to[3]

$$T_F(x,x) = \frac{1}{2M}\int \frac{d\xi^- d\zeta^-}{(2\pi)^2} e^{ixP^+\xi^-} \times \langle P, S_T|\bar{\psi}(0)\gamma^+ igF^{+i}(\zeta^-)\psi(\xi^-)|P, S_T\rangle(i\varepsilon_T^{ij}S_T^j), \tag{20}$$

$$\tilde{g}(x) = \frac{1}{2M}\int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-}(S_T^i)\langle P, S_T|\bar{\psi}(0)\gamma_5\gamma^+ \times \left(iD_T^i - ig\int_0^\infty d\zeta^- F^{+i}(\zeta^-)\right)\psi(\xi^-)|P, S_T\rangle, \tag{21}$$

with $F^{\mu\nu}$ representing the gluon field strength tensor, and $D^\mu = \partial^\mu - igA^\mu$ the covariant derivative. Equations (20) and (21) hold in the light-cone gauge $A^+ = 0$, while in a general gauge Wilson lines need to be inserted between the field operators.

It is important that T_F and $\tilde{g}$ are related to moments of transverse momentum dependent parton distributions. To be explicit, one has [22,28,41,42]

$$\pi T_F(x,x) = -\int d^2k_T \frac{\vec{k}_T^2}{2M^2} f_{1T}^{\perp}(x, \vec{k}_T^2)|_{\mathrm{DIS}}, \tag{22}$$

$$\tilde{g}(x) = \int d^2k_T \frac{\vec{k}_T^2}{2M^2} g_{1T}(x, \vec{k}_T^2), \tag{23}$$

where we use the conventions of Refs. [63–65] for the transverse momentum dependent correlators $f_{1T}^{\perp}$ and g_{1T}. In Eq. (14) we take into account that the Sivers function $f_{1T}^{\perp}$ [43] depends on the process in which it is probed [66,67]. In order to obtain numerical estimates for σ_{UT} and σ_{LT} we will exploit the relations in (22) and (23).

IV. NUMERICAL ESTIMATES

Now we move on to discuss numerical estimates for the polarization observables. To this end we consider the three spin asymmetries A_{LL}, A_{UT}, and A_{LT}, whose definitions are repeated here for convenience,

$$A_{LL} = \frac{\sigma_{LL}}{\sigma_{UU}}, \qquad A_{UT} = \frac{\sigma_{UT}}{\sigma_{UU}}, \qquad A_{LT} = \frac{\sigma_{LT}}{\sigma_{UU}}. \tag{24}$$

To compute σ_{UU} we use the unpolarized parton distributions from the CTEQ5-parametrization [68]. The helicity distributions entering σ_{LL} are taken from the parametrization [69]. For the ETQS matrix element T_F we explore two choices: (1) we use the relation (22) between T_F and the Sivers function, and take $f_{1T}^{\perp}$ from the recent fit provided in Ref. [51]; (2) we use T_F from the parametrization obtained in [9]—taking into consideration the recently discovered sign change [70]—by fitting transverse SSAs measured in hadronic collisions.

In the case of σ_{LT} one needs input for g_T and $\tilde{g}$. For g_T we resort to the frequently used Wandzura-Wilczek approximation [71] (see [72] for a recent study of the quality of this approximation)

$$g_T(x) \approx \int_x^1 \frac{dy}{y} g_1(y), \tag{25}$$

whereas for $\tilde{g}$ we use (23) and a Wandzura-Wilczek-type approximation for the particular k_T moment of g_{1T} in (23) [73], leading to

$$\tilde{g}(x) \approx x\int_x^1 \frac{dy}{y} g_1(y). \tag{26}$$

We mention that (26) and a corresponding relation between chiral-odd parton distributions were used in [74,75] in

[3] Note that in the literature different conventions for T_F exist.

order to estimate certain spin asymmetries in semi-inclusive DIS. The comparison to data discussed in [75] looks promising, though more experimental information is needed for a thorough test of approximate relations like the one in (26). Measuring the double spin asymmetry A_{LT}, which we consider in the present paper, may provide such a test.

For our numerical estimates we use leading order parton distributions, and take into account the three light quark flavors. The transverse momentum of the jet P_{JT} serves as the scale for the parton distributions. For the following reasons the scale dependence of all the asymmetries is rather weak: because the leading order evolution kernels for f_1 and g_1 are the same, A_{LL} is almost scale independent. This also applies to A_{LT} when using the approximations (25) and (26). Since both the parametrization of the Sivers function in [51] as well as the one for the ETQS matrix element T_F in [9,70] are related to the unpolarized distribution f_1, also the scale dependence of A_{UT} is quite mild.

Our results are for typical kinematics accessible at a potential future electron-ion collider [39,40]: we consider the energies $\sqrt{s} = 50$ GeV and $\sqrt{s} = 100$ GeV. The asymmetries are either presented as function of x_F for fixed P_{JT} or vice versa.

We start by discussing the twist-2 asymmetry A_{LL}. As shown in Fig. 4, this observable is relatively small (on the percent level). It is largest in the backward region (negative x_F), and rises with increasing P_{JT}. Despite the small effect, measuring A_{LL} could provide complementary information on the quark helicity distributions of the proton. On the other hand, for the longitudinal double spin asymmetry in $lp \to \text{jet}X$ one faces the same problems one has in inclusive DIS: quarks and antiquarks enter with equal weight, and a flavor separation is hardly possible. However, if instead one considers A_{LL} for inclusive hadron production these problems can, in principle, be circumvented like in semi-inclusive DIS. According to Fig. 4, A_{LL} clearly increases towards lower values of $\sqrt{s}$. Therefore, A_{LL} for $lp \to HX$ (at $\sqrt{s} < 50$ GeV) should definitely be a very interesting observable for studying the quark helicity structure of the proton.

Let us now turn to the transverse SSA A_{UT}, which is displayed in Fig. 5 and 6. For both parametrizations we obtain a healthy asymmetry in the forward region, with effects at the level 5–10%. (Note also that, for the parametrization taken from [51], our numerical results in the collinear approach are similar to those obtained in Ref. [52] by using factorization in terms of transverse momentum dependent parton correlators and the same input for the Sivers function.) The asymmetry drops with increasing P_{JT} and hardly changes when varying $\sqrt{s}$ (see Fig. 6). The weak energy-dependence of A_{UT} appears

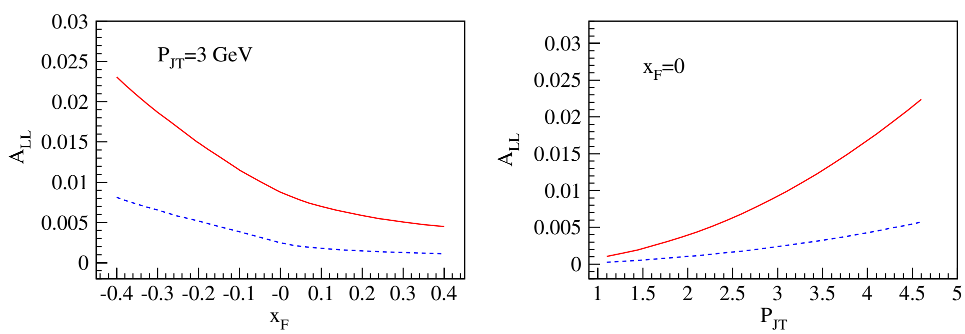

FIG. 4 (color online). A_{LL} as a function of x_F (left) and P_{JT} (right). Solid line: $\sqrt{s} = 50$ GeV; dashed line: $\sqrt{s} = 100$ GeV.

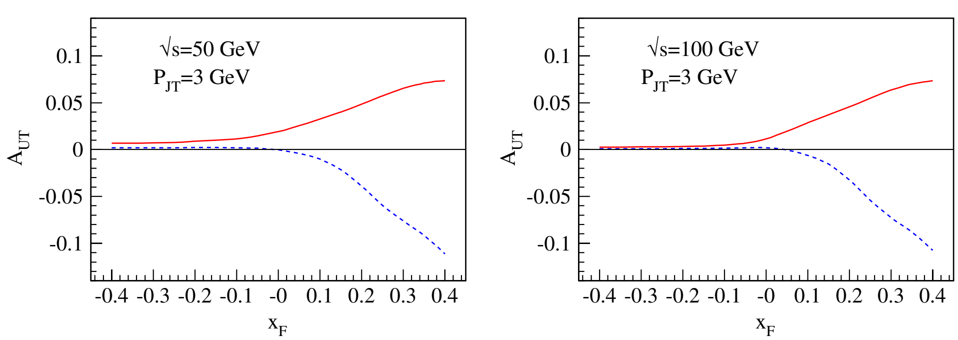

FIG. 5 (color online). A_{UT} as a function of x_F for $\sqrt{s} = 50$ GeV (left) and $\sqrt{s} = 100$ GeV (right). The solid line is for T_F taken from the fit of the Sivers function in [51] and using the relation in (22), while the dashed line is computed with T_F from [9,70].

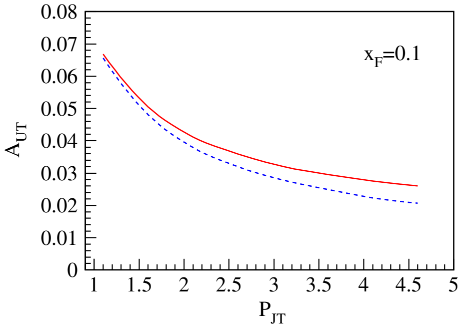

FIG. 6 (color online). A_{UT} as a function of P_{JT} at $x_F = 0.1$. Solid line: $\sqrt{s} = 50$ GeV; dashed line: $\sqrt{s} = 100$ GeV. The matrix element T_F is taken from the fit of the Sivers function in [51] and using the relation in (22).

because, according to (14), both σ_{UU} and σ_{UT} have the same hard scattering coefficient. From Fig. 5 it is obvious that the transverse SSA A_{UT} seems very promising in order to experimentally constrain the ETQS matrix element T_F and the Sivers function. Such a constraint is of utmost importance as the existing parametrizations even differ in sign [70]. If P_{JT} is sufficiently large, our leading order calculation should give a reliable estimate of the asymmetry. One may even be able to check whether $T_F(x, x)$ changes sign as function of x, as has been recently speculated [76].

Finally, our numerical estimates for A_{LT} are shown in Fig. 7. This asymmetry is apparently too small to be measured. The hard scattering coefficient of σ_{LT} is the same than the one for σ_{LL}. Therefore, A_{LT} decreases with increasing energy like A_{LL} does. The main reason for A_{LT} being even much smaller than A_{LL} can be traced back to the factor x showing up on the RHS of the Wandzura-Wilczek-type approximation (26). Because of this, an experimental study of A_{LT} can serve as an interesting check of the relation (26), as already pointed out above. In addition, like in the case of A_{LL}, measurable effects for A_{LT} in hadron production at lower values of $\sqrt{s}$ can be expected.

V. SUMMARY AND DISCUSSION

We have studied a complete set of polarization observables for the process $lp \to \mathrm{jet}X$. Neglecting parity violating contributions and transverse polarization of the lepton one can consider three spin asymmetries: A_{LL}, A_{UT}, and A_{LT}. We have computed these asymmetries at the level of Born diagrams in collinear factorization. Moreover, numerical estimates for typical kinematics of a potential future electron-ion collider have been provided. (We have explored the *cm* energies $\sqrt{s} = 50$ GeV and $\sqrt{s} = 100$ GeV.) In the following we summarize our findings and add some discussion:

(i) We have discussed in detail how to calculate the process systematically at higher orders in perturbation theory. It is important to extend our explicit calculations to the 1-loop level for two reasons. First, one must investigate how stable the various asymmetries are upon inclusion of NLO corrections, in particular, the logarithmically enhanced $\gamma - q$ channel as pointed out in Sec. II. Second, a 1-loop calculation for twist-3 observables, in which derivatives of quark-gluon-quark correlators (see dT_F/dx and $d\tilde{g}/dx$ in (14)) appear, has never been done before.

(ii) The numerical result for the double spin asymmetry A_{LL} (A_{LT}) is small (tiny)—on the percent level for A_{LL}. However, in both cases significant effects can be expected for lower values of $\sqrt{s}$, let us say around 10–20 GeV. Of course, in this region one cannot perform jet measurements but has rather to consider inclusive hadron production.

(iii) We find that in the forward region the transverse SSA A_{UT} can become of the order 5%–10%. This observable gives a direct handle on the ETQS twist-3 matrix element T_F—from a theoretical point of view it is one of the simplest observables for addressing T_F (to leading order neither soft fermion poles, nor trigluon correlations, nor hard gluon poles contribute)—and also, by means of (22), to the transverse momentum dependent Sivers

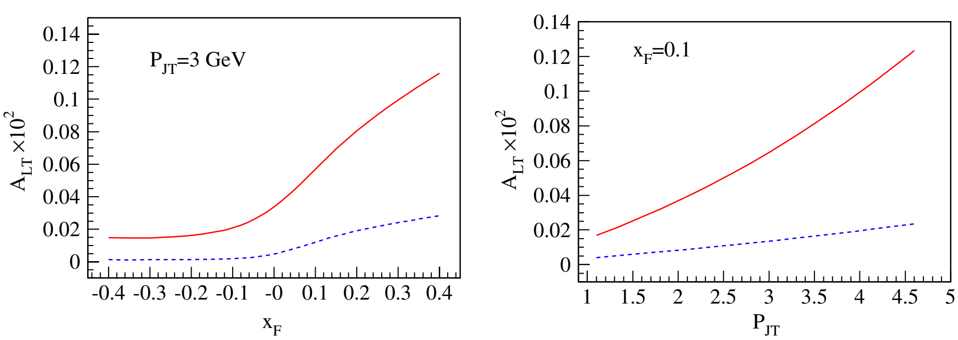

FIG. 7 (color online). A_{LT} as a function of x_F (left) and P_{JT} (right). Solid line: $\sqrt{s} = 50$ GeV; dashed line: $\sqrt{s} = 100$ GeV. Note that A_{LT} is scaled by a factor of 100.

function $f_{1T}^{\perp}$. Given the fact that at present even the sign of T_F is unclear [70], experimental information on this observable would be very valuable.

In general, we believe that there is sufficient justification for further exploration of the potential of high-energy lepton-nucleon scattering with an unidentified final-state lepton. This type of reaction may also constitute an interesting part of the physics program at a future electron-ion collider.

ACKNOWLEDGMENTS

We thank Naomi Makins, Werner Vogelsang, and Feng Yuan for helpful discussions. We are grateful to RIKEN, Brookhaven National Laboratory, and the U.S. Department of Energy (Contract No. DE-AC02-98CH10886) for providing the facilities essential for the completion of this work. A. M. acknowledges the support of the NSF under Grant No. PHY-0855501.

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