1^{++} nonet singlet-octet mixing angle, strange quark mass, and strange quark condensate

Kwei-Chou Yang*

Department of Physics, Chung Yuan Christian University, Chung-Li 320, Taiwan

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Two strategies are taken into account to determine the $f_1(1420)$-$f_1(1285)$ mixing angle θ. (i) First, using the Gell-Mann-Okubo mass formula together with the $K_1(1270)$-$K_1(1400)$ mixing angle $\theta_{K_1} = (-34 \pm 13)^\circ$ extracted from the data for $\mathcal{B}(B \to K_1(1270)\gamma)$, $\mathcal{B}(B \to K_1(1400)\gamma)$, $\mathcal{B}(\tau \to K_1(1270)\nu_\tau)$, and $\mathcal{B}(\tau \to K_1(1420)\nu_\tau)$, gave $\theta = (23^{+17}_{-23})^\circ$. (ii) Second, from the study of the ratio for $f_1(1285) \to \phi\gamma$ and $f_1(1285) \to \rho^0\gamma$ branching fractions, we have a twofold solution $\theta = (19.4^{+4.5}_{-4.6})^\circ$ or $(51.1^{+4.5}_{-4.6})^\circ$. Combining these two analyses, we thus obtain $\theta = (19.4^{+4.5}_{-4.6})^\circ$. We further compute the strange quark mass and strange quark condensate from the analysis of the $f_1(1420)$-$f_1(1285)$ mass difference QCD sum rule, where the operator-product-expansion series is up to dimension six and to $\mathcal{O}(\alpha_s^3, m_s^2\alpha_s^2)$ accuracy. Using the average of the recent lattice results and the θ value that we have obtained as inputs, we get $\langle\bar{s}s\rangle/\langle\bar{u}u\rangle = 0.41 \pm 0.09$.

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I. INTRODUCTION

The $f_1(1285)$ and $f_1(1420)$ mesons with quantum number $J^{PC} = 1^{++}$ are the members of the 1^3P_1 states in the quark model language and are mixtures of the pure octet f_8 and singlet f_1, where the mixing is characterized by the mixing angle θ. The *BABAR* results for the upper bounds of $B^- \to f_1(1285)K^-$, $f_1(1420)K^-$ were available recently [1]. The relative ratio of these two modes is highly sensitive to θ [2]. On the other hand, in the two-body B decay involving the K meson in the final state, the amplitude receives large corrections from the chiral enhancement a_6 term which is inversely proportional to the strange quark mass. The quark mass term mixes left- and right-handed quarks in the QCD Lagrangian. The spontaneous breaking of chiral symmetry from $SU(3)_L \times SU(3)_R$ to $SU(3)_V$ is further broken by the quark masses $m_{u,d,s}$ when the baryon number is added to the three commuting conserved quantities Q_u, Q_d, and Q_s, respectively, the numbers of $q - \bar{q}$ quarks for $q = u$, d, and s. The nonzero quark condensate which signals dynamical symmetry breaking is the important parameter in QCD sum rules [3], while the magnitude of the strange quark mass can result in the flavor symmetry breaking in the quark condensate. In an earlier study $\langle\bar{s}s\rangle/\langle\bar{u}u\rangle \sim 0.8 < 1$ was usually taken. However, very recently the Jamin-Lange approach [4] together with the lattice result for f_{B_s}/f_B [5] and also the Schwinger-Dyson equation approach [6] can give a central value larger than 1.

In this paper, we shall embark on the study of the $f_1(1420)$ and $f_1(1285)$ mesons to determine the mixing angle θ, strange quark mass, and strange quark condensate. In Sec. II, we shall present detailed discussions on the determination of the mixing angle θ. Substituting the $K_1(1270)$-$K_1(1400)$ mixing angle, which was extracted from the $B \to K_1\gamma$ and $\tau \to K_1\nu_\tau$ data, to the Gell-Mann-Okubo mass formula, we can derive the value of θ. Alternatively, from the analysis of the decay ratio for $f_1(1285) \to \phi\gamma$ and $f_1(1285) \to \rho^0\gamma$, we have a more accurate estimation for θ. In Sec. III we shall obtain the mass difference QCD sum rules for the $f_1(1420)$ and $f_1(1285)$ to determine the magnitude of the strange quark mass. From the sum rule analysis, we obtain the constraint ranges for m_s and θ as well as for $\langle\bar{s}s\rangle$. Many attempts have been made to compute m_s using QCD sum rules and finite energy sum rules [7–13]. The running strange quark mass in the $\overline{\mathrm{MS}}$ scheme at a scale of $\mu \approx 2$ GeV is $m_s = 101^{+29}_{-21}$ MeV given in the particle data group (PDG) average [14]. More precise lattice estimates have been recently obtained as $m_s(2\ \mathrm{GeV}) = 92.2(1.3)$ MeV in [15], $m_s(2\ \mathrm{GeV}) = 96.2(2.7)$ MeV in [16], and $m_s(2\ \mathrm{GeV}) = 95.1(1.1)(1.5)$ MeV in [17]. These lattice results agree with strange scalar/pseudoscalar sum rule results which are $m_s \simeq 95(15)$ MeV. In the present study, we study the m_s from a new frame, the $f_1(1420)$-$f_1(1285)$ mass difference sum rule, which may result in larger uncertainties due to the input parameters. Nevertheless, it can be a cross-check compared with the previous studies. Further using the very recent lattice result for $m_s(2\ \mathrm{GeV}) = 93.6 \pm 1.0$ MeV as the input, we obtain an estimate for the strange quark condensate.

II. SINGLET-OCTET MIXING ANGLE θ OF THE 1^{++} NONET

A. Definition

In the quark model, $a_1(1260)$, $f_1(1285)$, $f_1(1420)$, and K_{1A} are classified in 1^{++} multiplets, which, in terms of spectroscopic notation $n^{2S+1}L_J$, are 1^3P_1 p-wave mesons. Analogous to η and η', because of $SU(3)$ breaking effects, $f_1(1285)$ and $f_1(1420)$ are the mixing states of the pure octet f_8 and singlet f_1,

*kcyang@cycu.edu.tw

$$|f_1(1285)\rangle = |f_1\rangle \cos\theta + |f_8\rangle \sin\theta, \\ |f_1(1420)\rangle = -|f_1\rangle \sin\theta + |f_8\rangle \cos\theta. \tag{1}$$

In the present paper, we adopt

$$f_1 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s), \tag{2}$$

$$f_8 = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s), \tag{3}$$

where there is a relative sign difference between the $\bar{s}s$ contents of f_1 and f_8 in our convention. From the Gell-Mann-Okubo mass formula, the mixing angle θ satisfies

$$\cos^2\theta = \frac{3m^2_{f_1(1285)} - (4m^2_{K_{1A}} - m^2_{a_1})}{3(m^2_{f_1(1285)} - m^2_{f_1(1420)})}, \tag{4}$$

where

$$m^2_{K_{1A}} = \langle K_{1A}|\mathcal{H}|K_{1A}\rangle \\ = m^2_{K_1(1400)}\cos^2\theta_{K_1} + m^2_{K_1(1270)}\sin^2\theta_{K_1}, \tag{5}$$

with $\mathcal{H}$ being the Hamiltonian. Here θ_{K_1} is the $K_1(1400)$-$K_1(1270)$ mixing angle. The sign of the mixing angle θ can be determined from the mass relation [14]

$$\tan\theta = \frac{4m^2_{K_{1A}} - m^2_{a_1} - 3m^2_{f_1(1420)}}{3m^2_{18}}, \tag{6}$$

where $m^2_{18} = \langle f_1|\mathcal{H}|f_8\rangle \simeq (m^2_{a_1} - m^2_{K_{1A}})2\sqrt{2}/3 < 0$, we find $\theta > 0$. Because of the strange and nonstrange light quark mass differences, K_{1A} is not the mass eigenstate and it can mix with K_{1B}, which is one of the members in the 1^1P_1 multiplets. From the convention in [18] (see also discussions in [19,20]), we write the two physical states $K_1(1270)$ and $K_1(1400)$ in the following relations:

$$|K_1(1270)\rangle = |K_{1A}\rangle \sin\theta_K + |K_{1B}\rangle \cos\theta_K, \\ |K_1(1400)\rangle = |K_{1A}\rangle \cos\theta_K - |K_{1B}\rangle \sin\theta_K. \tag{7}$$

The mixing angle was found to be $|\theta_{K_1}| \approx 33°, 57°$ in [18] and $\approx \pm 37°, \pm 58°$ in [21]. A similar range $35° \lesssim |\theta_{K_1}| \lesssim 55°$ was obtained in [22]. The sign ambiguity for θ_{K_1} is due to the fact that one can add arbitrary phases to $|\bar{K}_{1A}\rangle$ and $|\bar{K}_{1B}\rangle$. This sign ambiguity can be removed by fixing the signs of decay constants $f_{K_{1A}}$ and $f^{\perp}_{K_{1B}}$, which are defined by

$$\langle 0|\bar{\psi}\gamma_\mu\gamma_5 s|\bar{K}_{1A}(P,\lambda)\rangle = -if_{K_{1A}} m_{K_{1A}} \epsilon^{(\lambda)}_\mu, \tag{8}$$

$$\langle 0|\bar{\psi}\sigma_{\mu\nu} s|\bar{K}_{1B}(P,\lambda)\rangle = if^{\perp}_{K_{1B}} \epsilon_{\mu\nu\alpha\beta}\epsilon^{\alpha}_{(\lambda)} P^\beta, \tag{9}$$

where $\epsilon^{0123} = -1$ and $\psi \equiv u$ or d. Following the convention in [20], we adopt $f_{K_{1A}} > 0$, $f^{\perp}_{K_{1B}} > 0$, so that θ_{K_1} should be negative to account for the observable $\mathcal{B}(B \to K_1(1270)\gamma) \gg \mathcal{B}(B \to K_1(1400)\gamma)$ [23,24]. Furthermore, from the data of $\tau \to K_1(1270)\nu_\tau$ and $K_1(1400)\nu_\tau$ decays together with the sum rule results for the K_{1A} and K_{1B} decay constants, the mixing angle $\theta_{K_1} = (-34 \pm 13)°$ was obtained in [24]. Substituting this value into (4), we then obtain $\theta^{\rm quad} = (23^{+17}_{-23})°$ [25], i.e., $\theta^{\rm quad} = 0° - 40°$.[1]

B. The determination of θ

Experimentally, since $K^*\bar{K}$ and $K\bar{K}\pi$ are the dominant modes of $f_1(1420)$, whereas $f_0(1285)$ decays mainly to the 4π states, this suggests that the quark content is primarily $s\bar{s}$ for $f_1(1420)$ and $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ for $f_1(1285)$. Therefore, the mixing relations can be rewritten to exhibit the $n\bar{n}$ and $s\bar{s}$ components which decouple for the ideal mixing angle $\theta_i = \tan^{-1}(1/\sqrt{2}) \simeq 35.3°$. Let $\bar{\alpha} = \theta_i - \theta$, we rewrite these two states in the flavor basis,[2]

$$f_1(1285) = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)\cos\bar{\alpha} + \bar{s}s\sin\bar{\alpha}, \\ f_1(1420) = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)\sin\bar{\alpha} - \bar{s}s\cos\bar{\alpha}. \tag{10}$$

Since the $f_1(1285)$ can decay into $\phi\gamma$, we know that $f_1(1285)$ has the $s\bar{s}$ content and θ deviates from its ideal mixing value. To have a more precise estimate for θ, we study the ratio of $f_1(1285) \to \phi\gamma$ and $f_1(1285) \to \rho^0\gamma$ branching fractions. Because the electromagnetic (EM) interaction Lagrangian is given by

$$\mathcal{L}_I = -A^\mu_{\rm EM}(e_u\bar{u}\gamma_\mu u + e_d\bar{d}\gamma_\mu d + e_s\bar{s}\gamma_\mu s) \\ = -A^\mu_{\rm EM}\bigg((e_u + e_d)\frac{\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d}{2} \\ + (e_u - e_d)\frac{\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d}{2} + e_s\bar{s}\gamma_\mu s\bigg), \tag{11}$$

with $e_u = 2/3e$, $e_d = -1/3e$, and $e_s = -1/3e$ being the electric charges of u, d, and s quarks, respectively, we obtain

[1]Replacing the meson mass squared m^2 by m throughout (4), we obtain $\theta^{\rm lin} = (23^{+17}_{-23})°$. The difference is negligible. Our result can be compared with that using $\theta_{K_1} = -57°$ into (4); one has $\theta^{\rm quad} = 52°$.

[2]In PDG [14], the mixing angle is defined as $\alpha = \theta - \theta_i + \pi/2$. Comparing it with our definition, we have $\alpha = \pi/2 - \bar{\alpha}$.

$$\frac{\mathcal{B}(f_1(1285)\to\phi\gamma)}{\mathcal{B}(f_1(1285)\to\rho^0\gamma)} = \left(\frac{\langle\phi|e_s\bar{s}\gamma_\mu s|f_1(1285)\rangle}{\langle\rho|(e_u-e_d)(\bar{u}\gamma_\mu u-\bar{d}\gamma_\mu d)/2|f_1(1285)\rangle}\right)^2 = \underbrace{\left(\frac{m_{f_1}^2-m_\phi^2}{m_{f_1}^2-m_\rho^2}\right)^3}_{\text{phase factor}}$$

$$= \underbrace{\left(\frac{-e/3}{2e/3+e/3}\right)^2}_{\text{EM factor}}\left(\frac{\langle\phi|\bar{s}\gamma_\mu s|f_1(1285)\rangle}{\langle\rho|(\bar{u}\gamma_\mu u-\bar{d}\gamma_\mu d)/2|f_1(1285)\rangle}\right)^2\underbrace{\left(\frac{m_{f_1}^2-m_\phi^2}{m_{f_1}^2-m_\rho^2}\right)^3}_{\text{phase factor}}$$

$$\approx \frac{4}{9}\left(\frac{m_\phi f_\phi}{m_\rho f_\rho}\right)^2\tan^2\bar{\alpha}\left(\frac{m_{f_1}^2-m_\phi^2}{m_{f_1}^2-m_\rho^2}\right)^3, \tag{12}$$

where $f_1 \equiv f_1(1285)$, and f_ϕ and f_ρ are the decay constants of ϕ and ρ, respectively. Here we have taken the single-pole approximation[3]:

$$\frac{\langle\phi|\bar{s}\gamma_\mu s|f_1(1285)\rangle}{\langle\rho|(\bar{u}\gamma_\mu u-\bar{d}\gamma_\mu d)/2|f_1(1285)\rangle} \approx \frac{m_\phi f_\phi g_{f_1\phi\phi}}{m_\rho f_\rho g_{f_1\rho\rho}/\sqrt{2}}\frac{\sin\bar{\alpha}}{\cos\bar{\alpha}/\sqrt{2}} \approx \frac{m_\phi f_\phi}{m_\rho f_\rho}\times 2\tan\bar{\alpha}. \tag{13}$$

Using $f_\rho = 209 \pm 1$ MeV, $f_\phi = 221 \pm 3$ MeV [27], and the current data $\mathcal{B}(f_1(1285)\to\phi\gamma) = (7.4\pm 2.6)\times 10^{-4}$ and $\mathcal{B}(f_1(1285)\to\rho^0\gamma) = (5.5\pm 1.3)\%$ [14] as inputs, we obtain $\bar{\alpha} = \pm(15.8^{+4.5}_{-4.6})^\circ$, i.e., the twofold solution $\theta = (19.4^{+4.5}_{-4.6})^\circ$ or $(51.1^{+4.5}_{-4.6})^\circ$. Combining with the analysis $\theta = (0\text{–}40)^\circ$ given in Sec. II A, we thus find that $\theta = (19.4^{+4.5}_{-4.6})^\circ$ is much preferred and can explain experimental observables well.

III. MASS OF THE STRANGE QUARK

We proceed to evaluate the strange quark mass from the mass difference sum rules of the $f_1(1285)$ and $f_1(1420)$ mesons. We consider the following two-point correlation functions:

$$\Pi_{\mu\nu}(q^2) = i\int d^4x e^{iqx}\langle 0|T(j_\mu(x)j_\nu^\dagger(0))|0\rangle = -\Pi_1(q^2)g_{\mu\nu} + \Pi_2(q^2)q_\mu q_\nu, \tag{14}$$

$$\Pi'_{\mu\nu}(q^2) = i\int d^4x e^{iqx}\langle 0|T(j'_\mu(x)j_\nu'^\dagger(0))|0\rangle = -\Pi'_1(q^2)g_{\mu\nu} + \Pi'_2(q^2)q_\mu q_\nu. \tag{15}$$

The interpolating currents satisfying the relations,

$$\langle 0|j_\mu^{(\prime)}(0)|f_1^{(\prime)}(P,\lambda)\rangle = -if_{f_1^{(\prime)}}m_{f_1^{(\prime)}}\epsilon_\mu^{(\lambda)}, \tag{16}$$

are

$$j_\mu = \cos\theta j_\mu^{(1)} + \sin\theta j_\mu^{(8)}, \tag{17}$$

$$j'_\mu = -\sin\theta j_\mu^{(1)} + \cos\theta j_\mu^{(8)}, \tag{18}$$

where

$$j_\mu^{(1)} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s), \tag{19}$$

$$j_\mu^{(8)} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s), \tag{20}$$

and we have used the shorthand notations for $f_1 \equiv f_1(1285)$ and $f'_1 \equiv f_1(1420)$. In the massless quark limit, we have $\Pi_1 = q^2\Pi_2$ and $\Pi'_1 = q^2\Pi'_2$ if one neglects the axial-vector anomaly.[4] Here we focus on $\Pi_1^{(\prime)}$ since it receives contributions only from axial-vector ($^3P_1$) mesons, whereas $\Pi_2^{(\prime)}$ contains effects from pseudoscalar mesons. The lowest-lying $f_1^{(\prime)}$ meson contribution can be approximated via the dispersion relation as

$$\frac{m_{f_1^{(\prime)}}^2 f_{f_1^{(\prime)}}^2}{m_{f_1^{(\prime)}}^2 - q^2} = \frac{1}{\pi}\int_0^{s_0^{f^{(\prime)}}} ds\frac{\mathrm{Im}\Pi_1^{(\prime)\mathrm{OPE}}(s)}{s-q^2}, \tag{21}$$

where $\Pi_1^{(\prime)\mathrm{OPE}}$ is the QCD operator-product-expansion (OPE) result of $\Pi_1^{(\prime)}$ at the quark-gluon level [20], and $s_0^{f_1^{(\prime)}}$ is the threshold of the higher resonant states. Note that the subtraction terms on the right-hand side of (21), which are polynomials in q^2, are neglected since they have no contributions after performing the Borel transformation. The four-quark condensates are expressed as

$$\langle 0|\bar{q}\Gamma_i\lambda^a q\bar{q}\Gamma_i\lambda^a q|0\rangle = -a_2\frac{1}{16N_c^2}\mathrm{Tr}(\Gamma_i\Gamma_i) \times \mathrm{Tr}(\lambda^a\lambda^a)\langle\bar{q}q\rangle^2, \tag{22}$$

[3]The following approximation was used in [26]:

$$\frac{\langle\phi|\bar{s}\gamma_\mu s|f_1(1285)\rangle}{\langle\rho|(\bar{u}\gamma_\mu u-\bar{d}\gamma_\mu d)/2|f_1(1285)\rangle} \approx 2\tan\bar{\alpha}.$$

[4]Considering the anomaly, the singlet axial-vector current is satisfied with

$$\partial^\mu j_\mu^{(1)} = \frac{1}{\sqrt{3}}(m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s) + \frac{3\alpha_s}{4\pi}G\tilde{G}.$$

where $a_2 = 1$ corresponds to the vacuum saturation approximation. In the present work, we have $\Gamma = \gamma_\mu$ and $\gamma_\mu\gamma_5$, for which we allow the variation $a_2 = -2.9$–3.1 [9,28,29]. For $\Pi_1^{(\prime)\mathrm{OPE}}$, we take into account the terms with dimension ≤ 6, where the term with dimension $= 0$ ($D = 0$) is up to $\mathcal{O}(\alpha_s^3)$, with $D = 2$ (which is proportional to m_s^2) up to $\mathcal{O}(\alpha_s^2)$ and with $D = 4$ up to $\mathcal{O}(\alpha_s^2)$. Note that such radiative corrections for terms can read from [30–32]. We do not include the radiative correction to the $D = 6$ terms since all the uncertainties can be lumped into a_2. Note that such radiative corrections for terms with dimensions $= 0$ and 4 are the same as the vector meson case and can read from [30,31].

Further applying the Borel (inverse-Laplace) transformation,

$$\mathbf{B}[f(q^2)] = \lim_{\substack{n\to\infty \\ -q^2\to\infty \\ -q^2/n^2=M^2\,\mathrm{fixed}}} \frac{1}{n!}(-q^2)^{n+1}\left[\frac{d}{dq^2}\right]^n f(q^2), \tag{23}$$

to both sides of (21) to improve the convergence of the OPE series and further suppress the contributions from higher resonances, the sum rules thus read

$$\begin{aligned} f_{f_1}^2 m_{f_1}^2 e^{-m_{f_1}^2/M^2} = {}& \int_0^{s_0^{f_1}} \frac{sdse^{-s/M^2}}{4\pi^2}\left[1 + \frac{\alpha_s(\sqrt{s})}{\pi} + F_3\frac{\alpha_s^2(\sqrt{s})}{\pi^2} + (F_4 + F_4'\cos^2\theta)\frac{\alpha_s^3(\sqrt{s})}{\pi^3}\right] - (\cos\theta - \sqrt{2}\sin\theta)^2[\bar m_s(\mu_\circ)]^2 \\ & \times \int_0^{s_0^{f_1}} ds\frac{1}{2\pi^2}e^{-s/M^2}\left[1 + \left(H_1\ln\frac{s}{\mu_\circ^2} + H_2\right)\frac{\alpha_s(\mu_\circ)}{\pi} + \left(H_{3a}\ln^2\frac{s}{\mu_\circ^2} + H_{3b}\ln\frac{s}{\mu_\circ^2} + H_{3c} - \frac{H_{3a}\pi^2}{3}\right)\right. \\ & \times \left.\left(\frac{\alpha_s(\mu_\circ)}{\pi}\right)^2\right] - \frac{1}{12}\left(1 - \frac{11}{18}\frac{\alpha_s(M)}{\pi}\right)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle - \left[\frac{4}{27}\frac{\alpha_s(M)}{\pi} + \left(-\frac{257}{486} + \frac{4}{3}\zeta(3) - \frac{2}{27}\beta_1\gamma_E\right)\frac{\alpha_s^2(M)}{\pi^2}\right] \\ & \times \sum_{q_i\equiv u,d,s}\langle\bar m_i\bar q_i q_i\rangle + \frac{1}{3}(\sqrt{2}\cos\theta + \sin\theta)^2\left[2a_1\bar m_q\langle\bar q q\rangle - \frac{352\pi\alpha_s}{81M^2}a_2\langle\bar q q\rangle^2\right] + \frac{1}{3}(\cos\theta - \sqrt{2}\sin\theta)^2 \\ & \times\left[2a_1\bar m_s\langle\bar s s\rangle - \frac{352\pi\alpha_s}{81M^2}a_2\langle\bar s s\rangle^2\right], \end{aligned} \tag{24}$$

$$\begin{aligned} f_{f_1'}^2 m_{f_1'}^2 e^{-m_{f_1'}^2/M^2} = {}& \int_0^{s_0^{f_1'}} \frac{sdse^{-s/M^2}}{4\pi^2}\left[1 + \frac{\alpha_s(\sqrt{s})}{\pi} + F_3\frac{\alpha_s^2(\sqrt{s})}{\pi^2} + (F_4 + F_4'\sin^2\theta)\frac{\alpha_s^3(\sqrt{s})}{\pi^3}\right] + (\sin\theta + \sqrt{2}\cos\theta)^2[\bar m_s(\mu_\circ)]^2 \\ & \times \int_0^{s_0^{f_1'}} ds\frac{1}{2\pi^2}e^{-s/M^2}\left[1 + \left(H_1\ln\frac{s}{\mu_\circ^2} + H_2\right)\frac{\alpha_s(\mu_\circ)}{\pi} + \left(H_{3a}\ln^2\frac{s}{\mu_\circ^2} + H_{3b}\ln\frac{s}{\mu_\circ^2} + H_{3c} - \frac{H_{3a}\pi^2}{3}\right)\right. \\ & \times \left.\left(\frac{\alpha_s(\mu_\circ)}{\pi}\right)^2\right] - \frac{1}{12}\left(1 - \frac{11}{18}\frac{\alpha_s(M)}{\pi}\right)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle - \left[\frac{4}{27}\frac{\alpha_s(M)}{\pi} + \left(-\frac{257}{486} + \frac{4}{3}\zeta(3) - \frac{2}{27}\beta_1\gamma_E\right)\frac{\alpha_s^2(M)}{\pi^2}\right] \\ & \times \sum_{q_i\equiv u,d,s}\langle\bar m_i\bar q_i q_i\rangle + \frac{1}{3}(\sqrt{2}\sin\theta - \cos\theta)^2\left[2a_1\bar m_q\langle\bar q q\rangle - \frac{352\pi\alpha_s}{81M^2}a_2\langle\bar q q\rangle^2\right] + \frac{1}{3}(\sin\theta + \sqrt{2}\cos\theta)^2 \\ & \times\left[2a_1\bar m_s\langle\bar s s\rangle - \frac{352\pi\alpha_s}{81M^2}a_2\langle\bar s s\rangle^2\right], \end{aligned} \tag{25}$$

where

$$\begin{aligned} & F_3 = 1.9857 - 0.1153n_f \simeq 1.6398 \ \ \text{for } n_f = 3, \quad F_4 = -6.6368 - 1.2001n_f - 0.0052n_f^2 \simeq -10.2839 \ \ \text{for } n_f = 3, \\ & F_4' = -1.2395\Delta, \quad H_1 = -\frac{8}{81}\beta_1^2 = -2, \quad H_2 = \frac{2}{9}\beta_2 + 4\beta_2\left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_2}{\beta_2}\right) - \frac{8}{9}\beta_1^2 - 4\beta_1 \simeq 3.6667, \\ & H_{3a} = 4.2499, \quad H_{3b} = -23.1667, \quad H_{3c} = 29.7624, \\ & \bar m_q\langle\bar q q\rangle \equiv \frac{1}{2}(\bar m_u\langle\bar u u\rangle + \bar m_d\langle\bar d d\rangle), \quad \langle\bar q q\rangle^2 \equiv \frac{1}{2}(\langle\bar u u\rangle^2 + \langle\bar d d\rangle^2), \quad a_1 = 1 + \frac{7}{3}\frac{\alpha_s(M)}{\pi} + \left(\frac{85}{6} - \frac{7}{6}\beta_1\gamma_E\right)\frac{\alpha_s^2(M)}{\pi^2}, \end{aligned} \tag{26}$$

with $\beta_1 = (2n_f - 33)/6$, $\beta_2 = (19n_f - 153)/12$, $\gamma_1 = 2$, $\gamma_2 = 101/12 - 5n_f/18$, and $n_f = 3$ being the number of flavors and $\Delta = 1$, and 0 for f_1 (singlet) and f_8 (octet), respectively [32]. In the calculation the coupling constant $\alpha_s(\sqrt{s})$ in Eqs. (24) and (25) can be expanded in powers of $\alpha_s(M)$:

$$\frac{\alpha_s(\sqrt{s})}{\pi} = \frac{\alpha_s(M)}{\pi} + \frac{1}{2}\beta_1 \ln\frac{s}{M^2}\left(\frac{\alpha_s(M)}{\pi}\right)^2 + \left(\frac{1}{2}\beta_2 \ln\frac{s}{M^2} + \frac{1}{4}\beta_1^2\ln^2\frac{s}{M^2}\right)\left(\frac{\alpha_s(M)}{\pi}\right)^3 + \left(\frac{\beta_3}{2}\ln\frac{s}{M^2} + \frac{5}{8}\beta_1\beta_2\ln^2\frac{s}{M^2} + \frac{1}{8}\beta_1^3\ln^3\frac{s}{M^2}\right) \times \left(\frac{\alpha_s(M)}{\pi}\right)^4 + \cdots, \quad (27)$$

where $\beta_3 \simeq -20.1198$. Using the renormalization-group result for the m_s^2 term given in [31], we have expanded the contribution to the order $\mathcal{O}(\alpha_s^2 m_s^2)$ at the subtraction scale $\mu_\circ^2 = 2\ \mathrm{GeV}^2$ for which the series has better convergence than at the scale 1 GeV^2; however, the convergence of the series has no obvious change if using a higher reference scale. As in the case of flavor-breaking τ decay, the $D = 2$ series converges slowly; nevertheless, we have checked that this term, which intends to make the output m_s to be smaller in the fit, is suppressed due to the fact that the mass sum rules for $f_1(1285)$ and $f_1(1420)$ are obtained by applying the differential operator $M^4\partial\ln/\partial M^2$ to both sides of (24) and (25), respectively. Nevertheless, the differential operator will instead make the $D = 4$ term containing $m_s\langle\bar{s}s\rangle$ become much more important than the m_s^2 term in determining the $f_1(1285)$-$f_1(1420)$ mass difference although they are the same order in magnitude.

In the numerical analysis, we use $\Lambda_{\mathrm{QCD}}^{(3)\mathrm{NLO}} = 0.360$ GeV, corresponding to $\alpha_s(1\ \mathrm{GeV}) = 0.495$, $\Lambda_{\mathrm{QCD}}^{(4)\mathrm{NLO}} = 0.313$ GeV, and the following values (at the scale $\mu = 1$ GeV) [9,28,29,33]:

$$\begin{aligned} \left\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}\right\rangle &= (0.009 \pm 0.007)\ \mathrm{GeV}^4, \\ \langle\bar{m}_q\bar{q}q\rangle &= -f_{\pi^+}^2 m_{\pi^+}^2/4, \\ \langle\bar{q}q\rangle^2 &\simeq (-0.247)^6\ \mathrm{GeV}^6, \\ \langle\bar{s}s\rangle &= (0.30\text{–}1.3)\langle\bar{q}q\rangle, \\ a_2 &= -2.9\text{–}3.1, \end{aligned} \quad (28)$$

where the value of $\langle\bar{q}q\rangle^2$ corresponds to $(m_u + m_d) \times (1\ \mathrm{GeV}) \simeq 11$ MeV, and we have cast the uncertainty of $\langle\bar{q}q\rangle^2$ to a_2 in the $D = 6$ term. We do not consider the isospin breaking effect between $\langle\bar{u}u\rangle$ and $\langle\bar{d}d\rangle$ since $\langle\bar{d}d\rangle/\langle\bar{u}u\rangle - 1 \approx -0.007$ [34] is negligible in the present analysis. The threshold is allowed by $s_0^{f_1} = 2.70 \pm 0.15\ \mathrm{GeV}^2$ and determined by the maximum stability of the mass sum rule. For an estimate on the threshold difference, we parametrize in the form $(\sqrt{s_0^{f_1'}} - \sqrt{s_0^{f_1}})/\sqrt{s_0^{f_1}} = \delta \times (m_{f_1'} - m_{f_1})/m_{f_1}$, with $\delta = 1.0 \pm 0.3$. In other words, we assign a 30% uncertainty to the default value. We search for the allowed solutions for strange quark mass and the singlet-octet mixing angle θ under the following constraints: (i) Comparing with the observables, the errors for the mass sum rule results of the $f_1(1285)$ and $f_1(1420)$ in the Borel window $0.9\ \mathrm{GeV}^2 \leq M^2 \leq 1.3\ \mathrm{GeV}^2$ are constrained to be less than 3% on average. In this Borel window, the contribution originating from higher resonances (and the continuum), modeled by

$$\frac{1}{\pi}\int_{s_0^{f^{(\prime)}}}^{\infty} ds e^{-s/M^2}\,\mathrm{Im}\Pi_1^{(\prime)\mathrm{OPE}}(s), \quad (29)$$

is about less than 40% and the highest OPE term (with dimension six) at the quark level is no more than 10%. (ii) The deviation between the $f_1(1420)$-$f_1(1285)$ mass difference sum rule result and the central value of the data [14] is within 1σ error: $|(m_{f_1'} - m_{f_1})_{\text{sum rule}} - 144.6\ \mathrm{MeV}| \leq 1.5$ MeV. The detailed results are shown in Table I. We also check that if by further enlarging the uncertainties of $s_0^{f_1}$ and δ, e.g. 25%, the changes of results can be negligible. We obtain the strange quark mass with large uncertainty: $m_s(1\ \mathrm{GeV}) = 106.3 \pm 35.1$ MeV [i.e. $m_s(2\ \mathrm{GeV}) = 89.5 \pm 29.5$ MeV] and $\langle\bar{s}s\rangle/\langle\bar{u}u\rangle = 0.56 \pm 0.25$ corresponding to $\theta = (19.4^{+4.5}_{-4.6})^\circ$, where the values and m_s and $\langle\bar{s}s\rangle$ are strongly correlated.

Further accounting for the average of the recent lattice results [15–17]: $m_s(2\ \mathrm{GeV}) = 93.6 \pm 1.0$ MeV and using the θ value that we have obtained as the inputs, we get $\langle\bar{s}s\rangle/\langle\bar{u}u\rangle = 0.41 \pm 0.09$ which is less than 1 and in contrast to the Schwinger-Dyson equation approach in [6] where the ratio was obtained as $(1.0 \pm 0.2)^3$. Our prediction is consistent with the QCD sum rule result of studying the scalar/pseudoscalar two-point function in [35] where the authors obtained $\langle\bar{s}s\rangle/\langle\bar{u}u\rangle = 0.4$–$0.7$, depending on the value of the strange quark mass.

TABLE I. The fitting results in the $f_1(1284)$-$f_1(1420)$ mass difference sum rules. In fit II, we have taken the average of the recent lattice results for m_s, which is rescaled to 1 GeV as the input.

	$m_s(1\ \mathrm{GeV})$	$\langle\bar{s}s\rangle/\langle\bar{u}u\rangle$	$\langle(\alpha_s/\pi)G^2\rangle$	a_2
Fit I	106.3 ± 35.1	0.56 ± 0.25	0.0106 ± 0.0042	0.89 ± 0.62
Fit II	$[124.7 \pm 1.3]$	0.41 ± 0.09	0.0108 ± 0.0037	0.95 ± 0.45

IV. SUMMARY

We have adopted two different strategies for determining the mixing angle θ: (i) Using the Gell-Mann-Okubo mass formula and the $K_1(1270)$-$K_1(1400)$ mixing angle $\theta_{K_1} = (-34 \pm 13)^\circ$ which was extracted from the data for $\mathcal{B}(B \to K_1(1270)\gamma)$, $\mathcal{B}(B \to K_1(1400)\gamma)$, $\mathcal{B}(\tau \to K_1(1270)\nu_\tau)$, and $\mathcal{B}(\tau \to K_1(1420)\nu_\tau)$, the result is $\theta = (23^{+17}_{-23})^\circ$. (ii) On the other hand, from the analysis of the ratio of $\mathcal{B}(f_1(1285) \to \phi\gamma)$ and $\mathcal{B}(f_1(1285) \to \rho^0\gamma)$, we have $\bar{\alpha} = \theta_i - \theta = \pm(15.8^{+4.5}_{-4.6})^\circ$, i.e., $\theta = (19.4^{+4.5}_{-4.6})^\circ$ or $(51.1^{+4.5}_{-4.6})^\circ$. Combining these two analyses, we deduce the mixing angle $\theta = (19.4^{+4.5}_{-4.6})^\circ$.

We have estimated the strange quark mass and strange quark condensate from the analysis of the $f_1(1420)$-$f_1(1285)$ mass difference QCD sum rule. We have expanded the OPE series up to dimension six, where the term with dimension zero is up to $\mathcal{O}(\alpha_s^3)$, with dimension $= 2$ up to $\mathcal{O}(m_s^2\alpha_s^2)$ and with dimension $= 4$ terms up to $\mathcal{O}(\alpha_s^2)$. Further using the average of the recent lattice results and the θ value that we have obtained as the inputs, we get $\langle\bar{s}s\rangle/\langle\bar{u}u\rangle = 0.41 \pm 0.09$.

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