Three-parton contribution to pion form factor in k_T factorization

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(Received 5 May 2011; published 10 August 2011)

We set up a framework for the study of the power-suppressed three-parton contribution to the pion electromagnetic form factor in the k_T factorization theorem. It is first shown that the gauge dependence proportional to parton transverse momenta from the two-parton Fock state and the gauge dependence associated with the three-parton Fock state cancel each other. After verifying the gauge invariance, we derive the three-parton-to-three-parton k_T-dependent hard kernel at leading order of the coupling constant, and find that it leads to about 5% correction to the pion electromagnetic form factor in the whole range of experimentally accessible momentum transfer squared. This subleading contribution is much smaller than the leading-order twist-2, next-to-leading-order twist-2, and leading-order two-parton twist-3 ones, which have been calculated in the literature.

I. INTRODUCTION

Aspects of the k_T factorization theorem [1–6] in perturbative QCD have been investigated intensively. One of the important issues is about the derivation of a k_T-dependent hard kernel at subleading level, which is defined as the difference between QCD diagrams and effective diagrams for transverse-momentum-dependent (TMD) hadron wave functions. We have explained that partons in both sets of diagrams should remain off mass shell by k_T^2 in the k_T factorization theorem [7]. The same statement has been made in the application of the k_T factorization theorem to inclusive processes such as prompt photon production [8]. The off-shellness of partons may cause concern of the gauge invariance [9,10].[1] However, we have shown that the gauge dependence cancels between the above two sets of diagrams, and a k_T-dependent hard kernel is gauge invariant [11,13]. Following this prescription, the next-to-leading-order (NLO) correction to the pion transition (electromagnetic) form factor associated with the process $\pi\gamma^* \to \gamma(\pi)$ has been calculated at leading twist, i.e., twist 2 [13,14]. Here we shall study the power-suppressed three-parton contribution to the pion electromagnetic form factor in the k_T factorization theorem. The three-parton contribution in the collinear factorization theorem [15] to a simpler process, the ρ meson transition form factor, has been evaluated recently [16].

We shall first demonstrate the gauge invariance of the three-parton contribution to the pion electromagnetic form factor in the k_T factorization theorem. There are two sources of gauge dependence for this power correction [17]: the first source is proportional to parton transverse

momenta from the two-parton Fock state. The corresponding hadronic matrix element is written as

$$\langle 0|\bar{q}(z)\Gamma i\partial_\alpha q(0)|\pi\rangle, \tag{1}$$

where z is the coordinate of the antiquark field $\bar{q}$, and Γ represents a combination of gamma matrices. The second source is associated with the three-parton Fock state with an additional valence gluon. The corresponding matrix element is given by

$$\langle 0|\bar{q}(z)\Gamma gT^a A_\alpha^a(z')q(0)|\pi\rangle, \tag{2}$$

with a color matrix T^a, and the gluon field A_α^a at the coordinate z'. The gauge dependences from the above two sources cancel each other, when Eqs. (1) and (2) are combined to form the gauge-invariant matrix element

$$\langle 0|\bar{q}(z)\Gamma iD_\alpha(z')q(0)|\pi\rangle, \tag{3}$$

with the covariant derivative $iD_\alpha \equiv i\partial_\alpha + gT^a A_\alpha^a$. The cancellation of the gauge dependences is similar to that occurring in the collinear factorization theorem [17]. In the k_T factorization theorem we just keep the transverse momentum dependence in denominators of particle propagators [7]. Hence, it is natural that the gauge dependence disappears at higher twists in the same way as in the collinear factorization theorem.

Our formalism implies that contributions proportional to transverse momenta in numerators of hard kernels must be combined with contributions from three-parton Fock states in order to guarantee gauge invariance. Therefore, there is concern on the study of the pion transition form factor in [18], where only the former was included. It has been pointed out explicitly that the leading-order (LO) hard kernel for the pion electromagnetic form factor becomes gauge-dependent, if one simply considers parton transverse momenta in numerators [10]. However, the contribution from the three-parton Fock state was still missing

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[1]Criticisms raised in [9,10] have been addressed in [11,12].

in [10], such that the false postulation on the gauge dependence of the k_T factorization theorem was made.

Since both the initial- and final-state pions involve higher-twist matrix elements like that in Eq. (3), the three-parton contribution to the pion electromagnetic form factor is suppressed at least by $1/Q^2$, Q^2 being the momentum transfer squared. After examining the gauge invariance, we calculate the k_T-dependent hard kernel for the three-parton-to-three-parton scattering in the Feynman gauge, and convolute it with the three-parton pion wave functions. We observe that the diagrams with a four-gluon vertex dominate this power correction. It will be shown that the three-parton contribution is only about 5% of the sum of those which have been investigated before, including the LO twist-2, NLO twist-2, and LO two-parton twist-3 ones [14]. That is, the three-parton contribution is not crucial for accommodating experimental data of the pion electromagnetic form factor. At the same power of $1/Q^2$, one should also take into account the scattering of two (four) partons into four (two) partons in principle. This piece has been analyzed in light-cone sum rules [19], and found to be less important than other contributions. With this work, we conclude that the chirally enhanced two-parton twist-3 correction is the most important $1/Q^2$ correction to the pion electromagnetic form factor.

In Sec. II we verify the gauge invariance of the LO three-parton contribution to the pion electromagnetic form factor by combining the gauge-dependent hard kernels corresponding to Eqs. (1) and (2). The k_T-dependent hard kernel is then derived from the three-parton-to-three-parton scattering diagrams, and convoluted with the three-parton pion wave functions numerically in Sec. III. Section IV is the conclusion. Detailed calculations of the gauge-dependent hard kernels corresponding to Eq. (2) are presented in Appendix A, and the expressions of the three-parton-to-three-parton hard kernels are collected in Appendix B.

II. GAUGE INVARIANCE

Consider the pion electromagnetic form factor involved in the process $\pi(P_1)\gamma^* \to \pi(P_2)$, whose LO diagrams are displayed in Fig. 1. The momentum P_1 of the initial-state pion and P_2 of the final-state pion are parametrized as

$$P_1 = (P_1^+, 0, \mathbf{0}_T) = \frac{Q}{\sqrt{2}}(1, 0, \mathbf{0}_T),$$

$$P_2 = (0, P_2^-, \mathbf{0}_T) = \frac{Q}{\sqrt{2}}(0, 1, \mathbf{0}_T), \tag{4}$$

with $Q^2 = -q^2$, $q = P_2 - P_1$ being the virtual photon momentum. The gluon propagator of momentum l is written as

$$\frac{-i}{l^2}\left(g^{\sigma\nu} - \lambda\frac{l^\sigma l^\nu}{l^2}\right), \tag{5}$$

in the covariant gauge, where the parameter λ will be used to identify sources of gauge dependence. We assume that

the antiquarks in the initial- and final-state pions, represented by lower fermion line, carry the parton momenta

$$k_1 = (x_1 P_1^+, 0, \mathbf{k}_{1T}), \qquad k_2 = (0, x_2 P_2^-, \mathbf{k}_{2T}), \tag{6}$$

respectively, x_1 and x_2 being the momentum fractions. It is understood that the components k_1^- and k_2^+ have been dropped in hard kernels, and integrated out of the TMD pion wave functions.

We employ the Fierz identity

$$I_{ij}I_{lk} = \frac{1}{4}I_{ik}I_{lj} + \frac{1}{4}(\gamma_5)_{ik}(\gamma_5)_{lj} + \frac{1}{4}(\gamma^\alpha)_{ik}(\gamma_\alpha)_{lj}$$
$$+ \frac{1}{4}(\gamma_5\gamma^\alpha)_{ik}(\gamma_\alpha\gamma_5)_{lj} + \frac{1}{8}(\sigma^{\alpha\beta}\gamma_5)_{ik}(\sigma_{\alpha\beta}\gamma_5)_{lj}, \tag{7}$$

to factorize the fermion flow, where I denotes the 4×4 identity matrix. The structure $\gamma_\alpha\gamma_5$ in the above identity contributes at twist 2 and higher twists, and γ_5 and $\sigma_{\alpha\beta}\gamma_5$ contribute at twist 3 and higher twists at two-parton level. The matrix Γ in Eqs. (1) and (2) can pick up one of the above structures, among which we focus on the first one $\gamma_\alpha\gamma_5$ as an example below. We also insert the identity

$$I_{ij}I_{lk} = \frac{1}{N_c}I_{lj}I_{ik} + 2(T^c)_{lj}(T^c)_{ik}, \tag{8}$$

to factorize the color flow, where $N_c = 3$ is the number of colors, I denotes the 3×3 identity matrix, and T^c is a color matrix. The first (second) term in Eq. (8) will be associated with a color-singlet (color-octet) state of the valence quark and antiquark.

The first source of gauge dependence is extracted from the diagrams in Fig. 1, where the quark and antiquark pair forms a color-singlet state. Combining the decompositions in Eqs. (7) and (8), we sandwich Fig. 1 with the structures

$$\frac{1}{4N_c}\gamma_\alpha\gamma_5, \qquad \frac{1}{4N_c}\gamma_5\gamma_\beta, \tag{9}$$

from the initial and final states, respectively, where the subscripts α and β can take arbitrary components. The LO hard kernel from Fig. 1(a) contains the gauge-dependent piece

$$H^{a\lambda} = -ieg^2\lambda\frac{C_F}{16N_c}\frac{\text{tr}[\gamma^\sigma\gamma_5\gamma_\beta\gamma_\mu(P_1 - k_2)\gamma^\nu\gamma_\alpha\gamma_5]}{(P_1 - k_2)^2(k_1 - k_2)^2}$$
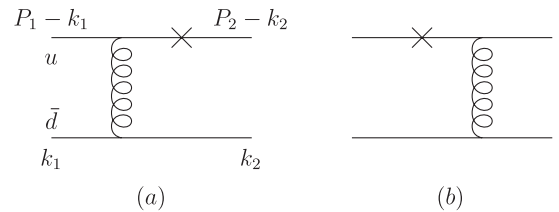$$\times \frac{(k_1 - k_2)_\sigma(k_1 - k_2)_\nu}{(k_1 - k_2)^2}, \tag{10}$$

FIG. 1.   LO diagrams for the pion electromagnetic form factor, where the symbol $\times$ represents the virtual photon vertex.

with C_F being a color factor. The above expression diminishes with the substitution $k_1 = x_1 P_1$, $k_2 = x_2 P_2$, $\gamma_\alpha = \gamma^-$ (proportional to P_1), and $\gamma_\beta = \gamma^+$ (proportional to P_2) into the numerator, implying that Eq. (10) does not contribute at leading power in the k_T factorization theorem.

To obtain the gauge-dependent hard kernel from Eq. (10) at $1/Q^2$, we insert the identity $(k_1 - k_2)_\nu = (P_1 - k_2)_\nu - (P_1 - k_1)_\nu$. It can be shown that the contribution from the $(P_1 - k_2)_\nu$ term is canceled by the corresponding one in Fig. 1(b). The second term, with $P_1 - k_1$ being the momentum of the incoming valence quark, corresponds to the matrix element with the derivative of the quark field in the initial-state pion. This term can be picked up by differentiating Eq. (10) with respect to k_1. Once Eq. (1) for the initial-state pion is identified, we pick up the $k_{2\sigma}$ term in $(k_1 - k_2)_\sigma$ via differentiation, which corresponds to the derivative of the valence antiquark field in the final-state pion. Note that denominators of particle propagators, depending on k_1 and k_2, will be differentiated too. However, their differentiation gives rise to even higher-twist matrix elements, and can be neglected. We then extract

$$H_{TT}^{a\lambda}(k_1, k_2) \equiv \frac{\partial^2 H^{a\lambda}}{\partial k_{1\alpha} \partial k_{2\beta}}$$

$$= ieg^2 \lambda \frac{C_F}{16 N_c}$$

$$\times \frac{\mathrm{tr}[\gamma^\beta \gamma_5 \gamma_\beta \gamma_\mu (P_1 - x_2 P_2) \gamma^\alpha \gamma_\alpha \gamma_5]}{(P_1 - k_2)^2 (k_1 - k_2)^4}, \quad (11)$$

associated with $\langle 0 | \bar{q}(z) \gamma_5 \gamma^\alpha i \partial_\alpha q(0) | \pi(P_1) \rangle$ and the similar matrix element for the final-state pion.

The LO hard kernel from Fig. 1(b) contains

$$H^{b\lambda} = -ieg^2 \lambda \frac{C_F}{16 N_c} \frac{\mathrm{tr}[\gamma^\sigma \gamma_5 \gamma_\beta \gamma^\nu (P_2 - \not{k}_1) \gamma_\mu \gamma_\alpha \gamma_5]}{(P_2 - k_1)^2 (k_1 - k_2)^2}$$

$$\times \frac{(k_1 - k_2)_\sigma (k_1 - k_2)_\nu}{(k_1 - k_2)^2}, \quad (12)$$

whose differentiation with respect to $k_{1\alpha}$ and $k_{2\beta}$ leads to

$$H_{TT}^{b\lambda}(k_1, k_2)$$

$$= ieg^2 \lambda \frac{C_F}{16 N_c} \frac{\mathrm{tr}[\gamma^\alpha \gamma_5 \gamma_\beta \gamma^\beta (P_2 - x_1 P_1) \gamma_\mu \gamma_\alpha \gamma_5]}{(P_2 - k_1)^2 (k_1 - k_2)^4}. \quad (13)$$

This hard kernel corresponds to the matrix element of the initial-state pion with the derivative of the antiquark field $\bar{q}(z)$. Equations (11) and (13) represent the gauge dependence in LO two-parton-to-two-parton scattering at power of $1/Q^2$, which was also observed in [10].

The second source of gauge dependence arises from Fig. 2, where three partons appear in the initial state as indicated by Eq. (2). All possible attachments of the additional valence gluon to the lines other than the valence
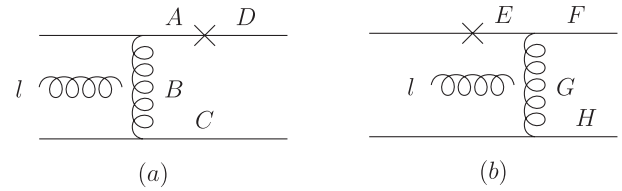

FIG. 2. Diagrams with three partons from the initial state, where letters $A, B, \cdots$ denote the attachments of the additional valence gluon.

quark and antiquark in the initial state are labeled by letters $A, B, \cdots$. In principle, the diagrams with the attachments to the valence quark and antiquark in the initial state should be included in order to respect $U(1)$ gauge symmetry for the electromagnetic interaction.[2] These diagrams contribute to higher Gegenbauer terms in the two-parton twist-3 pion distribution amplitudes. Equations of motion can then be constructed to relate the coefficients of the higher Gegenbauer terms in the two-parton twist-3 and three-parton twist-3 pion distribution amplitudes [20]. Hence, one should pay attention to the consistency between the models for these two sets of distribution amplitudes in a numerical analysis. We shall adopt the nonasymptotic models for both sets of distribution amplitudes in Sec. III, when estimating the importance of the three-parton contribution relative to other two-parton contributions in the pion electromagnetic form factor.

According to Eq. (8), we absorb a color matrix T^c and the coupling constant g associated with an attachment into the matrix element for the initial-state pion, and another T^c goes into the evaluation of gauge-dependent hard kernels. For example, the color factor corresponding to the attachment A of the valence gluon to the virtual quark line is given by

$$\mathrm{tr}[T^b T^a T^b T^c] = -\frac{1}{4 N_c} \delta^{ac}, \quad (14)$$

where the color matrix T^a (T^b) comes from the valence (hard) gluon vertex. After summing over c, the tensor δ^{ac} sets $c = a$ in the matrix element for the initial-state pion, leading to Eq. (2). Including the coefficient 2 in Eq. (8), we adopt the structure $\gamma_\alpha \gamma_5 / 2$ for the initial state in the calculation of Fig. 2, whose details can be found in Appendix A. The results are collected as follows:

$$H_{AT}^\lambda = -ieg^2 \lambda \frac{1}{32 N_c^2}$$

$$\times \frac{\mathrm{tr}[\gamma^\beta \gamma_5 \gamma_\beta \gamma_\mu (P_1 - x_2 P_2) \gamma^\alpha \gamma_\alpha \gamma_5]}{(P_1 - k_2)^2 (k_1 - k_2)^4}, \quad (15)$$

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[2] We thank V. Braun for pointing out this $U(1)$ gauge symmetry.

$$H_{BT}^\lambda = ieg^2\lambda \frac{1}{32} \frac{\mathrm{tr}[\gamma^\beta \gamma_5 \gamma_\beta \gamma_\mu (P_1 - x_2 P_2)\gamma^\alpha \gamma_\alpha \gamma_5]}{(P_1 - k_2)^2 (k_1 - k_2)^4},$$
(16)

$$H_{CT}^\lambda = 0,$$
(17)

$$H_{DT}^\lambda = -ieg^2\lambda \frac{1}{32N_c^2}$$
$$\times \frac{\mathrm{tr}[\gamma^\beta \gamma_5 \gamma_\beta \gamma^\alpha (P_2 - x_2 P_2 - y_1 P_1)\gamma_\mu \gamma_\alpha \gamma_5]}{(P_2 - k_2 - l_1)^2 (k_1 - k_2)^4},$$
(18)

$$H_{ET}^\lambda = -ieg^2\lambda \frac{C_F}{16N_c}$$
$$\times \frac{\mathrm{tr}[\gamma^\beta \gamma_5 \gamma_\beta \gamma^\alpha (P_2 - x_1 P_1 - y_1 P_1)\gamma_\mu \gamma_\alpha \gamma_5]}{(P_2 - k_1 - l_1)^2 (k_1 - k_2)^4},$$
(19)

$$H_{FT}^\lambda = ieg^2\lambda \frac{1}{32N_c^2}$$
$$\times \frac{\mathrm{tr}[\gamma^\beta \gamma_5 \gamma_\beta \gamma^\alpha (P_2 - x_2 P_2 - y_1 P_1)\gamma_\mu \gamma_\alpha \gamma_5]}{(P_2 - k_2 - l_1)^2 (k_1 - k_2)^4}$$
$$- ieg^2\lambda \frac{1}{32N_c^2}$$
$$\times \frac{\mathrm{tr}[\gamma^\beta \gamma_5 \gamma_\beta \gamma^\alpha (P_2 - x_1 P_1 - y_1 P_1)\gamma_\mu \gamma_\alpha \gamma_5]}{(P_2 - k_1 - l_1)^2 (k_1 - k_2)^4},$$
(20)

$$H_{GT}^\lambda = ieg^2\lambda \frac{1}{32}$$
$$\times \frac{\mathrm{tr}[\gamma^\beta \gamma_5 \gamma_\beta \gamma^\alpha (P_2 - x_1 P_1 - y_1 P_1)\gamma_\mu \gamma_\alpha \gamma_5]}{(P_2 - k_1 - l_1)^2 (k_1 - k_2)^4}$$
$$+ ieg^2\lambda \frac{1}{32}$$
$$\times \frac{\mathrm{tr}[\gamma^\alpha \gamma_5 \gamma_\beta \gamma^\beta (P_2 - x_1 P_1 - y_1 P_1)\gamma_\mu \gamma_\alpha \gamma_5]}{(P_2 - k_1 - l_1)^2 (k_1 + l_1 - k_2)^4},$$
(21)

$$H_{HT}^\lambda = -ieg^2\lambda \frac{1}{32N_c^2}$$
$$\times \frac{\mathrm{tr}[\gamma^\alpha \gamma_5 \gamma_\beta \gamma^\beta (P_2 - x_1 P_1 - y_1 P_1)\gamma_\mu \gamma_\alpha \gamma_5]}{(P_2 - k_1 - l_1)^2 (k_1 + l_1 - k_2)^4},$$
(22)

with the gluon momentum fraction $y_1 = l_1^+/P_1^+$.

Summing the above expressions, we arrive at

$$\sum_{i=A}^{H} H_{iT}^\lambda = H_{gT}^{a\lambda} + H_{gT}^{b\lambda},$$
(23)

with

$$H_{gT}^{a\lambda} = H_{TT}^{a\lambda}(k_1, k_2), \qquad H_{gT}^{b\lambda} = H_{TT}^{b\lambda}(k_1 + l_1, k_2). \quad (24)$$

The contributions from the attachments A and B are added into $H_{gT}^{a\lambda}$ with the desired color factor C_F. The contribution from the attachment D and the first term in the attachment F cancel each other. The second term from the attachment F and the first term from the attachment G are combined into the expression with the color factor C_F, which then cancels the contribution from the attachment E. The second term of G and the contribution from the attachment H are added into $H_{gT}^{b\lambda}$. Note that $H_{gT}^{a\lambda}$ does not depend on the valence gluon momentum l_1, which can then be integrated out of the matrix element, giving $\langle 0|\bar{q}(z)\gamma_5\gamma^\alpha gT^a A_\alpha^a(0)q(0)|\pi(P_1)\rangle$. The hard kernel $H_{gT}^{b\lambda}$, depending on the combination $k_1 + l_1$, corresponds to the matrix element $\langle 0|\bar{q}(z)\gamma_5\gamma^\alpha gT^a A_\alpha^a(z)q(0)|\pi(P_1)\rangle$. Because of the symmetry under the exchange of the initial- and final-state kinematic variables, the gauge-dependent hard kernels with three partons from the final state are written as

$$H_{Tg}^{a\lambda} = H_{TT}^{a\lambda}(k_1, k_2 + l_2), \qquad H_{Tg}^{b\lambda} = H_{TT}^{b\lambda}(k_1, k_2), \quad (25)$$

where l_2 is the momentum carried by the outgoing valence gluon.

At last, we extract the gauge dependence from the three-parton-to-three-parton diagrams. In this case the indices α and β, associated with the initial and final valence gluons, respectively, must be carried by gluon vertices, instead of by parton momenta. Focusing on the gauge-dependent piece, we can apply the Ward identity to hard gluons. It is easy to find that Fig. 3, where the two valence gluons scatter via a three-gluon vertex, does not contribute: if the gauge dependence arises from the lower hard gluon, the results, being proportional to the parton momenta after applying the Ward identity, should be dropped. If the gauge dependence arises from the upper hard gluon, the Ward identity diminishes the amplitude for a similar reason. It is also easy to see that Fig. 8(a) with a four-gluon vertex does not contribute to a gauge-dependent hard kernel. If both the valence gluons attach to the quark line, the
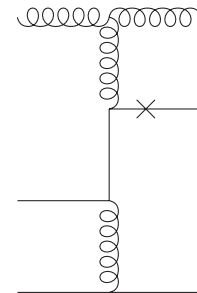

FIG. 3. One of the three-parton-to-three-parton diagrams, where the two valence gluons scatter via a three-gluon vertex.
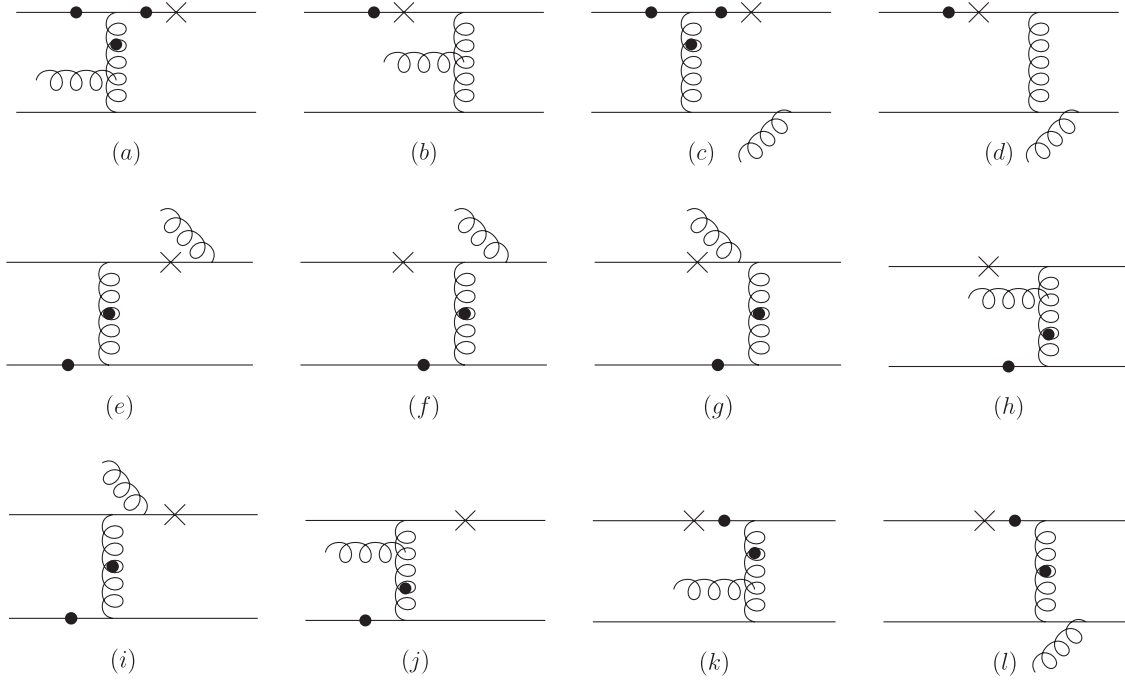
FIG. 4. Three-parton-to-three-parton diagrams, where the valence gluon from the initial state is shown, and the possible attachments of the valence gluon from the final state are represented by dots.

gauge-dependent contribution vanishes because of the Ward identity applied to the antiquark line. If both the valence gluons attach to the antiquark line, the gauge-dependent contribution vanishes too.

The other diagrams are classified into several sets, which are formed by all possible attachments of a hard gluon as displayed in Figs. 4(a)–4(l). Cancellation occurs in each set of diagrams after applying the Ward identity. Neglecting those pieces proportional to the parton momenta, we have

$$(a) + (b) = (c) + (d) = 0,$$
$$(e) + (f) + (g) + (h) = 0. \tag{26}$$

A finite gauge-dependent contribution comes only from Figs. 4(i)–4(l), given by

$$(i) + (j) = H_{gg}^{a\lambda} = H_{TT}^{a\lambda}(k_1, k_2 + l_2),$$
$$(k) + (l) = H_{gg}^{b\lambda} = H_{TT}^{b\lambda}(k_1 + l_1, k_2). \tag{27}$$

Because of the same expressions of $H_{TT}^{a\lambda}$ and $H_{gT}^{a\lambda}$, their corresponding matrix elements are combined into

$$\langle 0|\bar{q}(z)\gamma_5 i\gamma^\alpha D_\alpha(0)q(0)|\pi(P_1)\rangle = 0, \tag{28}$$

with the equation of motion for the quark field, $\not{D}(0)q(0) = 0$. That is, the gauge invariance holds, when the contributions from Figs. 1 and 2 are combined. The combination of the matrix elements for $H_{Tg}^{a\lambda}$ and $H_{gg}^{a\lambda}$ also vanishes according to Eq. (28). A similar reasoning applies to the gauge-dependent hard kernels $H_{TT}^{b\lambda}$ and $H_{Tg}^{b\lambda}$ and to $H_{gT}^{b\lambda}$ and $H_{gg}^{b\lambda}$: the combination of their matrix elements

vanishes due to the equation of the motion for the quark field in the final-state pion. This observation completes the proof of the gauge invariance of the LO three-parton contribution to the pion electromagnetic form factor at the power of $1/Q^2$ in the k_T factorization theorem. The extension of the proof to all orders can follow the steps outlined in [13]. Note that our proof applies to the collinear factorization theorem too: simply neglecting transverse momenta in denominators, one can show the gauge invariance of the three-parton contribution to the pion electromagnetic form factor in the collinear factorization theorem.

III. THREE-PARTON CONTRIBUTION

In this section we calculate the three-parton contribution to the pion electromagnetic form factor. Start with the gauge-invariant twist-3 matrix element

$$\langle 0|\bar{q}(z)\sigma^+{}_{\alpha'}\gamma_5 iD_\alpha(z')q(0)|\pi(P_1)\rangle, \tag{29}$$

where the subscript α is associated with the vertex the valence gluon attaches to. The power behavior will not be changed, and the gauge invariance will not be broken by inserting another covariant derivative D^+. We then exchange D^+ and D_α, take the difference of D^+D_α and $D_\alpha D^+$, and apply the identity $[D^+, D_\alpha] = -igG_\alpha^+$. It is then equivalent to employing the following alternative matrix element [21], which defines the three-parton twist-3 pion wave function $T(z, z')$,
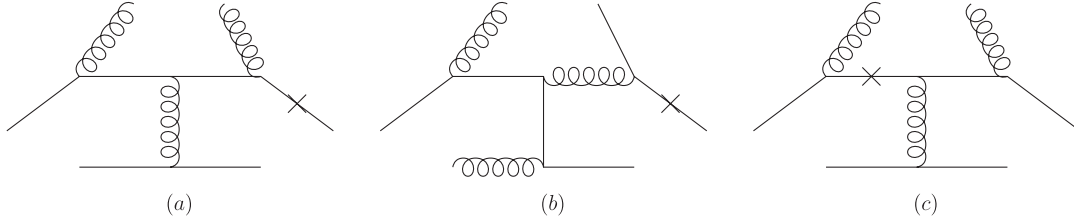
FIG. 5.   Three-parton-to-three-parton diagrams in category A, where both valence gluons attach to the same quark or antiquark line.

$$\langle 0|\bar{q}(z)\sigma^+{}_{\alpha'}\gamma_5 gG^+{}_\alpha(z')q(0)|\pi(P_1)\rangle$$
$$= if_\pi m_0(P_1^+)^2 g^T_{\alpha\alpha'} T(z, z'), \qquad (30)$$

with the chiral scale $m_0 = m_\pi^2/(m_u + m_d)$, m_π, m_u, and m_d being the pion, u quark, and d quark masses, respectively. The operators with other spin structures contribute at higher twists: for example, the operator $\gamma_\mu\gamma_5 G_{\alpha\beta}$ gives a three-parton twist-4 contribution, and $\gamma_5 G_{\alpha\beta}$ does not contribute [20,22,23]. With Eq. (30), one may verify the gauge invariance of a hard kernel in the k_T factorization theorem by demonstrating the cancellation between the gauge-dependent contributions from the operators $\partial^+ A_\alpha$ and $\partial_\alpha A^+$ [21].

Below we derive the hard kernels from the three-parton-to-three-parton diagrams corresponding to Eq. (30) in the Feynman gauge ($\lambda = 0$). Choosing this gauge, the operator $\partial_\alpha A^+$ does not contribute, so only $\partial^+ A_\alpha$ is relevant. The three momenta $P_1 - k_1 - l_1$, k_1, and l_1 are assigned to the initial-state quark, antiquark, and gluon, respectively, and $P_2 - k_2 - l_2$, k_2, and l_2 to the final-state quark, antiquark, and gluon, respectively. We have the structures for the initial- and final-state pions

$$\frac{1}{4}\sigma^{-\alpha'}\gamma_5\frac{i}{l_1^+}if_\pi m_0(P_1^+)^2 g^T_{\alpha\alpha'} = -\frac{i}{4y_1}P_1\gamma^T_\alpha\gamma_5 f_\pi m_0,$$

$$-\gamma_5\frac{1}{4}\sigma^{+\beta'}\frac{-i}{l_2^-}(-i)f_\pi m_0(P_2^-)^2 g^T_{\beta\beta'} = \frac{i}{4y_2}\gamma_5 P_2\gamma^T_\beta f_\pi m_0,$$
$$(31)$$

where the gluon momentum fraction y_2 is defined by $y_2 = l_2^-/P_2^-$, and the gamma matrix γ^T involves only transverse components.

There are totally 196 diagrams for the three-parton-to-three-parton scattering, which can be divided into four categories.[3] Category A contains 20 quark-gluon configurations, in which neither of the valence gluons attaches to the hard gluon line. Each configuration allows 6 different attachments for the photon line. We further divide this category into two groups as shown in Fig. 5, where both valence gluons attach to the same quark or antiquark line, and in Fig. 6, where one valence gluon attaches to the quark line and another to the antiquark line. Only the diagrams giving nonvanishing contributions are displayed. As

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[3]We thank the referee for suggesting this classification of diagrams.

observed in Appendix B, the amplitudes with both the valence gluons attaching to the quark line are power-suppressed. Category B contains 8 quark-gluon configurations, in which one of the valence gluons attaches to the hard gluon line. Each configuration allows 5 different attachments for the photon line, among which those with nonvanishing contributions are displayed in Fig. 7. Category C contains 4 quark-gluon configurations, where both the valence gluons are connected to the hard gluon. Each configuration allows 4 different attachments for the photon line, among which those with nonvanishing contributions are displayed in Fig. 8. Category D contains 4 quark-gluon configurations, where the two valence gluons scatter via a three-gluon vertex as shown in Fig. 3. Each configuration allows 5 different attachments for the photon line. Since this category of diagrams does not contribute, we shall not discuss them further. Besides, when a valence gluon attaches to a valence quark, the diagram should be regarded as being from an effective two-parton Fock state, and will not be calculated.

We extract the hard kernels proportional to the final-state momentum $P_{2\mu}$. The hard kernels proportional to $P_{1\mu}$ can be obtained by exchanging the kinetic variables of the initial- and final-state pions. Adopting the electric charge e, instead of the quark charge e_u or e_d, we have taken into account the diagrams with the virtual photon attaching to the antiquark line. Figure 8(a) with a four-gluon vertex gives the dominant three-parton contribution

$$H_{8a} = ieg^2\frac{f_\pi^2 m_0^2}{16y_1 y_2}\frac{N_c^2}{8(N_c^2 - 1)}$$
$$\times\frac{\text{tr}[\gamma^\sigma\gamma_5 P_2\gamma^{T\beta}\gamma_\mu(P_1 - k_2 - l_2)\gamma^\nu P_1\gamma^{T\alpha}\gamma_5]}{(P_1 - k_2 - l_2)^2(k_1 - k_2)^2(k_1 - k_2 + l_1 - l_2)^2}$$
$$\times(g_{\alpha\nu}g_{\beta\sigma} + g_{\alpha\sigma}g_{\beta\nu} - 2g_{\alpha\beta}g_{\sigma\nu})$$
$$= -ieg^2\frac{f_\pi^2 m_0^2}{16y_1 y_2}\frac{N_c^2}{N_c^2 - 1}$$
$$\times\frac{\text{tr}[P_2\gamma_\mu P_2 P_1]}{(P_1 - P_2)^2(k_1 - k_2 + l_1 - l_2)^2(k_1 - k_2)^2}. \qquad (32)$$

To arrive at the second expression, we have made an approximation according to the power counting $Q^2 \gg xQ^2$, $yQ^2 \gg k_T^2$ [14], under which the TMD term in the following denominator is neglected:
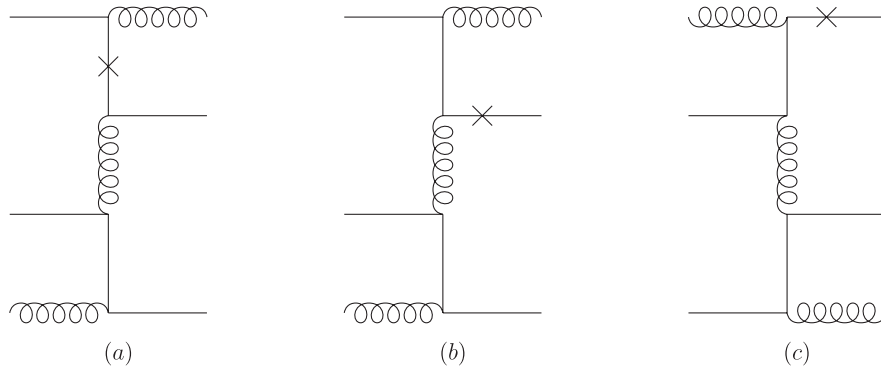
FIG. 6.   Three-parton-to-three-parton diagrams in category A, where one valence gluon attaches to the quark line and another to the antiquark line.
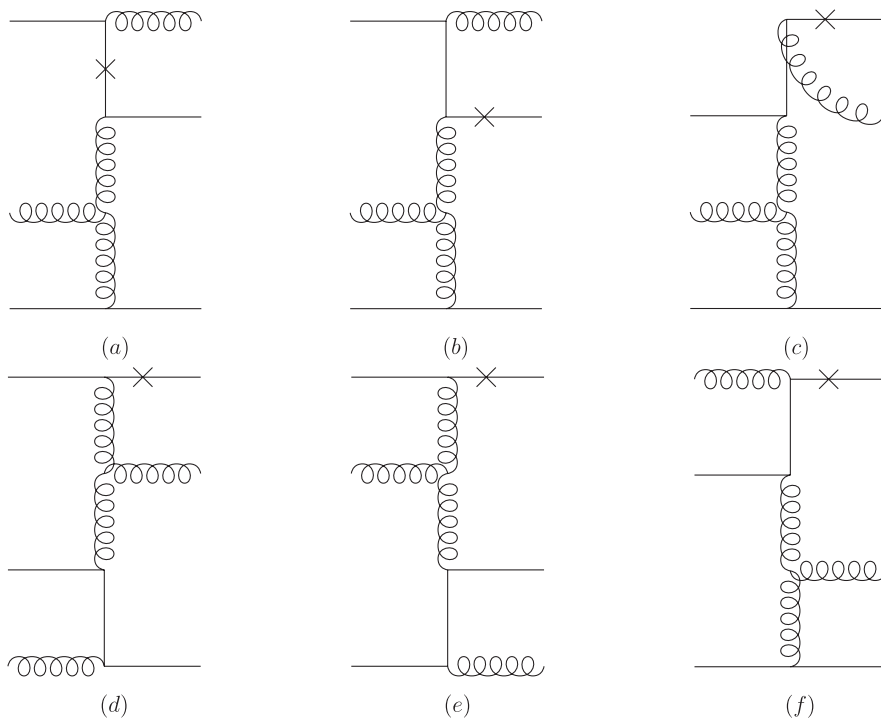

FIG. 7.   Three-parton-to-three-parton diagrams in category B, where one valence gluon attaches to the hard gluon line.
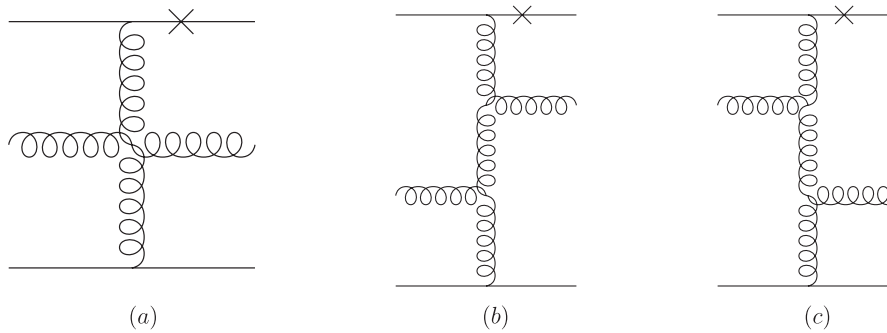

FIG. 8.   Three-parton-to-three-parton diagrams in category C, where both valence gluons attach to the hard gluon line.

$$\frac{(k_2 - l_2)}{(P_1 - k_2 - l_2)^2} = \frac{(k_2 + l_2)^- \gamma^+}{-2P_1^+(k_2 + l_2)^-} = \frac{P_2^- \gamma^+}{-2P_1^+ P_2^-}$$

$$= \frac{P_2}{(P_1 - P_2)^2}. \tag{33}$$

The expressions for other three-parton-to-three-parton diagrams are collected in Appendix B.

Since the Sudakov factor for exclusive QCD processes was derived in the space of impact parameters [4,5], we Fourier transform Eq. (32). The k_T factorization formula for the pion electromagnetic pion form factor from Fig. 8(a) is then written as

$$F_{8a}(Q^2) = \pi \alpha_s f_\pi^2 m_0^2 \frac{N_c^2}{N_c^2 - 1} \int_0^1 dx_1 \int_0^{1-x_1} \frac{dy_1}{y_1}$$

$$\times \int_0^1 dx_2 \int_0^{1-x_2} \frac{dy_2}{y_2} \Phi(x_1, y_1) \Phi(x_2, y_2)$$

$$\times K(\sqrt{(x_1 + y_1)(x_2 + y_2)} Q) K(\sqrt{x_1 x_2} Q), \tag{34}$$

where the three-parton pion distribution amplitude $\Phi(x_1, y_1)$ corresponds to $T(z, z')$ in Eq. (30) in the space of momentum fractions. The functions K, arising from the Fourier transformation of the TMD denominators $(k_1 - k_2 + l_1 - l_2)^2$ and $(k_1 - k_2)^2$ in Eq. (32), are defined by

$$K(t) = \int_0^{1/\Lambda} b \, db \, K_0(tb) \exp[-s(P_1^+, b)], \tag{35}$$

in which K_0 is the modified Bessel function, and the explicit expression of the Sudakov exponent $s(P_1^+, b)$ is referred to [5,24]. We have kept only the most effective piece of the Sudakov evolution in the small x region, that results from the gluon exchanges between the energetic valence quark and the Wilson line associated with it. Because the Sudakov factor $\exp[-s(P_1^+, b)]$ diminishes at $b = 1/\Lambda$, with the QCD scale $\Lambda \approx 0.3$ GeV, the upper bound of the integration variable has been set to $1/\Lambda$ in Eq. (35). For an order-of-magnitude estimate and for demonstrating the smallness of the three-parton contribution, we do not consider the renormalization-group evolution from the low scale, at which Φ is defined, to the scale of the hard kernel. The coupling constant is also assumed to be a constant $\alpha_s = 0.5$.

Fourier transforming the hard kernels in Appendix B into the impact-parameter space, we construct the corresponding k_T factorization formulas similar to Eq. (34). We then add all the contributions to the pion electromagnetic form factor, and employ the model of the three-parton twist-3 pion distribution amplitude [20,22]

$$\Phi(x_1, y_1) = 360 \eta_3 x_1 (1 - x_1 - y_1) y_1^2 \left[1 + \frac{\omega_3}{2} (7y_1 - 3) \right], \tag{36}$$
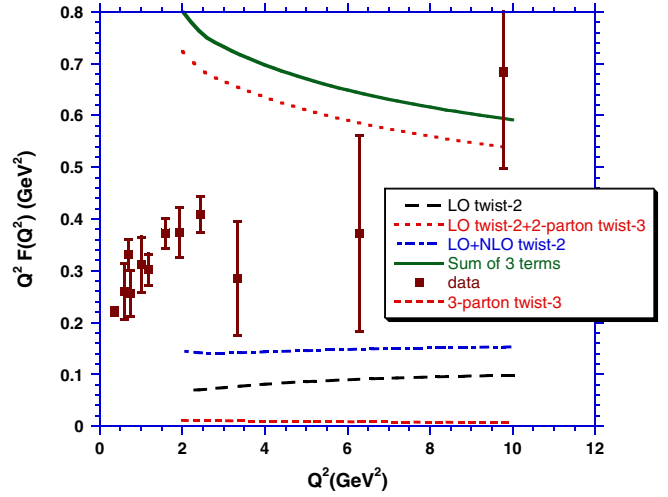

FIG. 9 (color online). Q^2 dependence of the three-parton contribution to the pion electromagnetic form factor. The LO twist-2, NLO twist-2, and LO two-parton twist-3 contributions are quoted from Fig. 7 in [14], which were derived using the nonasymptotic models for the pion distribution amplitudes.

for a numerical analysis, with the parameters $\eta_3 = 0.015$ and $\omega_3 = -3$, and $1 - x_1 - y_1$ being the momentum fraction of the valence quark. The total three-parton contribution to $Q^2 F(Q^2)$, $F(Q^2)$ being the pion electromagnetic form fact, is displayed in Fig. 9. The curve exhibits a decrease in Q^2 (though not obvious in the figure) compared to the LO and NLO twist-2 contributions, indicating that this contribution is power-suppressed. It is only about 5% of the sum of those evaluated in [14,25], including the LO twist-2, NLO twist-2, and LO two-parton twist-3 pieces. That is, the three-parton contribution is not crucial for accommodating the experimental data of $Q^2 F(Q^2)$ [26,27] in the whole accessible range of Q^2 up to 10 GeV^2. The only important subleading contribution to the pion electromagnetic form factor that has been investigated so far comes from the chirally enhanced two-parton twist-3 one.

IV. CONCLUSION

In this paper we have applied the k_T factorization theorem to the study of the power-suppressed three-parton contribution to the pion electromagnetic form factor. It was demonstrated that the gauge invariance of the k_T-dependent hard kernel holds for this power correction: the gauge dependence proportional to parton transverse momenta from the two-parton Fock state and the gauge dependence associated with the three-parton Fock state cancel each other. We have calculated the three-parton-to-three-parton hard kernel at LO, and found that the three-parton contribution is about 5% of the sum of the LO twist-2, NLO twist-2, and LO two-parton twist-3 ones in the whole range of experimentally accessible Q^2. Our

analysis shows that the power expansion for this exclusive process might be reliable in the k_T factorization theorem. At the same power of $1/Q^2$, the two-parton twist-4 contribution should be taken into account, which has been studied in the framework of light-cone sum rules [19]. We shall calculate this correction in the k_T factorization theorem in the future, which involves a twist-2 distribution amplitude from one side and a two-parton twist-4 distribution amplitude from the other side. We shall also extend our framework to exclusive B meson decays, for which three-parton contributions have been analyzed in light-cone sum rules [28], in the QCD (collinear) factorization [29], and in the soft-collinear effective theory [30].

APPENDIX A: GAUGE DEPENDENCE

In this appendix we present the detailed derivation of the gauge-dependent hard kernels from Fig. 2. The attachment A contains, with the color factor in Eq. (14), the gauge-dependent piece

$$H_A^\lambda = ieg^2\lambda\frac{1}{32N_c^2}\frac{\text{tr}[(k_1-k_2)\gamma_5\gamma_\beta\gamma_\mu(P_1-k_2)\gamma^\alpha(P_1-l_1-k_2)(k_1-k_2)\gamma_\alpha\gamma_5]}{(P_1-k_2)^2(P_1-l_1-k_2)^2(k_1-k_2)^4}. \tag{A1}$$

Inserting the identity $k_1 - k_2 = (P_1 - l_1 - k_2) - (P_1 - l_1 - k_1)$ into Eq. (A1), we obtain

$$H_A^\lambda = ieg^2\lambda\frac{1}{32N_c^2}\Bigg\{\frac{\text{tr}[(k_1-k_2)\gamma_5\gamma_\beta\gamma_\mu(P_1-k_2)\gamma^\alpha\gamma_\alpha\gamma_5]}{(P_1-k_2)^2(k_1-k_2)^4}$$
$$-\frac{\text{tr}[(k_1-k_2)\gamma_5\gamma_\beta\gamma_\mu(P_1-k_2)\gamma^\alpha(P_1-l_1-k_2)(P_1-k_1-lP_1-k_1-l_1)\gamma_\alpha\gamma_5]}{(P_1-k_2)^2(P_1-l_1-k_2)^2(k_1-k_2)^4}\Bigg\}. \tag{A2}$$

The second term, proportional to the momentum $P_1 - k_1 - l_1$ of the incoming valence quark, should be dropped, since there is the valence gluon A^α from the initial state already. Taking the derivative of the first term with respect to $k_{2\beta}$, and then substituting $k_1 = x_1 P_1$ and $k_2 = x_2 P_2$ into the numerator, we have Eq. (15).

The diagram with the attachment B of the valence gluon to the hard gluon line produces the gauge-dependent hard kernel

$$H_B^\lambda = -eg^2\frac{1}{8N_c}\frac{\text{tr}[\gamma_{\delta'}\gamma_5\gamma_\beta\gamma_\mu(P_1-k_2)\gamma_{\nu'}\gamma^\alpha\gamma_5]}{(P_1-k_2)^2}\text{tr}(T^dT^bT^c)\Gamma_{\delta\nu\alpha}^{dba}\Bigg[\lambda\frac{g^{\delta\delta'}}{(k_1-k_2)^2}\frac{(k_1-k_2+l_1)^\nu(k_1-k_2+l_1)^{\nu'}}{(k_1-k_2+l_1)^4}$$
$$+\lambda\frac{(k_1-k_2)^\delta(k_1-k_2)^{\delta'}}{(k_1-k_2)^4}\frac{g^{\nu\nu'}}{(k_1-k_2+l_1)^2}-\lambda^2\frac{(k_1-k_2)^\delta(k_1-k_2)^{\delta'}}{(k_1-k_2)^4}\frac{(k_1-k_2+l_1)^\nu(k_1-k_2+l_1)^{\nu'}}{(k_1-k_2+l_1)^4}\Bigg], \tag{A3}$$

with the triple-gluon vertex,

$$\Gamma_{\delta\nu\alpha}^{dba} = f^{dba}[g_{\alpha\nu}(2l_1+k_1-k_2)_\delta + g_{\nu\delta}(2k_2-2k_1-l_1)_\alpha + g_{\delta\alpha}(k_1-k_2-l_1)_\nu], \tag{A4}$$

f^{dba} being a antisymmetric tensor. Using the identity

$$\text{tr}(T^dT^bT^c) = \frac{1}{4}(d^{dbc}+if^{dbc}), \qquad d^{dbc}f^{dba}=0, \qquad f^{dbc}f^{dba}=N_c\delta^{ac}, \tag{A5}$$

d^{dbc} being a symmetric tensor, the above amplitude becomes

$$H_B^\lambda = -ieg^2\frac{1}{32}\frac{\text{tr}[\gamma_{\delta'}\gamma_5\gamma_\beta\gamma_\mu(P_1-k_2)\gamma_{\nu'}\gamma^\alpha\gamma_5]}{(P_1-k_2)^2}[g_{\alpha\nu}(2l_1+k_1-k_2)_\delta + g_{\nu\delta}(2k_2-2k_1-l_1)_\alpha$$
$$+g_{\delta\alpha}(k_1-k_2-l_1)_\nu]\Bigg[\lambda\frac{g^{\delta\delta'}}{(k_1-k_2)^2}\frac{(k_1-k_2+l_1)^\nu(k_1-k_2+l_1)^{\nu'}}{(k_1-k_2+l_1)^4}+\lambda\frac{(k_1-k_2)^\delta(k_1-k_2)^{\delta'}}{(k_1-k_2)^4}\frac{g^{\nu\nu'}}{(k_1-k_2+l_1)^2}$$
$$-\lambda^2\frac{(k_1-k_2)^\delta(k_1-k_2)^{\delta'}}{(k_1-k_2)^4}\frac{(k_1-k_2+l_1)^\nu(k_1-k_2+l_1)^{\nu'}}{(k_1-k_2+l_1)^4}\Bigg]. \tag{A6}$$

The λ^2 term leads to

$$ieg^2 \frac{1}{32} \lambda^2 \frac{\text{tr}[(k_1 - k_2)\gamma_5\gamma_\beta\gamma_\mu(P_1 - k_2)(k_1 - k_2 + l_1)\gamma_\alpha\gamma_5]}{(P_1 - k_2)^2(k_1 - k_2)^4(k_1 - k_2 + l_1)^4}[(k_1 - k_2) \cdot (2l_1 + k_1 - k_2)(k_1 - k_2 + l_1)^\alpha$$

$$+ (k_1 - k_2) \cdot (k_1 - k_2 + l_1)(2k_2 - 2k_1 - l_1)^\alpha + (k_1 - k_2 - l_1) \cdot (k_1 - k_2 + l_1)(k_1 - k_2)^\alpha]. \tag{A7}$$

Inserting the identity $k_1 - k_2 + l_1 = (P_1 - k_2) - (P_1 - k_1 - l_1)$, it is easy to see that the first term of the identity gives an expression which is canceled by the corresponding one from the attachment G. The second term, being proportional to the momentum $P_1 - k_1 - l_1$ of the incoming valence quark, should be neglected.

We then consider the λ terms. The first λ term in Eq. (A6) gives

$$- ieg^2 \frac{1}{32} \lambda \frac{\text{tr}[(2l_1 + k_1 - k_2)\gamma_5\gamma_\beta\gamma_\mu(P_1 - k_2)(k_1 - k_2 + l_1)\gamma_\alpha\gamma_5]}{(P_1 - k_2)^2(k_1 - k_2)^2(k_1 - k_2 + l_1)^4}(k_1 - k_2 + l_1)^\alpha$$

$$- ieg^2 \frac{1}{32} \lambda \frac{\text{tr}[(k_1 - k_2 + l_1)\gamma_5\gamma_\beta\gamma_\mu(P_1 - k_2)(k_1 - k_2 + l_1)\gamma_\alpha\gamma_5]}{(P_1 - k_2)^2(k_1 - k_2)^2(k_1 - k_2 + l_1)^4}(2k_2 - 2k_1 - l_1)^\alpha \tag{A8}$$

$$- ieg^2 \frac{1}{32} \lambda \frac{\text{tr}[\gamma^\alpha\gamma_5\gamma_\beta\gamma_\mu(P_1 - k_2)(k_1 - k_2 + l_1)\gamma_\alpha\gamma_5]}{(P_1 - k_2)^2(k_1 - k_2)^2(k_1 - k_2 + l_1)^4}(k_1 - k_2 + l_1) \cdot (k_1 - k_2 - l_1),$$

which is negligible for the same reason as for the λ^2 term. Hence, H_B^λ receives a contribution only from the second λ term,

$$H_B^\lambda = -ieg^2 \frac{1}{32} \lambda \frac{\text{tr}[(k_1 - k_2)\gamma_5\gamma_\beta\gamma_\mu(P_1 - k_2)\gamma^\alpha\gamma_\alpha\gamma_5]}{(P_1 - k_2)^2(k_1 - k_2)^4(k_1 - k_2 + l_1)^2}(k_1 - k_2) \cdot (2l_1 + k_1 - k_2)$$

$$= -ieg^2 \frac{1}{32} \lambda \frac{\text{tr}[(k_1 - k_2)\gamma_5\gamma_\beta\gamma_\mu(P_1 - k_2)\gamma^\alpha\gamma_\alpha\gamma_5]}{(P_1 - k_2)^2(k_1 - k_2)^4}. \tag{A9}$$

To arrive at the second line, the higher-power term $l_1^2 = -l_{1T}^2$ has been added, so we have $(k_1 - k_2) \cdot (2l_1 + k_1 - k_2) + l_1^2 = (k_1 - k_2 + l_1)^2$. The differentiation of the above expression with respect to $k_{2\beta}$ leads to Eq. (16). The diagrams with other attachments in Fig. 2 can be calculated in a similar way, so the detail will not be presented here.

APPENDIX B: HARD KERNELS

In this appendix we collect the expressions of the three-parton-to-three-parton hard kernels for the pion electromagnetic form factor in the Feynman gauge. Start with category A defined in Sec. III. When the attachments of the two valence gluons are arranged in the way that the hard gluon vertices sandwich the spin structures associated with the pions, the contribution diminishes because of $\gamma^\nu P_1 \gamma_\alpha^T \gamma_\nu = 0$ or $\gamma^\nu P_2 \gamma_\beta^T \gamma_\nu = 0$. The nonvanishing amplitudes come from Figs. 5 and 6, which are written as

$$H_{5a} = -ieg^2 \frac{N_c^2 + 1}{N_c^2(N_c^2 - 1)} \frac{\text{tr}(P_2\gamma_\mu P_2 P_1 P_2 P_1)}{(P_1 - P_2)^2(P_1 - k_2 - l_2)^2(P_1 - k_1 - l_1 - P_2)^2(k_1 - k_2)^2}, \tag{B1}$$

$$H_{5b} = ieg^2 \frac{N_c^2 + 1}{N_c^2(N_c^2 - 1)} \frac{\text{tr}(P_2\gamma_\mu P_2 P_1 l_2 l_1)}{(P_1 - P_2)^2(k_1 - k_2 + l_1 - l_2)^2(k_1 - l_2)^2(k_2 - l_1)^2}, \tag{B2}$$

$$H_{5c} = ieg^2 \frac{N_c^2 + 1}{N_c^2(N_c^2 - 1)} \frac{\text{tr}(P_2 P_1 P_2 \gamma_\mu P_2 P_1)}{(P_2 - k_2 - l_2 - P_1)^2(P_2 - k_1 - l_1)^2(P_1 - k_1 - l_1 - P_2)^2(k_1 - k_2)^2}, \tag{B3}$$

$$H_{6a} = ieg^2 \frac{1}{N_c^2(N_c^2 - 1)} \frac{\text{tr}(P_2 l_1 P_2 \gamma_\mu P_2 P_1)}{(P_1 - k_1 - l_1 - P_2)^2(k_1 - k_2 + l_1)^2(l_1 - k_2)^2(P_2 - k_1 - l_1)^2}, \tag{B4}$$

$$H_{6b} = -ieg^2 \frac{1}{N_c^2(N_c^2 - 1)} \frac{\text{tr}(P_2\gamma_\mu P_2 P_1 P_2 P_1)}{(P_1 - P_2)^2(k_1 - k_2 + l_1)^2(l_1 - k_2)^2(P_1 - k_1 - l_1 - P_2)^2}, \tag{B5}$$

$$H_{6c} = ieg^2 \frac{1}{N_c^2(N_c^2 - 1)} \frac{\text{tr}(P_2\gamma_\mu P_2 P_1 P_2 P_1)}{(P_1 - P_2)^2(k_1 - k_2)^2(k_1 - l_2)(P_1 - k_2 - l_2)^2}. \tag{B6}$$

It is observed that the results from Figs. 5(a) and 5(c) are suppressed by a power of $1/Q$ compared to Fig. 5(b). That is, when both the valence gluons attach to the quark line, the contribution is power-suppressed. The hard kernels from Fig. 6 are of the same power as Eq. (B2).

Figure 7 from category B gives the hard kernels

$$H_{7a} = -\frac{i}{2}eg^2 \frac{1}{N_c^2 - 1} \frac{\text{tr}[P_2(\not{k}_1 - \not{l}_1)P_2\gamma_\mu P_2 P_1]}{(P_2 - k_1 - l_1)^2(k_1 - k_2 + l_1)^2(k_1 - k_2)^2(P_1 - k_1 - l_1 - P_2)^2}, \tag{B7}$$

$$H_{7b} = \frac{i}{2}eg^2 \frac{1}{N_c^2 - 1} \frac{\text{tr}[P_2\gamma_\mu P_2(\not{k}_1 - \not{l}_1)P_2 P_1]}{(P_1 - P_2)^2(k_1 - k_2 + l_1)^2(k_1 - k_2)^2(P_1 - k_1 - l_1 - P_2)^2}, \tag{B8}$$

$$H_{7c} = \frac{i}{2}eg^2 \frac{\text{tr}(P_2\gamma_\mu P_2 P_1)}{(P_1 - P_2)^2(k_1 - k_2 + l_1)^2(k_1 - k_2)^2}, \tag{B9}$$

$$H_{7d} = -\frac{i}{2}eg^2 \frac{1}{N_c^2 - 1} \frac{\text{tr}[P_2\gamma_\mu P_2 P_1(\not{k}_2 + 2\not{l}_2)\not{l}_1]}{(P_1 - P_2)^2(k_1 - k_2 + l_1)^2(l_1 - k_2)^2(k_1 - k_2 + l_1 - l_2)^2}, \tag{B10}$$

$$H_{7e} = -\frac{i}{2}eg^2 \frac{1}{N_c^2 - 1} \frac{\text{tr}[P_2\gamma_\mu P_2 P_1\not{l}_1(\not{k}_1 + 2\not{l}_1)]}{(P_1 - P_2)^2(k_1 - k_2 + l_1 - l_2)^2(k_1 - k_2)^2(k_1 - k_2 - l_2)^2}, \tag{B11}$$

$$H_{7f} = -\frac{i}{2}eg^2 \frac{1}{N_c^2 - 1} \frac{\text{tr}[P_2\gamma_\mu P_2 P_1(\not{k}_2 - \not{l}_2)P_1]}{(P_1 - P_2)^2(k_1 - k_2)^2(k_1 - k_2 - l_2)^2(P_1 - k_2 - l_2)^2}. \tag{B12}$$

Figure 8 from category C contributes the hard kernels

$$H_{8a} = -ieg^2 \frac{N_c^2}{N_c^2 - 1} \frac{\text{tr}(P_2\gamma_\mu P_2 P_1)}{(P_1 - P_2)^2(k_1 - k_2 + l_1 - l_2)^2(k_1 - k_2)^2}, \tag{B13}$$

$$H_{8b} = -\frac{i}{2}eg^2 \frac{N_c^2}{N_c^2 - 1} \frac{(k_1 - k_2 - 2l_2 + l_1) \cdot (k_1 - k_2 - l_1)\,\text{tr}(P_2\gamma_\mu P_2 P_1)}{(P_1 - P_2)^2(k_1 - k_2 + l_1)^2(k_1 - k_2)^2(k_1 - k_2 + l_1 - l_2)^2}, \tag{B14}$$

$$H_{8c} = -\frac{i}{2}eg^2 \frac{N_c^2}{N_c^2 - 1} \frac{(k_1 - k_2 + l_2) \cdot (k_1 - k_2 + 2l_1 - l_2)\,\text{tr}(P_2\gamma_\mu P_2 P_2 P_1)}{(P_1 - P_2)^2(k_1 - k_2 + l_1 - l_2)^2(k_1 - k_2 - l_2)^2(k_1 - k_2)^2}. \tag{B15}$$

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