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Tetramixing of vector and pseudoscalar mesons: A source of intrinsic quarks

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The tetramixing of pseudoscalar mesons π-η-η'-η_c and vector mesons ω-ρ-ϕ-J/ψ are studied in the light-cone constituent quark model, and such mixing of four mesons provides a natural source for the intrinsic charm $c\bar{c}$ components of light mesons. By mixing with the light mesons, the charmonium states J/ψ and η_c could decay into light mesons more naturally, without introducing gluons or a virtual photon as intermediate states. Thus, the introduction of light quark components into J/ψ is helpful to reproduce the new experimental data of J/ψ decays. The mixing matrices and the Q^2 behaviors of the transition form factors are also calculated and compared with experimental data.

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The mixing of mesons has been widely investigated since the 1960s, when the concept of a mixing state of the ρ-ω mesons was proposed [1] by considering that the electromagnetic interaction does not conserve isospin. Later, the ω-ϕ mixing and η-η' mixing were introduced [2] to explain the deviation of the meson mass from the Gell-Mann-Okubo mass formula [3,4]. It was also pointed out that the difference between the u and d quark masses introduces the π-η mixing [5]. Then, the trimixing of π-η-η' [6,7] and ρ-ω-ϕ [7,8] were proposed, and their effects were studied in different methods. On the other hand, the $c\bar{c}$ contribution to the η and η' mesons was considered [9], and the trimixing η-η'-η_c was studied [10]. As a further extension in this paper, we try to combine the above two types of trimixing by considering the tetramixing of pseudoscalar mesons π-η-η'-η_c. The mixing of gluon component gg and η-η' were also studied [11,12]. As the mixing of η and η' is still not completely clear right now, we think that the charm and gluon components may both be possible to mix with these mesons, and it is worthwhile to study both of them carefully.

The recent CLEO experiment [13] of the charmonium decays $J/\psi \to \gamma\pi$, $\gamma\eta$, and $\gamma\eta'$ also motivates us to extend our tetramixing to the vector mesons ω-ρ-ϕ-J/ψ. According to the pure valence $c\bar{c}$ structure of charmonia in the naive quark model, these decay modes of charmonium $\psi(nS)$ must happen via the annihilation of the heavy quark constituents into gluons or a virtual photon [13,14], because of the Okubo-Zweig-Iizuka rule, which postulates a suppression of transitions between hadrons without valence quarks in common [14]. Moreover, the mechanisms of these decays are not completely clear yet, and there are various ways to describe them, such as $\psi(nS) \to \gamma gg \to \gamma P$, $\psi(nS) \to ggg \to q\bar{q}\gamma$, $\psi(nS) \to \gamma^* \to q\bar{q}\gamma$, and so on [13]. However, with the model of ω-ρ-ϕ-J/ψ mixing, the above-mentioned decays of J/ψ could occur more naturally through the direct transition from its light quark components to light mesons such as π, η, or η' without introducing intermediate gluons or a virtual photon, and the $c\bar{c}$ components of these light pseudoscalar mesons also allow J/ψ to decay to them. The mixing of ω-ρ-ϕ-J/ψ is thus helpful to reproduce the new experimental data of J/ψ decays.

For the light vector mesons ω, ρ, and ϕ, the existence of $c\bar{c}$ states in them may be interpreted as a support to the theory of intrinsic charm [15] in these mesons. Different from the extrinsic quarks, which are generated on a short time scale in a reaction process with large momentum transfers, the intrinsic quarks are intrinsic nonperturbatively to the hadron wave function and exist over a time scale independent of any probe momentum [15,16]. The postulation of intrinsic $c\bar{c}$ components in ρ and π offers a possible solution of the "$\rho\pi$ puzzle" by allowing direct transitions between $J/\psi(\psi')$ and $\rho(\pi)$ through the rearrangement of the valence and the intrinsic $c\bar{c}$ components of $\rho(\pi)$ [14]. Now the tetramixing of ω-ρ-ϕ-J/ψ introduces the intrinsic $c\bar{c}$ components into all three light vector mesons ω, ρ, and ϕ, and J/ψ can decay to them in a similar way, without annihilation of the quark constituents. This applies to the pseudoscalar mesons π, η, and η', too, as they mix with the charmonium η_c in our model. The intrinsic $c\bar{c}$ component in η' was also studied in Refs. [17,18], in which the intrinsic charm content of the η' meson $f^c_{\eta'}$ was evaluated, and we shall compare our result of $f^c_{\eta'}$ with previous results in Refs. [17,18] and other works at the end of this paper.

We adopt the light-cone constituent quark model [19–21] to study the mixing of mesons. The light-cone constituent quark model is a convenient and effective model to treat the nonperturbative aspect of QCD, and

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the mixing of mesons in this model has been studied [22,23].

The mixing of pseudoscalar mesons and vector mesons could be described by two $SO(4)$ rotation matrices M_v and M_s, respectively:

$$\begin{pmatrix} \omega \\ \rho \\ \phi \\ J/\psi \end{pmatrix} = M_v \begin{pmatrix} \omega_I \\ \rho_I \\ \phi_I \\ J/\psi_I \end{pmatrix}, \qquad \begin{pmatrix} \pi \\ \eta \\ \eta' \\ \eta_c \end{pmatrix} = M_s \begin{pmatrix} \pi_I \\ \eta_q \\ \eta_s \\ \eta_{c0} \end{pmatrix}, \tag{1}$$

in which the unmixed states are

$$\omega_I = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\varphi_{\omega_I}, \qquad \rho_I = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})\varphi_{\rho_I},$$
$$\phi_I = -s\bar{s}\varphi_{\phi_I}, \qquad J/\psi_I = c\bar{c}\varphi_{J/\psi_I},$$
$$\pi_I = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})\varphi_{\pi_I}, \qquad \eta_q = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\varphi_{\eta_q},$$
$$\eta_s = s\bar{s}\varphi_{\eta_s}, \qquad \eta_{c0} = c\bar{c}\varphi_{\eta_{c0}}, \tag{2}$$

where φ is the momentum space wave function of the corresponding meson.

Since the rotation group $SO(4) = SO(3) \otimes SO(3)$, the $SO(4)$ mixing matrix M can be written as

$$M = R_+ R_-, \tag{3}$$

where the matrices R_+ and R_- are generated by the $SO(3)$ generators, and each of them could be parameterized by three independent rotation angles as

$$R_+(\theta_1, \theta_2, \theta_3) = \begin{pmatrix} \cos\frac{\alpha}{2} & -\frac{\theta_1}{\alpha}\sin\frac{\alpha}{2} & -\frac{\theta_2}{\alpha}\sin\frac{\alpha}{2} & -\frac{\theta_3}{\alpha}\sin\frac{\alpha}{2} \\ \frac{\theta_1}{\alpha}\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} & -\frac{\theta_3}{\alpha}\sin\frac{\alpha}{2} & \frac{\theta_2}{\alpha}\sin\frac{\alpha}{2} \\ \frac{\theta_2}{\alpha}\sin\frac{\alpha}{2} & \frac{\theta_3}{\alpha}\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} & -\frac{\theta_1}{\alpha}\sin\frac{\alpha}{2} \\ \frac{\theta_3}{\alpha}\sin\frac{\alpha}{2} & -\frac{\theta_2}{\alpha}\sin\frac{\alpha}{2} & \frac{\theta_1}{\alpha}\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} \end{pmatrix}, \tag{4}$$

$$R_-(\theta_4, \theta_5, \theta_6) = \begin{pmatrix} \cos\frac{\beta}{2} & \frac{\theta_4}{\beta}\sin\frac{\beta}{2} & \frac{\theta_5}{\beta}\sin\frac{\beta}{2} & \frac{\theta_6}{\beta}\sin\frac{\beta}{2} \\ -\frac{\theta_4}{\beta}\sin\frac{\beta}{2} & \cos\frac{\beta}{2} & -\frac{\theta_6}{\beta}\sin\frac{\beta}{2} & \frac{\theta_5}{\beta}\sin\frac{\beta}{2} \\ -\frac{\theta_5}{\beta}\sin\frac{\beta}{2} & \frac{\theta_6}{\beta}\sin\frac{\beta}{2} & \cos\frac{\beta}{2} & -\frac{\theta_4}{\beta}\sin\frac{\beta}{2} \\ -\frac{\theta_6}{\beta}\sin\frac{\beta}{2} & -\frac{\theta_5}{\beta}\sin\frac{\beta}{2} & \frac{\theta_4}{\beta}\sin\frac{\beta}{2} & \cos\frac{\beta}{2} \end{pmatrix}, \tag{5}$$

where

$$\alpha = \sqrt{\theta_1^2 + \theta_2^2 + \theta_3^2}, \qquad \beta = \sqrt{\theta_4^2 + \theta_5^2 + \theta_6^2}, \tag{6}$$

and thus the mixing matrix M is parameterized as six independent rotation angles $(\theta_1, \theta_2, \ldots, \theta_6)$. Our detailed procedure of obtaining the matrix form of R_+ and R_- [Eqs. (4) and (5)] is given in Appendix A.

When referring to the mixing of specific types of mesons, M stands for M_v or M_s, and the parameters change to $(\theta_1^v, \theta_2^v, \ldots)$ or $(\theta_1^s, \theta_2^s, \ldots)$ correspondingly.

During the numerical calculation, we also used a more compact form of M with eight real parameters under constraints, and the detailed procedure is given in Appendix B.

The decay constants and transition form factors also mix as [7]

$$\begin{pmatrix} f_\omega \\ f_\rho \\ f_\phi \\ f_{J/\psi} \end{pmatrix} = M_v \begin{pmatrix} f_{\omega_I} \\ f_{\rho_I} \\ f_{\phi_I} \\ f_{J/\psi_I} \end{pmatrix},$$
$$\begin{pmatrix} F_{\pi\to\gamma\gamma^*}(Q^2) \\ F_{\eta\to\gamma\gamma^*}(Q^2) \\ F_{\eta'\to\gamma\gamma^*}(Q^2) \\ F_{\eta_c\to\gamma\gamma^*}(Q^2) \end{pmatrix} = M_s \begin{pmatrix} F_{\pi_I\to\gamma\gamma^*}(Q^2) \\ F_{\eta_q\to\gamma\gamma^*}(Q^2) \\ F_{\eta_s\to\gamma\gamma^*}(Q^2) \\ F_{\eta_{c0}\to\gamma\gamma^*}(Q^2) \end{pmatrix}, \tag{7}$$

$$\begin{pmatrix} F_{\omega\to\pi\gamma^*}(Q^2) \\ F_{\omega\to\eta\gamma^*}(Q^2) \\ F_{\eta'\to\omega\gamma^*}(Q^2) \\ F_{\eta_c\to\omega\gamma^*}(Q^2) \\ F_{\rho\to\pi\gamma^*}(Q^2) \\ F_{\rho\to\eta\gamma^*}(Q^2) \\ F_{\eta'\to\rho\gamma^*}(Q^2) \\ F_{\eta_c\to\rho\gamma^*}(Q^2) \\ F_{\phi\to\pi\gamma^*}(Q^2) \\ F_{\phi\to\eta\gamma^*}(Q^2) \\ F_{\phi\to\eta'\gamma^*}(Q^2) \\ F_{\eta_c\to\phi\gamma^*}(Q^2) \\ F_{J/\psi\to\pi\gamma^*}(Q^2) \\ F_{J/\psi\to\eta\gamma^*}(Q^2) \\ F_{J/\psi\to\eta'\gamma^*}(Q^2) \\ F_{J/\psi\to\eta_c\gamma^*}(Q^2) \end{pmatrix} = (M_v \otimes M_s) \begin{pmatrix} F_{\omega_I\to\pi_I\gamma^*}(Q^2) \\ F_{\omega_I\to\eta_q\gamma^*}(Q^2) \\ 0 \\ 0 \\ F_{\rho_I\to\pi_I\gamma^*}(Q^2) \\ F_{\rho_I\to\eta_q\gamma^*}(Q^2) \\ 0 \\ 0 \\ 0 \\ 0 \\ F_{\phi_I\to\eta_q\gamma^*}(Q^2) \\ 0 \\ 0 \\ 0 \\ 0 \\ F_{J/\psi_I\to\eta_{c0}\gamma^*}(Q^2) \end{pmatrix}. \tag{8}$$

The above decay constants and transition form factors are defined as [23,24]

$$\langle 0|j_\mu|V(p, S_z)\rangle = M_V f_V \epsilon_\mu(S_z), \tag{9}$$

$$\langle \gamma(p-q)|J^\mu|P(p,\lambda)\rangle = ie^2 F_{P\to\gamma\gamma^*}(Q^2)\varepsilon^{\mu\nu\rho\sigma} p_\nu \epsilon_\rho(p-q,\lambda) q_\sigma, \tag{10}$$

$$\langle P(p')|J^\mu|V(p,\lambda)\rangle = ieF_{V\to P\gamma}(Q^2)\varepsilon^{\mu\nu\rho\sigma}\epsilon_\nu(p,\lambda)p'_\rho p_\sigma. \tag{11}$$

To calculate them, we use the light-cone quark model with the Fock state expansions of the unmixed mesons [the right-hand side of Eq. (1)]:

$$|M\rangle = \sum |q\bar{q}\rangle\psi_{q\bar{q}} + \sum |q\bar{q}g\rangle\psi_{q\bar{q}g} + \cdots, \tag{12}$$

and to simplify the problem, we adopt the lowest order of the above expansions, which takes only the quark-antiquark valence states of the unmixed mesons into consideration.

The wave function of an unmixed meson in the light-cone formalism is [19,25]

$$|M(P^+, \mathbf{P}_\perp, S_z)\rangle = \int \frac{dx d^2\mathbf{k}_\perp}{\sqrt{x(1-x)}16\pi^3} \varphi(x, \mathbf{k}_\perp) \chi_M^{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2), \quad (13)$$

where φ is the momentum space wave function, described by the Brodsky-Huang-Lepage prescription [19,25]:

$$\varphi(x, \mathbf{k}_\perp) = \varphi_{\mathrm{BHL}}(x, \mathbf{k}_\perp) = A \exp\left[-\frac{1}{8\beta^2}\left(\frac{m_1^2 + \mathbf{k}_\perp^2}{x} + \frac{m_2^2 + \mathbf{k}_\perp^2}{1-x}\right)\right] \quad (14)$$

(A and β are the parameters of the meson, and m_1 and m_2 are masses of the constituent quarks), and $\chi_M^{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)$ is the light-cone spin wave function, which is related to the instant-form spin wave function by the Melosh-Wigner rotation [26–28]

$$\chi_i^\uparrow(T) = w_i[(k_i^+ + m_i)\chi_i^\uparrow(F) - k_i^R \chi_i^\downarrow(F)];$$
$$\chi_i^\downarrow(T) = w_i[(k_i^+ + m_i)\chi_i^\downarrow(F) + k_i^L \chi_i^\uparrow(F)], \quad (15)$$

where $w_i = 1/\sqrt{2k_i^+(k^0 + m_i)}$, $k^{R,L} = k^1 \pm k^2$, $k^+ = k^0 + k^3 = x\mathcal{M}$, and $\mathcal{M} = \sqrt{(\mathbf{k}_\perp^2 + m_1^2)/x + (\mathbf{k}_\perp^2 + m_2^2)/(1-x)}$. The Melosh-Wigner rotation is an important ingredient of the light-cone quark model and plays an essential role in explaining the "proton spin puzzle" [28,29]. The detailed formulas for calculating the decay constants and transition form factors of mesons were listed in Ref. [23], and the examples of applying them to set meson parameters and to calculate the decay constants and transition form factors numerically can be found in Ref. [30].

The values of the meson parameters A, β, m_1, and m_2 and the parameters of the mixing matrices $(a_v, b_v, \ldots)$ and $(a_s, b_s, \ldots)$ can be chosen by fitting the light-cone constituent quark model results of the meson decay constants and transition form factors (at $Q^2 = 0$) to experimental data. The $Q^2 \to \infty$ limiting behavior of $Q^2 F_{P\to\gamma\gamma^*}$ is also considered as a constraint to set the parameters [21,31]:

$$\lim_{Q^2\to\infty} Q^2 F_{P\to\gamma\gamma^*}(Q^2) = 2c_P f_P = \frac{2c_P^2}{4\pi^2 F_{P\to\gamma\gamma^*}(0)}, \quad (16)$$

TABLE I. Experimental data and the light-cone constituent quark model fitting results of the meson decay constants and transition form factors. The experimental data (unmarked) are from Ref. [32], and the experimental data (marked with daggers) are from Ref. [13]. The data in the fourth column (unmarked) are from Ref. [7], and the data (marked with stars) are calculated assuming that J/ψ and η_c do not mix.

Decay constants or form factors	Experimental data (GeV)	Theoretical fitting of tetramixing (GeV)	Theoretical fitting of tetramixing (GeV)
$F_{\pi\to\gamma\gamma^*}(0)$	0.2744 ± 0.0082	0.2909	0.279
$F_{\eta\to\gamma\gamma^*}(0)$	0.2726 ± 0.0074	0.2891	0.277
$F_{\eta'\to\gamma\gamma^*}(0)$	0.3423 ± 0.0101	0.3187	0.334
$F_{\eta_c\to\gamma\gamma^*}(0)$	0.0806 ± 0.0004	0.0568	0.0810*
$f_\omega(\omega \to e^+e^-)$	0.0466 ± 0.0005	0.0502	0.04556
$f_\rho(\rho \to e^+e^-)$	0.1549 ± 0.0009	0.1815	0.1603
$f_\phi(\phi \to e^+e^-)$	0.0758 ± 0.0005	0.0729	0.075
$f_{J/\psi}(J/\psi \to e^+e^-)$	0.2768 ± 0.0044	0.2734	0.2749*
$F_{\omega\to\pi\gamma^*}(0)$	2.2978 ± 0.0403	2.4639	2.382
$F_{\omega\to\eta\gamma^*}(0)$	0.4494 ± 0.0197	0.4285	0.454
$F_{\eta'\to\omega\gamma^*}(0)$	0.4260 ± 0.0355	0.4528	0.461
$F_{\eta_c\to\omega\gamma^*}(0)$	?	−0.0895	0
$F_{\rho\to\pi\gamma^*}(0)$	0.8237 ± 0.0549	0.9207	0.84
$F_{\rho\to\eta\gamma^*}(0)$	1.5687 ± 0.0525	1.6124	1.50
$F_{\eta'\to\rho\gamma^*}(0)$	1.3175 ± 0.0327	1.3815	1.39
$F_{\eta c\to\rho\gamma^*}(0)$	?	−0.0537	0
$F_{J/\psi\to\pi\gamma^*}(0)$	$0.0006 \pm 0.000\,03^\dagger$	0.0006	0
$F_{J/\psi\to\eta\gamma^*}(0)$	$0.0035 \pm 0.000\,07^\dagger$	0.0035	0
$F_{J/\psi\to\eta'\gamma^*}(0)$	$0.0085 \pm 0.0002^\dagger$	0.0083	0
$F_{J/\psi\to\eta_c\gamma^*}(0)$	0.6583 ± 0.0787	0.5991	0.6545*

TABLE II. The meson parameters A and β (GeV) and the masses (GeV) of constituent quarks determined from the fitting process.

A_{ω_I}	A_{ρ_I}	A_{ϕ_I}	A_{J/ψ_I}	A_{π_I}	A_{η_q}	A_{η_s}	$A_{\eta_{c0}}$	$m_{u(d)}$	m_s
41.4712	38.1430	63.1638	31.1724	47.3635	38.7860	95.4496	125.8099	0.198	0.556
β_{ω_I}	β_{ρ_I}	β_{ϕ_I}	β_{J/ψ_I}	β_{π_I}	β_{η_q}	β_{η_s}	$\beta_{\eta_{c0}}$	m_c	
0.4319	0.4318	0.4757	0.9781	0.4112	0.4887	0.4887	0.7373	1.270	

where $c_P = (c_{\pi_I}, c_{\eta_q}, c_{\eta_s}) = (1, 5/3, \sqrt{2}/3)$. With the similar method as Ref. [17], we also obtain $c_{\eta_{c0}} = 4\sqrt{2}/3$. All these requirements are taken as constraints to determine the parameters of mesons and parameters of the mixing matrices. During our calculation, we first use the decay constants and the radii of π^+ and K^+ as the constraints to locate the values of A_π, β_π, m_u, and m_s, assuming that the wave function parameters of $\pi^\pm$ are the same as those of π_I, and the constituent quark mass $m_d \approx m_u$ (their difference could be ignored compared with m_c) [7]. The other parameters are then determined under the left constraints.

Our numerical calculation gives the mixing matrices of vector and pseudoscalar mesons:

$$M_v = \begin{pmatrix} 0.9886 & -0.0122 & -0.1429 & 0.0076 \\ 0.0299 & 0.9910 & 0.1221 & -0.0025 \\ 0.1400 & -0.1250 & 0.9808 & 0.0258 \\ -0.0111 & 0.0058 & -0.0239 & 0.9986 \end{pmatrix}, \tag{17}$$

$$M_s = \begin{pmatrix} 0.9895 & 0.0552 & -0.1119 & 0.0342 \\ -0.1082 & 0.8175 & -0.5614 & -0.0259 \\ 0.0590 & 0.5696 & 0.8160 & 0.0452 \\ -0.0395 & -0.0065 & -0.0478 & 0.9960 \end{pmatrix}. \tag{18}$$

We see that some of the entries of the mixing matrices are small; for example, one of the entries in the first row of M_v is 0.0076. But this nonzero entry means a charm component in the ω meson, which allows η_c to decay to ω directly in our model. Other entries of the mixing matrices have the same meaning, and it is such entries that are helpful to reproduce the experimental decay data of J/ψ and other meson decays.

The results of fitting light-cone constituent quark model results to experimental data are shown in Table I. The fourth column contains the results of trimixing model

TABLE III. Parameters of the mixing matrices M_v and M_s determined from the fitting process.

θ_1^v	θ_2^v	θ_3^v	θ_4^v	θ_5^v	θ_6^v
$-0.2181°$	$7.9190°$	$-7.6665°$	$-2.6511°$	$8.4010°$	$-6.5877°$
θ_1^s	θ_2^s	θ_3^s	θ_4^s	θ_5^s	θ_6^s
$-7.8710°$	$4.6505°$	$32.4605°$	$2.1461°$	$5.8085°$	$36.7524°$

π-η-η' and ρ-ω-ϕ, while J/ψ and η_c do not mix with other meson states, with the values of their parameters (MeV) set as $A_{J/\psi} = 31.1660$, $\beta_{J/\psi} = 0.9777$, $A_{\eta_c} = 125.7935$, and $\beta_{\eta_c} = 0.7524$ to fit the experimental data. The most apparent differences between tetramixing and trimixing results are in the last four rows, which show that the trimixing formalism do not explain the nonzero decay width of $J/\psi \to \pi$, η, and η', while the tetramixing formalism can well reproduce these experimental decay data. The parameters of the mesons and the mixing matrices determined during the fitting process are listed in Tables II and III.

The Q^2 behaviors of the form factors of π, η, and η' are shown in Figs. 1–3, and we see that they are generally in agreement with the experimental data. The Q^2 behavior of the form factor of η_c is shown in Fig. 4, and, by comparing with theoretical data from another model, the calculated curve fits well in most of the lower Q^2 region. We can also obtain the Q^2 behavior of the transition form factors in the timelike region, either by making the substitution $q_\perp \to iq_\perp$ [33] or by parameterizing the transition form factors as explicit functions of q^2 in the spacelike region and then extending them through analytic continuum to the timelike region [34]. The results are shown in Figs. 5–10, among which Figs. 5 and 6 are compared with the experimental data, while Figs. 7–10 could be considered as our predictions of the Q^2 behaviors of J/ψ transition form factors.

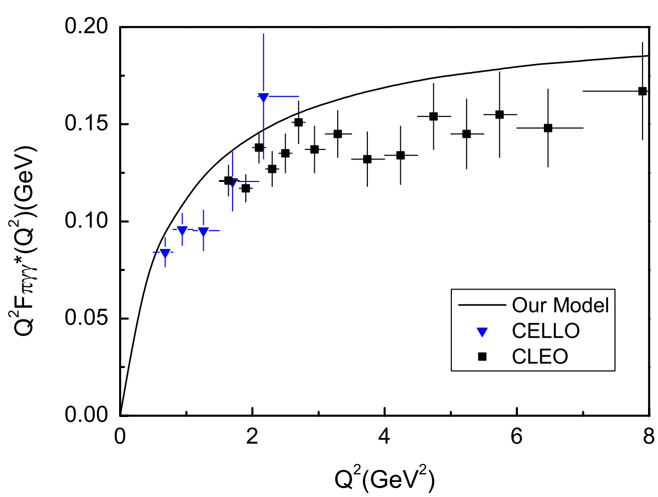

FIG. 1 (color online). The Q^2 behavior of the form factor $Q^2 F_{\pi\to\gamma\gamma^*}(Q^2)$ compared with experimental data [42,43].

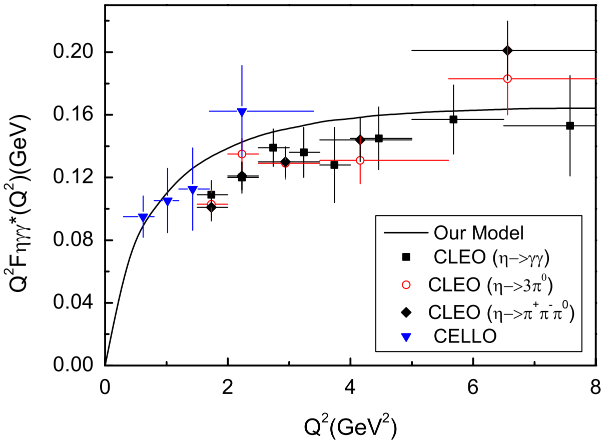

FIG. 2 (color online). The Q^2 behavior of the form factor $Q^2F_{\eta\to\gamma\gamma^*}(Q^2)$ compared with experimental data [42,43].

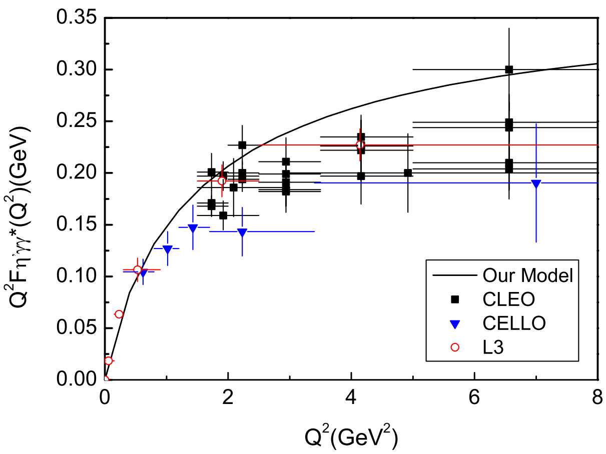

FIG. 3 (color online). The Q^2 behavior of the form factor $Q^2F_{\eta'\to\gamma\gamma^*}(Q^2)$ compared with experimental data [42–44].

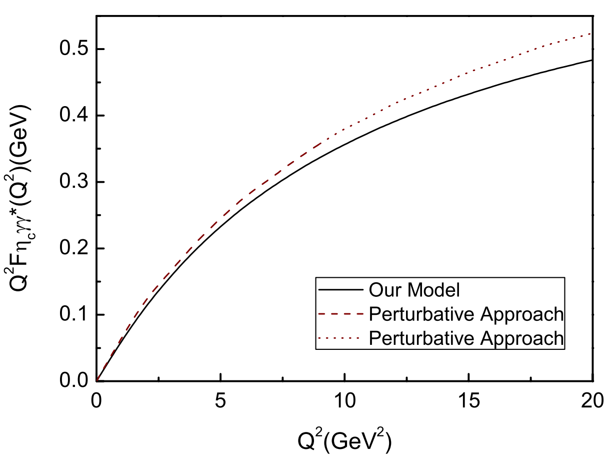

FIG. 4 (color online). Prediction of the Q^2 behavior of the form factor $Q^2F_{\eta_c\to\gamma\gamma^*}(Q^2)$, compared with the predictions in the leading order of the perturbative approach [45]. The dotted curve of the perturbative approach indicates the Q^2 region where QCD corrections may alter the predictions slightly.

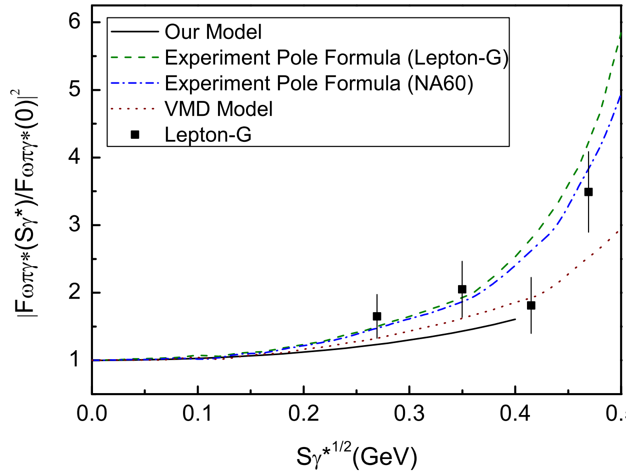

FIG. 5 (color online). The Q^2 behavior of the form factor $Q^2F_{\omega\to\pi\gamma^*}(Q^2)$ compared with experimental data [46,47].

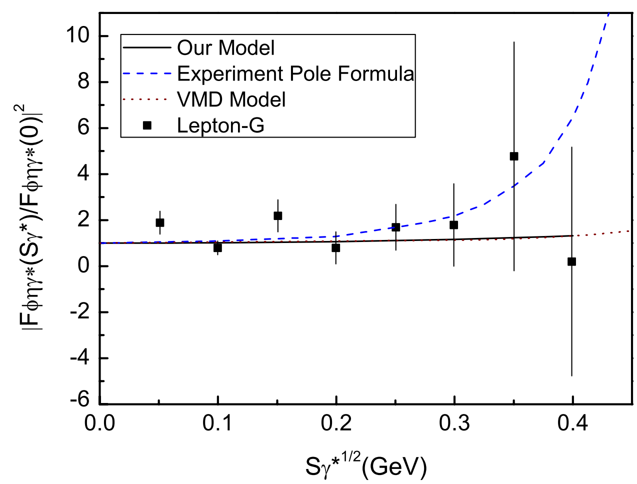

FIG. 6 (color online). The Q^2 behavior of the form factor $Q^2F_{\phi\to\eta\gamma^*}(Q^2)$ compared with experimental data [48].

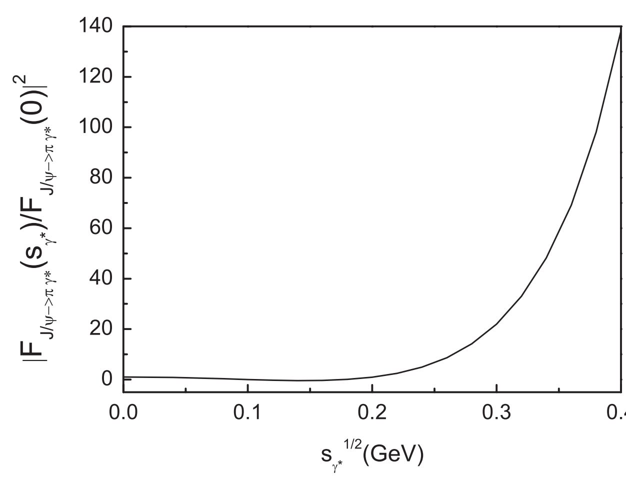

FIG. 7. Prediction of the Q^2 behavior of the form factor $Q^2F_{J/\psi\to\pi\gamma^*}(Q^2)$.

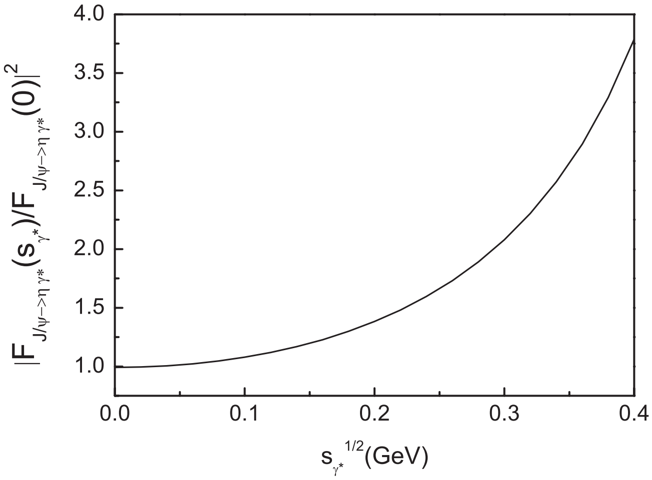

FIG. 8. Prediction of the Q^2 behavior of the form factor $Q^2F_{J/\psi\to\eta\gamma^*}(Q^2)$.

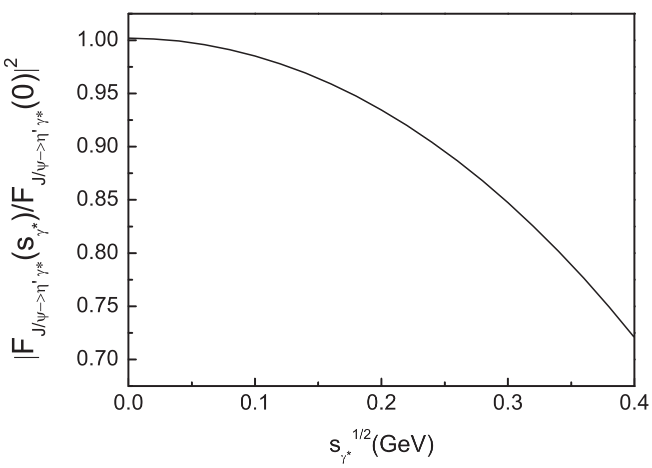

FIG. 9. Prediction of the Q^2 behavior of the form factor $Q^2F_{J/\psi\to\eta'\gamma^*}(Q^2)$.

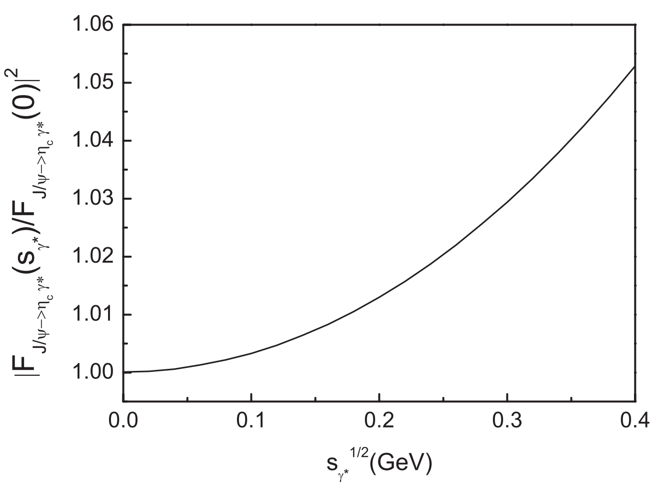

FIG. 10. Prediction of the Q^2 behavior of the form factor $Q^2F_{J/\psi\to\eta_c\gamma^*}(Q^2)$.

We can further use our results to learn the properties of the intrinsic $c\bar{c}$ component in the light pseudoscalar mesons. With the $F_{P\to\gamma\gamma^*}(0)$ (where $P=\pi,\eta,\eta',\eta_c$) in Table I and the mixing matrix M_s, we obtain the transition form factors of unmixed mesons $F_{P_I\to\gamma\gamma^*}(0)$ (where $P_I=\pi_I,\eta_q,\eta_s,\eta_{c0}$). Taking them into Eq. (16), we have the values of f_π, f_q, f_s, and f_c:

$$\begin{aligned} f_\pi &= 0.0984\ \text{GeV}, \qquad f_q = 0.0976\ \text{GeV}, \\ f_s &= 0.1298\ \text{GeV}, \qquad f_c = 0.4874\ \text{GeV}. \end{aligned} \tag{19}$$

Then we have [7]

$$\begin{aligned} &\begin{pmatrix} f_\pi^\pi & f_\pi^q & f_\pi^s & f_\pi^c \\ f_\eta^\pi & f_\eta^q & f_\eta^s & f_\eta^c \\ f_{\eta'}^\pi & f_{\eta'}^q & f_{\eta'}^s & f_{\eta'}^c \\ f_{\eta_c}^\pi & f_{\eta_c}^q & f_{\eta_c}^s & f_{\eta_c}^c \end{pmatrix} \\ &= M_s \begin{pmatrix} f_\pi & 0 & 0 & 0 \\ 0 & f_q & 0 & 0 \\ 0 & 0 & f_s & 0 \\ 0 & 0 & 0 & f_c \end{pmatrix} \\ &= \begin{pmatrix} 0.0974 & 0.0054 & -0.0145 & 0.0168 \\ -0.0106 & 0.0798 & -0.0729 & -0.0127 \\ 0.0058 & 0.0556 & 0.1059 & 0.0219 \\ -0.0039 & -0.0006 & -0.0062 & 0.4855 \end{pmatrix}. \end{aligned} \tag{20}$$

We see that $f_{\eta'}^c = 0.0219$ GeV $= 21.9$ MeV. It is compared with previous results in Table IV. $f_{\eta'}^c$ could be considered as the reflection of the intrinsic charm content of the η' meson [17], and we see from Table IV that our result of $f_{\eta'}^c$ is in the similar region with most of the previous results.

In summary, we use the light-cone constituent quark model to study the tetramixing of pseudoscalar mesons π-η-η'-η_c and vector mesons ω-ρ-ϕ-J/ψ. The parameters of mixing matrices and meson parameters are determined by fitting our theoretical model results of the meson decay constants and transition form factors (at $Q^2=0$) to the experimental data. We also calculate the Q^2 behaviors of the meson transition form factors, and these results are generally in agreement with the experimental data or results from other models. Our results of the Q^2 behaviors of transition from factors of J/ψ decaying into pseudoscalar mesons could be regarded as the predictions of our model, as there are no experimental data at present. The introduction of light quark components in J/ψ and η_c not only allows them to decay into the light mesons directly without intermediate gluons or virtual photon but is also helpful for us to understand the structures of charmonium states better. Considering that the mixing introduces a $c\bar{c}$ component into the light mesons, and such a $c\bar{c}$ component is intrinsic

TABLE IV. The value of $f^c_{\eta'}$ (MeV) in different models.

Our model	Feldmann and Kroll [35]	Halperin and Zhitnitsky [36]	Cheng and Tseng [37]	Cao, Cao, Huang, and Ma [17]	Yuan and Chao [38]
21.9	−65–15	50–180	−50	Around −15	40

to the wave functions and exists over a time scale independent of any probe momentum, we could naturally interpret it as the intrinsic charm of these mesons. Our result of the intrinsic charm content of the η' meson $f^c_{\eta'}$ is also comparable with predictions from other models.

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APPENDIX A

The $SO(4)$ group elements can be written in terms of the $SO(3)$ group generators (A_k and B_k):

$$M = R_+ R_-, \tag{A1}$$

$$R_+ = e^{-i\theta_k A_k}, \qquad R_- = e^{-i\theta_{k+2} B_k} \qquad (k = 1, 2, 3). \tag{A2}$$

The generators A_k and B_k obey the commuting relations of $SO(3)$ generators [39]:

$$[A_i, A_j] = i\varepsilon_{ijk} A_k, \qquad [B_i, B_j] = i\varepsilon_{ijk} B_k, \qquad [A_i, B_j] = 0. \tag{A3}$$

We see that the groups have the relation $SO(4) = SO(3) \otimes SO(3)$, and the generators A_k (as well as B_k) ($k = 1, 2, 3$) could be seen as the angular momentum operators in each of the three directions.

One form of A_k and B_k is [40]

$$A_1 = \frac{i}{2}\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad A_2 = \frac{i}{2}\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad A_3 = \frac{i}{2}\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \tag{A4}$$

$$B_1 = \frac{i}{2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad B_2 = \frac{i}{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad B_3 = \frac{i}{2}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}. \tag{25}$$

Then

$$R_+ = e^{-i\theta_k A_k} = e^{-i\alpha \mathbf{n}\cdot\mathbf{A}} = e^{-i\alpha A_n}, \tag{A6}$$

where

$$\alpha = \sqrt{\theta_1^2 + \theta_2^2 + \theta_3^2}, \qquad \mathbf{n} = \frac{1}{\alpha}(\theta_1, \theta_2, \theta_3). \tag{A7}$$

From the matrix form of A_k, we have

$$A_n = \frac{\theta_k A_k}{\alpha} = \frac{i}{2\alpha}\begin{pmatrix} 0 & -\theta_1 & -\theta_2 & -\theta_3 \\ \theta_1 & 0 & -\theta_3 & \theta_2 \\ \theta_2 & \theta_3 & 0 & -\theta_1 \\ \theta_3 & -\theta_2 & \theta_1 & 0 \end{pmatrix}. \tag{A8}$$

A_n is the angular momentum component of the direction $\mathbf{n}$. In fact, the matrix form of A_n in Eq. (A8) can be diagonalized as

$$A'_n = \begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}, \tag{A9}$$

which is the expression of angular momentum operator A_n in its eigenstate representation, with the eigenvalues of A_n being $(-1/2, -1/2, 1/2, 1/2)$. The two matrix forms are related as $A'_n = S^\dagger A_n S$, with the transformation matrix

$$S = \frac{1}{\sqrt{2}\alpha}\begin{pmatrix} \frac{\theta_1\theta_2+i\theta_3\alpha}{\sqrt{\theta_2^2+\theta_3^2}} & i\theta_2 & \frac{\theta_1\theta_2-i\theta_3\alpha}{\sqrt{\theta_2^2+\theta_3^2}} & -i\theta_2 \\ \frac{\theta_1\theta_3-i\theta_2\alpha}{\sqrt{\theta_2^2+\theta_3^2}} & i\theta_3 & \frac{\theta_1\theta_3+i\theta_2\alpha}{\sqrt{\theta_2^2+\theta_3^2}} & -i\theta_3 \\ 0 & \alpha & 0 & \alpha \\ \sqrt{\theta_2^2+\theta_3^2} & -i\theta_1 & \sqrt{\theta_2^2+\theta_3^2} & i\theta_1 \end{pmatrix}. \tag{A10}$$

In the eigenstate representation of angular momentum A_n, the matrix form of R_+ is

$$R'_+ = e^{-i\alpha A_n} = \begin{pmatrix} e^{i\alpha/2} & 0 & 0 & 0 \\ 0 & e^{i\alpha/2} & 0 & 0 \\ 0 & 0 & e^{-i\alpha/2} & 0 \\ 0 & 0 & 0 & e^{-i\alpha/2} \end{pmatrix}, \tag{A11}$$

and then we have

$$R_+ = SR'_+S^\dagger = \begin{pmatrix} \cos\frac{\alpha}{2} & -\frac{\theta_1}{\alpha}\sin\frac{\alpha}{2} & -\frac{\theta_2}{\alpha}\sin\frac{\alpha}{2} & -\frac{\theta_3}{\alpha}\sin\frac{\alpha}{2} \\ \frac{\theta_1}{\alpha}\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} & -\frac{\theta_3}{\alpha}\sin\frac{\alpha}{2} & \frac{\theta_2}{\alpha}\sin\frac{\alpha}{2} \\ \frac{\theta_2}{\alpha}\sin\frac{\alpha}{2} & \frac{\theta_3}{\alpha}\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} & -\frac{\theta_1}{\alpha}\sin\frac{\alpha}{2} \\ \frac{\theta_3}{\alpha}\sin\frac{\alpha}{2} & -\frac{\theta_2}{\alpha}\sin\frac{\alpha}{2} & \frac{\theta_1}{\alpha}\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} \end{pmatrix}, \tag{A12}$$

which is the expression of R_+ given in Eq. (4). Following the same procedure, we also obtain the expression of R_- given in Eq. (5).

APPENDIX B

During the numerical calculation, we write M in a more compact form with eight real parameters ($a, b, c, d, p, q, r,$ and s) under the constraints $a^2 + b^2 + c^2 + d^2 = 1$ and $p^2 + q^2 + r^2 + s^2 = 1$ [41]:

$$M = \begin{pmatrix} a & -b & -c & -d \\ b & a & -d & c \\ c & d & a & -b \\ d & -c & b & a \end{pmatrix}\begin{pmatrix} p & -q & -r & -s \\ q & p & s & -r \\ r & -s & p & q \\ s & r & -q & p \end{pmatrix} \tag{B1}$$

$$= \begin{pmatrix} ap-bq-cr-ds & -aq-bp+cs-dr & -ar-bs-cp+dq & -as+br-cq-dp \\ bp+aq-dr+cs & -bq+ap+ds+cr & -br+as-dp-cq & -bs-ar-dq+cp \\ cp+dq+ar-bs & -cq+dp-as-br & -cr+ds+ap+bq & -cs-dr+aq-bp \\ dp-cq+br+as & -dq-cp-bs+ar & -dr-cs+bp-aq & -ds+cr+bq+ap \end{pmatrix}. \tag{B2}$$

These parameters are related to the six rotation angles as

$$a = \cos\frac{\alpha}{2}, \qquad b = \frac{\theta_1}{\alpha}\sin\frac{\alpha}{2}, \qquad c = \frac{\theta_2}{\alpha}\sin\frac{\alpha}{2}, \qquad d = \frac{\theta_3}{\alpha}\sin\frac{\alpha}{2}, \tag{B3}$$

$$p = \cos\frac{\beta}{2}, \qquad q = -\frac{\theta_4}{\beta}\sin\frac{\beta}{2}, \qquad r = -\frac{\theta_5}{\beta}\sin\frac{\beta}{2}, \qquad s = -\frac{\theta_6}{\beta}\sin\frac{\beta}{2}. \tag{B4}$$

When referring to the mixing of specific types of mesons, the parameters $(a, b, \ldots)$ change to $(a_v, b_v, \ldots)$ or $(a_s, b_s, \ldots)$ correspondingly.

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