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First nonperturbative calculation in light-front QED for an arbitrary covariant gauge

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This work is the first check of gauge invariance for nonperturbative calculations in light-front QED. To quantize QED in an arbitrary covariant gauge, we use a light-front analog of the equal-time Stueckelberg quantization. Combined with a Pauli-Villars regularization, where massive, negative-metric photons and fermions are included in the Lagrangian, we are then able to construct the light-front QED Hamiltonian and the associated mass eigenvalue problem in a Fock-space representation. The formalism is applied to the dressed-electron state, with a Fock-space truncation to include at most one photon. From this eigenstate, we compute the anomalous magnetic moment. The result is found to be gauge independent, to an order in α consistent with the truncation.

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I. INTRODUCTION

Any calculation in a gauge theory should be checked for its gauge dependence. Unfortunately, nonperturbative calculations in light-front QED [1] have been limited to a single gauge, usually light-cone gauge. This is due to the need to solve the constraint equation for the nondynamical part of the fermion field, which is entangled with the photon field. A careful use of Pauli-Villars (PV) regularization [2] has been shown [3] to allow the use of Feynman gauge, by providing cancellation of the photon-field dependence in the constraint equation. What is remarkable, however, is that this cancellation is actually not unique to Feynman gauge but holds for any gauge. Thus, nonperturbative calculations can be done in any gauge, provided the free-photon part of the Hamiltonian can be constructed. Here we provide such a construction for an arbitrary covariant gauge and apply the formalism to a calculation of the dressed-electron eigenstate and its anomalous magnetic moment, in order to investigate the gauge invariance of the result.

This builds on earlier work on Yukawa theory [4] and QED [3,5–9], where PV particles are used to regulate a light-front Hamiltonian and the eigenstates of the Hamiltonian are computed in one or more charge sectors of the theory. The eigenstate is expanded in a truncated Fock basis. The eigenvalue problem becomes a coupled set of integral equations for the wave functions, which are the coefficients of the Fock states in the expansion. Truncation keeps the coupled set finite in size, and the PV regularization keeps the integrations finite. For severe truncations, the coupled set can be solved analytically. In general, the set is solved numerically [6,8]. The renormalization can be handled in a standard way, with the bare parameters of the original Lagrangian fixed by physical constraints, or by a sector-dependent parametrization, where the bare parameters become dependent on the Fock sectors connected by the terms in the Hamiltonian [7,10–14]. However, in a weakly coupled theory such as QED, such an approach cannot be expected to compete with high-order perturbation theory, due to numerical errors. Consideration of QED is a test for a method intended for strongly coupled theories.

Light-cone coordinates [1,15] are used in order to have well-defined Fock-state expansions, which are at the heart of the method. We define these coordinates as $x^+ = t + z$ for time and $\underline{x} = (x^- = t - z, \vec{x}_\perp)$ for space, with $\vec{x}_\perp = (x, y)$. The light-cone energy is $p^- = E - p_z$ and momentum, $\underline{p} = (p^+ = E + p_z, \vec{p}_\perp)$. The mass-shell condition $p^2 = m^2$ relates these as $p^- = (m^2 + p_\perp^2)/p^+$. The positivity of p^+ keeps the vacuum simple and prevents vacuum contributions to the Fock expansions, except for the possibility of zero modes [16]. These modes of zero p^+ can be neglected in theories where symmetry breaking does not occur.

Our new construction of the Hamiltonian is based on a light-front analog of the equal-time Stueckelberg quantization of a massive vector field [17].[1] The Stueckelberg quantization is known to allow for a zero-mass limit. It is also useful as a way to treat the physical and PV photons on an equal footing, consistent with the need to maintain the PV regularization. The quantization adds a fourth (unphysical) polarization to the three physical polarizations. The unphysical polarization is the only one that does not satisfy the Lorentz gauge condition $\partial \cdot A = 0$. However, it does satisfy the Euler-Lagrange field equation, because its four-momentum is placed on a different, gauge-dependent mass shell, chosen in just such a way as to satisfy the field equation. The key to the light-front analog is that the chosen mass shell is invoked for the minus component of the momentum, rather than the zero component. The details of this can be found below, in Sec. II.

With this new quantization, we can formulate mass eigenvalue problems for the eigenstates of QED in an arbitrary covariant gauge and test for the gauge invariance

[1]For an alternative construction for the massless case, which uses the canonical Dirac constraint procedure, see [18].

of physical quantities computed from the eigenstates. We expect that gauge invariance will be broken by approximations made in solving the eigenproblems. One such approximation is the Fock-space truncation used to reduce the eigenproblem to a finite size. Another is retention of finite values for the regulating PV masses; the regularization is constructed with use of flavor-changing currents that explicitly break gauge invariance [3] and are removed only in the infinite-PV-mass limit, which may not be possible in a numerical calculation.

As a first test, we apply this formalism to a calculation of the dressed-electron eigenstate. Fock space is truncated to include only the bare-electron state and the one-electron/one-photon states, plus their PV analogs. This leads to an analytically solvable problem, reduced to an effective 2×2 matrix problem in the one-electron sector. From the solution, the anomalous moment can be computed, from the zero-momentum-transfer limit of the spin-flip transition amplitude.

To obtain meaningful results, it is important to maintain the chiral symmetry of the massless-electron limit. This is achieved by adjusting the coupling strengths of the PV photons, to ensure that the dressed mass is zero when the bare mass is zero. As will be seen below, in Sec. III, this requires two PV-photon flavors in any gauge. However, one flavor is sufficient if the limit of infinite PV-fermion mass is taken. This also holds for the sector-dependent approach [13], as we show in Sec. III. In general, the constraint of chiral-symmetry restoration and the PV-photon couplings are gauge dependent.

There are, of course, other nonperturbative methods. Lattice gauge theory [19] is particularly successful, and use of Dyson-Schwinger equations [20] has produced notable results. However, these lack the direct access to wave functions in Minkowski space, which light-front Hamiltonian methods provide [1]. Thus, the methods are quite complementary, particularly now that light-front calculations can be done in an arbitrary gauge. There are also a light-front lattice method, the transverse lattice [21]; a light-front approach in terms of effective fields [22]; and a supersymmetric formulation for discrete light-front Hamiltonians specifically for supersymmetric theories [23].

The remainder of the paper contains the following sections. The general formalism for an arbitrary covariant gauge is given in Sec. II. The dressed-electron eigenproblem is solved in Sec. III and used there to compute the anomalous moment. Section IV provides a summary of the method and of the results obtained. Some details are left to three Appendices.

II. LIGHT-FRONT QED IN AN ARBITRARY COVARIANT GAUGE

We begin with the QED Lagrangian for Lorentz gauge with an arbitrary gauge parameter ζ and additional PV fields:

$$\mathcal{L} = \sum_{i=0}^{2} (-1)^i \Big[-\frac{1}{4} F_i^{\mu\nu} F_{i,\mu\nu} + \frac{1}{2} \mu_i^2 A_i^\mu A_{i\mu} - \frac{1}{2} \zeta (\partial^\mu A_{i\mu})^2 \Big] + \sum_{i=0}^{2} (-1)^i \bar{\psi}_i (i\gamma^\mu \partial_\mu - m_i) \psi_i - e \bar{\psi} \gamma^\mu \psi A_\mu. \tag{2.1}$$

Here

$$\psi = \sum_{i=0}^{2} \sqrt{\beta_i}\, \psi_i, \qquad A_\mu = \sum_{i=0}^{2} \sqrt{\xi_i}\, A_{i\mu},$$
$$F_{i\mu\nu} = \partial_\mu A_{i\nu} - \partial_\nu A_{i\mu}, \tag{2.2}$$

$i = 0$ corresponds to a physical field, and $i = 1$ and 2, to PV fields. The photon fields have mass μ_i, and the zero-mass limit $\mu_0 \to 0$ for the physical photon is to be taken later.

The coupling coefficients β_i and ξ_i satisfy constraints. To keep e as the charge of physical fermion, we set $\beta_0 = 1$ and $\xi_0 = 1$. To regulate ultraviolet divergences that come from loop integrals, we arrange cancellations for each internal line summed over physical and PV fields, by imposing the constraints

$$\sum_{i=0}^{2} (-1)^i \xi_i = 0, \qquad \sum_{i=0}^{2} (-1)^i \beta_i = 0. \tag{2.3}$$

Two remaining coefficients, say ξ_2 and β_2, are fixed by requiring chiral-symmetry restoration in the massless-electron limit [5] and a zero photon eigenmass [9].

The dynamical fields are ψ_{i+} and $A_{i\mu}$. We quantize the dynamical fermion fields in the usual way

$$\psi_{i+} = \frac{1}{\sqrt{16\pi^3}} \sum_s \int d\underline{k} \chi_s [b_{is}(\underline{k}) e^{-i\underline{k}\cdot\underline{x}} + d^\dagger_{i,-s}(\underline{k}) e^{i\underline{k}\cdot\underline{x}}], \tag{2.4}$$

with

$$\{b_{is}(\underline{k}), b^\dagger_{i's'}(\underline{k}')\} = (-1)^i \delta_{ii'} \delta_{ss'} \delta(\underline{k} - \underline{k}'), \tag{2.5}$$

$$\{d_{is}(\underline{k}), d^\dagger_{i's'}(\underline{k}')\} = (-1)^i \delta_{ii'} \delta_{ss'} \delta(\underline{k} - \underline{k}'). \tag{2.6}$$

For the vector fields, we apply a light-front analog of Stueckelberg quantization [17].

Consider the Lagrangian of a free massive vector field:

$$\mathcal{L} = -\tfrac{1}{4} F^2 + \tfrac{1}{2} \mu A^2 - \tfrac{1}{2} \zeta (\partial \cdot A)^2. \tag{2.7}$$

The Euler-Lagrange field equation is

$$(\Box + \mu^2) A_\mu - (1 - \zeta) \partial_\mu (\partial \cdot A) = 0. \tag{2.8}$$

This equation is satisfied by the Fourier expansion

$$A_\mu(x) = \int \frac{d\underline{k}}{\sqrt{16\pi^3 k^+}} \Big\{ \sum_{\lambda=1}^{3} e_\mu^{(\lambda)}(\underline{k})[a_\lambda(\underline{k}) e^{-ik\cdot x} + a_\lambda^\dagger(\underline{k}) e^{ik\cdot x}] + e_\mu^{(0)}(\underline{k})[a_0(\underline{k}) e^{-i\tilde{k}\cdot x} + a_0^\dagger(\underline{k}) e^{i\tilde{k}\cdot x}] \Big\}, \tag{2.9}$$

with $\tilde{k}$ a four-vector associated with a different mass $\tilde{\mu} \equiv \mu/\sqrt{\zeta}$, such that

$$\tilde{k} = \underline{k}, \qquad \tilde{k}^- = (k_\perp^2 + \tilde{\mu}^2)/k^+. \tag{2.10}$$

The polarization vectors are defined by

$$e^{(1,2)}(\underline{k}) = (0, 2\hat{e}_{1,2} \cdot \vec{k}_\perp/k^+, \hat{e}_{1,2}), \tag{2.11}$$

$$e^{(3)}(\underline{k}) = ((k_\perp^2 - \mu^2)/k^+, k^+, \vec{k}_\perp)/\mu, \tag{2.12}$$

$$e^{(0)}(\underline{k}) = \tilde{k}/\mu, \tag{2.13}$$

and satisfy $k \cdot e^{(\lambda)} = 0$ and $e^{(\lambda)} \cdot e^{(\lambda')} = -\delta_{\lambda\lambda'}$ for $\lambda, \lambda' = 1, 2, 3$. The first term in A_μ satisfies $(\Box + \mu^2)A_\mu = 0$ and $\partial \cdot A = 0$ separately. The $\lambda = 0$ term violates each, but the field equation is satisfied. The gauge condition $\partial \cdot A = 0$ is to be satisfied by the projection of states onto a physical subspace.

The light-front Hamiltonian density is

$$\mathcal{H} = \mathcal{H}|_{\zeta=1} + \tfrac{1}{2}(1-\zeta)(\partial \cdot A)(\partial \cdot A - 2\partial_- A^- - 2\partial_\perp \cdot \vec{A}_\perp), \tag{2.14}$$

with the Feynman-gauge piece being

$$\mathcal{H}|_{\zeta=1} = \frac{1}{2} \sum_{\mu=0}^{3} \epsilon^\mu [(\partial_\perp A^\mu)^2 + \mu^2 (A^\mu)^2]. \tag{2.15}$$

The metric of each field component is defined by $\epsilon^\mu = (-1, 1, 1, 1)$. The light-front Hamiltonian for the free massive field is then found to be

$$\mathcal{P}^- = \int d\underline{x} \mathcal{H}|_{x^+=0} = \int d\underline{k} \sum_\lambda \epsilon^\lambda \frac{k_\perp^2 + \mu_\lambda^2}{k^+} a_\lambda^\dagger(\underline{k}) a_\lambda(\underline{k}), \tag{2.16}$$

with $\mu_\lambda = \mu$ for $\lambda = 1, 2, 3$, but $\mu_0 = \tilde{\mu} = \mu/\sqrt{\zeta}$. The nonzero commutators are

$$[a_\lambda(\underline{k}), a_{\lambda'}^\dagger(\underline{k}')] = \epsilon^\lambda \delta_{\lambda\lambda'} \delta(\underline{k} - \underline{k}'). \tag{2.17}$$

Thus, the Hamiltonian for the free photon field takes the usual form except that the mass of the fourth polarization is different and gauge dependent and that the metric of this polarization is opposite that of the other polarizations. In Feynman gauge, this reduces to the usual Gupta-Bleuler quantization [24].

The nondynamical components of the fermion fields satisfy the constraints ($i = 0, 1, 2$)

$$i(-1)^i \partial_- \psi_{i-} + eA_- \sqrt{\beta_i} \sum_j \psi_{j-} = (i\gamma^0\gamma^\perp)\Big[(-1)^i \partial_\perp \psi_{i+} - ieA_\perp \sqrt{\beta_i} \sum_j \psi_{j+}\Big] - (-1)^i m_i \gamma^0 \psi_{i+}. \tag{2.18}$$

Ordinarily, light-cone gauge ($A_- = 0$) is chosen to make the constraint explicitly invertible. However, the interaction Lagrangian has been arranged in just such a way that the A-dependent terms can be canceled between the three constraints [3]. Multiplication by $(-1)^i\sqrt{\beta_i}$ and a sum over i yields

$$i\partial_- \psi_- = (i\gamma^0\gamma^\perp)\partial_\perp \psi_+ - \gamma^0 \sum_i \sqrt{\beta_i}\, m_i \psi_{i+}, \tag{2.19}$$

as the constraint for the composite field that appears in the interaction Lagrangian. This constraint is the same as the free-fermion constraint, in any gauge, and the interaction Hamiltonian can be constructed from the free-field solution.

Without this cancellation of A-dependent terms, the constraint would generate four-point interactions between fermion and photon fields, the instantaneous-fermion interactions [1]. The addition of the PV-fermion fields has, in effect, factorized these interactions into flavor-changing photon emission and absorption three-point vertices. The instantaneous interactions are recovered in the limit of infinite PV-fermion masses because the light-cone energy denominator with an intermediate PV fermion cancels the PV-mass factors in the emission and absorption vertices, as illustrated in Fig. 1.

The light-front Hamiltonian, without antifermion terms, is

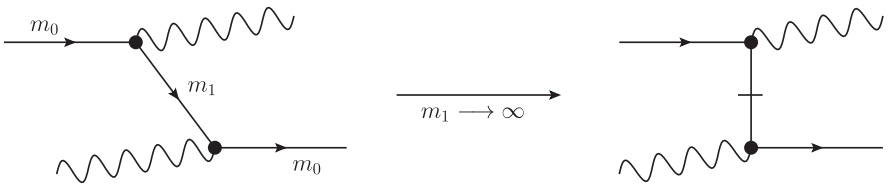

FIG. 1. The infinite-PV-mass limit of a tree graph involving an intermediate PV fermion of mass m_1. The external fermions are the physical ones, with mass m_0. The limit yields an instantaneous interaction, denoted by the bar through the intermediate line.

$$\mathcal{P}^- = \sum_{is}\int d\underline{p}\frac{m_i^2+p_\perp^2}{p^+}(-1)^i b_{is}^\dagger(\underline{p})b_{is}(\underline{p}) + \sum_{l\lambda}\int d\underline{k}\frac{\mu_{l\lambda}^2+k_\perp^2}{k^+}(-1)^l\epsilon^\lambda a_{l\lambda}^\dagger(\underline{k})a_{l\lambda}(\underline{k}) + \sum_{ijls\lambda}\sqrt{\beta_i\beta_j\xi_l}\int d\underline{p}d\underline{q}\{b_{is}^\dagger(\underline{p})[b_{js}(\underline{q})V_{ij,2s}^\mu(\underline{p},\underline{q}) + b_{j,-s}(\underline{q})U_{ij,-2s}^\mu(\underline{p},\underline{q})]\epsilon_{l\mu}^{(\lambda)}(\underline{q}-\underline{p})a_{l\lambda}^\dagger(\underline{q}-\underline{p}) + \text{H.c.}\}. \tag{2.20}$$

The instantaneous-photon terms associated with light-cone gauge do not appear. The polarization vectors $e_l^{(\lambda)}$ have an additional flavor index l, because they depend on the mass of the photon flavor. The vertex functions are given by [3]

$$V_{ij\pm}^0(\underline{p},\underline{q}) = \frac{e}{\sqrt{16\pi^3}}\frac{\vec{p}_\perp\cdot\vec{q}_\perp \pm i\vec{p}_\perp\times\vec{q}_\perp + m_im_j + p^+q^+}{p^+q^+\sqrt{q^+-p^+}},$$
$$V_{ij\pm}^3(\underline{p},\underline{q}) = \frac{-e}{\sqrt{16\pi^3}}\frac{\vec{p}_\perp\cdot\vec{q}_\perp \pm i\vec{p}_\perp\times\vec{q}_\perp + m_im_j - p^+q^+}{p^+q^+\sqrt{q^+-p^+}},$$
$$V_{ij\pm}^1(\underline{p},\underline{q}) = \frac{e}{\sqrt{16\pi^3}}\frac{p^+(q^1\pm iq^2)+q^+(p^1\mp ip^2)}{p^+q^+\sqrt{q^+-p^+}},$$
$$V_{ij\pm}^2(\underline{p},\underline{q}) = \frac{e}{\sqrt{16\pi^3}}\frac{p^+(q^2\mp iq^1)+q^+(p^2\pm ip^1)}{p^+q^+\sqrt{q^+-p^+}}, \tag{2.21}$$

and

$$U_{ij\pm}^0(\underline{p},\underline{q}) = \frac{\mp e}{\sqrt{16\pi^3}}\frac{m_j(p^1\pm ip^2)-m_i(q^1\pm iq^2)}{p^+q^+\sqrt{q^+-p^+}},$$
$$U_{ij\pm}^3(\underline{p},\underline{q}) = \frac{\pm e}{\sqrt{16\pi^3}}\frac{m_j(p^1\pm ip^2)-m_i(q^1\pm iq^2)}{p^+q^+\sqrt{q^+-p^+}},$$
$$U_{ij\pm}^1(\underline{p},\underline{q}) = \frac{\pm e}{\sqrt{16\pi^3}}\frac{m_iq^+-m_jp^+}{p^+q^+\sqrt{q^+-p^+}},$$
$$U_{ij\pm}^2(\underline{p},\underline{q}) = \frac{ie}{\sqrt{16\pi^3}}\frac{m_iq^+-m_jp^+}{p^+q^+\sqrt{q^+-p^+}}. \tag{2.22}$$

The extension to include antifermion terms is straightforward [9]. We now apply this formalism to a nonperturbative calculation of the dressed-electron state and its anomalous magnetic moment.

III. DRESSED-ELECTRON EIGENSTATE

A. Eigenvalue problem

We wish to solve the light-front eigenvalue problem $\mathcal{P}^-|\psi(\underline{P})\rangle = \frac{M^2}{P^+}|\psi(\underline{P})\rangle$. The eigenstate $|\psi(\underline{P})\rangle$ is expanded in a Fock basis where P^+ is diagonal and $\vec{P}_\perp$ is zero. Here we consider the lowest order truncation, to include the bare-electron state and the one-electron/one-photon states. For a total $J_z = \pm\frac{1}{2}$, the eigenstate is of the form

$$|\psi^\pm(\underline{P})\rangle = \sum_i z_i^\pm b_{i\pm}^\dagger(\underline{P})|0\rangle + \sum_{ijs\lambda}\int d\underline{k}C_{ijs}^{\lambda\pm}(\underline{k})b_{is}^\dagger(\underline{P}-\underline{k})a_{j\lambda}^\dagger(\underline{k})|0\rangle. \tag{3.1}$$

The normalization condition is

$$\langle\psi^{\sigma'}(\underline{P}')|\psi^\sigma(\underline{P})\rangle = \delta(\underline{P}'-\underline{P})\delta_{\sigma'\sigma}. \tag{3.2}$$

For this truncation, we can remove the second PV-fermion flavor, since it plays no role in the regularization or chiral-symmetry restoration [5]. We let $m_2\to\infty$, $\beta_2\to 0$, and $\beta_1\to 1$.

The eigenvalue problem for this state reduces to a set of coupled equations for the bare-electron amplitudes and the two-body wave functions:

$$[M^2-m_i^2]z_i^\pm = \int (P^+)^2dyd^2k_\perp\sum_{jl\mu}\sqrt{\xi_l}(-1)^{j+l}\epsilon^\lambda e_{l\mu}^{(\lambda)}(\underline{k}) \times [V_{ji\pm}^{\mu*}(\underline{P}-\underline{k},\underline{P})C_{jl\pm}^{\lambda\pm}(\underline{k}) + U_{ji\pm}^{\mu*}(\underline{P}-\underline{k},\underline{P})C_{jl\mp}^{\lambda\pm}(\underline{k})], \tag{3.3}$$

and

$$\left[M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_{l\lambda}^2+k_\perp^2}{y}\right]C_{jls}^{\lambda\pm}(\underline{k}) = \sqrt{\xi_l}\sum_{i'}(-1)^{i'}z_{i'}^\pm P^+e_{l\mu}^{(\lambda)}(\underline{k})[V_{ji'\pm}^\mu(\underline{P}-\underline{k},\underline{P})\delta_{s\pm} + U_{ji'\pm}^\mu(\underline{P}-\underline{k},\underline{P})\delta_{s\mp}], \tag{3.4}$$

with $y\equiv k^+/P^+$ being the photon's longitudinal momentum fraction. The second equation (3.4) is trivially inverted to find the two-body wave functions:

$$C_{jls}^{\lambda\pm}(\underline{k}) = P^+\sqrt{\xi_l}\sum_{i'}(-1)^{i'}z_{i'}^\pm e_{l\mu}^{(\lambda)}(\underline{k}) \times\frac{[V_{ji'\pm}^\mu(\underline{P}-\underline{k},\underline{P})\delta_{s\pm}+U_{ji'\pm}^\mu(\underline{P}-\underline{k},\underline{P})\delta_{s\mp}]}{[M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_{l\lambda}^2+k_\perp^2}{y}]}. \tag{3.5}$$

Substitution into the first equation (3.3) yields a 2×2 matrix eigenvalue problem for the one-body amplitudes $z_i^\pm$:

$$(M^2-m_i^2)z_i^\pm = 2e^2\sum_{i'}(-1)^{i'}z_{i'}^\pm[J+\Delta J+m_im_{i'}(I_0+\Delta I_0) - 2(m_i+m_{i'})(I_1+\Delta I_1)], \tag{3.6}$$

with

$$I_n(M^2) = \int\frac{dydk_\perp^2}{16\pi^2}\sum_{jl}\frac{(-1)^{j+l}\xi_l}{M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_l^2+k_\perp^2}{y}}\frac{m_j^n}{y(1-y)^n}, \tag{3.7}$$

$$J(M^2) = \int \frac{dydk_\perp^2}{16\pi^2} \sum_{jl} \frac{(-1)^{j+l}\xi_l}{M^2 - \frac{m_j^2+k_\perp^2}{1-y} - \frac{\mu_l^2+k_\perp^2}{y}} \frac{m_j^2+k_\perp^2}{y(1-y)^2}, \quad (3.8)$$

and the gauge-dependent parts

$$\Delta I_0(M^2) = -\frac{1-\zeta}{32\pi^2\zeta} \sum_{jl} (-1)^{j+l}\xi_l \int \frac{dydk_\perp^2}{y^2(1-y)^2} \times \frac{m_j^2 y^2 + k_\perp^2}{(M^2 - \frac{m_j^2+k_\perp^2}{1-y} - \frac{\mu_l^2+k_\perp^2}{y})(M^2 - \frac{m_j^2+k_\perp^2}{1-y} - \frac{\tilde{\mu}_l^2+k_\perp^2}{y})}, \quad (3.9)$$

$$\Delta I_1(M^2) = \frac{1-\zeta}{32\pi^2\zeta} \sum_{jl} (-1)^{j+l} \frac{m_j\xi_l}{2} \int \frac{dydk_\perp^2}{y^2(1-y)^2} \times \frac{M^2 y(1-y) - m_j^2 y - k_\perp^2}{(M^2 - \frac{m_j^2+k_\perp^2}{1-y} - \frac{\mu_l^2+k_\perp^2}{y})(M^2 - \frac{m_j^2+k_\perp^2}{1-y} - \frac{\tilde{\mu}_l^2+k_\perp^2}{y})}, \quad (3.10)$$

$$\Delta J(M^2) = -\frac{1-\zeta}{32\pi^2\zeta} \sum_{jl} (-1)^{j+l}\xi_l \int \frac{dydk_\perp^2}{y^2(1-y)^2} \times \frac{(M^2 - \frac{m_j^2}{1-y})^2(1-y)^2 + m_j^2 k_\perp^2}{(M^2 - \frac{m_j^2+k_\perp^2}{1-y} - \frac{\mu_l^2+k_\perp^2}{y})(M^2 - \frac{m_j^2+k_\perp^2}{1-y} - \frac{\tilde{\mu}_l^2+k_\perp^2}{y})}. \quad (3.11)$$

For ΔJ we have taken advantage of the fact that $\sum_{jl}(-1)^{j+l}\xi_l = 0$ to simplify the expression by elimination from the numerator terms that are proportional to the denominator.

For $\mu_0 < M - m_0$, there is a line of poles in the $(y, k_\perp^2)$ plane, in an arc between the points at $y = y_\pm \equiv [(M^2 - m_0^2 + \mu_0^2) \pm \sqrt{(M^2 - m_0^2 + \mu_0^2)^2 - 4M^2\mu_0^2}]/(2M^2)$ on the longitudinal axis. For $\mu_0 = 0$, these reduce to $y_- = 0$ and $y_+ = 1 - m_0^2/M^2$, as considered previously [3]. As in that case, we define integrals with these poles as principal values. Also, the same considerations hold for poles associated with denominators containing $\tilde{\mu}_0$.

B. Analytic solution

The matrix problem (3.6) can be solved analytically, in terms of the defined integrals. The solution is facilitated by the identity $J + \Delta J = M^2(I_0 + \Delta I_0)$, which was shown for Feynman gauge in [5] and is extended to an arbitrary gauge in Appendix A.

The analytic solutions are

$$\alpha_\pm = \frac{(M \pm m_0)(M \pm m_1)}{8\pi(m_1 - m_0)[2(I_1 + \Delta I_1) \pm M(I_0 + \Delta I_0)]}, \quad (3.12)$$

with

$$z_1 = \frac{M \pm m_0}{M \pm m_1} z_0, \quad (3.13)$$

and $M = m_e$, the physical electron mass. The solution with the lower sign is the physical one, because $M = m_0$ when $\alpha_- = 0$.

We fix ξ_2 by requiring chiral-symmetry restoration, that is, $M = 0$ for $m_0 = 0$ when α_- is equal to the physical value of α. This implies that $(I_1 + \Delta I_1)$ must be zero. From earlier work [5], we know that

$$I_1(0)|_{m_0=0} = \frac{m_1}{16\pi^2} \sum_l (-1)^l \xi_l \frac{\mu_l^2/m_1^2}{1 - \mu_l^2/m_1^2} \ln(\mu_l^2/m_1^2), \quad (3.14)$$

so that we only need to evaluate

$$\Delta I_1(0)|_{m_0=0} = -\frac{m_1(1-\zeta)}{64\pi^2\zeta} \sum_l (-1)^l \xi_l \times \int \frac{(m_1^2 y + k_\perp^2) dydk_\perp^2}{(m_1^2 y + \mu_l^2(1-y) + k_\perp^2)(m_1^2 y + \tilde{\mu}_l^2(1-y) + k_\perp^2)}. \quad (3.15)$$

Next, we write $k_\perp^2$ in the numerator as $k_\perp^2 + m_1^2 - m_1^2$ and use the technique in Appendix A to conclude that the $k_\perp^2 + m_i^2$ combination integrates to zero.[2] After the separation of the denominator into two terms, we can easily perform the remaining $k_\perp^2$ integration, to obtain

$$\Delta I_1(0)|_{m_0=0} = \frac{m_1^3}{64\pi^2} \sum_l \frac{(-1)^l \xi_l}{\mu_l^2} \times \int dy \ln\left(\frac{m_1^2 y + \mu_l^2(1-y)}{m_1^2 y + \tilde{\mu}_l^2(1-y)}\right). \quad (3.16)$$

The integral over y yields

$$\Delta I_1(0)|_{m_0=0} = \frac{m_1^3}{64\pi^2} \sum_l (-1)^l \xi_l \times \frac{(\zeta-1)m_1^2 \ln(m_1^2/\mu_l^2) - (m_1^2 - \mu_l^2)\ln\zeta}{\zeta(m_1^2 - \mu_l^2)(m_1^2 - \mu_l^2/\zeta)}. \quad (3.17)$$

Therefore, the constraint from chiral-symmetry restoration is

$$\frac{m_1}{16\pi^2} \sum_l (-1)^l \xi_l \left\{ \frac{\mu_l^2/m_1^2}{1 - \mu_l^2/m_1^2} \ln(\mu_l^2/m_1^2) - \frac{m_1^2}{4} \frac{(\zeta-1)m_1^2 \ln(m_1^2/\mu_l^2) - (m_1^2 - \mu_l^2)\ln\zeta}{\zeta(m_1^2 - \mu_l^2)(m_1^2 - \mu_l^2/\zeta)} \right\} = 0. \quad (3.18)$$

[2] In general, $k_\perp^2 + m_i^2$ is replaced by $M^2(1-y)^2$, but here $M = 0$.

Thus, in any covariant gauge, two PV-photon flavors are required to maintain the chiral limit.

In the limit of infinite mass for the PV electron, and with use of $\sum_l (-1)^l \xi_l = 0$, the general constraint reduces to

$$\frac{\zeta - 1}{\zeta} \sum_l (-1)^l \xi_l \ln(\mu_l^2/\mu^2) = 0, \tag{3.19}$$

with μ any mass scale. Given $\xi_0 = 1$ and $\xi_1 = 1 + \xi_2$, this is solved by

$$\xi_2 = -\frac{\ln(\mu_0/\mu_1)}{\ln(\mu_2/\mu_1)} > 0. \tag{3.20}$$

If the limit $\mu_2 \to \infty$ can be taken, ξ_2 is reduced to zero and the second PV-photon flavor is removed.

For the sector-dependent approach, the analogous quantity to consider in Yukawa theory is $\mathcal{A}(M^2)$, defined in Eq. (B1) of [13]. The combination of various pieces used in this definition yields, in the notation used there,

$$\mathcal{A}(M^2) = -\frac{1}{64\pi^2} \int \frac{dx dR_\perp^2}{x(1-x)} \sum_{ij} \frac{(-1)^{i+j} m_i}{\frac{m_i^2 + R_\perp^2}{1-x} + \frac{\mu_j^2 + R_\perp^2}{x} - M^2}. \tag{3.21}$$

A comparison with our (3.7) shows that $\mathcal{A}(M^2) = \frac{1}{4} I_1(M^2)|_{\xi_2 = 0}$. When $M = 0$ and $m_0 = 0$, the $i = 1$ term of $\mathcal{A}$ is not zero, unless $m_1 \to \infty$. Therefore, δm_2 in (29b) of [13] is also not zero, in contradiction of chiral-symmetry restoration. Thus, the sector-dependent approach requires a second PV-photon flavor in Yukawa theory. For QED, the situation is not materially different.

To complete the analysis of the eigensolution, we need to consider the gauge dependence and infrared dependence of the integral combination

$$\Delta \equiv 2\Delta I_1(M^2) - M \Delta I_0(M^2), \tag{3.22}$$

which enters the denominator of (3.12). The combination $2I_1(M^2) - M I_0(M^2)$ also appears there, but is gauge-independent by definition and is known to be infrared safe [3]. From the definitions (3.9) of ΔI_0 and ΔI_1, we can obtain, after eliminating $k_\perp^2$ from the numerator in the same manner as before,

$$\Delta = \frac{1-\zeta}{32\pi^2 \zeta} \sum_{jl} (-1)^{j+l} \xi_l \int \frac{dy dk_\perp^2}{y^2(1-y)} \times \frac{(M - m_j)^2 [M(1-y) + m_j]}{(M^2 - \frac{m_j^2 + k_\perp^2}{1-y} - \frac{\mu_l^2 + k_\perp^2}{y})(M^2 - \frac{m_j^2 + k_\perp^2}{1-y} - \frac{\tilde{\mu}_l^2 + k_\perp^2}{y})}. \tag{3.23}$$

The $k_\perp^2$ integral yields

$$\Delta = \frac{1}{32\pi^2} \sum_{jl} (-1)^{j+l} \frac{\xi_l}{\mu_l^2} (M - m_j)^2 \int dy [M(1-y) + m_j] \times \ln\left\{ \left| \frac{m_j^2 y + \mu_l^2 (1-y) - M^2 y(1-y)}{m_j^2 y + \tilde{\mu}_l^2 (1-y) - M^2 y(1-y)} \right| \right\}. \tag{3.24}$$

The $j = 0$ term is of order $(M - m_0)^2 \propto \alpha^2$, and the $j = 1$ term is of order $1/m_1$, after invocation of the chiral constraint (3.18) to eliminate the leading, order-m_1 term. Thus, Δ breaks gauge invariance only in ways to be expected; gauge invariance can be attained only without truncations and then only in the $m_1 \to \infty$ limit. The high-order α correction is a signal of a truncation effect, and, of course, the $1/m_1$ contribution disappears as $m_1 \to \infty$.

To study the dependence on the IR mass scale μ_0, we consider the $\mu_0 \to 0$ limit of the $j = l = 0$ term in (3.24), which is

$$\frac{\zeta - 1}{32\pi^2 \zeta} (M - m_0)^2 \int_0^1 dy \frac{M(1-y) + m_0}{y[m_0^2 - M^2(1-y)]}. \tag{3.25}$$

The behavior near $y = 0$ is such that the term has a log divergence multiplied by $(\zeta - 1)(M - m_0)$. Thus, this contribution is of order α, not α^2, and, of course, is absent in Feynman gauge. The order of the contribution is, however, still consistent with being a truncation error. Also, the presence of an IR divergence is consistent with the presence of other, UV divergences, which are uncanceled due to truncation, as discussed in [3] and as discussed below, with respect to normalization of the eigenstate.

C. Anomalous magnetic moment

We compute the anomalous magnetic moment of the dressed electron from the spin-flip matrix element of the electromagnetic current J [25]. The plus component of the current is used because, in the absence of vacuum polarization, it is not renormalized [5,26]. In general, the transition amplitude for absorption of a photon of momentum q by a dressed electron is given by

$$\langle \psi^\sigma(\underline{P} + \underline{q}) | \frac{J^+(0)}{P^+} | \psi^\pm(\underline{P}) \rangle = 2\delta_{\sigma\pm} F_1(q^2) \pm \frac{q^1 \pm i q^2}{M} \delta_{\sigma\mp} F_2(q^2), \tag{3.26}$$

where F_1 and F_2 are the usual Dirac and Pauli form factors. The anomalous moment is $a_e = F_2(0)$; normalization of the state is equivalent to $F_1(0) = 1$. As described in [25], the limit of zero momentum transfer for F_2 can be written as

$$a_e = \mp M \sum_{jls\lambda} \int d\underline{k} \epsilon^\lambda (-1)^{j+l} C_{jls}^{\lambda\pm *}(\underline{k}) y \left(\frac{\partial}{\partial k^1} \pm i \frac{\partial}{\partial k^2} \right) C_{jls}^{\lambda\mp}(\underline{k}). \tag{3.27}$$

This form assumes complete separation of the internal and external momentum variables in the wave functions $C_{jls}^{\lambda\pm}$, which does occur for components in terms of polarizations. The sum over polarizations λ does not include the gauge projection, because gauge invariance has already been broken by both the truncation and the flavor-changing currents. The normalization condition (3.2), or $F_1(0) = 1$, becomes

$$1 = (z_0^{\pm})^2 - (z_1^{\pm})^2 + \sum_{jls\lambda} \int d\underline{k} \epsilon^{\lambda} (-1)^{j+l} |C_{jls}^{\lambda\pm}(\underline{k})|^2, \tag{3.28}$$

which determines $z_0^{\pm}$.

The reduction of the expression for the anomalous moment is given in Appendix B. The result in the limit of infinite PV electron mass is given in (B7). The normalization condition is evaluated in Appendix C, with the same infinite mass limit yielding (C8). From these expressions, the anomalous moment can be computed for various values of the UV scale μ_1, the IR scale μ_0, and the gauge parameter ζ. Sample results are given in Figs. 2–4. Because truncation errors do not allow either $\mu_0 \to 0$, except in Feynman gauge, or $\mu_1 \to \infty$, we look for regions in each where the physical quantity is relatively flat. In Fig. 4, we then look for sensitivity to the gauge parameter

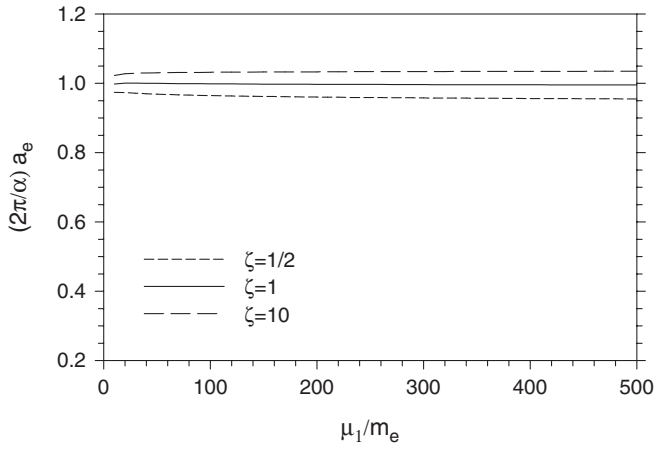

FIG. 2. The anomalous magnetic moment a_e for the dressed-electron state truncated to include at most the one-electron/one-photon Fock states, as a function of the PV-photon mass μ_1. The IR mass scale μ_0 is $0.001 m_e$, and the gauge parameter ζ is 1/2, 1, and 10.

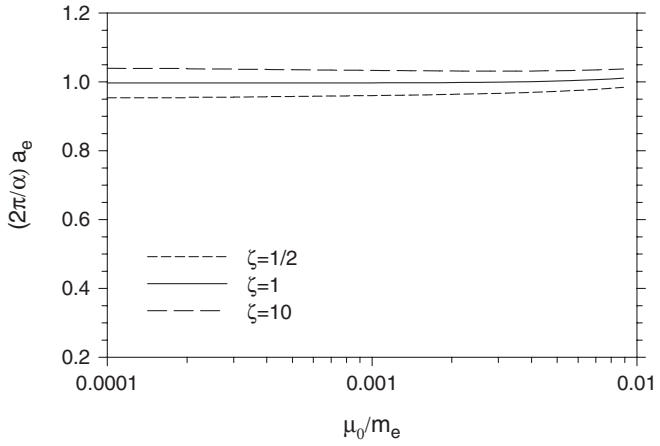

FIG. 3. Same as Fig. 2 but as a function of the IR mass scale μ_0, with $\mu_1 = 200 m_e$.

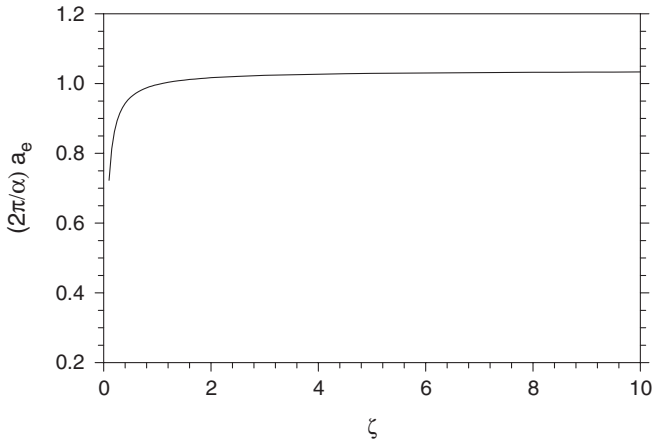

FIG. 4. Same as Fig. 2 but as a function of the gauge parameter ζ, with $\mu_0 = 0.001 m_e$ and $\mu_1 = 200 m_e$.

when μ_0 and μ_1 have values in such regions. The plot shows little sensitivity and, therefore, approximate gauge independence, except for small values of ζ. For small ζ, the theory is near the singular limit where the gauge-fixing term is removed from the Lagrangian (2.1) and truncation errors are amplified.

The remaining gauge dependence can be seen to be consistent with the order of the truncation in the calculation. Because $M - m_0$ is of order α, we have, for the leading term in a_e, as given by (B7),

$$M \int_0^1 dy \frac{y(1-y)My}{m_0^2 y - M^2 y(1-y)} = \frac{1}{2} + \mathcal{O}(\alpha), \tag{3.29}$$

and, therefore,

$$a_e = \frac{\alpha}{2\pi} z_0^2 + \mathcal{O}(\alpha^2, 1/\mu_1^2). \tag{3.30}$$

Up to normalization, the Schwinger result [27] of $\alpha/2\pi$ is recovered, and the gauge-dependent contributions, as well as some physical nonperturbative contributions, are higher order in α, consistent with the truncation to one photon in the Fock basis.

To complete the analysis, we must consider the normalization factor z_0, as given in (C8). At the present order of truncation, a value other than 1 for z_0 represents a truncation error, in the sense that contributions occurring at the same order in α for the numerator of the expectation value for the anomalous moment have been left out by the truncation; therefore, the gauge dependence of z_0 must be due to truncation errors.

In the IR limit $\mu_0 \to 0$, the normalization z_0 does have a singular contribution of the form $3\frac{\alpha}{4\pi}\frac{1-\zeta}{\zeta}\int\frac{dy}{y}$. This comes from a combination of the last two terms of the curly bracket in (C8) for $\mu_0 \to 0$ and $l = 0$. Therefore, the normalization contains an IR divergence in addition to its usual UV divergence, except in Feynman gauge, where there is only a UV divergence.

These divergences are the characteristic ''uncanceled divergences'' caused by Fock-space truncation [3]. They arise in both the standard and sector-dependent parametrizations, although in the latter case the IR divergence is present even in Feynman gauge [7]. For the standard parametrization used here, we find the divergences in the normalization factor. For the sector-dependent parametrization, the divergence is most easily seen by considering the probability for the one-electron/one-photon sector. This should be between zero and one, but the renormalization of the sector-dependent coupling absorbs the divergence in the normalization factor $1/z_0^2$ and allows the probability of the one-electron/one-photon sector to diverge. In this case, the overall norm of the eigenstate is maintained only because the probability for the bare-electron sector goes to negative infinity in such a way that the sum of probabilities is formally one.

Because of the uncanceled divergences, not all of the PV masses can be taken to infinity and, except for our standard parameterization in Feynman gauge, the physical photon mass cannot be taken to zero. As argued in [3], the errors introduced by these limitations are to be minimized by seeking ranges of mass values over which results do not change significantly. This strikes a balance between the errors caused by the presence of unphysical PV fields and a nonzero photon mass and the errors associated with Fock-space truncation. The former decrease with increasing PV masses (as the PV fields are removed from the spectrum) and decreasing photon mass; the latter, the truncation errors, increase with increasing PV masses as the uncanceled divergences assert themselves.

IV. SUMMARY

We have developed a formalism whereby nonperturbative calculations can be done for light-front QED in an arbitrary covariant gauge. The formalism combines a light-front Stueckelberg quantization for the free photon field with a Pauli-Villars regularization that simplifies the constraint equation for the nondynamical part of the fermion field. The Stueckelberg quantization allows the physical and PV photons to be handled in the same way, which facilitates the regularization and the preservation of symmetries. In Feynman gauge, this quantization is equivalent to the Gupta-Bleuler quantization used previously [3,5,8].

As a first application of the formalism, we have studied the dressed electron in a Fock space truncated to include at most one photon and no positrons. In particular, we have investigated the gauge invariance of the mass shift and the anomalous magnetic moment. In both cases, the residual gauge dependence can be ascribed to errors induced by the Fock-space truncation. The dependence of the anomalous moment on the gauge parameter ζ is illustrated in Fig. 4 in the case where the PV-fermion mass m_1 is infinite. If the PV-fermion mass is kept finite, there is also gauge dependence of order $1/m_1$, due to fermion-flavor-mixing currents. The strong dependence as $\zeta \to 0$ is to be expected, because in this limit the gauge-fixing term is removed from the Lagrangian and the theory becomes undefined. We have also found that two PV-photon flavors are required to maintain the chiral symmetry of the massless-electron limit; this extends the earlier Feynman-gauge result [5] to arbitrary gauges.

The formalism can be applied to higher-order truncations and to other charge sectors. For high-order truncations, where calculations are done numerically [8], the PV-fermion mass m_1 must usually be kept finite; however, with m_1 large enough, the gauge-dependent effects should be small.

ACKNOWLEDGMENTS

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APPENDIX A: AN INTEGRAL IDENTITY

From [5], we have already that $J = M^2 I_0$. Thus, to show that $J + \Delta J = M^2(I_0 + \Delta I_0)$, we need only consider the gauge-dependent parts. If we write the common denominators in (3.9) as the difference of two terms, we obtain

$$\Delta J - M^2 \Delta I_0 = -\frac{1}{32\pi^2} \sum_{jl} (-1)^{j+l} \xi_l \frac{M^2 - m_j^2}{\mu_l^2} \int \frac{dy dk_\perp^2}{y} \times \left[M^2 - \frac{m_j^2 + k_\perp^2}{(1-y)^2} \right] \left(\frac{1}{M^2 - \frac{m_j^2 + k_\perp^2}{1-y} - \frac{\mu_l^2 + k_\perp^2}{y}} - \frac{1}{M^2 - \frac{m_j^2 + k_\perp^2}{1-y} - \frac{\tilde{\mu}_l^2 + k_\perp^2}{y}} \right), \tag{A1}$$

where we have used $1/(\tilde{\mu}_l^2 - \mu_l^2) = \zeta/(1-\zeta)\mu_l^2$ to simplify the leading factors. For the terms that contain $m_j^2 + k_\perp^2$, we change the integration variable y to

$$x = (1-y) \frac{\mu_l^2 + k_\perp^2}{m_j^2 y + \mu_l^2 (1-y) + k_\perp^2}, \tag{A2}$$

or

$$\tilde{x} = (1-y) \frac{\tilde{\mu}_l^2 + k_\perp^2}{m_j^2 y + \tilde{\mu}_l^2 (1-y) + k_\perp^2}, \tag{A3}$$

depending on whether μ_l or $\tilde{\mu}_l$ appears in the denominator of the integrand. As discussed in [5], these variables range between 0 and 1, though in reverse order, and satisfy

$$\frac{m_j^2 + k_\perp^2}{1-y} + \frac{\mu_l^2 + k_\perp^2}{y} = \frac{m_j^2 + k_\perp^2}{1-x} + \frac{\mu_l^2 + k_\perp^2}{x}, \tag{A4}$$

and the analogous expression for $\tilde{x}$. With use of this identity and differentiation of (A2), we obtain

$$(m_j^2+k_\perp^2)\frac{dy}{y(1-y)^2}=\frac{dx}{x}\Big(M^2-\frac{m_j^2+k_\perp^2}{1-x}-\frac{\mu_l^2+k_\perp^2}{x}-M^2\Big). \tag{A5}$$

The analogous expression holds for $d\tilde{x}$. Substitution into (A1) and replacement of x and $\tilde{x}$ by y, to have a common integration variable for all terms, yields

$$\Delta J - M^2\Delta I_0 = -\frac{1}{32\pi^2}\sum_{jl}(-1)^{j+l}\xi_l\frac{M^2-m_j^2}{\mu_l^2}\int\frac{dydk_\perp^2}{y}\Bigg\{M^2\Big(\frac{1}{M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_l^2+k_\perp^2}{y}}-\frac{1}{M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\tilde{\mu}_l^2+k_\perp^2}{y}}\Big)$$
$$+\Bigg[\Big(1-\frac{M^2}{M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_l^2+k_\perp^2}{y}}\Big)-\Big(1-\frac{M^2}{M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\tilde{\mu}_l^2+k_\perp^2}{y}}\Big)\Bigg]\Bigg\}. \tag{A6}$$

Here we have taken into account the reversed order of limits for x and $\tilde{x}$ by changing the sign of the terms in the square brackets. Clearly, the sum of terms in the curly brackets is zero, and, therefore, $\Delta J = M^2\Delta I_0$.

APPENDIX B: EVALUATION OF THE ANOMALOUS-MOMENT FORMULA

On substitution of the two-body wave functions (3.5) and use of the Kronecker deltas in spin, the expression (3.27) for the anomalous moment becomes

$$a_e = -M\sum_{jl\lambda}\sum_{i'i''}\epsilon^\lambda(-1)^{j+l+i'+i''}\xi_l z_{i'}^+ z_{i''}^-\int dydk_\perp^2\,\frac{y\pi(P^+)^3}{M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_{l\lambda}^2+k_\perp^2}{y}}\Bigg\{e^{(\lambda)}_{l\mu'}(\underline{k})V^{\mu'*}_{ji'+}(\underline{P}-\underline{k},\underline{P})\Big(\frac{\partial}{\partial k^1}+i\frac{\partial}{\partial k^2}\Big)$$
$$\times\frac{e^{(\lambda)}_{l\mu}(\underline{k})U^{\mu}_{ji''-}(\underline{P}-\underline{k},\underline{P})}{M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_{l\lambda}^2+k_\perp^2}{y}}+e^{(\lambda)}_{l\mu'}(\underline{k})U^{\mu'*}_{ji'+}(\underline{P}-\underline{k},\underline{P})\Big(\frac{\partial}{\partial k^1}+i\frac{\partial}{\partial k^2}\Big)\frac{e^{(\lambda)}_{l\mu}(\underline{k})V^{\mu}_{ji''-}(\underline{P}-\underline{k},\underline{P})}{M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_{l\lambda}^2+k_\perp^2}{y}}\Bigg\}, \tag{B1}$$

with the vertex functions specified in (2.21) and (2.22). The terms generated by differentiation of the denominators cancel. Simplification of the remaining terms, summed over polarizations λ, yields

$$a_e = -M\frac{e^2}{8\pi^2}\sum_{jl}\sum_{i'i''}(-1)^{j+l+i'+i''}\xi_l z_{i'}^+ z_{i''}^-\int dydk_\perp^2\Bigg\{\frac{m_{i'}(1-y)-m_j}{y(1-y)}\frac{2}{(M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_l^2+k_\perp^2}{y})^2}$$
$$+\frac{m_{i''}-m_j}{y(1-y)}\frac{2+(1-\zeta)/\zeta}{(M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_l^2+k_\perp^2}{y})^2}+\frac{m_{i''}-m_j}{\mu_l^2(1-y)}\Big(\frac{m_j m_{i'}y}{1-y}+\frac{\mu_l^2}{y\zeta}\Big)\Big(\frac{1}{(M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\tilde{\mu}_l^2+k_\perp^2}{y})^2}-\frac{1}{(M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_l^2+k_\perp^2}{y})^2}\Big)\Bigg\}. \tag{B2}$$

The $k_\perp^2$ integrals have double poles, if $\mu_0 < M - m_0$ or $\tilde{\mu}_0 < M - m_0$; following the earlier convention in [3], we define the integrals by

$$\int\frac{f(x)dx}{(x-a)^2}=\lim_{\eta\to 0}\frac{1}{2\eta}\Big[\mathcal{P}\int\frac{f(x)dx}{x-a-\eta}-\mathcal{P}\int\frac{f(x)dx}{x-a+\eta}\Big]. \tag{B3}$$

With or without the pole, we find

$$\int\frac{dk_\perp^2}{(M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_l^2+k_\perp^2}{y})^2}=\frac{y^2(1-y)^2}{m_j^2y+\mu_l^2(1-y)-M^2y(1-y)}. \tag{B4}$$

These leave the anomalous moment in the form

$$a_e=\frac{\alpha}{\pi}M\sum_{jl}\sum_{i'i''}(-1)^{j+l+i'+i''}\xi_l z_{i'}^+ z_{i''}^-\int dy\frac{y(1-y)[m_{i'}y-(m_{i'}+m_{i''}-2m_j)]}{m_j^2y+\mu_l^2(1-y)-M^2y(1-y)}+\Delta a_e, \tag{B5}$$

with Δa_e a gauge-dependent part given by

$$\Delta a_e = -\frac{1-\zeta}{\zeta}\frac{\alpha}{2\pi}M\sum_{jl}\sum_{i'i''}(-1)^{j+l+i'+i''}\xi_l z_{i'}^{+}z_{i''}^{-}(m_{i''}-m_j)\int dy y(1-y)\Big\{\frac{1}{m_j^2 y+\mu_l^2(1-y)-M^2y(1-y)} - \frac{m_j m_{i'}y^2+\mu_l^2(1-y)/\zeta}{[m_j^2y+\mu_l^2(1-y)-M^2y(1-y)][m_j^2y+\tilde{\mu}_l^2(1-y)-M^2y(1-y)]}\Big\}. \tag{B6}$$

Except for the normalization factors $z_i^{\pm}$, this expression is IR safe; the photon mass μ_0 can be set to zero. We first take the limit $m_1 \to \infty$, which, with use of $z_i^{+} = z_i^{-} \equiv z_i$, implies $\sum_i(-1)^i z_i \to z_0$ and $\sum_i(-1)^i m_i z_i \to M z_0$. Next, the second PV-photon flavor is removed by the limit $\mu_2 \to \infty$, in which $\xi_2 \to 0$ and $\xi_1 \to 1$, to obtain

$$a_e \to \frac{\alpha}{\pi}Mz_0^2\sum_{l=0}^{1}(-1)^l\int dy\frac{y(1-y)[My-2(M-m_0)]}{m_0^2y+\mu_l^2(1-y)-M^2y(1-y)} + \Delta a_e \tag{B7}$$

and

$$\Delta a_e \to -\frac{1-\zeta}{\zeta}\frac{\alpha}{2\pi}M(M-m_0)z_0^2\sum_{l=0}^{1}(-1)^l\int dy\Big\{\frac{y(1-y)}{m_0^2y+\mu_l^2(1-y)-M^2y(1-y)} - \frac{y(1-y)[m_0My^2+\mu_l^2(1-y)/\zeta]}{[m_0^2y+\mu_l^2(1-y)-M^2y(1-y)][m_0^2y+\tilde{\mu}_l^2(1-y)-M^2y(1-y)]}\Big\}. \tag{B8}$$

APPENDIX C: EVALUATION OF THE NORMALIZATION CONDITION

On substitution of the two-body wave functions (3.5), the normalization condition (3.28) becomes

$$1 = (z_0^{\pm})^2 - (z_1^{\pm})^2 + \sum_{jl}\sum_{i'i''}(-1)^{j+l+i'+i''}\xi_l z_{i'}^{\pm}z_{i''}^{\pm}\sum_{\lambda}\epsilon^{\lambda}\int\frac{\pi(P^+)^3dydk_\perp^2}{(M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_{l_\lambda}^2+k_\perp^2}{y})^2}e_{l\mu'}^{(\lambda)}(\underline{k})e_{l\mu}^{(\lambda)}(\underline{k}) \times[V_{ji'\pm}^{\mu'*}(\underline{P}-\underline{k},\underline{P})V_{ji''\pm}^{\mu}(\underline{P}-\underline{k},\underline{P}) + U_{ji'\pm}^{\mu'*}(\underline{P}-\underline{k},\underline{P})U_{ji''\pm}^{\mu}(\underline{P}-\underline{k},\underline{P})]. \tag{C1}$$

To simplify the expression, we first add and subtract the $\lambda = 0$ term with the denominator replaced by the denominator of the $\lambda \neq 0$ terms. We then have

$$1 = (z_0^{\pm})^2 - (z_1^{\pm})^2 + \sum_{jl}\sum_{i'i''}(-1)^{j+l+i'+i''}\xi_l z_{i'}^{\pm}z_{i''}^{\pm}\int\pi(P^+)^3dydk_\perp^2\Big\{\frac{1}{(M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_l^2+k_\perp^2}{y})^2}\sum_{\lambda}\epsilon^{\lambda}e_{l\mu'}^{(\lambda)}(\underline{k})e_{l\mu}^{(\lambda)}(\underline{k}) + \Big(\frac{1}{(M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_l^2+k_\perp^2}{y})^2}-\frac{1}{(M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\tilde{\mu}_l^2+k_\perp^2}{y})^2}\Big)e_{l\mu'}^{(0)}(\underline{k})e_{l\mu}^{(0)}(\underline{k})\Big\}[V_{ji'\pm}^{\mu'*}(\underline{P}-\underline{k},\underline{P})V_{ji''\pm}^{\mu}(\underline{P}-\underline{k},\underline{P}) + U_{ji'\pm}^{\mu'*}(\underline{P}-\underline{k},\underline{P})U_{ji''\pm}^{\mu}(\underline{P}-\underline{k},\underline{P})]. \tag{C2}$$

On use of the vertex functions in (2.21) and (2.22) and of $z_i^{+} = z_i^{-} \equiv z_i$, we obtain

$$1 = z_0^2 - z_1^2 + \frac{e^2}{16\pi^2}\sum_{jl}\sum_{i'i''}(-1)^{j+l+i'+i''}\xi_l z_{i'}z_{i''}\int dydk_\perp^2\Big\{\frac{1}{(M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_l^2+k_\perp^2}{y})^2} \times\Big[2\frac{m_j^2-2m_j(m_{i'}+m_{i''})(1-y)+m_{i'}m_{i''}(1-y)^2+k_\perp^2}{y(1-y)^2} - \frac{1-\zeta}{\zeta}\Big(\frac{m_j(m_{i'}+m_{i''})}{y(1-y)}+\frac{2k_\perp^2}{y^3(1-y)}+\frac{1+\zeta}{\zeta}\frac{\mu_l^2}{y^3}\Big)\Big] + \frac{1}{\mu_l^2}\Big[\frac{1}{(M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_l^2+k_\perp^2}{y})^2}-\frac{1}{(M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\tilde{\mu}_l^2+k_\perp^2}{y})^2}\Big]\Big[\frac{m_j^2m_{i'}m_{i''}y}{(1-y)^2}+\frac{k_\perp^4}{y^3(1-y)^2}+\frac{\tilde{\mu}_l^4}{y^3}+\frac{(m_j^2+m_{i'}m_{i''})k_\perp^2}{y(1-y)^2} + \frac{\tilde{\mu}_l^2m_j(m_{i'}+m_{i''})}{y(1-y)}+\frac{2\tilde{\mu}_l^2k_\perp^2}{y^3(1-y)}\Big]\Big\}. \tag{C3}$$

The $k_\perp^2$ integrals are defined by (B3) when a pole is present. The integrals needed are given by (B4),

$$\int\frac{k_\perp^2dk_\perp^2}{(M^2-\frac{m_j^2+k_\perp^2}{1-y}-\frac{\mu_l^2+k_\perp^2}{y})^2} = -y^2(1-y)^2\ln[|m_j^2y+\mu_l^2(1-y)-M^2y(1-y)|], \tag{C4}$$

and

$$\int \frac{k_\perp^4\, dk_\perp^2}{(M^2 - \frac{m_j^2+k_\perp^2}{1-y} - \frac{\mu_l^2+k_\perp^2}{y})^2} = y^2(1-y)^2(m_j^2 y + \mu_l^2(1-y) - M^2 y(1-y))(2\ln[|m_j^2 y + \mu_l^2(1-y) - M^2 y(1-y)|] + 1), \tag{C5}$$

where we have dropped infinite terms that cancel in the final expressions, due to either $\sum_l (-1)^l \xi_l = 0$ or the difference between the two denominators, $(M^2 - \frac{m_j^2+k_\perp^2}{1-y} - \frac{\mu_l^2+k_\perp^2}{y})^2$ and $(M^2 - \frac{m_j^2+k_\perp^2}{1-y} - \frac{\tilde{\mu}_l^2+k_\perp^2}{y})^2$. Substitution of these integrals yields

$$\begin{aligned}
1 = {} & z_0^2 - z_1^2 + \frac{e^2}{16\pi^2} \sum_{jl} \sum_{i'i''} (-1)^{j+l+i'+i''} \xi_l z_{i'} z_{i''} \int dy \Big\{ \frac{1}{m_j^2 y + \mu_l^2(1-y) - M^2 y(1-y)} \Big[2y(m_j^2 - 2m_j(m_{i'} + m_{i''})(1-y) \\
& + m_{i'} m_{i''}(1-y)^2) - \frac{1-\zeta}{\zeta} \Big(m_j(m_{i'} + m_{i''}) y(1-y) + \frac{1+\zeta}{\zeta} \frac{\mu_l^2(1-y)^2}{y} \Big) \Big] - 2\Big(y - \frac{1-\zeta}{\zeta} \frac{1-y}{y} \Big) \\
& \times \ln[|m_j^2 y + \mu_l^2(1-y) - M^2 y(1-y)|] + \frac{1}{\mu_l^2} \Big[\frac{1}{m_j^2 y + \mu_l^2(1-y) - M^2 y(1-y)} - \frac{1}{m_j^2 y + \tilde{\mu}_l^2(1-y) - M^2 y(1-y)} \Big] \\
& \times \Big[m_j^2 m_{i'} m_{i''} y^3 + \frac{\tilde{\mu}_l^4(1-y)^2}{y} + \tilde{\mu}_l^2 m_i(m_{i'} + m_{i''}) y(1-y) \Big] + \frac{1}{\mu_l^2 y} [(m_j^2 y + \mu_l^2(1-y) \\
& - M^2 y(1-y)(2\ln[|m_j^2 y + \mu_l^2(1-y) - M^2 y(1-y)|] + 1) - (m_j^2 y + \tilde{\mu}_l^2(1-y) \\
& - M^2 y(1-y)(2\ln[|m_j^2 y + \tilde{\mu}_l^2(1-y) - M^2 y(1-y)|] + 1)] + \frac{1}{\mu_l^2} \Big[y(m_j^2 + m_{i'} m_{i''}) + \frac{2\tilde{\mu}_l^2(1-y)}{y} \Big] \\
& \times \ln\Big(\Big| \frac{m_j^2 y + \tilde{\mu}_l^2(1-y) - M^2 y(1-y)}{m_j^2 y + \mu_l^2(1-y) - M^2 y(1-y)} \Big| \Big) \Big\}.
\end{aligned} \tag{C6}$$

Collecting terms with logarithms, we find that multipliers containing $1/y$ cancel. The remaining terms containing $1/y$ are not singular in the full expression, which can be arranged explicitly by adding $\frac{1-\zeta}{\zeta} \frac{1-y}{y}$ to the curly brackets of (C6). This additional piece makes no contribution to the sum over l, because $\sum_l (-1)^l \xi_l = 0$. The resulting expression for the normalization condition is

$$\begin{aligned}
1 = {} & z_0^2 - z_1^2 + \frac{e^2}{16\pi^2} \sum_{jl} \sum_{i'i''} (-1)^{j+l+i'+i''} \xi_l z_{i'} z_{i''} \int dy \Big\{ \frac{1}{m_j^2 y + \mu_l^2(1-y) - M^2 y(1-y)} \Big[2y(m_j^2 - 2m_j(m_{i'} + m_{i''})(1-y) \\
& + m_{i'} m_{i''}(1-y)^2) - \frac{1-\zeta}{\zeta} m_j(m_{i'} + m_{i''}) y(1-y) \Big] - 2y \ln[|m_j^2 y + \mu_l^2(1-y) - M^2 y(1-y)|] \\
& + \frac{1}{\mu_l^2} \Big[\frac{1}{m_j^2 y + \mu_l^2(1-y) - M^2 y(1-y)} - \frac{1}{m_j^2 y + \tilde{\mu}_l^2(1-y) - M^2 y(1-y)} \Big] [m_j^2 m_{i'} m_{i''} y^3 + \tilde{\mu}_l^2 m_j(m_{i'} + m_{i''}) y(1-y)] \\
& - \Big[\frac{1}{m_j^2 y + \mu_l^2(1-y) - M^2 y(1-y)} - \frac{1}{\zeta} \frac{1}{m_j^2 y + \tilde{\mu}_l^2(1-y) - M^2 y(1-y)} \Big] (1-y)[m_j^2 - M^2(1-y)] \\
& + \frac{1}{\mu_l^2} \Big[y(m_j^2 + m_{i'} m_{i''}) + 2[m_j^2 - M^2(1-y)] \Big] \ln\Big(\Big| \frac{m_j^2 y + \tilde{\mu}_l^2(1-y) - M^2 y(1-y)}{m_j^2 y + \mu_l^2(1-y) - M^2 y(1-y)} \Big| \Big) \Big\}.
\end{aligned} \tag{C7}$$

In the $m_1 \to \infty$ limit, this becomes

$$1 = z_0^2 + \frac{\alpha}{4\pi}\sum_l (-1)^l \xi_l z_0^2 \int dy \Bigg\{ \frac{1}{m_0^2 y + \mu_l^2(1-y) - M^2 y(1-y)} \Bigg[2y(m_0^2 - 4m_0 M(1-y) + M^2(1-y)^2) - \frac{1-\zeta}{\zeta} 2m_0 M y(1-y) \Bigg]$$
$$- 2y \ln[|m_0^2 y + \mu_l^2(1-y) - M^2 y(1-y)|] + \frac{1}{\mu_l^2}\Bigg[\frac{1}{m_0^2 y + \mu_l^2(1-y) - M^2 y(1-y)} - \frac{1}{m_0^2 y + \tilde{\mu}_l^2(1-y) - M^2 y(1-y)} \Bigg]$$
$$\times [m_0^2 M^2 y^3 + 2\tilde{\mu}_l^2 m_0 M y(1-y)] - \Bigg[\frac{1}{m_0^2 y + \mu_l^2(1-y) - M^2 y(1-y)} - \frac{1}{\zeta} \frac{1}{m_0^2 y + \tilde{\mu}_l^2(1-y) - M^2 y(1-y)} \Bigg]$$
$$\times (1-y)[m_0^2 - M^2(1-y)] + \frac{1}{\mu_l^2}[y(m_0^2 + M^2) + 2[m_0^2 - M^2(1-y)]] \ln\Bigg(\Bigg| \frac{m_0^2 y + \tilde{\mu}_l^2(1-y) - M^2 y(1-y)}{m_0^2 y + \mu_l^2(1-y) - M^2 y(1-y)} \Bigg| \Bigg) \Bigg\}. \tag{C8}$$

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