Predictions of selected flavor observables within the standard model

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This paper gathers a selection of standard model predictions issued from the metrology of the Cabibbo-Kobayashi-Maskawa parameters performed by the CKMfitter group. The selection includes purely leptonic decays of neutral and charged B, D, and K mesons. In the light of the expected measurements from the LHCb experiment, special attention is given to the radiative decay modes of B mesons as well as to the B-meson mixing observables, in particular the semileptonic charge asymmetries $a_{\rm SL}^{d,s}$ which have been recently investigated by the D0 experiment at Tevatron. Constraints arising from rare kaon decays are addressed, in light of both current results and expected performances of future rare kaon experiments. All results have been obtained with the CKMfitter analysis package, featuring the frequentist statistical approach and using Rfit to handle theoretical uncertainties.

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I. INTRODUCTION

In the standard model (SM), the weak charged-current transitions mix quarks of different generations, which is encoded in the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix [1,2]. In the case of three generations of quarks, the physical content of this matrix reduces to four real parameters, among which one phase, the only source of CP violation in the SM (we do not consider minute CP-violating effects from the strong-interaction θ term or from the masses of neutrinos). One can define these four real parameters as

$$\lambda^2 = \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2}, \qquad A^2\lambda^4 = \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2}, \qquad \bar\rho + i\bar\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}, \tag{1}$$

and exploit the unitarity of the CKM matrix to determine all its elements (and when needed, to obtain their expansion in powers of λ) [3].

Extracting information on these parameters from data is a challenge for both experimentalists and theorists, since the SM depends on a large set of parameters which are not predicted within its framework, and must be determined experimentally. A further problem comes from the presence of the strong interaction that binds quarks into hadrons and is still difficult to tackle theoretically, leading to most of the theoretical uncertainties discussed when extracting the CKM matrix parameters. The CKMfitter group follows this goal using a standard χ^2-like frequentist approach, in addition to the Rfit scheme to treat theoretical uncertainties, aiming at combining a large set of constraints from flavor physics [3,4].

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TABLE I. Constraints used for the global fit, and the main inputs involved (more information can be found in Ref. [4]). The lattice inputs are our own averages obtained as described in the text.

CKM	Process	Observables	Theoretical inputs
$\lvert V_{ud}\rvert$	$0^+ \to 0^+$ transitions	$\lvert V_{ud}\rvert_{\rm nucl} = 0.974\,25 \pm 0.000\,22$ [5]	Nuclear matrix elements
$\lvert V_{us}\rvert$	$K \to \pi\ell\nu$	$\lvert V_{us}\rvert_{\rm semi} f_+(0) = 0.2163 \pm 0.0005$ [6]	$f_+(0) = 0.9632 \pm 0.0028 \pm 0.0051$
	$K \to e\nu_e$	$\mathcal{B}(K \to e\nu_e) = (1.584 \pm 0.0020) \times 10^{-5}$ [7]	$f_K = 156.3 \pm 0.3 \pm 1.9$ MeV
	$K \to \mu\nu_\mu$	$\mathcal{B}(K \to \mu\nu_\mu) = 0.6347 \pm 0.0018$ [6]	
	$\tau \to K\nu_\tau$	$\mathcal{B}(\tau \to K\nu_\tau) = 0.006\,96 \pm 0.000\,23$ [7]	
$\lvert V_{us}\rvert/\lvert V_{ud}\rvert$	$K \to \mu\nu/\pi \to \mu\nu$	$\frac{\mathcal{B}(K\to\mu\nu_\mu)}{\mathcal{B}(\pi\to\mu\nu_\mu)} = (1.3344 \pm 0.0041) \times 10^{-2}$ [6]	$f_K/f_\pi = 1.205 \pm 0.001 \pm 0.010$
	$\tau \to K\nu/\tau \to \pi\nu$	$\frac{\mathcal{B}(\tau\to K\nu_\tau)}{\mathcal{B}(\tau\to\pi\nu_\tau)} = (6.53 \pm 0.11) \times 10^{-2}$ [8]	
$\lvert V_{cd}\rvert$	$D \to \mu\nu$	$\mathcal{B}(D \to \mu\nu) = (3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$ [9]	$f_{D_s}/f_D = 1.186 \pm 0.005 \pm 0.010$
$\lvert V_{cs}\rvert$	$D_s \to \tau\nu$	$\mathcal{B}(D_s \to \tau\nu) = (5.29 \pm 0.28) \times 10^{-2}$ [10]	$f_{D_s} = 251.3 \pm 1.2 \pm 4.5$ MeV
	$D_s \to \mu\nu$	$\mathcal{B}(D_s \to \mu\nu_\mu) = (5.90 \pm 0.33) \times 10^{-3}$ [10]	
$\lvert V_{ub}\rvert$	Semileptonic decays	$\lvert V_{ub}\rvert_{\rm semi} = (3.92 \pm 0.09 \pm 0.45) \times 10^{-3}$ [10]	Form factors, shape functions
	$B \to \tau\nu$	$\mathcal{B}(B \to \tau\nu) = (1.68 \pm 0.31) \times 10^{-4}$ [4]	$f_{B_s} = 231 \pm 3 \pm 15$ MeV $f_{B_s}/f_B = 1.209 \pm 0.007 \pm 0.023$
$\lvert V_{cb}\rvert$	Semileptonic decays	$\lvert V_{cb}\rvert_{\rm semi} = (40.89 \pm 0.38 \pm 0.59) \times 10^{-3}$ [10]	Form factors, OPE matrix elements
α	$B \to \pi\pi, \rho\pi, \rho\rho$	Branching ratios, CP asymmetries [10]	Isospin symmetry
β	$B \to (c\bar{c})K$	$\sin(2\beta)_{[c\bar{c}]} = 0.678 \pm 0.020$ [10]	
γ	$B \to D^{(*)}K^{(*)}$	Inputs for the 3 methods [10]	GGSZ, GLW, ADS methods
$V_{tq}^* V_{tq'}$	Δm_d	$\Delta m_d = 0.507 \pm 0.005$ ps $-$ 1 [10]	$\hat{B}_{B_s}/\hat{B}_{B_d} = 1.01 \pm 0.01 \pm 0.03$
	Δm_s	$\Delta m_s = 17.77 \pm 0.12$ ps $-$ 1 [11]	$\hat{B}_{B_s} = 1.28 \pm 0.02 \pm 0.03$
$V_{tq}^* V_{tq'}$, $V_{cq}^* V_{cq'}$	ϵ_K	$\lvert\epsilon_K\rvert = (2.229 \pm 0.010) \times 10^{-3}$ [7]	$\hat{B}_K = 0.730 \pm 0.004 \pm 0.036$ $\kappa_\epsilon = 0.940 \pm 0.013 \pm 0.023$

Not all the observables in flavor physics can be used as inputs for these constraints, due to limitations on our experimental and/or theoretical knowledge on these quantities. The list of inputs of the global fit is indicated in Table I: they fulfill the double requirement of a satisfying control of the attached theoretical uncertainties and a good experimental accuracy of their measurements. In addition, we only take as inputs the quantities that provide reasonably tight constraints on the CKM parameters A, λ, $\bar{\rho}$, and $\bar{\eta}$. This selects, in particular, leptonic and semileptonic decays of mesons yielding information on moduli of CKM matrix elements, nonleptonic two-body B decays related to angles of the CKM matrix, and B and K-mixing parameters.

The current situation of the global fit in the $(\bar{\rho}, \bar{\eta})$ is indicated in Fig. 1. Some comments are in order before discussing the metrology of the parameters. There exists a unique preferred region defined by the entire set of observables under consideration in the global fit. This region is represented by the yellow surface inscribed by the red contour line for which the values of $\bar{\rho}$ and $\bar{\eta}$ correspond to C.L. $< 95.45\%$. The goodness of the fit can be addressed in the simplified case where all the inputs uncertainties are taken as Gaussian, with a p value found to be 14% (i.e., 1.5σ; a rigorous derivation of the p value in the general case is beyond the scope of this paper [12]). One obtains

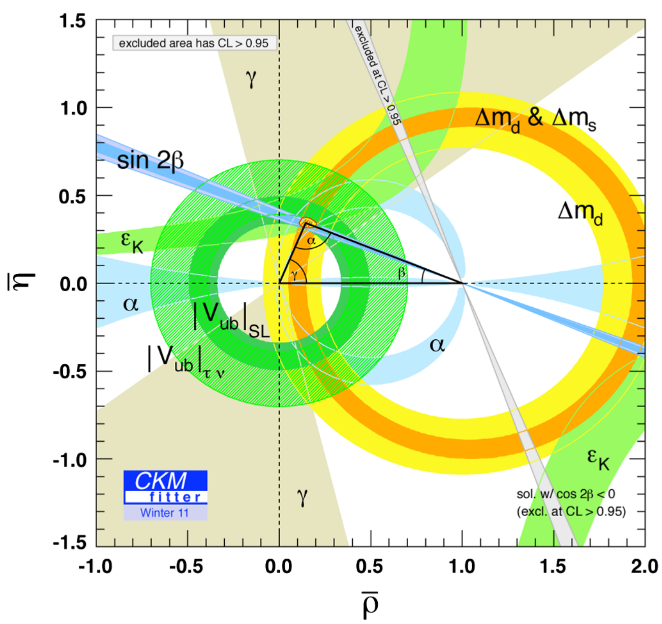

FIG. 1 (color online). Constraints on the CKM $(\bar{\rho}, \bar{\eta})$ coordinates from the global SM CKM fit. Regions outside the colored areas have C.L. $> 95.45\%$. For the combined fit the yellow area inscribed by the contour line represents points with C.L. $< 95.45\%$. The shaded area inside this region represents points with C.L. $< 68.3\%$.

the following values (at 1σ) for the four parameters describing the CKM matrix:

$$A = 0.816^{+0.011}_{-0.022}, \qquad \lambda = 0.225\,18^{+0.000\,36}_{-0.000\,77}, \tag{2}$$

$$\bar{\rho} = 0.144^{+0.028}_{-0.019}, \qquad \bar{\eta} = 0.342^{+0.015}_{-0.014}. \tag{3}$$

At this stage, it is fair to say that the SM hypothesis has passed the statistical test of the global consistency of all observables embodied in the fit, although some discrepancies are detailed in the following sections. We are therefore allowed to perform the metrology of the CKM parameters and to give predictions for any CKM-related observable within the SM. Let us add that the existence of a C.L. $< 95.45\%$ region in the $(\bar{\rho}, \bar{\eta})$ plane is not equivalent to the statement that each individual constraint lies in the global range of C.L. $< 95.45\%$. One of the interests of SM

TABLE II. Comparison between prediction and measurement of some flavor observables in the SM. The first column describes the observables. The second and third columns give the measurement and the prediction from the global fit (not including the measurement of the quantity considered), respectively. The fourth column expresses the departure of the prediction to the measurement, when available.

Observable	Measurement	Prediction	Pull (σ)
	Charged leptonic decays		
$\mathcal{B}(B^+ \to \tau^+ \nu_\tau)$	$(16.8 \pm 3.1) \times 10^{-5}$ [4]	$(7.57^{+0.98}_{-0.61}) \times 10^{-5}$	2.8
$\mathcal{B}(B^+ \to \mu^+ \nu_\mu)$	$<10^{-6}$ [10]	$(3.74^{+0.44}_{-0.38}) \times 10^{-7}$	...
$\mathcal{B}(D_s^+ \to \tau^+ \nu_\tau)$	$(5.29 \pm 0.28) \times 10^{-2}$ [10]	$(5.44^{+0.05}_{-0.17}) \times 10^{-2}$	0.5
$\mathcal{B}(D_s^+ \to \mu^+ \nu_\mu)$	$(5.90 \pm 0.33) \times 10^{-3}$	$(5.39^{+0.21}_{-0.22}) \times 10^{-3}$	1.3
$\mathcal{B}(D^+ \to \mu^+ \nu_\mu)$	$(3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$ [9]	$(4.18^{+0.13}_{-0.20}) \times 10^{-4}$	0.6
	Neutral leptonic B decays		
$\mathcal{B}(B_s^0 \to \tau^+ \tau^-)$	...	$(7.73^{+0.37}_{-0.65}) \times 10^{-7}$	...
$\mathcal{B}(B_s^0 \to \mu^+ \mu^-)$	$<32 \times 10^{-9}$ [10]	$(3.64^{+0.17}_{-0.31}) \times 10^{-9}$	...
$\mathcal{B}(B_s^0 \to e^+ e^-)$	$<2.8 \times 10^{-7}$ [10]	$(8.54^{+0.40}_{-0.72}) \times 10^{-14}$	...
$\mathcal{B}(B_d^0 \to \tau^+ \tau^-)$	$<4.1 \times 10^{-3}$ [10]	$(2.36^{+0.12}_{-0.21}) \times 10^{-8}$	...
$\mathcal{B}(B_d^0 \to \mu^+ \mu^-)$	$<6 \times 10^{-9}$ [10]	$(1.13^{+0.06}_{-0.11}) \times 10^{-10}$	...
$\mathcal{B}(B_d^0 \to e^+ e^-)$	$<8.3 \times 10^{-9}$ [10]	$(2.64^{+0.13}_{-0.24}) \times 10^{-15}$	...
	$B_q - \bar{B}_q$ mixing observables		
$\Delta\Gamma_s/\Gamma_s$	$0.092^{+0.051}_{-0.054}$ [10]	$0.179^{+0.067}_{-0.071}$	0.5
$a_{\rm SL}^d$	$(-47 \pm 46) \times 10^{-4}$ [10]	$(-6.5^{+1.9}_{-1.7}) \times 10^{-4}$	0.8
$a_{\rm SL}^s$	$(-17 \pm 91^{+12}_{-23}) \times 10^{-4}$ [13]	$(0.29^{+0.09}_{-0.08}) \times 10^{-4}$	0.2
$a_{\rm SL}^s - a_{\rm SL}^d$	...	$(6.8^{+1.9}_{-1.7}) \times 10^{-4}$	...
$\sin(2\beta)$	0.678 ± 0.020 [10]	$0.832^{+0.013}_{-0.033}$	2.7
$2\beta_s$	$[0.04; 1.04] \cup [2.16; 3.10]$ [14]	$0.0363^{+0.0016}_{-0.0015}$	...
	$0.76^{+0.36}_{-0.38} \pm 0.02$ [15]		
	Radiative B decays		
$\mathcal{B}(B_d \to K^*(892)\gamma)$	$(43.3 \pm 1.8) \times 10^{-6}$ [10]	$(64^{+22}_{-21}) \times 10^{-6}$	1.2
$\mathcal{B}(B^- \to K^{*-}(892)\gamma)$	$(42.1 \pm 1.5) \times 10^{-6}$ [10]	$(66^{+21}_{-20}) \times 10^{-6}$	1.1
$\mathcal{B}(B_s \to \phi\gamma)$	$(57^{+21}_{-18}) \times 10^{-6}$ [10]	$(65^{+31}_{-24}) \times 10^{-6}$	0.1
$\mathcal{B}(B \to X_s\gamma)/\mathcal{B}(B \to X_c \ell \nu)$	$(3.346 \pm 0.247) \times 10^{-3}$ [10]	$(3.03^{+0.34}_{-0.32}) \times 10^{-3}$	0.2
	Rare K decays		
$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$	$(1.75^{+1.15}_{-1.05}) \times 10^{-10}$ [16]	$(0.854^{+0.116}_{-0.098}) \times 10^{-10}$	0.8
$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})$	...	$(0.277^{+0.028}_{-0.035}) \times 10^{-10}$	...

predictions is that each comparison between the prediction issued from the fit and the corresponding measurement constitutes a null test of the SM hypothesis. Indeed, we will see that discrepancies actually do exist among the present set of observables considered in this paper (the corresponding pulls are reported in Table II).

We predict observables that were not used as input constraints, either because they are not measured with a sufficient accuracy yet, e.g., $\mathcal{B}(B_s \to \ell^+\ell^-)$, or because the control on the theoretical uncertainties remains controversial, e.g., $\Delta\Gamma_s/\Gamma_s$. The corresponding predictions can then be directly compared with their experimental measurements (when they are available). We also consider some particularly interesting observables used as an input of the fit, e.g., $\mathcal{B}(B \to \tau\nu)$. In this last case, we must compare the measurement of the observable with the outcome of the fit without including the observable among the inputs, so that the experimental information is used only once.

Following this procedure, we do not take the following quantities as inputs, but we predict their values: the semileptonic asymmetries $a^s_{\rm SL}$ and $a^d_{\rm SL}$, the weak phase in the B^0_s mixing β_s, the branching ratios of the dileptonic decays of neutral B mesons $\mathcal{B}(B_{d,s} \to \ell^+\ell^-)$, the branching ratio of (exclusive and inclusive) radiative $b \to s$ transitions, and rare $K \to \pi\nu\bar{\nu}$ decays. The first three observables all have in common to provide only loose constraints on the CKM parameters, while the two latter, though fulfilling the requirement of a good control of their related theoretical uncertainties, are so far out of reach of the current experiments. The LHCb experiment should bring a breakthrough in that respect very soon and these quantities will be included in the global fit once the required measurement accuracy is achieved [17]. The experimental situation is pretty similar for the semileptonic asymmetries related to neutral-meson mixing, with the additional drawback that these observables suffer from large theoretical uncertainties. The exclusive radiative $b \to s$ transitions suffer from significant uncertainties and are thus only considered for predictions. On the contrary, the inclusive $B \to X_s\gamma$, which have been measured and are well controlled theoretically, will be added as input of the global fit [12], but are kept for the present paper among the predictions. Finally, rare kaon decays have not been measured yet or provide only loose constraints on the CKM matrix elements.

In the following sections, we first discuss the main sources of theoretical uncertainty, before spelling out some of the fundamental formulas used for our predictions within the SM. We then collect the results obtained and compare them with their measurements (when available).

II. STRONG-INTERACTION PARAMETERS

The first category of theoretical uncertainties in flavor analyses arises from matrix elements that encode the effects of strong interaction in the nonperturbative regime. These matrix elements boil down to decay constants, form factors, and bag parameters for most of the observables under scrutiny in the present note, and all our predictions are subjected to and limited by the uncertainties in the determination of these observables. These uncertainties must be controlled with care since their misassessment or underestimation would affect the statements that we will make on flavor observables.

Among the different methods used to estimate nonperturbative QCD parameters, quark models, sum rules, and lattice QCD (LQCD) simulations are tools of choice. We opt for the latter whenever possible, as they provide well-established methods to compute these observables not only with a good accuracy at the present time, but also with a theoretical framework allowing for a systematic improvement on the theoretical control of the uncertainties. Over the past few years, many new estimates of the decay constants and the bag factors have been issued by different lattice collaborations, with different ways to address the uncertainties. A part of the uncertainties has a clear statistical interpretation: lattice simulations evaluate Green functions in an Euclidean metric expressed as path integrals using Monte Carlo methods, whose accuracy depends on the size of the sample of gauge configurations used for the computation in a straightforward way. But systematics are also present and depend on the strategies of computation chosen by competing lattice collaborations: discretization methods used to describe gauge fields and fermions on a lattice, interpolating fields, parameters of the simulations, such as the size of the (finite) volumes and lattice spacing, the masses of the quarks that can be simulated, and the number of dynamical flavors included as sea quarks (2 and 2 + 1 being the most frequent, after a long period where only quenched simulations were available). These simulations must be extrapolated to obtain physical quantities (relying, in particular, on effective theories such as chiral perturbation theory and heavy-quark effective theory).

The combination of lattice values with different approaches to address the uncertainty budget is a critical point of most global analyses of the flavor physics data, even though the concept of the theoretical uncertainty for such quantities is ill defined (and hence is the combination of them). Several approaches have been proposed to perform such a combination. We have collected the relevant LQCD calculations of the decay constants f_{B_d}, f_{B_s}, f_{D_s}, f_D, f_K, and f_π, as well as the bag parameters $\mathcal{B}_{B_d}$, $\mathcal{B}_{B_s}$, and $\mathcal{B}_K$, and the $K_{\ell 3}$ form factor $f_+(0)$. In addition we designed an averaging method aiming at providing a systematic, reproducible, and to some extent conservative scheme [18]. These lattice averages are the input parameters used in the fits presented in this paper.

In the specific case of decay constants, the $SU(3)$-flavor breaking ratios f_K/f_π, f_{D_s}/f_D, and f_{B_s}/f_{B_d} are better determined than the individual decay constants. We will therefore take these ratios as well as the strange-meson decay constants as reference quantities for our inputs. In the same spirit, it is more relevant to consider the predictions of the ratio $K_{\ell 2}/\pi_{\ell 2}$ of the kaon and pion leptonic partial widths, as well as $\mathcal{B}(\tau \to K\nu_\tau)/\mathcal{B}(\tau \to \pi\nu_\tau)$ instead of the individual branching ratios.

A comment is in order concerning the second category of theoretical uncertainties that are not directly related to LQCD. As far as global fit inputs are concerned, this is the case for the inclusive and exclusive determinations of $|V_{ub}|$ and $|V_{cb}|$, which involve nonperturbative inputs of different nature. We use the latest HFAG results [10] for each of these determinations and combine inclusive and exclusive determinations following the same scheme as for the combination of lattice quantities. We refer the reader to Refs. [19,20] for a more detailed discussion of each constraint, whereas the related hadronic inputs can be found in Ref. [4].

III. NEUTRAL B-MESON LEPTONIC DECAYS

Dileptonic decays of B_d and B_s mesons are among the most appealing laboratories to study scalar couplings in addition to the SM couplings. The current experimental limits set by the Tevatron and LHCb experiments on the dimuonic branching ratios [21,22] are already giving significant constraints on scenarios beyond the standard model. The main limitation in the current predictions arises from the knowledge of the decay constants f_{B_d} and f_{B_s}. The master formula for the branching ratios reads as

$$\mathcal{B}[B_{d,s} \to \ell^+\ell^-]_{\rm SM} = \frac{G_F^2 \alpha_{em}^2 f_{B_{d,s}}^2 m_\ell^2 m_{B_{d,s}} \tau_{B_{s,d}}}{16\pi^3 \sin^4\theta_W^{\rm eff}} \times \sqrt{1 - \frac{4m_\ell^2}{m_{B_{d,s}}^2}} |V_{tb}^* V_{t(d,s)}|^2 Y^2\left(\frac{\bar{m}_t^2}{m_W^2}\right), \quad (4)$$

where Y is the next-to-leading order (NLO) Inami-Lim function [23,24]. $\bar{m}_t$ is the top quark mass defined in the $\overline{\rm MS}$ scheme, related to the pole mass $m_t^{\rm pole}$ determined at the Tevatron as $\bar{m}_t(\bar{m}_t) = 0.957 m_t^{\rm pole}$ at next-to-leading order of QCD. G_F is the Fermi constant, $\sin^2\theta_W^{\rm eff}$ the electroweak mixing angle, and α_{em} the electromagnetic coupling constant at the Z pole. We vary the renormalization scale between $m_t/2$ and $2m_t$.

IV. CHARGED MESON LEPTONIC DECAYS

The decay of a charged meson M ($= \pi, K, D, D_s, B$) into a leptonic pair $\ell\nu_\ell$ is mediated in the SM by a charged weak boson, with the branching ratio:

$$\mathcal{B}[M \to \ell\nu_\ell]_{\rm SM} = \frac{G_F^2 m_M m_\ell^2}{8\pi}\left(1 - \frac{m_\ell^2}{m_M^2}\right)^2 |V_{q_u q_d}|^2 f_M^2 \tau_M (1 + \delta_{em}^{M\ell 2}), \quad (5)$$

where q_u (q_d) stands for the uplike (downlike) valence quark of the meson, respectively, $V_{q_u q_d}$ is the relevant CKM matrix element, f_M is the decay constant of the meson M, and τ_M its lifetime. A similar formula holds for τ decays into a single light meson (pion or kaon). The corrective factor $\delta_{EM}^{M\ell 2}$ stands for channel-dependent electromagnetic radiative corrections. They are taken into account in the case of the lighter mesons (π and K), where their impact is estimated to be at the level of 2%–3% [25], and for the D mesons, where the effect is 1% [26]. In the case of B mesons, no dedicated corrections are supposed here, and we assume that all the corrections from soft photons will be taken into account through the Monte Carlo analyses of the experiments (see Ref. [27] for a discussion of the corrections due to soft-photon emission).

V. RADIATIVE B-MESON DECAYS

The radiative transitions $b \to s(d)\gamma^{(*)}$ provide very interesting probes of New Physics, as they are mediated by penguin transitions which are directly related in the SM with B_d and B_s mixing (from the CKM point of view), but can be affected differently by additional particles/couplings. A convenient framework for their analysis is provided by the effective Hamiltonian where all degrees of freedom heavier than the b quark have been integrated out, leading to Wilson coefficients C_i (encoding short-distance physics) multiplied by operators with light degrees of freedom (describing long-distance physics).

Hadronic uncertainties may be significant for the exclusive decays $\mathcal{B}(B_d \to K^*(892)\gamma)$ and $\mathcal{B}(B_s \to \phi\gamma)$ due to the form factors and the long-distance gluon exchanges:

$$\mathcal{B}[\bar{B} \to V\gamma]_{\rm SM} = \frac{\tau_B}{c_V^2} \frac{G_F^2 \alpha_{em} m_B^3 m_b^2}{32\pi^4}\left(1 - \frac{m_V^2}{m_B^2}\right)^3 \times [T_1^{B\to V}(0)]^2 \sum_{X=L,R} \left| \sum_{U=u,c} V_{Us}^* V_{Ub} a_{7X}^U \right|^2, \quad (6)$$

where c_V is a Clebsch-Gordan coefficient, α_{em} the electromagnetic coupling constant at vanishing momentum, the index $X = L, R$ corresponds to the photon polarization, and $a_{7R}^{c,u}$ is m_s/m_b suppressed compared to $a_{7L}^{c,u} = C_7 + O(\alpha_s, 1/m_b)$, where C_7 is the Wilson coefficient of the electromagnetic dipole operator (corrections can be estimated using a $1/m_b$ expansion). In Eq. (9), U denotes any up-type quark. We follow Ref. [28] for the prediction of the branching ratio and the analysis of hadronic uncertainties [however, we use results from light-cone QCD sum rules and do not perform any rescaling of the tensor form factor $T_1^{B\to V}(0)$].

The inclusive transition $b \to s\gamma$ can be treated relying on the quark-hadron duality and using a heavy-quark expansion, so that the prediction for this transition suffers from fewer theoretical uncertainties (mostly related to the precise values of the quark masses and the higher orders in both α_s and $1/m_b$ expansions). However, this observable is not fully inclusive as a cut in the photon energy is required. The corresponding expression is [29]

$$\mathcal{B}[\bar{B} \to X_s \gamma]_{\mathrm{SM},E_\gamma > E_0} = \mathcal{B}[\bar{B} \to X_c e \bar{\nu}] \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{em}}{\pi C} [P(E_0) + N(E_0)], \tag{7}$$

where C is a factor related to the choice of $b \to c$ transition as a normalization for the computation, $N(E_0)$ collects the estimate from nonperturbative $1/m_b$-suppressed contributions, and $P(E_0)$ has been estimated up to next-to-next-to-leading order (NNLO) using an interpolation on the charm quark mass, leading to a formula of the form $P(E_0) = \sum_{i,j} C_i C_j K_{ij}(E_0)$, where the K_{ij} are perturbative kernels. For the present analysis, we use the parametrization described in detail in Ref. [20].

The (exclusive and inclusive) radiative decays $b \to s\ell^+\ell^-$ provide more observables, which are already experimentally accessible, but they are out of the scope of this short note [12].

VI. *CP*-VIOLATING *B*-MIXING OBSERVABLES

The mixing of the B_d and B_s mesons can be described upon introducing the mass and decay matrices, $M^q = M^{q\dagger}$ and $\Gamma^q = \Gamma^{q\dagger}$. These matrices are involved in the evolution operator for the quantum-mechanical description of the $B_q - \bar{B}_q$ oscillations (with $q = d$ or $q = s$). Their diagonalization defines the physical eigenstates $|B_H^q\rangle$ and $|B_L^q\rangle$ with masses M_H^q, M_L^q and decay rates Γ_H^q, Γ_L^q. One can reexpress these quantities in terms of three parameters: $|M_{12}^q|$, $|\Gamma_{12}^q|$, and the relative phase $\phi_q = \arg(-M_{12}^q/\Gamma_{12}^q)$.

Oscillation frequencies, which feature the CKM elements directly, can be predicted in a theoretically clean way, though the precision is severely limited by the knowledge of the decay constants and bag parameters [19]. Here we would like to concentrate on the prediction of the four *CP*-violating observables β_q (mixing phases) and a_{SL}^q (semileptonic asymmetries), with $q = d$ or $q = s$. The two first observables are CKM angles of the B_d and B_s unitarity triangles, respectively, and read as functions of the CKM elements:

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \tag{8}$$

$$\beta_s = -\arg\left(-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*}\right). \tag{9}$$

These angles (which should not be confused with the relative phases ϕ_q introduced above) measure *CP* violation arising in the interference between mixing and decay in $b \to c\bar{c}s$ and hence exhibit a strong hierarchy between the d and s quarks. On the contrary, the semileptonic asymmetry probes *CP* violation in the mixing, and can be written as

$$a_{\mathrm{SL}}^q = 2\left(1 - \left|\frac{q}{p}\right|\right) = \mathrm{Im}\frac{\Gamma_{12}^q}{M_{12}^q} = \frac{|\Gamma_{12}^q|}{|M_{12}^q|}\sin\phi_q = \frac{\Delta\Gamma_q}{\Delta M_q}\tan\phi_q. \tag{10}$$

The ingredients needed to predict these asymmetries are hence the matrix elements M_{12} and Γ_{12}. The dispersive term M_{12}^q is mainly driven by box diagrams involving virtual top quarks, and it is related to the effective $|\Delta B| = 2$ Hamiltonian $H_q^{|\Delta B|=2}$ as

$$M_{12}^q = \frac{\langle B_q | H_q^{|\Delta B|=2} | \bar{B}_q \rangle}{2M_{B_q}}. \tag{11}$$

The SM expression for $H_q^{|\Delta B|=2}$ is [23]

$$H_q^{|\Delta B|=2} = (V_{tq}^* V_{tb})^2 C Q + \mathrm{H.c.} \tag{12}$$

with the four-quark operator $Q = \bar{q}_L \gamma_\mu b_L \bar{q}_L \gamma^\mu b_L$ and the Wilson coefficient C:

$$C^{\mathrm{SM}} = \frac{G_F^2}{4\pi^2} M_W^2 \hat{\eta}_B S\left(\frac{\bar{m}_t^2}{M_W^2}\right) \tag{13}$$

and the Inami-Lim function S is calculated from the box diagram with two internal top quarks. The absorptive term Γ_{12}^q is dominated by on-shell charmed intermediate states, and it can be expressed as a two-point correlator of the $|\Delta B| = 1$ Hamiltonian $H_q^{|\Delta B|=1}$. By performing a $1/m_b$ expansion of this two-point correlator, one can express Γ_{12}^q in terms of Q and $\tilde{Q}_S = \bar{q}_L^\alpha b_R^\beta \bar{q}_L^\beta b_R^\alpha$, where S stands for "scalar" and α, $\beta = 1, 2, 3$ are color indices [19]. The matrix elements are expressed in terms of the bag parameters:

$$\langle B_q | Q | \bar{B}_q \rangle = \tfrac{2}{3} M_{B_q}^2 f_{B_q}^2 \tilde{\mathcal{B}}_{B_q}, \tag{14}$$

$$\langle B_q | \tilde{Q}_S | \bar{B}_q \rangle = \frac{1}{12} M_{B_q}^2 f_{B_q}^2 \tilde{\mathcal{B}}'_{S,B_q}. \tag{15}$$

One has also to consider further bag parameters $\mathcal{B}_{R,B_q}$ which parameterize the matrix elements of the subleading operators in the heavy quark expansion of the *CP*-violating observables (only rough estimates are available for these bag parameters) [19]. The SM predictions of the mixing phases and semileptonic asymmetries for the neutral mesons in the SM are collected in Table II.

VII. THE RARE KAON DECAYS $K^+ \to \pi^+ \nu\bar{\nu}$ AND $K_L \to \pi^0 \nu\bar{\nu}$

Theoretically clean constraints on the CKM matrix can be obtained from rare kaon decays with neutrinos in the final state, as they can only arise via second-order weak transitions (Z penguins and box) within the SM, and light-quark loops are strongly GIM suppressed. Within the SM, the $K^+ \to \pi^+ \nu\bar{\nu}$ decay rate is given by [30–33]

$$\mathcal{B}[K^+ \to \pi^+ \nu\bar{\nu}]_{\mathrm{SM}} = \kappa_+(1 + \Delta_{em})\left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X_t\right)^2 + \left(\frac{\mathrm{Re}\lambda_c}{\lambda}(P_c + \delta P_{c,u}) + \frac{\mathrm{Re}\lambda_t}{\lambda^5} X_t\right)^2\right], \tag{16}$$

where the isospin-breaking parameter κ_+ can be extracted from semileptonic K decays with a correction Δ_{em} for the photon cutoff dependence, the X_t functions comprise the top quark contributions, and the light quark contributions are given by the P_c and $\delta P_{c,u}$ parameters, which are the dominant theoretical uncertainties. Similarly for the $K_L \to \pi^0 \nu\bar{\nu}$ mode, the SM decay rate is given by [30]

$$\mathcal{B}[K_L \to \pi^0 \nu\bar{\nu}]_{\mathrm{SM}} = \kappa_L\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X_t\right)^2, \tag{17}$$

with only small residual uncertainties from the isospin-breaking parameter κ_L and scale invariance. In terms of CKM parameters, a measurement of the $K^+ \to \pi^+ \nu\bar{\nu}$ provides a quasielliptical constraint in the $(\bar{\rho}, \bar{\eta})$ plane, centered close to the vertex of the unitarity triangle located at (1, 0). The measurement of the branching ratio for $K_L \to \pi^0 \nu\bar{\nu}$ would provide a clean constraint on $\bar{\eta}^2$.

VIII. DISCUSSION

This paper collects a selection of SM predictions driven by the global fit of the CKM parameters, in view of related recent or foreseeable experimental measurements. The main outcome is summarized in Table II, gathering the SM predictions using the inputs collected in Table I. The third column of Table II shows the agreement between the measurement and the prediction as a pull. The latter is computed from the χ^2 difference with and without the measurement of this observable, interpreted with the appropriate number of degrees of freedom, and converted in the number of equivalent standard deviations (the lack of an updated average for β_s between the Tevatron experiments explains the presence of two distinct measurements as well as the absence of a pull).

The largest departures of the measurements from the SM predictions are found for two observables: $\mathcal{B}(B^+ \to \tau^+ \nu_\tau)$ and $\sin(2\beta)$. It is remarkable that this discrepancy can be accommodated by a very simple extension of the SM allowing for the presence of New Physics in B mixing, as discussed extensively in Ref. [19]. One can also notice that the $D_s \to \mu\nu$ decay exhibits only a mild discrepancy between prediction and measurement, due to the recent improvements in both lattice simulations and experimental measurements.

Concerning neutral-meson mixing, and following the outstanding success of the B factories in their measurements of $\sin(2\beta)$, one of the main goals and challenges for the LHCb experiment will consist in characterizing the B-meson mixing properties through the measurement of all the relevant observables. Each of these measurements in LHCb but $\sin(2\beta)$ will provide a null test of the SM hypothesis. In the present experimental context, two of these observables are particularly interesting: the $B_s^0\bar{B}_s^0$ mixing weak phase β_s and the difference of the semileptonic asymmetries for the B_d and B_s mesons. The former is predicted very accurately:

$$2\beta_s = 0.0363^{+0.0016}_{-0.0015}. \tag{18}$$

Significant constraints have already been set on this phase by the Tevatron experiments [14,15]. The LHCb experiment should in the near future settle its value, as suggested by the promising exploratory work with the first data described in Ref. [34].

The semileptonic asymmetries are determined far less precisely by the global fit of the CKM parameters. Their prediction suffers from notable strong-interaction uncertainties (in particular bag parameters). Yet, following a recent D0 measurement of the dimuon asymmetry which departs from the SM by 3.2σ [35], the measurement by the LHCb experiment of the difference of the semileptonic asymmetries $a^s_{\mathrm{SL}} - a^d_{\mathrm{SL}}$ is eagerly awaited. The prediction of the difference in the SM is

$$a^s_{\mathrm{SL}} - a^d_{\mathrm{SL}} = (6.8^{+1.9}_{-1.7}) \times 10^{-4}. \tag{19}$$

Among the null tests of the SM hypothesis, the Z-penguin decay rate $\mathcal{B}(B_s^0 \to \mu^+\mu^-)$ is specially appealing. Its NLO prediction from the global fit reads

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) = (3.64^{+0.17}_{-0.31}) \times 10^{-9}. \tag{20}$$

We would like to conclude this discussion with observables which can uniquely be measured at super-B factories. The important role of $\mathcal{B}(B^+ \to \tau^+ \nu_\tau)$ onto the global fit has been already underlined in this paper, and its SM prediction is

$$\mathcal{B}(B^+ \to \tau^+ \nu_\tau) = (7.57^{+0.98}_{-0.61}) \times 10^{-5}. \tag{21}$$

An improved precision of the measurement can only be achieved at high-luminosity B factories. The branching ratio of the muonic mode, predicted to be

$$\mathcal{B}(B^+ \to \mu^+ \nu_\mu) = (3.74^{+0.44}_{-0.38}) \times 10^{-7}, \quad (22)$$

is a further experimental target.

Let us finally add that this short paper has collected the SM predictions for some salient observables in flavor physics, in view of the running or foreseen experimental programs here. This obviously does not exhaust the discussion of the inputs, predictions, and methods dealt with the CKMfitter package, but we leave this subject for a more extensive forthcoming publication [12].

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