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Latent heat of single flavor color superconductivity in a magnetic field

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We calculate the energy release associated with first-order phase transition between different types of single-flavor color superconductivity in a magnetic field.

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At sufficiently high baryon density, a hadronic matter will be squeezed into a degenerate system of quarks, which becomes a color superconductor (CSC) at sufficiently low temperature [1].

The most plausible place in the Universe where the color superconductivity may be implemented is the interior of compact stars. They have masses about $0.5M_{\odot}$–$2.7M_{\odot}$ and are believed to have radii of order 10 km. Near the surface their density is around normal nuclear density and raises to several times higher in the core region. There a quark matter of chemical potential $\mu \sim 400$–500 MeV may form.

The color superconductivity can influence the observable signature of a compact star in several ways. First, it will impact on the quark matter equation of state. Second, it affects the energy gap, as well as the Goldstone and/or Higgs modes associated with the long range interaction affects the transport properties of the star. Third, the presence of strong magnetic field renders the superconducting phase transition first-order. The latent heat release at the transition may lead to observable energy bursts which will be discussed in this report.

While the color-flavor-locked (CFL) phase, which pairs different quark flavors, is the ground state at ultra high baryon density (chiral limit), the situation is far more complicated at the moderate density in a compact star. The mass of s quarks and the electrical neutrality constraint create a substantial mismatch of the Fermi momenta among different flavors which in turn reduces the available phase space of Cooper pairing. The single-flavor CSC (pairing within each flavor), however, is free from such a limitation. The smallness of the single-flavor gaps because of the nodal lines of the pairing force, makes the single-flavor phases relevant for late age stars, when the temperature is sufficiently low. Among the four canonical phases, the spherical color-spin-lock (CSL) and nonspherical planar, polar and A [2–4], CSL is the most favored one in the absence of a magnetic field.

The presence of a magnetic field offsets the energy balance among the four canonical single-flavor pairings. Only the spherical CSL phase has electromagnetic Meissner effect [2] and nonspherical phases: polar, A and planar phases only shield part of the magnetic field. So if a quark matter cools down through the critical temperature of the single-flavor paring in a magnetic filed, forming CSL state will cost extra work to exclude magnetic fluxes from the bulk. Therefore, the magnetic contribution to the free energy may favor the nonspherical states. In a previous work [5], we have explored the consequences of the absence of the electromagnetic Meissner effect in a nonspherical CSC phase of single-flavor pairing and have obtained the phase diagram with respect to the temperature and the magnetic field. Computing the latent heat released across the phase boundaries is the main subject of the present work. We shall employ the same approximation in [5] by ignoring the masses of pairing quarks.

We work with a effective action like a Nambu-Jona-Lasinio whose Lagrangian density reads [6]:

$$\mathcal{L} = \bar{\psi}(-\gamma_\nu\partial_\nu + \mu\gamma_4)\psi + G\bar{\psi}\gamma_\nu T^l\psi\bar{\psi}\gamma_\nu T^l\psi \quad (1)$$

with $T^l = \frac{1}{2}\lambda^l$, $G > 0$ an effective coupling and introduce the condensate:

$$\Phi = \langle\bar{\psi}_C\Gamma^c\lambda^c\psi\rangle \quad (2)$$

where ψ is the quark field, $\psi_C = \gamma_2\psi^*$ is its charge conjugate, λ^c with $c = 2, 5, 7$ are antisymmetric Gell-Mann matrices and Γ^c is a 4×4 spinor matrix. We may choose $\Gamma^5 = \Gamma^7 = 0$ for the polar and A phases, $\Gamma^2 = 0$ for the planar phase but none of Γ^c's vanishes for the CSL phase. We find the pressure of each flavor under mean field approximation:

$$\begin{aligned} P = {}& -\frac{2}{\Omega}\sum_{\mathbf{k}}(k-\mu-E_{\mathbf{k}}) - \frac{1}{\Omega}\sum_{\mathbf{k}}(k-\mu-|k-\mu|) \\ & + \frac{2T}{\Omega}\sum_{\mathbf{k}}\ln\left(1+\exp\left(-\frac{|k-\mu|}{T}\right)\right) - \frac{9}{4G}\Delta^2 \\ & + \frac{4T}{\Omega}\sum_{\mathbf{k}}\ln\left(1+\exp\left(-\frac{E_{\mathbf{k}}}{T}\right)\right), \end{aligned} \quad (3)$$

where $E_{\mathbf{k}} = \sqrt{(k-\mu)^2 + \Delta^2 f^2(\theta)}$ with θ the angle between the momentum $\mathbf{k}$ and a prefixed spatial direction and Δ the gap parameter. The function $f(\theta)$ is given by

$$f(\theta) = \begin{cases} 1, & \text{for CSL phase} \\ \sqrt{\frac{3}{4}(1+\cos^2\theta)}, & \text{for planar phase} \\ \sqrt{\frac{3}{2}}\sin\theta, & \text{for polar phase} \\ \sqrt{3}\cos^2\frac{\theta}{2}. & \text{for A phase} \end{cases} \quad (4)$$

Maximizing the pressure with respect to Δ, we obtain the gap equation $(\frac{\partial P}{\partial \Delta^2})_\mu = 0$, which determines the temperature dependence of the gap, $\Delta(T)$. Substituting $\Delta(T)$ back to (3), we find that $P_n < P_A < P_{\rm polar} < P_{\rm planar} < P_{\rm CSL}$ up to the transition temperature T_c.

The diquark condensate (2) for CSL breaks the gauge symmetry $SU(3)_c \times U(1)_{\rm em}\chi$ completely. A nonspherical condensate, however, breaks the gauge symmetry partially and the Meissner effect is incomplete. Among the residual gauge group, which is the gauge transformation that leaves the diquark operator inside (2) invariant, there exists a U(1) transformation, $\psi \to e^{-(i/2)\lambda_8\theta - iq\phi}\psi$ with q the electric charge of ψ, $\theta = -2\sqrt{3}q\phi$ for the polar and A phases and $\theta = 4\sqrt{3}q\phi$ for the planar phase. The corresponding gauge field, $\mathcal{A}_\mu$ is identified with the electromagnetic field in the condensate and is related to the electromagnetic field A and the 8th component of the color field A^8 in the normal phase through a U(1) rotation

$$\begin{aligned} \mathcal{A}_\mu &= A_\mu \cos\gamma - A_\mu^8 \sin\gamma \\ \mathcal{V}_\mu &= A_\mu \sin\gamma + A_\mu^8 \cos\gamma \end{aligned} \quad (5)$$

where the mixing angle γ is given by $\tan\gamma_{{\rm polar},A} = 2\sqrt{3}q(e/g)$ and $\tan\gamma_{\rm planar} = 4\sqrt{3}q(e/g)$ with g the QCD running coupling constant. The second component of (5) $\mathcal{V} = 0$ because of the Meissner effect and thereby imposes a constraint inside a nonspherical CSC, $A_\mu^8 = -A_\mu \tan\gamma$, which implies that

$$\mathbf{B}^8 = -\mathbf{B}\tan\gamma \quad (6)$$

The thermal equilibrium in a magnetic field $H\hat{\mathbf{z}}$ is determined by minimizing the Gibbs free energy density,

$$\mathcal{G} = -P + \frac{1}{2}B^2 + \frac{1}{2}\sum_{l=1}^{8}(B^l)^2 - BH \quad (7)$$

with respect to Δ, B and B^l. Ignoring the induced magnetization of quarks, the pressure P is given by (3), with Δ given by the solution of the gap equation. For a nonspherical CSC pairing, the minimization with respect to B and B^l is subject to the constraint (6). For a hypothetical quark matter of one flavor only, we find that

$$\mathcal{G}_{\rm min.,j} = -P_j - \frac{1}{2}\eta_j H^2 \quad (8)$$

with $j = n$, CSL, polar, A and planar, where $\eta_n = 1$, $\eta_{\rm CSL} = 0$ and $\eta_j = \cos^2\gamma_j$ for a nonspherical CSC. The phase corresponding to minimum among $\mathcal{G}_{\rm min}$'s above wins the competition and transition from one phase to another is first-order below T_c.

For a quark matter of different flavors with Cooper pairing within each flavor, the Gibbs free energy density can be written as

$$\mathcal{G} = -P - \frac{1}{2}\eta H^2 \quad (9)$$

where P is the total pressure of all flavors. We have $\eta = 1$ if all flavors are normal and $\eta = 0$ if all flavors are CSL. If one flavor is in a nonspherical state and others are normal, $\eta = \cos^2\gamma$ with γ the mixing angle of the nonspherical phase. If more than one flavor is in nonspherical phases, $\eta = \cos^2\gamma$ with γ their common mixing angle. If their mixing angle were different, we would end up with $B = B^8 = 0$, in order to compromise the constraints (6) of all nonspherical states involved, making them less favored than CSL. It was shown in [5] that only four phases, I–IV (shown in Table I and Fig. 1), need to be considered in both two and three flavor quark matters. There is a first-order phase transition from one of them to another for $0 < T < T_c$.

The borders between two phases are determined by the equation

$$P_\alpha + \eta_\alpha \frac{H^2}{2} = P_\beta + \eta_\beta \frac{H^2}{2} \quad (10)$$

with the subscripts α and β labeling the four phases I–IV and the density of the latent heat released from the phase α to the phase β reads

$$Q_{\alpha\beta} = T[S_\alpha(T) - S_\beta(T)]. \quad (11)$$

The entropy density $S(T)$ is given by

$$S = \left(\frac{\partial P}{\partial T}\right)_\mu = \left(\frac{\partial P}{\partial T}\right)_{\Delta,\mu} + \left(\frac{\partial P}{\partial \Delta}\right)_{T,\mu}\left(\frac{\partial \Delta}{\partial T}\right)_\mu = \left(\frac{\partial P}{\partial T}\right)_{\Delta,\mu}$$

where the gap equation is employed in the last step.

The temperature dependence of the thermodynamic quantities, P and S can be written in a parametric form. In terms of the parameter $t = \frac{\Delta(T)}{T}$, the gap equation takes the form $\ln\frac{\Delta(0)}{\Delta(T)} = h(t)$ with

TABLE I. The possible phases involving spin-one CSC of a quark matter with two flavors or three flavors in a magnetic field.

	I	II	III	IV
2 flavor	CSL_u, CSL_d	$(\text{polar})_u$, $(\text{planar})_d$	$(\text{normal})_u$, $(\text{polar})_d$	$(\text{normal})_u$, $(\text{normal})_d$
3 flavor	CSL_u, $\text{CSL}_{d,s}$	$(\text{polar})_u$, $(\text{planar})_{d,s}$	$(\text{normal})_u$, $(\text{polar})_{d,s}$	$(\text{normal})_u$, $(\text{normal})_{d,s}$

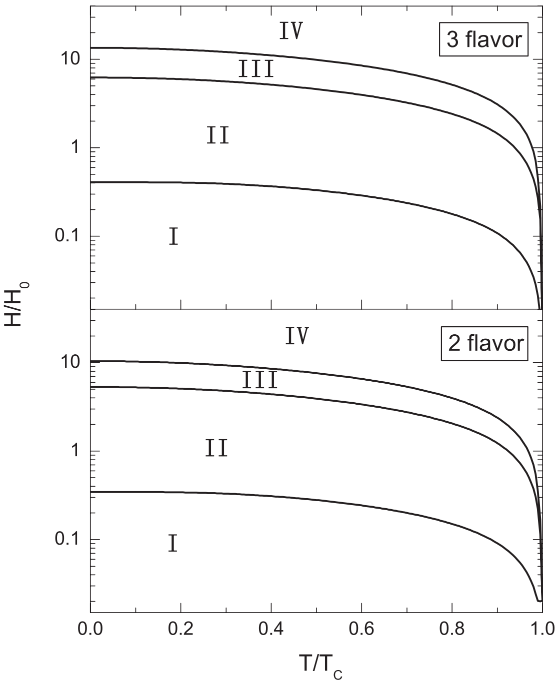

FIG. 1. The H-T phase diagram for two flavors and three flavors, where the reference magnetic field $H_0 = \mu\Delta_0/\pi$ with Δ_0 the CSL gap at $T = 0$.

$$h(t) = \int_0^{\pi} d\theta \sin\theta f^2(\theta) \times \int_0^{\infty} dx \frac{1}{\sqrt{x^2 + t^2 f^2(\theta)}[e^{\sqrt{x^2+t^2f^2(\theta)}} + 1]}. \quad (12)$$

It follows that

$$T = \frac{\Delta(0)}{t} e^{-h(t)}. \quad (13)$$

Introducing $P_s - P_n \equiv \rho(t)\frac{\mu^2\Delta_0^2}{2\pi^2}$ and $S_s - S_n = \frac{\mu^2\Delta_0^2}{2\pi^2} \times (\frac{d\rho}{dt} / \frac{dT}{dt})$ with s labeling different pairing states and $\Delta_0 \equiv \Delta_{\rm CSL}(0)$, we have

$$\rho(t) = e^{-2h(t)}\left[1 + 2h(t) + 4\frac{g(t)}{t^2} - \frac{2\pi^2}{3t^2}\right] \quad (14)$$

with

$$g(t) = \int_0^{\pi} d\theta \sin\theta \int_0^{\infty} dx \ln[e^{-\sqrt{x^2+t^2f^2(\theta)}} + 1] \quad (15)$$

and the curves $P(T)$ and $S(T)$ may be plotted parametrically without solving the gap equation for $T > 0$, as we did in [5].

Numerically, we identify Δ_0 with that of the one-gluon exchange [3,7] with $\mu = 500$ MeV and $\alpha_s = \frac{g^2}{4\pi} = 1$ as a calibration of the parameters in (1), We find $\Delta_0 \simeq 0.238$ MeV, $H_0 \simeq 5.44 \times 10^{14}$ G for two flavors, and $\Delta_0 \simeq 0.0864$ MeV, $H_0 \simeq 1.97 \times 10^{14}$ G for three flavors. As is shown in Fig. 1, the nonspherical phases occupy a significant portion of the H-T phase diagram for a magnitude of the magnetic field of order 10^{15} G. This magnitude of the magnetic field is plausible in a compact star.

The temperature dependence of the entropy density differences between all single-flavor CSC phases and that of the normal phase under the same chemical potential $\mu = 500$ MeV is shown in Fig. 2. The differences vanish at $T = 0$ since all entropies are zero. They also vanish at $T = T_c$ since the transition is second order there. The entropy density of a CSC phase is lower than that of the normal phase because of the long range order, so the differences are all negative for $0 < T < T_c$. The latent heat densities across the phase boundaries in Fig. 1 are displayed in Fig. 3, where adjustment of the chemical potential of each flavor is made to fulfill the requirement of the charge neutrality. The latent heat gets a peak value of about $0.67T_c$–$0.72T_c$ for all curves.

In the natural unit we are using, the entropy is dimensionless and the unit of latent heat can be MeV, the unit of latent heat density is $\mathrm{MeV}^4 = 2.088 \times 10^{41}$ erg/km^3. The units have been transformed to International System of Units in Figs. 2 and 3. For the biggest latent heat density we calculated in three flavor phase, from IV to III, this value is about 6.04×10^{44} erg/km^3. Then for a compact star with the radii of quark matter equal to R(km), the energy release is about $2.53 \times 10^{45} R^3$erg. Since $R < 10$ km typically, the latent heats released as the star cools through the phase

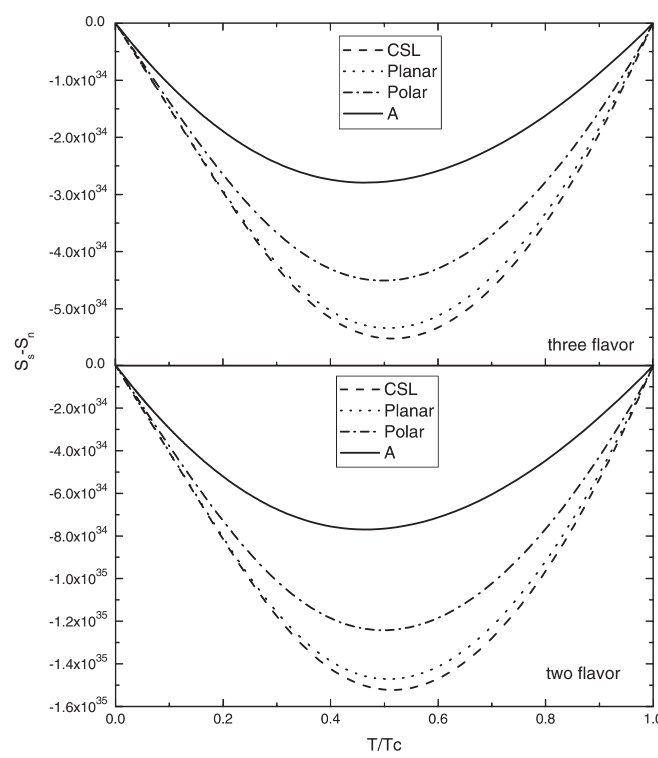

FIG. 2. The entropy density differences between single-flavor CSC phases and normal phase as a function of the temperature. The unit is erg/K/km^3.

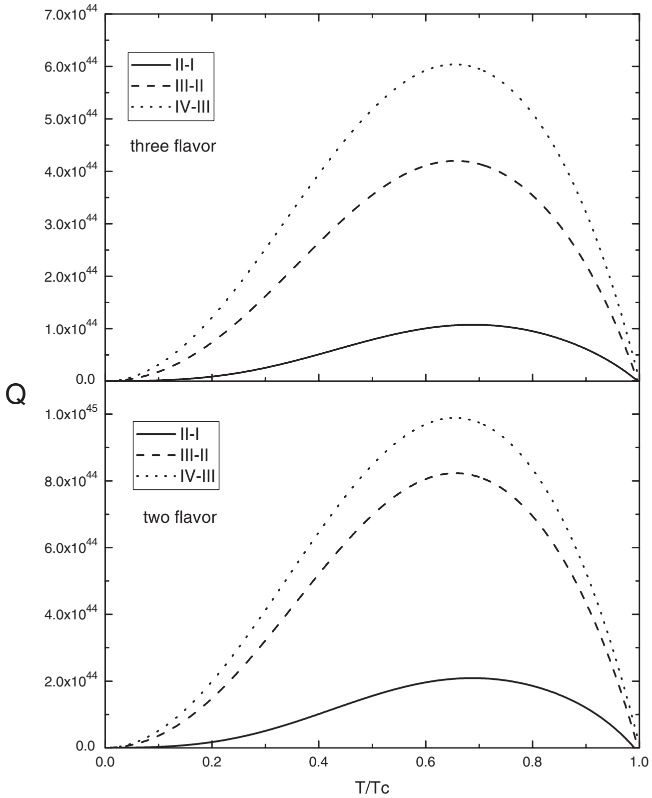

FIG. 3. The latent heat density dependence of temperature. We know that $S_s = S_n$ at T_c and $Q_{\alpha\beta} = T[S_\alpha(T) - S_\beta(T)]$, so latent heat also vanishes at T_0 and T_c. The unit of Q is erg/km^3.

boundaries we showed are smaller than the typical energy release of γ ray burst of order 10^{51} to 10^{54} erg [8,9]. But they may contribute to weaker energy bursts such as the x-ray radiation at the later stage of a compact star [10].

Finally, we would like to comment on the approximations employed. The two flavor case is certainly unrealistic since it assumes a large mass of s quarks, $m_s \gg \mu$. For three flavors, our approximation requires m_s to be sufficiently smaller than μ. On the other hand m_s cannot be too small in order for the single-flavor CSC to compete with multiflavor pairings under the Fermi momentum mismatch, which requires m_s to be of the same order of magnitude as $\sqrt{\mu\Delta(2\mathrm{SC})}$ or greater, with $\Delta(2\mathrm{SC})$ the 2SC gap at zero temperature without mismatch. The phase structure of 2SC in a magnetic field has been studied in [11]. Since $\Delta(2\mathrm{SC})$ is about 5–10 times lower than μ for the quark matter we considered, the approximation, though marginal, may not be too crude as is suggested by our recent analysis with a realistic m_s, 150 MeV, in the single-flavor pairing dynamics [12]. For the highest magnetic field in the phase diagram of Fig. 1, $H \simeq 5 \times 10^{15}$ G, which implies $eH/(\mu^2) \simeq 6 \times 10^{-5}$. Therefore, it is legitimate to ignore the impact of the magnetic field on the pairing dynamics, unlike the situation considered in [13].

Throughout the paper, we have assumed that the whole volume of the quark matter core of a compact star undergoes a phase transition at the same time. In reality the chemical potential, and consequently the gap and the critical temperature, changes with the distance from the center and the transition at the center and that at the edge of the core may not be simultaneous. For a typical compact star, the density at the center is at most two times larger than that on the edge [14] and the quark matter cools very rapidly and reaches isothermal condition in a few hours because of the high thermal conductivity [15]. As a crude estimation, we associate the chemical potential of 500 MeV to the center of the quark matter core and define its edge at the radius where the transition temperature drops by half. Then time scale for the transition at the center to that at the edge is about a few minutes to a few days depending on the direct Urca process [15]. This is much shorter than the typical age of a compact star (10^4–$10^6 y$), justifying our picture of the sudden release of the latent heat. The subject deserves further study to clarify more quantitatively the interplay between the phase transition and the structure and the evolution of a compact star.

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