Holographic (de)confinement transitions in cosmological backgrounds

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For type IIB supergravity with a running axio-dilaton, we construct bulk solutions which admit a cosmological background metric of Friedmann-Robertson-Walker type. These solutions include both a dark radiation term in the bulk as well as a four-dimensional (boundary) cosmological constant, while gravity at the boundary remains nondynamical. We holographically calculate the stress-energy tensor, showing that it consists of two contributions: The first one, generated by the dark radiation term, leads to the thermal fluid of $\mathcal{N} = 4$ SYM theory, while the second, the conformal anomaly, originates from the boundary cosmological constant. Conservation of the boundary stress-tensor implies that the boundary cosmological constant is time-independent, such that there is no exchange between the two stress-tensor contributions. We then study (de)confinement by evaluating the Wilson loop in these backgrounds. While the dark radiation term favors deconfinement, a negative cosmological constant drives the system into a confined phase. When both contributions are present, we find an oscillating universe with negative cosmological constant which undergoes periodic (de)confinement transitions as the scale of three-space expands and recontracts.

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I. INTRODUCTION

Gauge/gravity duality [1] has proved to be extremely successful in describing strongly coupled systems. This applies, in particular, to confining theories which can be modeled, for instance, using nontrivial dilaton flows [2–4]. In these models, the Wilson loop displays an area law. Moreover, gauge/gravity duality has also proved useful in describing deconfined finite temperature field theories which are naturally assumed to be dual to asymptotically Anti-de Sitter (AdS) black holes.

Both confinement and horizon formation also arise in quantum field theories on curved space backgrounds, in Anti-de Sitter and de Sitter geometries, respectively. In the gauge/gravity duality context, this has been investigated, for instance, in [5–12] by considering a boundary cosmological constant λ in the four-dimensional boundary quantum field theory. Holographic studies of strongly coupled quantum field theories in curved backgrounds are, however, not only interesting in their own right (e.g. in order to verify the properties of particle production phenomena such as the Unruh effect at strong coupling), but also from the point of view of Anti-de Sitter/conformal field theories (AdS/CFT) (dualities for time-dependent backgrounds. In particular, standard cosmological evolution in the presence of a cosmological constant can yield de Sitter or Anti-de Sitter geometries. Thus, from studying gauge theories in these backgrounds, we expect to learn about the properties of matter in the early universe (e.g. during inflation). With this situation in mind, in this paper we consider gravity duals of field theories on cosmological backgrounds where in the dual gravitational description a *bulk radiation term* is present in addition to a boundary cosmological constant. This term has first been considered in brane-world models in [13–16]. Because of its schematic form C/a^4, with a being the scale factor, it corresponds to a relativistic radiation contribution to the energy density. We discuss the interplay between this radiation and the boundary cosmological constant in the boundary energy-momentum tensor, as well as their effects on the temporal Wilson loop. We find that the combined effect of the dark radiation term and the boundary cosmological constant introduces an effective dynamics into the dual field theory, triggering (de)confinement transitions for the Wilson loop. For vanishing boundary cosmological constant and flat horizon topology, the dark radiation term gives just the Stefan-Boltzmann contribution $\rho \sim T^4$ to the boundary energy density. In the other cases, the relation to the temperature is more involved due to the time-dependence of the background geometry, as we discuss.

As a further ingredient, we consider a running axio-dilaton similarly to the model of Liu and Tseytlin [4]. The axio-dilaton introduces a finite gluon condensate which on flat space leads to confinement.

Our main results are explicit evaluations for Wilson loops in the field theories dual to the gravity solution with dark radiation term for the three cases of positive, vanishing, and negative boundary cosmological constant. The general intuition arising from the static quark-antiquark potential is that the dark radiation term always

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drives the system into a deconfined phase (with the Wilson loop displaying a perimeter law), since it acts similarly to a temperature in flat space. We find an interesting pattern for the (de)confining behavior of the Wilson loop, depending on the sign of the boundary cosmological constant:

(1) For positive cosmological constant, the theory is always in a deconfined state, even for vanishing dark radiation term, which is in accordance with the general expectation that de Sitter-like expansion tends to destabilize bound states.

(2) For vanishing cosmological constant, our bulk metrics are diffeomorphism equivalent to topological AdS black holes [17,18]. In this case, the running axio-dilaton as in [4] is crucial for determining the confinement properties. The Wilson loop shows confinement if the dark radiation constant is vanishing and deconfinement otherwise, both for flat and hyperbolic horizons. As discussed further below and in Sec. V, the nontrivial dilaton flow is essential for the Wilson loop confinement in this case, as in the absence of the running dilaton the quark-antiquark potential would be screened for a gravity dual with hyperbolic topological black hole [19] at all temperatures.

(3) For negative cosmological constant, we find an interesting (de)confinement transition which occurs due to the competition between the deconfining dark radiation term and the confining nature of Antide Sitter-like contraction: For small scale factors, the Wilson loop is deconfined, while for large scale factors it is confined. Intuitively, this can be thought of as the Wilson loop probing the (holographically defined) field theory vacuum whose energy-momentum VEV now has two components—the conformal anomaly component due to the boundary cosmological constant, and the dark radiation component which, in essence, behaves like thermal relativistic radiation, getting diluted by the scale factor as $a^{-4}(t)$.[1] Hence, close to the singularity the dark radiation component dominates, driving the system to deconfinement, while away from it the confining nature of the negative cosmological constant dominates.

In this work we mainly focus, for simplicity, on the Wilson loop as a measure of quark-antiquark (de)confinement. On curved space-times, other measures of confinement such as the density of states or the mass gap criterion do not necessarily coincide with the Wilson loop criterion: For example, in [22,23] it was argued that the Wilson loop confines on AdS spaces at any temperature, while [12] showed that Neumann boundary conditions on the boundary of AdS space allow for a large N deconfinement transition at finite temperature, with the deconfined phase being characterized by a $O(N^2)$ density of states at low energies. We thus leave a thorough investigation of the subtleties involved in relating these different criteria for confinement for future work, and rather focus on the Wilson loop as a criterion to characterize our holographic backgrounds. The reader should also consult Sec. V for a more in-depth discussion of the case of vanishing cosmological constant and hyperbolic horizon: In this case, the Wilson loop and the density of states measure indeed do not agree, due to the presence of the gluon condensate (i.e. the running dilaton) and since the thermodynamical contributions of the axio-dilaton cancel each other in the Liu-Tseytlin ansatz.

On the technical side, we decouple the axio-dilaton dynamics from the five-dimensional metric in the same way as in [4]. We then solve the Einstein equations of five-dimensional Einstein-Hilbert gravity with a cosmological Ansatz for the metric already used in [15] in the context of brane-world cosmology. The Friedmann equation arises from the constraint equation of the bulk Einstein equations, and we include the dark radiation term into our analysis. By imposing the usual Dirichlet boundary conditions of holography, the four-dimensional boundary gravity remains nondynamical, as there is no four-dimensional Einstein-Hilbert action on the boundary. Besides the dark radiation term, we also allow for a boundary cosmological constant in our Friedmann equation. We find that requiring boundary diffeomorphism invariance leads to a time-independent boundary cosmological constant.

The remainder of this paper is structured as follows: In the next section, the holographic background is given and we discuss how the boundary cosmological constant arises. In Sec. III, the holographic interpretation of the dark radiation term in our approach is illuminated using the specific example of vanishing boundary cosmological constant. Section IV discusses the solution for finite boundary cosmological constant, and how boundary diffeomorphism invariance (i.e. conservation of the boundary stress-energy tensor) forces the cosmological constant to be actually time-independent. Section V then derives the main result of this paper: The Wilson loop expectation values are calculated and their (de)confinement properties are classified. Summary and discussions are given in the final Sec. VI.

II. THE BACKGROUND GEOMETRY

In this section, we first review the reduction of ten-dimensional type IIB supergravity to a five-dimensional dilaton gravity by a Freund-Rubin ansatz, which allows for

[1]This interpretation of the dark radiation constant has also been given in the gauge/gravity context in [17,18,20]. Furthermore, the works [17,18] write the AdS-Schwarzschild black hole (without a running dilaton) in a cosmological foliation, which is possible in our construction as well. However, our setup is more involved due to the presence of the boundary cosmological constant and the running dilaton (see Sec. V). Finally, a related study [21] for $k = 0$ and without dark radiation found an interplay between boundary cosmological constant and the tension of an IR brane sourcing the geometry.

a nontrivial axio-dilaton. This ansatz, first employed in [3,4,24], links the axion with the dilaton in a way which allows to describe 1/4 supersymmetric D3-D(-1) solutions. Supersymmetry can then be broken by introducing finite temperature. In IIb, we then solve the five-dimensional Einstein equations with a time-dependent ansatz for the metric along the lines of [15] and find holographic backgrounds describing a cosmological evolution at the boundary. In this course, we identify the boundary cosmological constant, driving the cosmological evolution of the boundary metric, when solving the constraint equations in the bulk.

A. Five-dimensional dilaton gravity from IIB Supergravity

We start from the ten-dimensional type IIB supergravity retaining the dilaton Φ, axion χ, and self-dual five form field strength $F_{(5)}$,

$$S=\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left(R-\frac{1}{2}(\partial\Phi)^2+\frac{1}{2}e^{2\Phi}(\partial\chi)^2-\frac{1}{4\times 5!}F^2_{(5)}\right), \tag{1}$$

where other fields are consistently set to zero, and χ is Wick rotated [24]. Under the Freund-Rubin ansatz for $F_{(5)}$, $F_{\mu_1\cdots\mu_5}=-\sqrt{\Lambda}/2\epsilon_{\mu_1\cdots\mu_5}$ [3,4], and for the $10d$ metric taken as $M_5\times S^5$,

$$ds^2_{10}=g_{MN}dx^Mdx^N+g_{ab}dx^adx^b,$$
$$M,N=0,\ldots,4,\qquad a,b=5,\ldots,9,$$

the equations of motion of the noncompact five-dimensional part M_5 become[2]

$$R_{MN}=\frac{1}{2}(\partial_M\Phi\partial_N\Phi-e^{2\Phi}\partial_M\chi\partial_N\chi)-\Lambda g_{MN} \tag{3}$$

$$\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}g^{MN}\partial_N\Phi)=-e^{2\Phi}g^{MN}\partial_M\chi\partial_N\chi, \tag{4}$$

$$\partial_M(\sqrt{-g}e^{2\Phi}g^{MN}\partial_N\chi)=0 \tag{5}$$

These equations have a supersymmetric solution when the following ansatz is imposed for the axion χ [3,24]:

$$\chi=-e^{-\Phi}+\chi_0. \tag{6}$$

In this case, using the ansatz (6) in (3)–(5) gives rise to the two equations:

$$R_{MN}=-\Lambda g_{MN} \tag{7}$$

and

$$\partial_M(\sqrt{-g}g^{MN}\partial_N e^{\Phi})=0, \tag{8}$$

where (4) and (5) now may be shown to coincide using (8). The latter set of equations is also useful for finding finite temperature solutions in which supersymmetry is broken.

B. Solution with dark radiation

We examine here time-dependent solutions which include a "dark radiation" term [15,16] (also known as "mirage energy density" [20]). To find this term, we change the radial coordinate r to y, where $r/R=\mu r=e^{\mu y}$ and $\mu=1/R=\sqrt{\Lambda}/2$, and we consider the following Einstein frame metric,

$$ds^2_{\mathrm{E}}=-n^2(t,y)dt^2+a(t,y)^2\gamma_{i,j}dx^idx^j+dy^2,\qquad i,j=1,\ldots,3. \tag{9}$$

In this metric, we obtain from the Einstein equation for the tt and yy components [15]

$$\left(\frac{\dot{a}}{na}\right)^2+\frac{k}{a^2}=-\frac{\Lambda}{4}+\left(\frac{a'}{a}\right)^2+\frac{C}{a^4}, \tag{10}$$

where $\dot{a}=\partial a/\partial t$ and $a'=\partial a/\partial y$. Note that this is a first-order equation, integrated from the second-order Einstein equations (see [15] for their explicit form). It then turns out that without any additional matter in the bulk, the integration constant C must be a constant with respect to both y and t in order to satisfy both the tt and yy components of Einstein's equations. This constant C appears in the Eq. (10) in the form $\frac{C}{a^4}$, which is usually referred to as "dark radiation" term, since it behaves exactly as a component of relativistic radiation which, in the context of brane-world models, leaks from the bulk into the uv brane [16].

It also needs to be checked whether the Bianchi identities and the ij and ty components of Einstein's equations are satisfied with the above ansatz. As shown in [15], the first two are satisfied upon the use of Eq. (10), while the latter relates the free function $n(t,y)$ to $a(t,y)$ up to a time-dependent integration constant,

$$0=\frac{n'}{n}\frac{\dot{a}}{a}-\frac{\dot{a}'}{a}. \tag{11}$$

This last equation is solved by setting the following ansatz [15,16],

$$n(t,y)=\frac{\dot{a}(t,y)}{\dot{a}_0(t)},\qquad a=a_0(t)A(t,y). \tag{12}$$

Then, the equation for $A(t,y)$ is obtained from (10) as

$$\left(\frac{\dot{a}_0}{a_0}\right)^2+\frac{k}{a_0^2}=-\frac{\Lambda}{4}A^2+(A')^2+\frac{C}{a_0^4A^2}, \tag{13}$$

[2]The five-dimensional part M_5 of the solution is obtained by solving the following reduced Einstein frame 5d action,

$$S=\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(R+3\Lambda-\frac{1}{2}(\partial\Phi)^2+\frac{1}{2}e^{2\Phi}(\partial\chi)^2\right). \tag{2}$$

The opposite sign of the kinetic term of χ is due to the fact that the Euclidean version is considered here [24].

where $A' = \partial A/\partial y$. Looking at Eq. (13), we recognize its left-hand side as part of the Friedmann equation from standard cosmology. More precisely, it is the part of Friedmann's equation without the cosmological constant term. In particular, since the left-hand side of Eq. (13) is a function of time only, the right-hand side of (13) must also be only a function of time, i.e. independent of the radial coordinate y. The right-hand side of (13) thus effectively acts as a time-dependent vacuum energy "source term" for the cosmological evolution at the boundary, described by the left-hand side. Introducing a time-dependent boundary cosmological "constant" $\lambda(t)$, we can thus separate Eq. (13) into its left and right-hand sides, yielding two independent equations. This procedure is similar to separation of variables when solving differential equations: For general time-dependent $\lambda(t)$ the above replacement of Eq. (13) by the two Eqs. (14) and (15) does not affect the solution space, since every solution of (14) and (15) will be a solution of (13), and *vice versa*. Doing so, the left-hand side of (13) then becomes the four-dimensional Friedmann equation with a four-dimensional boundary cosmological term $\lambda(t)$,

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{k}{a_0^2} = \lambda(t), \tag{14}$$

where $k = \pm 1$ or 0. From standard cosmology, (14) only yields universes with spherical ($k = +1$) topology for $\lambda > 0$, while for $k = 0$ the allowed choices are $\mathrm{sgn}\lambda = 0, +1$, and for negative spatial curvature $k = -1$ even a spatially homogeneous and isotropic universe of constant negative curvature is allowed, i.e. $\mathrm{sgn}\lambda = -1, 0, +1$ are possible choices.

For any $\lambda(t)$, $A(t, y)$ can then be solved for by the following first-order differential equation in the variable y,

$$\lambda(t) = -\frac{\Lambda}{4}A^2 + (A')^2 + \frac{C}{a_0^4 A^2}, \tag{15}$$

using the solution $a_0(t)$ of (14). In the above treatment of Eq. (13) we introduced an *a priori* time-dependent function $\lambda(t)$. In an evolving universe, a time-dependent cosmological constant, however, would have to be sourced by additional energy-momentum sources at the boundary or in the bulk, which generate the relevant piece in the energy-momentum tensor that ensures energy-momentum conservation. Since in the holographic context with standard Dirichlet boundary condition, gravity at the boundary is not dynamical (i.e. the background metric for the dual field theory is a fixed background field), no boundary matter source can influence the boundary metric. The holographic energy-momentum tensor itself thus has to be conserved. We will calculate the holographic energy-momentum tensor in Sec. IV, but quote the result here already and argue that stress-energy conservation forces the cosmological constant to actually be time-independent.

The general solution of Eq. (15), which will be analyzed in more detail in Sec. IV, is

$$A = \frac{r}{R}\left(\left(1 - \frac{\lambda(t)R^2}{4}\frac{R^2}{r^2}\right)^2 + \frac{CR^2}{4a_0^4(t)}\frac{R^4}{r^4}\right)^{1/2}. \tag{16}$$

Using standard holographic techniques (for details see Sec. IV D 2), the vacuum expectation value of the boundary stress-energy tensor is found to be of perfect fluid form,

$$\langle T^\mu_\nu \rangle = \mathrm{diag}(-\rho, p, p, p), \qquad \alpha = \frac{4R^3}{16\pi G_N^{(5)}}, \tag{17}$$

$$\begin{aligned} \rho &= 3\alpha\left(\frac{C}{4R^2 a_0^4(t)} + \frac{\lambda(t)^2}{16}\right), \\ p &= \alpha\left(\frac{C}{4R^2 a_0^4(t)} - \frac{3\lambda(t)^2}{16}\right). \end{aligned} \tag{18}$$

If we now impose the holographic stress-energy tensor to be conserved, $\nabla_\mu \langle T^\mu{}_\nu \rangle = 0$, we actually require a continuity equation for pressure and energy density,

$$0 = \dot{\rho} + 3H(\rho + p). \tag{19}$$

In order to satisfy this continuity equation, the boundary cosmological constant $\lambda(t)$ then has to be a constant,

$$\dot{\lambda}(t) = 0. \tag{20}$$

Physically speaking, requiring the stress-energy tensor to be conserved amounts to requiring the holographically defined generating functional to be invariant under boundary diffeomorphisms. The conservation equation $\nabla_\mu \langle T^\mu{}_\nu \rangle = 0$ then is the one-point function diffeomorphism Ward-Takahashi identity. In holographic renormalization, this is a natural outcome since both the regularized on-shell action as well as the counterterms are constructed in a manifestly boundary diffeomorphism invariant way. A possible nonconservation of the bulk part of $\langle T_{\mu\nu} \rangle$ can then only be cancelled by additional boundary terms which change the chosen boundary conditions from Dirichlet to Neumann or mixed ones [25], hence inducing additional dynamical degrees of freedom into the boundary theory. We thus conclude that to constitute a physically meaningful holographic background, the boundary cosmological constant must be an actual constant in time. One should note that restricting the solution space of Eqs. (13) or (14) and (15) does not influence the argument given above concerning the equivalence of the (restricted) solution space of both sets of equations.

We will show in the following (in particular, in section V) that the solution $A(t, y)$ of this equation encodes important dynamical properties of the gauge theory in a Friedmann-Robertson-Walker universe. As will be shown in Sec. V, the behavior of Wilson loop expectation values as calculated from a minimal string world sheet, and hence the (de)confinement properties of the vacuum show an interesting competition between the dark radiation constant $C > 0$, and the boundary cosmological constant λ.

Finally, we would like to comment on the physical situation concerning the dark radiation term and the boundary cosmological constant in brane-world models [26,27], which is slightly different. In these models, due to the fact that the uv brane sits at a finite cutoff and due to the chosen boundary conditions, an energy exchange between the bulk and the brane is possible. In particular, the value of λ is always tuned by the bulk metric and the five-dimensional cosmological constant Λ, and the dark radiation term $\frac{C}{a_0^4 A^2}$ is considered to be an energy flux between brane and bulk. In the holographic setup we consider here, due to the standard Dirichlet boundary conditions chosen in holography, no bulk-boundary energy exchange is possible, and the boundary cosmological constant can be freely tuned. The holographic setup is thus less rigid compared to the brane-world models.

III. HOLOGRAPHIC INTERPRETATION OF DARK RADIATION

Above, we have shown how to obtain a consistent background solution describing a boundary metric undergoing cosmological evolution under the influence of both a boundary cosmological constant and a dark radiation term in the bulk. Here, we concentrate on the holographic interpretation of the dark radiation term for the simplest case, i.e. the case of vanishing boundary cosmological constant. We find that the dark radiation term introduces a temperature for the boundary $\mathcal{N} = 4$ field theory.

A. Solution for vanishing boundary cosmological constant

For the case of vanishing boundary cosmological constant, a solution of (14) is given by $\lambda = 0$, $k = 0$, and $a_0(t) = 1$, and $A(t, y) = A(y)$ is obtained by solving (15) with $\lambda = 0$. This gives

$$A = e^{\mu y}(1 + \tilde{c}_0 e^{-4\mu y})^{1/2}, \tag{21}$$

where $\tilde{c}_0 = C/(4\mu^2 a_0^4) = C/(4\mu^2)$ since $a_0 = 1$. From Eq. (12), we obtain

$$n = A - \frac{1}{A}\frac{2C}{\lambda}e^{-2\mu\tilde{y}} = e^{\mu y}\frac{1 - \tilde{c}_0 e^{-4\mu y}}{\sqrt{1 + \tilde{c}_0 e^{-4\mu y}}}. \tag{22}$$

Then, using $r/R = e^{\mu y}$, the full Einstein metric is given by

$$ds_{10}^2 = \frac{r^2}{R^2}(-\bar{n}^2 dt^2 + \bar{A}^2 (dx^i)^2) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2, \tag{23}$$

$$\bar{A} = \left(1 + \tilde{c}_0\left(\frac{R}{r}\right)^4\right)^{1/2}, \qquad \bar{n} = \frac{1 - \tilde{c}_0(\frac{R}{r})^4}{\sqrt{1 + \tilde{c}_0(\frac{R}{r})^4}}. \tag{24}$$

For the dilaton (8) we find, using the above metric,

$$e^{\Phi} = 1 + \frac{q}{2\tilde{c}_0 R^4}\log\frac{1 + \tilde{c}_0(R/r)^4}{1 - \tilde{c}_0(R/r)^4}. \tag{25}$$

Here, the integral constant q corresponds to the gauge condensate $\langle \mathrm{tr} F^2\rangle$, and the boundary condition $e^{\Phi} \to 1$ for $r \to \infty$ is imposed. For $\tilde{c}_0 = 0$, this solution reduces to the supersymmetric one used in [28], and e^{Φ} diverges at $r = 0$ if $\tilde{c}_0 > 0$.

B. Holographic interpretation of dark radiation

In order to give an interpretation to dark radiation constant C, we rewrite the solution Eq. (23) as a planar AdS-Schwarzschild black hole. The five-dimensional part of the metric in the Einstein frame can be brought into that form,

$$ds_{(5)}^2 = \frac{\tilde{r}^2}{R^2}(-f(\tilde{r})dt^2 + (dx^i)^2) + \frac{R^2 d\tilde{r}^2}{\tilde{r}^2 f(\tilde{r})}, \quad f(\tilde{r}) = 1 - \frac{\tilde{r}_0^4}{\tilde{r}^4}, \tag{26}$$

by the coordinate redefinition

$$\tilde{r} = r\sqrt{1 + \frac{R^4}{r^4}\tilde{c}_0} \Rightarrow \tilde{r}_0 = (CR^6)^{1/4}. \tag{27}$$

Thus, the dark radiation constant C has to be positive, and sets the horizon radius of the AdS-Schwarzschild black hole. The dark radiation constant is nothing but the mass of the AdS-Schwarzschild black hole and, applying standard holographic renormalization [29–31] we find the dual stress-energy tensor to be of perfect fluid form

$$\langle T_{\mu\nu}^{(0)}\rangle = \frac{CR}{16\pi G_N^{(5)}}\mathrm{diag}(3, 1, 1, 1). \tag{28}$$

We thus conclude that in holography the dark radiation constant defines a temperature for the fields of the dual field theory. For a nonexpanding cosmology ($k = \lambda = 0$ as in this case) this directly leads to a field theory (in this case $\mathcal{N} = 4$ with gluon and instanton condensate) at finite (Hawking) temperature

$$T_{H_0} = \frac{(4\tilde{c}_0)^{1/4}}{\pi R}. \tag{29}$$

Using this temperature and the energy-momentum tensor IIIb, we can, in particular, confirm the Stefan-Boltzmann law for the energy density[3] $\rho = -\langle T^0{}_0\rangle$,

$$\rho = \frac{4R^3}{16\pi G_N^{(5)}}\left(3\frac{\tilde{c}_0}{R^4}\right) = \frac{3N^2}{8}\pi^2 T_H^4 \tag{30}$$

where we used $G_N^{(5)} = 8\pi^3\alpha'^4 g_s^2/R^5$ and $R^4 = 4\pi N\alpha'^2 g_s$. This expression reproduces the known results of [32,33]. In brane-world models, the dark radiation term has been interpreted as the radiation of the bulk gravitons which

[3] The background metric in this special case is just the flat Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$.

transfer the energy of the fields in the brane to the bulk. In [16,34] it was noted that the dark radiation constant corresponds to the mass of the bulk AdS-Schwarzschild black hole. In their contexts, gravity is dynamical on the UV brane, and the dark radiation term appears in the Friedmann equation on the brane. As noted already in Sec. II, the holographic setup considered in this work is different due to the Dirichlet boundary conditions imposed. Here, gravity is not dynamical at the boundary of space-time. Instead, in our case the dark radiation term is dual to the energy density of the $\mathcal{N} = 4$ $U(N)$ SYM fields in a thermal state, as evident from the Stefan-Boltzmann law (30). The dark radiation constant C, which appears as an integral constant when solving Einstein's equations [15], sets the temperature of the dual field theory. To the best of our knowledge, such a holographic interpretation of the bulk radiation term has not yet been given in the literature before. This interpretation will qualitatively also hold in the time-dependent cosmologies considered in Secs. IV and V: We find that in all cases the dark radiation constant contributes in a thermal manner to the holographic stress-energy tensor of the system, with a time-dependent prefactor $a_0(t)^{-4}$ associated with the dilution of relativistic radiation due to expansion or contraction of the (boundary) universe. On the other hand, the boundary cosmological constant yields a conformal anomaly contribution to the stress-energy tensor. We will see that both contributions can compete, giving rise to interesting dynamics.

IV. HOLOGRAPHY FOR BOUNDARY (A)dS$_4$ SPACE-TIMES

Above, we saw that for vanishing boundary cosmological constant, the dark radiation constant corresponds to a temperature for the $\mathcal{N} = 4$ fields. In this section, we treat the case of finite boundary cosmological constant, and discuss, in particular, the boundary stress-energy tensor. We show that the dark radiation term induces a relativistic radiation contribution to the boundary stress-energy tensor, varying in time with the well-known $a_0(t)^{-4}$ dependence during cosmological expansion. Furthermore, we find that stress-energy conservation in the boundary theory forces the boundary cosmological constant to be time-independent.

A. Solution for finite boundary cosmological constant

A solution of (15) for finite $\lambda(t)$ is

$$A = e^{\mu y}\left(\left[1 - \frac{\lambda(t)}{4\mu^2}e^{-2\mu y}\right]^2 + \tilde{c}_0(t)e^{-4\mu y}\right)^{1/2}, \qquad (31)$$

where $\tilde{c}_0 = C/(4\mu^2 a_0(t)^4)$. Here, we have chosen asymptotic boundary conditions

$$A(y = \infty) = e^{\mu y} = r/R, \qquad (32)$$

where $\mu = 1/R$, i.e. we require the asymptotic form of the metric to be AdS$_5$. We find that A has time-dependence through $a_0(t)$ in $\tilde{c}_0$ and also $\lambda(t)$. This point is important to determine the structure of the metric below.

From Eq. (12), we obtain

$$n = \frac{e^{2\mu y}}{A(t, y)}\left(\left[1 - \frac{\lambda(t)}{4\mu^2}e^{-2\mu y}\right]^2 - \tilde{c}_0(t)e^{-4\mu y}\right). \qquad (33)$$

With $r/R = e^{\mu y}$, the full Einstein frame metric is then given by

$$ds^2_{10} = \frac{r^2}{R^2}(-\bar{n}^2 dt^2 + \bar{A}^2 a_0^2(t)\gamma^2(x)(dx^i)^2) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2, \qquad (34)$$

where

$$\bar{A} = \left(\left(1 - \frac{\lambda}{4\mu^2}\left(\frac{R}{r}\right)^2\right)^2 + \tilde{c}_0\left(\frac{R}{r}\right)^4\right)^{1/2},$$
$$\bar{n} = \frac{(1 - \frac{\lambda}{4\mu^2}(\frac{R}{r})^2)^2 - \tilde{c}_0(\frac{R}{r})^4}{\sqrt{(1 - \frac{\lambda}{4\mu^2}(\frac{R}{r})^2)^2 + \tilde{c}_0(\frac{R}{r})^4}}. \qquad (35)$$

The above metric has no naked singularities for time-independent λ, as we checked by calculating R, $R_{\mu\nu}R^{\mu\nu}$ and the Kretschmann scalar $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$.[4] We will see in Sec. IVd that λ also needs to be time-independent in order to ensure boundary energy-momentum conservation. Note that we use a coordinate system in which the constant curvature three-space has the metric

$$d\Omega_k^2 = \frac{d\vec{x}^2}{(1 + k\vec{x}^2/4)^2}. \qquad (36)$$

These are simply the standard spherical coordinates on the isotropic and homogenous three-space, with a conformal factor.

B. Almost constant scale factor and adiabatic expansion

Quantum fields in an expanding space usually are not in thermal equilibrium, not even locally, unless the expansion rate is slow compared to the equilibration time of the system. This should be the case for very small but nonzero boundary cosmological constant λ, in which case the scale factor $a_0(t)$ would still be changing with time, but with a very slow rate. In other words, the Hubble rate $H = \dot{a}_0/a_0$ is small. In this case, we can still make statements about the "slowly varying" temperature of the system, corresponding to the adiabatic regime.[5] We find the horizon as the zero of the g_{tt} metric coefficient in (34), which is at

[4]Recently, it was noted in a similar but not identical construction [35] that naked singularities might appear when deviating from pure dS expansion. Their singularity so far cannot be shielded by a horizon. In contrast, the Einstein frame curvature singularities in the backgrounds considered here are always behind the horizon $g_{tt} = 0$, and coincide with the cosmological singularities $a_0(t) = 0$.

[5]The system evolves adiabatically, starting from t_0, roughly for a time span $\sqrt{|\lambda|}|t - t_0| \ll 1$.

$$r_H = R\sqrt{\tilde{c}_0^{1/2} + \frac{\lambda}{4\mu^2}} \tag{37}$$

for $\lambda > -(4\mu^2)\tilde{c}_0^{1/2} = \lambda_c$. If the scale factor a_0 is slowly changing, it is possible to approximately satisfy (14) with a time-independent a_0, by taking $k = 1$ for $\lambda > 0$ and $k = -1$ for $\lambda < 0$ in the Friedmann Eq. (14),

$$a_0 \approx 1/|\lambda|^{1/2}, \qquad \gamma(x) = \left(1 + k\frac{x_i^2}{4}\right)^{-1}. \tag{38}$$

In this case, $\partial_\tau r_H \approx 0$, and from the near-horizon geometry

$$ds^2 \simeq 8\left(\frac{r_H}{R}\right)^2 \epsilon^2 d\tau^2 + R^2 d\epsilon^2 + \cdots. \tag{39}$$

a (slowly varying) Hawking temperature can be found for $\lambda > -(4\mu^2)\sqrt{\tilde{c}_0}$, reading

$$T_H = \frac{\sqrt{\lambda/(2\mu^2) + (4\tilde{c}_0)^{1/2}}}{\pi R}. \tag{40}$$

We thus find that negative (positive) λ decreases (increases) the effective temperature for the dual field theory. Furthermore we observe that the regime $\lambda < -(4\mu^2)\sqrt{\tilde{c}_0}$ is special: Formally, the Hawking temperature calculation does not apply to that case even if dark radiation is present, since g_{tt} has no real zero any more, i.e. there is no horizon. We will see in Sec. V that in this regime the Wilson loop shows a confining area law behavior. The situation is thus similar to the Sakai-Sugimoto model [36], where the gravity dual of the confined phase is a cigar-shaped geometry which smoothly caps off instead of admitting a black hole horizon.

C. Dilaton solution

In addition to the metric considered above, the other important field in our system is the running dilaton, whose exponential is related to the gauge coupling in the dual field theory. The solution to the dilaton equation of motion reads

$$e^\Phi = \frac{q}{2\tilde{c}_0(1+\frac{\lambda^2}{16\tilde{c}_0\mu^4})} \times\left\{\log\frac{1+\tilde{c}_0(R/r)^4 + (\lambda R/(4\mu^2 r))^2((R/r)^2 - 8\mu^2/\lambda)}{1-\tilde{c}_0(R/r)^4 + (\lambda R/(4\mu^2 r))^2((R/r)^2 - 8\mu^2/\lambda)} + \frac{\lambda}{2\tilde{c}_0^{1/2}\mu^2}\left(\tan^{-1}\beta + \tanh^{-1}\beta - \frac{1-i}{2}\pi\right)\right\} + \gamma, \tag{41}$$

where q and γ are the integration constants and

$$\beta = \frac{(r/R)^2 - \lambda/(4\mu^2)}{\tilde{c}_0^{1/2}} \tag{42}$$

We notice the following points for the above solution (41):

(1) The above expression (41) seems to be complex due to the factor $\frac{1-i}{2}\pi$ in the second line of (41). However, this is necessary to cancel the imaginary part of $\tanh^{-1}\beta$, which has a constant imaginary part $i\pi/2$ for $\beta > 1$. The condition of $\beta > 1$ is realized for $r > r_H$ (all $r > 0$) in the case of $\lambda > -4\mu^2\tilde{c}_0^{1/2}$ ($\lambda < -4\mu^2\tilde{c}_0^{1/2}$), and hence we only give the solution of e^Φ in this regime, and explicitly display the factor $\frac{1-i}{2}\pi$. As a result, the above expression (41) is real.

(2) The factor arctanh (β) diverges for $\beta \to 1$, which is realized for $r \to r_H = R\sqrt{\tilde{c}_0^{1/2} + \frac{\lambda}{4\mu^2}}$. The same logarithmic divergence comes from the first logarithmic term in the Eq. (41). This divergence can be seen in case (c) of Fig. 1.

(3) For the case of $\lambda \leq -4\mu^2\tilde{c}_0^{1/2}$, the solutions extend to $r = 0$ and there is no divergence at any point in radial direction. The value at $r = 0$ is given by

$$e^{\Phi(0)} = \gamma + \frac{8q\mu^4}{\lambda^2 + 16\mu^4\tilde{c}_0}\left(\log\frac{\lambda^2 + 16\mu^4\tilde{c}_0}{\lambda^2 - 16\mu^4\tilde{c}_0} + \frac{\lambda}{2\tilde{c}_0^{1/2}\mu^2} \times\left(\tan^{-1}\beta_0 + \tanh^{-1}\beta_0 - \frac{1-i}{2}\pi\right)\right), \tag{43}$$

where $\beta_0 = \lambda/(4\tilde{c}_0^{1/2}\mu^2)$. The important point is that $e^{\Phi(0)}$ is finite. We plot its numerical value in Fig. 2 for an appropriate parameter set as a function of $|\lambda|$.

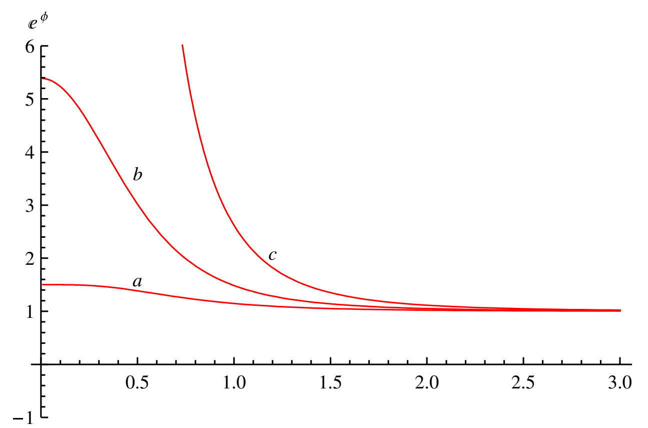

FIG. 1 (color online). Plots of e^ϕ vs r for (a) $\lambda = -1 - 4\mu^2\tilde{c}_0^{1/2}$, (b) $\lambda = -4\mu^2\tilde{c}_0^{1/2}$ and (c) $\lambda = 1 - 4\mu^2\tilde{c}_0^{1/2}$. Cases (a) and (c) are taken as examples for $\lambda < -4\mu^2\tilde{c}_0^{1/2}$ and $\lambda > -4\mu^2\tilde{c}_0^{1/2}$, respectively. Other parameters are set as $1/\mu = R = 1$, $q = 2$, and $\tilde{c}_0 = 0.1$. In the case of (c), e^ϕ diverges at the horizon $r_H = R\sqrt{\tilde{c}_0^{1/2} + \frac{\lambda}{4\mu^2}}$, which is 0.5 in this case. Note that in general, $\tilde{c}_0$ is explicitly time-dependent, these curves represent snapshots of the dilaton solution at constant time.

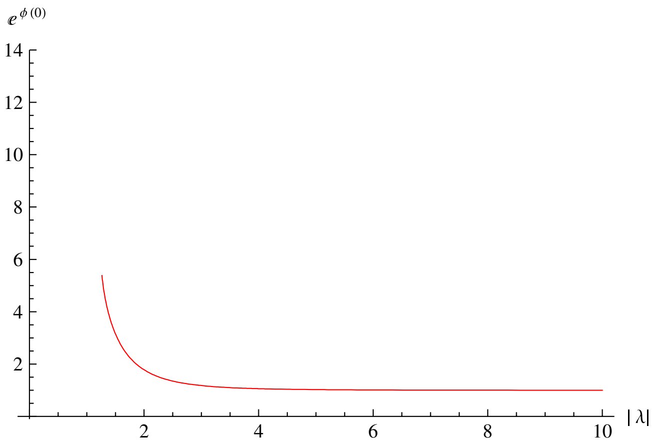

FIG. 2 (color online). Plot of $e^{\phi(0)}$ vs $|\lambda|$, where the parameters are set as in Fig. 1, yielding $4\mu^2\tilde{c}_0^{1/2} = 1.265$. We notice that $e^{\phi(0)}$ is real for $|\lambda| > 4\mu^2\tilde{c}_0^{1/2} = 1.265$, which corresponds to case (a) in Fig. 1, with no horizon present. For $|\lambda| < 4\mu^2\tilde{c}_0^{1/2} = 1.265$, case (c) of Fig. 1, the dilaton becomes complex in the region hidden behind the horizon. Moreover, $e^{\phi(0)} \to 1$ for $|\lambda| \to \infty$. Note that since in general $\tilde{c}_0$ is explicitly time-dependent, this curve should be understood at a given instance in time.

It is interesting to note that the asymptotic value of $e^{\Phi(0)}$ at $\lambda = \pm\infty$ is given by γ. Thus, in the limit of asymptotically large positive or negative cosmological constants, there is no running of the coupling due to the gluon condensate, but only due to the conformal anomaly induced by the background. It would be interesting to further investigate this fact from a field-theoretic point of view.

(4) Further, if we set $\gamma = 1$, we obtain the following asymptotic form

$$e^{\Phi} \simeq 1 + q/r^4 + \cdots \tag{44}$$

as $r \to \infty$. This is the standard AdS/CFT expansion for a scalar dual to a $\Delta = 4$ operator, which in this case is the gluon condensate $\mathrm{Tr}F^2$. The integration constant γ corresponds to the nonnormalizable mode, while q encodes the vacuum expectation value $\langle \mathrm{Tr}F^2 \rangle$.

Thus, while the ultraviolet behavior of e^{Φ} does not depend on λ, the behavior near the infrared region is very sensitive to the boundary cosmological constant. For $\lambda > -(4\mu^2)\sqrt{\tilde{c}_0}$, e^{Φ} diverges at the horizon of the black hole configuration. On the other hand, for $\lambda \leq -(4\mu^2)\sqrt{\tilde{c}_0}$, e^{Φ} approaches a constant at $r = 0$, and $\partial_r e^{\Phi}|_{r=0} = 0$. In this case, then, the Yang-Mills coupling constant reaches at an ir fixed point for $\lambda \leq -(4\mu^2)\sqrt{\tilde{c}_0}$. We should, however, note that conformal invariance of the boundary theory is still broken due to the gravity contribution to the conformal anomaly. However, this will not affect on the renormalization group equation for the Yang-Mills part.[6] We thus would naively expect quark confinement for $\lambda > -(4\mu^2)\sqrt{\tilde{c}_0}$ due to the strong infrared coupling. However, in this case the Wilson loop calculation of Sec. V shows that the quarks are not confined: The Wilson loop deconfines due to the presence of the horizon at $g_{tt} = 0$. We find confinement for $\lambda \leq -(4\mu^2)\sqrt{\tilde{c}_0}$ instead, where the coupling constant is finite and not so large, but where the horizon is absent. We thus conclude that similarly to the situation in the Sakai-Sugimoto model [36], the main factor controlling the confinement dynamics in this setup is not the coupling constant (i.e. the running dilaton) but the presence of a horizon in the bulk. The dilaton running is, however, important at zero temperature ($C = 0$) and leads to Wilson loop confinement e.g. in the hyperbolic case $k = -1$, as will be discussed in Sec. V.

D. Boundary energy-momentum tensor and boundary diffeomorphism invariance

1. The VEV of the boundary energy-momentum tensor

Next, we calculate the four-dimensional stress-tensor from holography. The Fefferman-Graham expansion of the metric (34) reads

$$ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{g_{\mu\nu}(\rho, x^\mu)dx^\mu dx^\nu}{\rho}, \qquad \mu, \nu = 0, \ldots, 3, \tag{45}$$

$$g_{\mu\nu}(\rho, x^\mu) = g_{(0)\mu\nu} + g_{(2)\mu\nu}\rho + \rho^2(g_{(4)\mu\nu} + h_{1(4)\mu\nu}\log\rho + h_{2(4)\mu\nu}(\log\rho)^2) + \cdots, \tag{46}$$

$$g_{(0)\mu\nu} = (g_{(0)00}, g_{(0)ij}) = (-1, a_0(t)^2\gamma_{ij}), \qquad g_{(2)\mu\nu} = -\frac{\lambda}{2R^2\mu^2}g_{(0)\mu\nu}, \tag{47}$$

and

$$g_{(4)00} = \frac{48\tilde{c}_0 - \lambda^2/\mu^2}{16R^4}, \qquad g_{(4)ij} = \frac{16\tilde{c}_0 + \lambda^2/\mu^2}{16R^4}g_{(0)ij}. \tag{48}$$

Then, by using the general formula [29]

[6] More exactly, the RG equation for the effective action Γ reads

$$\mu\frac{\partial}{\partial\mu}\Gamma + \sum_i \beta^i\partial_i\Gamma = \int d^4x\sqrt{-g}(cC_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - a\epsilon^{\mu\nu\alpha\beta}\epsilon^{\rho\sigma\gamma\delta}R_{\mu\nu\rho\sigma}R_{\alpha\beta\gamma\delta}).$$

The right-hand side vanishes since the Euler density $\epsilon^{\mu\nu\alpha\beta}\epsilon^{\rho\sigma\gamma\delta}R_{\mu\nu\rho\sigma}R_{\alpha\beta\gamma\delta}$ is topological and vanishes when integrated over space-time, and the Weyl tensor $C_{\mu\nu\rho\sigma} = 0$ since the cosmological backgrounds are conformally flat. Hence, if no operators except the gluon condensate are present and $\beta_{YM} \to 0$ in the ir, the theory approaches an ir fixpoint when the dilaton approaches a constant in the infrared. This is the case in the Liu-Tseytlin like backgrounds considered here, since [37] there the beta function vanishes in spite of the presence of a gluon condensate.

$$\langle T_{\mu\nu}\rangle = \frac{4R^3}{16\pi G_N}\bigg(g_{(4)\mu\nu} - \frac{1}{8}g_{(0)\mu\nu}((\mathrm{Tr}g_{(2)})^2 - \mathrm{Tr}g_{(2)}^2) - \frac{1}{2}(g_{(2)}^2)_{\mu\nu} + \frac{1}{4}g_{(2)\mu\nu}\mathrm{Tr}g_{(2)}\bigg), \tag{49}$$

we find the holographic stress-energy tensor

$$\langle T_{\mu\nu}\rangle = \langle \tilde{T}^{(0)}_{\mu\nu}\rangle + \frac{4R^3}{16\pi G_N^{(5)}}\bigg(\frac{-3\lambda^2}{16}g_{(0)\mu\nu}\bigg), \tag{50}$$

$$\langle \tilde{T}^{(0)}_{\mu\nu}\rangle = \frac{4R^3}{16\pi G_N^{(5)}}\frac{\tilde{c}_0}{R^4}(3, g_{(0)ij}), \tag{51}$$

where $\langle \tilde{T}^{(0)}_{\mu\nu}\rangle$ is the "thermal" stress-tensor contribution, i.e. the contribution which would be thermal for the $\mathcal{N} = 4$ SYM fields if the universe was not expanding. The second term, which depends on λ, comes from the loop corrections of the SYM fields in a curved space-time, and is the conformal anomaly contribution. The first term does not contribute to the conformal anomaly as in the usual finite temperature case. The conformal anomaly then reads

$$\langle T^\mu_\mu\rangle = -\frac{3\lambda^2}{8\pi^2}N^2, \tag{52}$$

where we used $G_N^{(5)} = 8\pi^3\alpha'^4 g_s^2/R^5$ and $R^4 = 4\pi N\alpha'^2 g_s$. The conformal anomaly precisely matches the free field theory result which can be easily obtained using the general formulas given in [38,39]. This is expected, since the conformal anomaly is one-loop exact, and hence trivially interpolates from weak to strong coupling.

2. Time-independence of the boundary cosmological constant from diffeomorphism invariance

The stress-energy tensor (50) has, in flat space ($k = 0$), the form of a perfect fluid stress-energy tensor. As such, it obeys the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{53}$$

where ρ, p and H represent the energy density, pressure and the Hubble constant $H = \dot{a}_0/a_0$, and dot denotes the time derivative. From (50) we read off

$$\rho = 3\alpha\bigg(\frac{\tilde{c}_0}{R^4} + \frac{\lambda^2}{16}\bigg), \quad p = \alpha\bigg(\frac{\tilde{c}_0}{R^4} - 3\frac{\lambda^2}{16}\bigg), \quad \alpha = \frac{4R^3}{16\pi G_N^{(5)}}. \tag{54}$$

A priori, from the way how we introduced the boundary cosmological constant $\lambda(t)$ in Sec. IIb, it has to be considered as being time-dependent. The continuity equation then requires

$$\dot{\lambda} = 0, \tag{55}$$

where we used $\tilde{c}_0 = C/(4\mu^2 a_0^4(t))$. Continuity of energy density and pressure thus dictate the cosmological constant to be an actual constant in time.

The requirement that the boundary stress-energy tensor satisfies the continuity Eq. (53) is very natural from the point of view of diffeomorphism invariance of the boundary theory. Gauge/gravity duality provides, in particular, a way to holographically calculate the generating functional of correlators in the dual field theory via the Gubser-Klebanov-Polyakov-Witten relation [1] from the appropriately renormalized on-shell gravity action. Since the regularized on-shell action as well as the holographic counterterms are invariant under boundary diffeomorphisms $\xi^\mu(x^\rho)$ by construction, the resulting generating functional respects this invariance in the absence of external sources. The boundary diffeomorphisms are the ones compatible with the Fefferman-Graham expansion of the metric, and hence can only depend on boundary coordinates x^ρ. In particular, only the leading piece of the expansion, the boundary metric, transforms under boundary diffeomorphisms as $g^{(0)}_{\mu\nu} \mapsto g^{(0)}_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$, but all the subleading coefficients such as the stress-energy tensor VEV $g^{(4)}_{\mu\nu}$ will be invariant. The source-operator coupling $\int d^p x g^{(0)\mu\nu}T_{\mu\nu}$ then enforces the Ward identities

$$0 = \nabla^\mu\langle T_{\mu\nu}\ldots\rangle \tag{56}$$

upon transformation of the effective action by such a boundary diffeomorphism. This identity should hold for correlators involving the stress-energy tensor. In particular, the holographic stress-energy tensor should be conserved,

$$0 = \nabla_\mu\langle T^\mu{}_\nu\rangle. \tag{57}$$

For a perfect fluid $T^\mu{}_\nu = \mathrm{diag}(-\rho(t), p(t), p(t), p(t))$, Eq. (57) is equivalent to the continuity Eq. (53). The time-independence of λ, Eq. (55), thus follows directly from the requirement of stress-energy conservation.

In a more general setup one would, however, expect the boundary cosmological constant to change with time due to energy exchange between bulk and boundary. As noted before, this is not possible in the holographic setup, due to the imposed Dirichlet boundary conditions imposed at the boundary, and due to the fact that gravity decouples from the gauge theory in this case as the uv cutoff is taken to infinity [34]. This dictates that the energy-momentum contributions to the holographic stress-energy tensor coming from the bulk (the dark radiation part) as well as from the nontrivial boundary geometry (the boundary cosmological constant part) cannot mix with each other in a nontrivial (time-dependent) way. Hence, each of them is conserved by itself, yielding (55).

V. DARK RADIATION, BOUNDARY COSMOLOGICAL CONSTANT AND QUARK CONFINEMENT

In this section, we consider the combined effect of both the dark radiation term and the boundary cosmological constant on infinitely heavy quarks, i.e. test quarks, in the

Super Yang-Mills theory, by holographically evaluating the static quark-antiquark potential from a Wilson loop vacuum expectation value. One way to introduce (supersymmetric) quarks in the present context is through probe D7 branes [40]. The test quark-antiquark pair is then described by a string worldsheet ending on a prescribed space-time contour on the D7 brane. If the D7 brane corresponds to an infinitely massive embedding, and barring special issues such as the presence of gauge field charge on the brane, the brane embedding then will coincide with the asymptotic boundary of our space-time, and we can consider a string worldsheet ending on a contour at this boundary. The static quark-antiquark potential is then calculated from the energy of the string, evaluated on a minimal surface with the boundary condition prescribed by the contour [41]. Usually, the string worldsheet then has two possible configurations:

(1) A pair of parallel strings, which stretch between the boundary and the horizon. This configuration describes a free quark-antiquark pair, and corresponds to a deconfined situation in which the Wilson loop shows a perimeter law.

(2) A U-shaped string whose two end-points are on the boundary, but which does not touch any black hole horizon or singularity in the bulk. If the g_{xx} component of the string frame metric has a minimum, the string will be stuck there and the energy will depend linearly (for large separations) on the separation of the end-points, and show an area law for the Wilson loop. This configuration describes a confined quark-antiquark pair.

These two types of configurations are seen to compete thermodynamically in the finite temperature gauge theory [11], as well as for the theory in dS_4 [9], with the deconfined configuration being thermodynamically preferred in both cases.

A. The wilson loop in cosmological evolution

Following [41], we consider the Nambu-Goto string dynamics with the string world volume in (t, x) plane. The energy E of this state is then obtained as a function of the proper distance L between the quark and antiquark as follows [9]: Choosing a gauge $X^0 = t = \tau$ and $X^1 = x^1 = \sigma$ for the world sheet coordinates (τ, σ) of the Nambu-Goto action, the Nambu-Goto Lagrangian in the present background (23) becomes

$$L_{\rm NG} = -\frac{1}{2\pi\alpha'}\int d\sigma e^{\Phi/2}\bar{n}(r)\sqrt{r'^2 + \left(\frac{r}{R}\right)^4(\bar{A}(r)a_0(t)\gamma(x))^2}, \tag{58}$$

where only the radial coordinate $r(x)$ is assumed to depend on x, and prime denotes the derivative with respect to x. The functions $\bar{n}$, $\bar{A}$ are defined in (34) and (35).

The energy of the string configuration, which is nothing but the static quark-antiquark potential, is obtained from (58) as

$$E = -L_{\rm NG} = \frac{1}{2\pi\alpha'}\int d\tilde{\sigma}|n_s|\sqrt{1 + \left(\frac{R^2}{r^2\bar{A}}\partial_{\tilde{\sigma}} r\right)^2}, \tag{59}$$

where

$$\tilde{\sigma} = a_0(t)\int d\sigma\gamma(\sigma) = a_0(t)\int d\sigma\frac{1}{1 + k\sigma^2/4}, \tag{60}$$

and

$$n_s = e^{\Phi/2}\left(\frac{r}{R}\right)^2|\bar{A}\,\bar{n}\,|. \tag{61}$$

We should note that $\tilde{\sigma}$ measures the physical length (proper distance) in the present case, while $\sigma = x$ measures the distance in comoving coordinates, which do not change with expanding scale. We will consider the static quark-antiquark potential as a function of proper distance.

The quark-antiquark potential (59) shows a scaling with distance if $n_s(r)$ has a finite minimum at some distance $r = r^*$ outside the horizon. This is the case for $\lambda \le -4\mu^2\tilde{c}_0^{1/2}$. In this case, the dilaton in $n_s(r)$ varies very slowly and monotonically (see the discussion around Fig. 1), and we can estimate the minimum of n_s by neglecting the dilaton dependence, $e^{\Phi/2} \approx 1$, in $n_s(r)$, i.e. taking

$$n_s \approx \left(\frac{r}{R}\right)^2|\bar{A}\,\bar{n}\,| = \left(\frac{r}{R}\right)^2\left|\left(1 - \frac{\lambda}{4\mu^2}\left(\frac{R}{r}\right)^2\right)^2 - \tilde{c}_0\left(\frac{R}{r}\right)^4\right|. \tag{62}$$

We then find the minimum of $n_s(r)$ at

$$r^* = R\left(\left(\frac{\lambda}{4\mu^2}\right)^2 - \tilde{c}_0\right)^{1/4}. \tag{63}$$

We see that the minimum is at a finite value of r^* for $\lambda^2 > (4\mu^2)^2\tilde{c}_0 > 0$, since $\tilde{c}_0 > 0$ is necessary in order to have a positive temperature contribution of the Yang-Mills fields to the holographic energy-momentum tensor (51). There are thus two regimes, $\lambda > 4\mu^2\sqrt{\tilde{c}_0}$ and $\lambda < -4\mu^2\sqrt{\tilde{c}_0}$. In the former case, however, the dilaton e^Φ diverges at a finite radius, and cannot be neglected any more in (61). We are thus left with considering the case $\lambda < -4\mu^2\sqrt{\tilde{c}_0}$. For $\lambda < -4\mu^2\sqrt{\tilde{c}_0}$, the value of n_s at the minimum is

$$n_s(r^*) = \frac{\lambda}{2\mu^2} - 2\sqrt{\left(\frac{\lambda}{4\mu^2}\right)^2 - \tilde{c}_0} \ge 0. \tag{64}$$

This is finite since we are considering the case of $\lambda < -4\mu^2\tilde{c}_0^{1/2}$. Note that $n_s(r^*) \ge 0$ since $\tilde{c}_0 \ge 0$. Then the energy E is approximated as [9]

$$E \sim \frac{n_s(r^*)}{2\pi\alpha'}L, \tag{65}$$

where

$$L = 2\int_{\tilde{\sigma}_{\min}}^{\tilde{\sigma}_{\max}} d\tilde{\sigma} \tag{66}$$

is the proper distance between the string end-points, and $\tilde{\sigma}_{\min}$ ($\tilde{\sigma}_{\max}$) is the value at $r_{\min}$ ($r_{\max}$) of the string configuration [10]. The potential $V = E$ thus grows linearly in proper distance as long as $n_s(r^*) > 0$, and the string tension is given by

$$\tau_{q\bar{q}} = \frac{n(r^*)}{2\pi\alpha'}. \tag{67}$$

On the other hand, for the case of $\lambda > -4\mu^2\tilde{c}_0^{1/2}$, we do not find such a finite minimum of $n_s(r)$. In this case $n_s(r_H) = 0$ at the horizon r_H defined by $\bar{n}(r_H) = 0$. The exact behavior of n_s, including the dilaton dependence, is shown in the numerically obtained plot Fig. 3 for all cases of relevance. The numerical results support the approximations made above. In particular, there is no linear potential for $\lambda > -4\mu^2\tilde{c}_0^{1/2}$. The quarks are deconfined in this case, which can qualitatively be understood as the effect of the cosmological constant not being sufficiently negative to overcome the thermal screening of the quark-antiquark force. This is particularly interesting for negative λ, in which case we find a possible deconfined phase even for AdS backgrounds if only the finite "temperature" screening of the quark-antiquark potential, set in this case by the dark radiation constant C, is strong enough. This seems to be a novel phenomenon at strong coupling, since the Wilson loop in AdS spaces previously was expected to confine [22,23] due to the diverging gravitational potential in AdS space (for a more thorough discussion see Sec. Vb.).

We obtained the relation $E(L)$, as shown in Fig. 4, and the tension $\tau_{q\bar{q}}$ in the following way: Since the Lagrangian

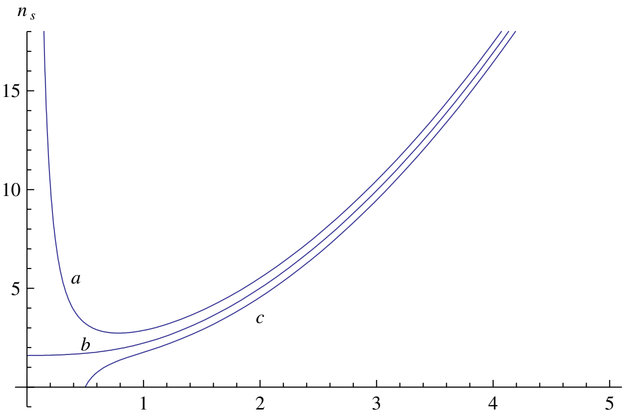

FIG. 3 (color online). Plots of n vs r for (a) $\lambda = -1 - 4\mu^2\tilde{c}_0^{1/2}$, (b) $\lambda = -4\mu^2\tilde{c}_0^{1/2}$ and (c) $\lambda = 1 - 4\mu^2\tilde{c}_0^{1/2}$. Cases (a) and (c) are taken as examples for $\lambda < -4\mu^2\tilde{c}_0^{1/2}$ and $\lambda > -4\mu^2\tilde{c}_0^{1/2}$, respectively. In agreement with (63), (a) and (b) show minima and hence confine. The parameters are taken to be $1/\mu = R = 1$, $k = -1$, and $\tilde{c}_0 = 0.2$. For (c), there is a horizon at $r = 0.5$. Note that for the parameter values chosen in this plot, $\lambda < 0$ in all cases and hence $k = -1$ is consistent when solving the Friedmann Eq. (14). k only enters the dilaton and hence the Wilson loop via the value of $\tilde{c}_0$. Furthermore, since in general $\tilde{c}_0$ is explicitly time-dependent, these curves represent snapshots at constant times.

in (58) does not explicitly depend on the coordinate $\sigma = x$, we find the following quantity conserved under σ-shifts,

$$e^{\Phi/2}\frac{1}{\sqrt{(r/R)^4\bar{A}^2(r) + (r')^2}}\left(\frac{r}{R}\right)^4\bar{n}\bar{A}^2(r) = H. \tag{68}$$

We can fix H at any point we like, so we fix it at $r = r_{\min}$. Then, choosing $H = e^{\Phi/2}(\frac{r}{R})^2\bar{n}(r)\bar{A}(r)|_{r_{\min}}$, we get

$$L = 2R^2\int_{r_{\min}}^{r_{\max}} dr\frac{1}{r^2\bar{A}(r)\sqrt{e^{\Phi(r)}r^4\bar{n}(r)^2\bar{A}(r)^2/(e^{\Phi(r_{\min})}r_{\min}^4\bar{n}(r_{\min})^2\bar{A}(r_{\min})^2) - 1}},$$
$$E = \frac{1}{\pi\alpha'}\int_{r_{\min}}^{r_{\max}} dr\frac{\bar{n}(r)e^{\Phi(r)/2}}{\sqrt{1 - e^{\Phi(r_{\min})}r_{\min}^4\bar{n}(r_{\min})^2\bar{A}(r_{\min})^2/(e^{\Phi(r)}r^4\bar{n}(r)^2\bar{A}(r)^2)}}. \tag{69}$$

Figure 4 shows the exact dependence of the energy E on the distance L for the values $q = 0$ (i.e. for constant dilaton) for $\lambda \le -4\mu^2\tilde{c}_0^{1/2}$ (curve A) and $\lambda > -4\mu^2\tilde{c}_0^{1/2}$ (curve B). In the former case, we find the linear potential at large L as expected, and we find a typical screening behavior in the latter case, similar to the one seen in the finite temperature deconfinement phase. The qualitative behavior of the Wilson loop for $q \neq 0$ are unchanged from the $q = 0$ case.

B. Classification of (De)confining behavior in cosmological backgrounds

In the above analysis, we found confining behavior for the Wilson loop if

$$\lambda \le -4\mu^2\sqrt{\tilde{c}_0} = -\frac{2\sqrt{C/R^2}}{a_0^2(t)}. \tag{70}$$

Thus, for vanishing dark radiation constant $C = 0$ (which would correspond to vanishing temperature in the static case) and negative boundary cosmological constant $\lambda < 0$, the system is always confined, in accordance with earlier results of [12,22]. On the other hand, the Wilson loop cannot have an area law for

$$\lambda > -4\mu^2\sqrt{\tilde{c}_0} = -\frac{2\sqrt{C/R^2}}{a_0^2(t)}. \tag{71}$$

This regime is deconfining. The right-hand side of these inequalities is generically time-dependent, and hence the

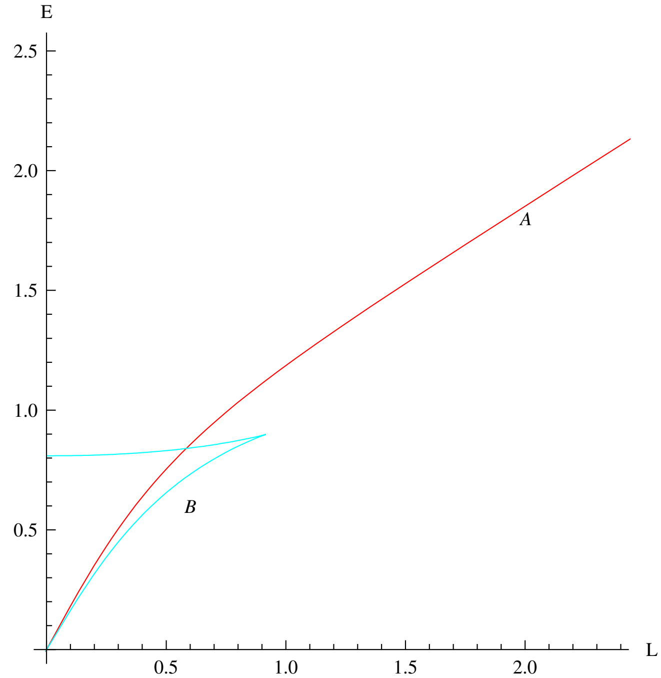

FIG. 4 (color online). Plots of E vs L for (A) $\lambda = -1 - 4\mu^2\tilde{c}_0^{1/2}$ and (B) $\lambda = +1 - 4\mu^2\tilde{c}_0^{1/2}$. Cases (A) and (B) are taken as examples for $\lambda < -4\mu^2\tilde{c}_0^{1/2}$ and $\lambda > -4\mu^2\tilde{c}_0^{1/2}$, respectively. Case (A) shows confinement, while case (B) shows typical screening behavior: In the latter case there exist two U-shaped configurations for small L, while for large separations the string breaks at the horizon and hence has vanishing energy (at $O(N^2)$). Other parameters are set as $q = 0$, $\mu = 1$, $R = 1$, $k = -1$, $\tilde{c}_0 = 1.0$, $r_{\max} = 3$ and $\alpha' = 1$. Note that in general $\tilde{c}_0$ is explicitly time-dependent, these curves represent snapshots of the dilaton solution at constant time.

inequalities will hold only, in particular, time intervals, in which the universe is larger/smaller than a critical value set by the boundary cosmological constant. There are three cases:

(i) Positive λ: In this case the system is in a deconfined phase. This could have been expected by the expanding nature of the de Sitter universe in this case: The expansion of space tends to destabilize bound states, leading to deconfinement even at zero dark radiation constant.

(ii) Vanishing λ: This case is similar to the (de)confinement properties of a planar AdS-Schwarzschild black hole. For any finite value of the dark radiation constant $C > 0$ (“finite temperature”), Eq. (71) is satisfied, and the system is in the deconfined phase. For vanishing dark radiation constant $C = 0$, however, Eq. (70) holds and the Wilson loop shows confining behavior.

(iii) The most interesting situation occurs for negative λ: Depending on the value of the scale factor $a_0(t)$ at any given time, either (70) or (71) can be satisfied, leading to transitions between confined and deconfined phases as the universe evolves. Physically, this transition is due to the competition between two effects: The screening effects of the thermal (dark radiation) energy always aims at driving the system into a deconfined phase, while the influence of the background cosmological evolution on the static quark-antiquark potential can be either confining (for $\lambda < 0$) or deconfining (for $\lambda > 0$). Thus for negative boundary cosmological constant thermal screening and background-induced confinement can compete and result into (de)confinement transitions.

For definiteness, let us consider the solution of (14) for $k = -1$ and $\lambda < 0$,

$$a_0(t) = \frac{\sin\sqrt{|\lambda|}t}{\sqrt{|\lambda|}}. \tag{72}$$

This solution thus describes an oscillating universe. Analyzing (70) and (71), we find two different regimes: Since the scale factor is bounded from above by $a_{0,\max} = 1/\sqrt{|\lambda|}$, a large enough dark radiation constant $C > R^2/4$ always satisfies (71), and hence quarks are always deconfined in this case. For $C < R^2/4$, however, the system oscillates between a deconfined phase at smaller scale factors $a_0(t)$, and a confined phase at larger values. For the marginal value $C = R^2/4$, the Wilson loop shows confining behavior only at maximal extension of the universe. Our holographic setup thus describes a (supersymmetric) plasma with the qualitative properties observed in the evolution of our universe: Near the big bang singularity $a_0 = 0$, the matter is in a deconfined state, and undergoes a confinement phase transition as the universe cools down. Figure 5 summarizes the situation.

These results, in particular, in the latter case, need to be compared to the results of [23], where it was argued that the Wilson loop is not a good measure for (de)confinement in AdS space. The arguments of [23] involve a conformal transformation between AdS space and half of the Einstein static universe (ESU), relating long distance behavior in AdS space to the (universal) short distance behavior in the ESU measured in turn by the Wilson loop. This argument fails in the backgrounds considered here since the conformal symmetry of $\mathcal{N} = 4$ SYM theory is broken by the gluon condensate, as well as by the conformal anomaly for $\lambda \neq 0$.[7] The results of Sec. V show that the Wilson loop is sensitive to both, and hence depends on the chosen conformal frame, measuring unambiguously the deconfinement properties of the chosen field theory

[7]This point was noted before in [42].

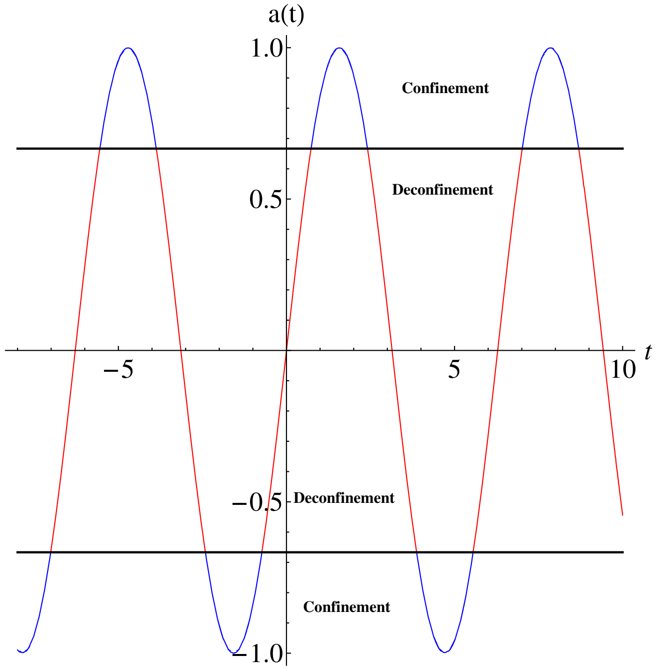

FIG. 5 (color online). The (de)confinement transition as seen in the Wilson loop behavior in an oscillating, open, universe ($\lambda < 0$, $k = -1$). During the periods depicted in blue, the Wilson loop shows confining behavior, while during the red periods (close to the big bang singularity), the plasma is in a deconfined phase. The transition line, shown in black, is fixed by the value of the dark radiation constant C.

state by coupling to the full energy-momentum tensor and to the gluon condensate.[8] Similarly, we do not expect the arguments of [22] for a weakly coupled meron gas disordering the Wilson to simply carry over to strong coupling.

The conformal anomaly is also responsible for the time-dependent nature of the holographic backgrounds presented here: The diffeomorphism relating these backgrounds to topological black holes [17,18] induces a conformal transformation on the boundary, which, due to the conformal anomaly, is not a quantum symmetry of the dual field theory. Thus, except for the case of vanishing boundary cosmological constant, observables calculated in the two different conformal frames will generically be different, and therefore have to be calculated in the time-dependent background itself. We cannot resort to the equivalence with topological black holes to define a thermodynamic ensemble or to consider phase transitions, as e.g. studied for compactifications of CFT's on dS space-times [43]. Our setup, hence, is time-dependent and describes genuine nonequilibrium physics. We can, however, characterize the properties of the nonequilibrium state by calculating observables such as the Wilson loop via the machinery of gauge-gravity duality.

For vanishing boundary cosmological constant, the conformal anomaly vanishes, and the phase structure of the AdS-Schwarzschild black hole with flat ($k = 0$) horizon, which has $O(N^2)$ entropy density at finite temperature and $O(1)$ at absolute zero [44], coincides exactly with our results for the Wilson loop. For the hyperbolic ($k = -1$) black hole, on the other hand, these two measures of confinement do not agree: From [17,18] it is clear that the black hole mass is given by $\mu = C$. The hyperbolic black hole has a nondegenerate horizon at $r_+ = L_{\rm AdS}$ for $\mu = 0$, and both its free energy and entropy are $O(N^2)$ at this point [45]. Our Wilson loop, on the other hand, is confined at $\mu = C = 0$. This is one of the rare examples where the density of states and the Wilson loop do not agree as measures of confinement, which can be traced back in this case to the effect of the gluon condensate which enters the Wilson loop via the string frame metric.

As discussed in the introduction, different measures of confinement leading to different answers in curved backgrounds are not uncommon, and since the field theory is known exactly,[9] this case appears to be a good playground for a future investigation of the interplay of different measures of confinement.[10] We plan to come back to this point in a future work.

Another interesting observation concerns the relation of the Wilson loop with the temperature (40): Although this temperature has been derived in a adiabatic approximation, assuming the cosmological constant to be sufficiently small, the Wilson loop feels exactly this temperature, without any approximation. The reason is that the Wilson loop calculation in this section is done ‘‘locally in time’’, i.e. by considering a string stretching into the fifth dimension at each fixed value in time, testing the presence of the horizon with (40). If the horizon is present the Wilson loop exhibits perimeter law, if not, the temperature (40) is zero or ill-defined, and the Wilson loop exhibits an area law. Thus,

[8]The gluon condensate is essential for confinement in the case $\lambda = 0$, $k = -1$, since the Wilson loop in the purely hyperbolic AdS-Schwarzschild black hole is screened [19] for all temperatures.

[9]The field theory is $\mathcal{N} = 4$ SYM theory with a gluon condensate [4]. This breaks conformal symmetry spontaneously, but not explicitly at the level of symmetry generators or Green functions, since the beta function still vanishes (see also [37]).

[10]This is particularly interesting in view of claims that the so-called ‘‘precursor’’ states [46], which create the $O(N^2)$ ground state entropy of the extremal hyperbolic black hole, are potentially relevant to the thermal screening of the quark-antiquark potential in the absence of the gluon condensate [19].

although (40) can only be considered as an approximation, it exactly reproduces the Wilson loop behavior.

VI. SUMMARY AND DISCUSSIONS

We have investigated properties of strongly coupled $\mathcal{N} = 4$ $U(N)$ supersymmetric Yang-Mills theory in the presence of a gluon condensate on cosmological spacetimes of Friedmann-Robertson-Walker type by applying methods of gauge/gravity duality. The dual gravity solutions are obtained from a Liu-Tseytlin Ansatz of IIB supergravity by solving the effective five-dimensional Einstein equations through a metric Ansatz first employed in the setup of brane-world cosmologies [15]. By introducing a single integration constant, the so-called ''dark radiation constant'' or ''mirage energy density'', Einstein's equations reduce to a single constraint equation. We solved this equation by introducing a boundary cosmological constant which *a priori* can be time-dependent. In this way the constraint is separated into a standard Friedmann equation and an equation for the sole undetermined function in the bulk metric. The resulting holographic background is dual to $\mathcal{N} = 4$ SYM theory in a FRW-type metric, and has a the gluon condensate. This bulk geometry determines the vacuum expectation value of the stress-tensor.

Holographically, the dark radiation term [17,18,20] induces a relativistic radiation component (of Stefan-Boltzmann form) in the stress-energy tensor. If the universe is static, this radiation component has the correct T^4 behavior and N^2 scaling in the large N limit. If the universe is nonstatic, it is modified by a factor $1/a_0^4(t)$, as expected for a gas of relativistic particles. For nonvanishing boundary cosmological constant there is also the familiar conformal anomaly contribution proportional to the square of the boundary cosmological constant λ^2.

Using the holographic stress-energy tensor, we have also clarified the holographic interpretation of a possible time-dependence of the cosmological constant $\lambda(t)$: The holographic stress-energy tensor is conserved if and only if the boundary cosmological constant $\lambda(t)$ is time-independent, $\dot{\lambda} = 0$. Requiring a conserved boundary stress-energy tensor is necessary to ensure that the dual field theory is boundary diffeomorphism invariant. In general relativist's terms, the coupling between field theory and curved background geometry does not spoil the equivalence principle. Hence, with the standard AdS/CFT Dirichlet boundary conditions, only a time-independent boundary cosmological constant $\lambda(t) = \lambda$ is holographically meaningful and consistent.

The main result of this paper is the behavior of the Wilson loop in the cosmological background geometries, which we take as the measure of the (de)confinement properties of quark-antiquark pairs. We find an interesting interplay between dark radiation and cosmological constant: The dark radiation component drives the Wilson loop to deconfining behavior, which may be understood as thermal screening of the quark-antiquark interaction. On the other hand, the boundary cosmological constant λ can have deconfining or confining effect: Positive λ (i.e. de Sitter-like expanding cosmologies) drives the system into deconfinement, while negative λ (i.e. antide Sitter-like cosmologies) drives it towards confinement. For negative λ, there exist periodically oscillating cosmologies in which the system undergoes periodic (de)confinement transitions: For dark radiation constants below a critical value set by the bulk AdS radius, the Wilson loop is deconfined when the universe is small (i.e. near the big bang singularity). After sufficient expansion it undergoes a transition to confining behavior. This is in qualitative agreement with the expected behavior in nature: Close to the big bang, i.e. at large temperatures, QCD matter should have been in a quark-gluon plasma state, and undergoes a confinement phase transition once the universe sufficiently expanded and cooled down.

It should be stressed that in contrast to previous works [17,18,20], the standard holographic Dirichlet boundary conditions employed in this paper forbid bulk-boundary energy exchange. The dual field theory lives on a curved but fixed background geometry with no propagating graviton in the boundary theory. The Friedmann equation is obtained from the bulk Einstein equations. It is then natural that boundary diffeomorphism invariance restricts the choice of bulk geometry.

As a follow-up and in view of the interesting (de) confinement properties of the system described, it appears to be worthwhile to investigate the dynamics of fundamental degrees of freedom. Such degrees of freedom can e.g. be introduced into these cosmological backgrounds by probe D7 branes [40]. This would for instance allow an investigation of the chiral symmetry breaking and its relation to Wilson loop (de)confinement, as well as possible chiral symmetry enhancement or suppression.—Applying holographic renormalisation to these backgrounds will further clarify the relation between the free energy, entropy, and the properties of the Wilson loop, and thus shed light on the relation between these different measures of confinement. In a similar way the existence of a mass gap can be inferred from a fluctuation analysis around the bulk geometry. We will come back to these questions in a future work.

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