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Splitting of folded strings in AdS_3

E. M. Murchikova*

Blackett Laboratory, Imperial College, London SW7 2AZ, U.K. and SINP, Moscow State University, Moscow, 119991, Russia

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In this paper we present semiclassical computations of the splitting of folded spinning strings in AdS_3, which may be of interest in the context of AdS/CFT duality. We start with a classical closed string and assume that it can split into two closed string fragments, if at a given time two points on it coincide in target space and their velocities agree. First we consider the case of the folded string with large spin. Assuming the formal large-spin approximation of the folded string solution in AdS_3, we can completely describe the process of splitting: compute the full set of charges and obtain the string solutions describing the evolution of the final states. We find that, in this limit, the world surface does not change in the process and the final states are described by the solutions of the same type as the initial string, i.e. the formal large-spin approximation of the folded string in AdS_3. Then we consider the general case—splitting of string given by the exact folded string solution. We find the expressions for the charges of the final fragments, the coordinate transformations diagonalizing them and, finally, their energies and spins. Because of the complexity of the initial string profile, we cannot find the solutions describing the evolution of the final fragments, but we can predict their qualitative behavior. We also generalize the results to include circular rotations and windings in S^5.

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I. INTRODUCTION

Decay properties of massive strings have been studied for a long time [1–15]. In this paper we present semiclassical computations of the splitting of folded spinning strings in AdS_3. Classical string solutions have proved to be a useful tool for exploring the AdS/CFT correspondence in the sector of large charges [16–23].

For flat Minkowski space, splitting of semiclassical strings was analyzed in detail in [12,13], for $R_t \times S^5$ space in [14,15]. There is an obvious lack of results in AdS space, and the purpose of the present paper is to fill this gap. Following the conventional approach, we start with a classical closed string and assume that it can split into two fragments, if at a given time τ_0 two points on it coincide in target space and their velocities agree. Closed string periodicity conditions are separately imposed on each of the two final pieces. Initial conditions are defined by the initial string at τ_0. The relations between the energies and spins of the cut fragments—together with "conservation laws" of splitting $E(E_{\rm I}, E_{\rm II}, \ldots)$, $S(S_{\rm I}, S_{\rm II}, \ldots)$, etc.—are completely determined by the charge conservation. Thus they may be found (at least parametrically) for the initial string solution of arbitrary complexity. Determining the evolution is much more complicated: one has to solve the string equations with the boundary conditions given by a part of the profile of the initial string. At the moment, this is possible only in the simplest cases.

The main purpose of this paper is to investigate splitting of folded spinning string in AdS_3 [17]

$$
\begin{aligned}
&Y_0 + iY_5 = \mathrm{dn}[\kappa\ell^{-1}\sigma, -\ell^2]e^{i\kappa\tau},\\
&Y_1 + iY_2 = \ell\,\mathrm{sn}[\kappa\ell^{-1}\sigma, -\ell^2]e^{i\omega\tau},\\
&\kappa = \frac{2}{\pi}\ell\mathbb{K}[-\ell^2], \qquad \frac{w^2}{\kappa^2} = 1 + \frac{1}{\ell^2},
\end{aligned}
\tag{1.1}
$$

where $\mathrm{sn}[z, m]$ and $\mathrm{dn}[z, m]$ are Jacobi elliptic functions, $\mathbb{K}[z]$ is the complete elliptic integral of the first kind. First we consider the limit of the folded string with large spin. Then solution (1.1) may be approximated by

$$
\begin{aligned}
&Y_0 + iY_5 = \cosh(\kappa\sigma)e^{i\omega\tau},\\
&Y_1 + iY_2 = \sinh(\kappa\sigma)e^{i\omega\tau},\\
&\kappa = \omega \gg 1.
\end{aligned}
\tag{1.2}
$$

In this simple case, we can completely describe the process of splitting: compute the full set of charges and find string solutions describing the evolution of the final states. It appeared that when such a string splits, the world surface does not change in the process and the final states are described by the solutions of the same type as (1.2):

$$
\begin{aligned}
&Y_{\rm I,II0} + iY_{\rm I,II5} = \cosh(\kappa_{\rm I,II}\sigma)e^{i\kappa_{\rm I,II}\tau},\\
&Y_{\rm I,II1} + iY_{\rm I,II2} = \sinh(\kappa_{\rm I,II}\sigma)e^{i\kappa_{\rm I,II}\tau},\\
&\kappa_{\rm I,II} = \kappa\frac{\pi \mp 2\sigma_0}{2\pi},
\end{aligned}
\tag{1.3}
$$

where σ_0 parameterizes the coordinate of the splitting point.

In the general case we find expressions for the charges of the final fragments, the coordinate transformations that diagonalize them and, at the end, their energies and spins

*e.murchikova@imperial.ac.uk

as the functions of ℓ and σ_0 (in the coordinate system where no non-Cartan components present). These are

$$E_{\rm I,II} = \frac{\sqrt{\lambda}}{2}\sqrt{(\kappa\mathbb{C}_{\rm I,II} + \omega\mathbb{S}_{\rm I,II})^2 - \mathbb{M}^2_{\rm I,II}(\omega+\kappa)^2} + \frac{\sqrt{\lambda}}{2}\sqrt{(\kappa\mathbb{C}_{\rm I,II} - \omega\mathbb{S}_{\rm I,II})^2 - \mathbb{M}^2_{\rm I,II}(\omega-\kappa)^2}$$
$$S_{\rm I,II} = \frac{\sqrt{\lambda}}{2}\sqrt{(\kappa\mathbb{C}_{\rm I,II} + \omega\mathbb{S}_{\rm I,II})^2 - \mathbb{M}^2_{\rm I,II}(\omega+\kappa)^2} - \frac{\sqrt{\lambda}}{2}\sqrt{(\kappa\mathbb{C}_{\rm I,II} - \omega\mathbb{S}_{\rm I,II})^2 - \mathbb{M}^2_{\rm I,II}(\omega-\kappa)^2}. \quad (1.4)$$

Here

$$\kappa\mathbb{C}_{\rm I,II} = \frac{1}{2}\mathcal{E}_{\rm fold} \mp \frac{\ell}{\pi}\mathbb{E}[{\rm am}[\kappa\ell^{-1}\sigma_0, -\ell^2], -\ell^2],$$
$$\omega\mathbb{S}_{\rm I,II} = \frac{1}{2}\mathcal{S}_{\rm fold} \mp \sqrt{1+\ell^2}\Big(-\frac{2}{\pi}\sigma_0\mathbb{K}[-\ell^2] + \mathbb{E}[{\rm am}[\kappa\ell^{-1}\sigma_0, -\ell^2], -\ell^2]\Big),$$
$$\mathbb{M}_{\rm I,II} = \pm\frac{\ell^2}{\kappa\pi}{\rm cn}[\kappa\ell^{-1}\sigma_0, -\ell^2], \quad (1.5)$$

where $E_{\rm fold} = \sqrt{\lambda}\mathcal{E}_{\rm fold}$ and $S_{\rm fold} = \sqrt{\lambda}\mathcal{S}_{\rm fold}$ are the energy and spin of the folded string (1.1); $\mathbb{E}[z]$ and $\mathbb{E}[z, m]$ are the complete and incomplete elliptic integrals of the second kind, respectively, and ${\rm cn}[z, m]$ is a Jacobi elliptic function. These relations parametrically encode the conservation laws of splitting, namely $E(E_{\rm I}, E_{\rm II})$, $S(S_{\rm I}, S_{\rm II})$, etc.

Because of the complexity of the folded string profile (1.1), we are unable to find the solutions describing the evolution of the final fragments explicitly. However, we can describe the evolution qualitatively. Let us examine the case of large but not infinitely large (as in (1.2)) spin, with the cut occurring far enough from the string ends for σ_0 to satisfy $\kappa(\pi/2 - \sigma_0) \gg 1$. In this limit one expects the final pieces to have almost the standard folded shape (1.1), disturbed by a kink moving along the string, similar to the one observed in flat Minkowski space [12]. The kink is a "correction" to the "leading" folded shape of the cut fragments, thus the angle of bending has to depend on the position of the kink. It may be substantial at the string ends but must be small close to the center.

The results obtained for the folded string in AdS_3 generalizes to include circular rotations and windings in S^5. We discuss such a generalization with the example of the string in $AdS_3 \times S^3$.

The rest of the paper is organized as follows. In Sec. II we introduce notations and discuss a general approach to studying splitting of classical bosonic closed strings in $AdS_5 \times S^5$. Section III is a review of the splitting of the folded strings in flat Minkowski space. Section IV is dedicated to the splitting of Gubser–Klebanov–Polyakov folded strings in AdS_3. The results obtained in AdS_3 are generalized to include circular rotations and windings in S^5 in Sec. V.

II. SPLITTING OF CLOSED STRINGS IN $AdS_5 \times S^5$. GENERAL FORMALISM

In this section we discuss a general approach to studying of splitting of classical closed bosonic strings in $AdS_5 \times S^5$.

The action for a bosonic string in $AdS_5 \times S^5$ reads

$$I_B = \frac{1}{2}T\int d\tau\int_0^{2\pi} d\sigma(L_{\rm AdS} + L_{\rm S}), \quad T = \frac{R^2}{2\pi\alpha'} = \frac{\sqrt{\lambda}}{2\pi}, \quad (2.1)$$

where

$$L_{\rm AdS} = -\partial_a Y_P\partial^a Y^P - \tilde{\Lambda}(Y_P Y^P + 1),$$
$$L_{\rm S} = -\partial_a X_M\partial^a X_M + \Lambda(X_M X_M - 1). \quad (2.2)$$

Here $X_M, M = 1, \ldots, 6$ and $Y_P, P = 0, \ldots, 5$ are embedding coordinates of R^6 with the Euclidean metric $\delta_{MN} = (+1, +1, +1, +1, +1, +1)$ in $L_{\rm S}$ and of $R^{2,4}$ with $\eta_{PQ} = (-1, +1, +1, +1, +1, -1)$ in $L_{\rm AdS}$, respectively, $(Y_P = \eta_{PQ}Y^Q)$. Λ and $\tilde{\Lambda}$ are the Lagrange multipliers imposing the two hypersurface conditions:

$$\eta_{PQ}Y^P Y^Q = -1 \qquad X_M X_M = 1. \quad (2.3)$$

The action (2.1) is supplemented with the conformal gauge constraints

$$\dot{Y}_P\dot{Y}^P + Y'_P Y'^P + \dot{X}_M\dot{X}_M + X'_M X'_M = 0,$$
$$\dot{Y}_P Y'^P + \dot{X}_M X'_M = 0 \quad (2.4)$$

and the closed string periodicity conditions

$$Y_P(\tau, \sigma + 2\pi) = Y_P(\tau, \sigma),$$
$$X_M(\tau, \sigma + 2\pi) = X_M(\tau, \sigma). \quad (2.5)$$

The classical equations of motion following from (2.1) are

$$\partial^a\partial_a Y_P - \tilde{\Lambda}Y_P = 0, \quad \tilde{\Lambda} = \partial^a Y_P\partial_a Y^P, \quad Y_P Y^P = -1,$$
$$\partial^a\partial_a X_M + \Lambda X_M = 0, \quad \Lambda = \partial^a X_M\partial_a X_M, \quad X_M X_M = 1. \quad (2.6)$$

The action is invariant under the $SO(2,4)$ and $SO(6)$ rotations with correspondent conserved (on-shell) charges

$$S_{PQ} = \sqrt{\lambda}\int_0^{2\pi}\frac{d\sigma}{2\pi}(Y_P\dot{Y}_Q - Y_Q\dot{Y}_P),$$
$$J_{MN} = \sqrt{\lambda}\int_0^{2\pi}\frac{d\sigma}{2\pi}(X_M\dot{X}_N - X_N\dot{X}_M). \quad (2.7)$$

We will be working with "spinning" string solutions which have nonzero values of these charges.

It is useful to solve the constraints (2.3) by choosing an explicit parametrization of the embedding coordinates Y_P and X_M, e.g.

$$\begin{aligned} Y_{05} &= Y_0 + iY_5 = \cosh\rho e^{it}, \\ Y_{12} &= Y_1 + iY_2 = \sinh\rho\cos\theta e^{i\phi_1}, \\ Y_{34} &= Y_3 + iY_4 = \sinh\rho\sin\theta e^{i\phi_2}; \end{aligned} \tag{2.8}$$

$$\begin{aligned} X_{12} &= X_1 + iX_2 = \sin\gamma\cos\psi e^{i\varphi_1}, \\ X_{34} &= X_3 + iX_4 = \sin\gamma\sin\psi e^{i\varphi_2}, \\ X_{56} &= X_5 + iX_6 = \cos\gamma e^{i\varphi_3}. \end{aligned} \tag{2.9}$$

The corresponding metrics take the form

$$ds^2_{AdS_5} = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho(d\theta^2 + \cos^2\theta d\phi_1^2 + \sin^2\theta d\phi_2^2) \tag{2.10}$$

$$ds^2_{S^5} = \cos^2\gamma d\varphi_3^2 + d\gamma^2 + \sin^2\gamma(d\psi^2 + \cos^2\psi d\varphi_1^2 + \sin^2\psi d\varphi_2^2). \tag{2.11}$$

The Cartan generators of $SO(2,4)$ corresponding to the three linear isometries of the AdS_5metric are the translations in the AdS-time t and two angles ϕ_1 and ϕ_2:

$$\begin{aligned} S_0 &\equiv S_{05} \equiv E = \sqrt{\lambda}\mathcal{E}, \\ S_1 &\equiv S_{12} = \sqrt{\lambda}\mathcal{S}_1, \\ S_2 &\equiv S_{34} = \sqrt{\lambda}\mathcal{S}_2. \end{aligned} \tag{2.12}$$

The Cartan generators of $SO(6)$ corresponding to the three linear isometries of the S^5 metric are the translations in the three angles φ_1, φ_2 and φ_3:

$$\begin{aligned} J_1 &\equiv J_{12} = \sqrt{\lambda}\mathcal{J}_1, \\ J_2 &\equiv J_{34} = \sqrt{\lambda}\mathcal{J}_2, \\ J_3 &\equiv J_{56} = \sqrt{\lambda}\mathcal{J}_3. \end{aligned} \tag{2.13}$$

Let us consider a string solution

$$X_M = X_{\mathrm{in}M}(\tau,\sigma), \qquad Y_P = Y_{inP}(\tau,\sigma) \tag{2.14}$$

with energy E and spins S_n, J_k. We assume that if at a given τ_0 two points on the string coincide in the target space

$$\begin{aligned} X_{\mathrm{in}M}(\tau_0,\sigma_1) &= X_{\mathrm{in}M}(\tau_0,\sigma_2) \\ Y_{\mathrm{in}P}(\tau_0,\sigma_1) &= Y_{\mathrm{in}P}(\tau_0,\sigma_2) \end{aligned} \tag{2.15}$$

and their velocities agree

$$\begin{aligned} \dot{X}_{\mathrm{in}M}(\tau_0,\sigma_1) &= \dot{X}_{\mathrm{in}M}(\tau_0,\sigma_2) \\ \dot{Y}_{\mathrm{in}P}(\tau_0,\sigma_1) &= \dot{Y}_{\mathrm{in}P}(\tau_0,\sigma_2), \end{aligned} \tag{2.16}$$

then the string can split into two pieces

$$\begin{aligned} &\text{fragment I: } \sigma \in (0,\sigma_1)\cup(\sigma_2,2\pi) \\ &\text{fragment II: } \sigma \in (\sigma_1,\sigma_2). \end{aligned} \tag{2.17}$$

The behavior of the cut fragments is governed by Eqs. (2.4) and (2.6) with the boundary conditions defined by the initial string at the moment of splitting:

$$\begin{aligned} &X_{\mathrm{I}M}(\tau_0,\sigma) = X_{\mathrm{in}M}(\tau_0,\sigma) \qquad \dot{X}_{\mathrm{I}M}(\tau_0,\sigma) = \dot{X}_{\mathrm{in}M}(\tau_0,\sigma) \\ &Y_{\mathrm{I}P}(\tau_0,\sigma) = Y_{\mathrm{in}P}(\tau_0,\sigma) \qquad \dot{Y}_{\mathrm{I}P}(\tau_0,\sigma) = \dot{Y}_{\mathrm{in}P}(\tau_0,\sigma) \\ &\qquad \sigma \in (0,\sigma_1)\cup(\sigma_2,2\pi); \\ &X_{\mathrm{II}M}(\tau_0,\sigma) = X_{\mathrm{in}M}(\tau_0,\sigma) \qquad \dot{X}_{\mathrm{II}M}(\tau_0,\sigma) = \dot{X}_{\mathrm{in}M}(\tau_0,\sigma) \\ &Y_{\mathrm{II}P}(\tau_0,\sigma) = Y_{\mathrm{in}P}(\tau_0,\sigma) \qquad \dot{Y}_{\mathrm{II}P}(\tau_0,\sigma) = \dot{Y}_{\mathrm{in}P}(\tau_0,\sigma) \\ &\qquad \sigma \in (\sigma_1,\sigma_2). \end{aligned} \tag{2.18}$$

The closed string periodicity conditions are imposed on each fragment separately:

$$\begin{aligned} &X_{\mathrm{I,II}M}(\tau,\sigma) = X_{\mathrm{I,II}M}(\tau,\sigma+2\pi_{\mathrm{I,II}}) \\ &Y_{\mathrm{I,II}P}(\tau,\sigma) = Y_{\mathrm{I,II}P}(\tau,\sigma+2\pi_{\mathrm{I,II}}), \\ &\text{where } 2\pi_{\mathrm{I}} = 2\pi-(\sigma_2-\sigma_1) \quad 2\pi_{\mathrm{II}} = \sigma_2-\sigma_1. \end{aligned} \tag{2.19}$$

Conditions (2.18) and (2.19) uniquely determine the final states. The relations between the energies ($E_{\mathrm{I,II}}$) and spins ($S_{\mathrm{I,II}n}$, $J_{\mathrm{I,II}k}$) of the cut fragments—together with "conservation laws" of splitting $E(E_{\mathrm{I}}, E_{\mathrm{II}},\ldots)$, $S(S_{\mathrm{I}}, S_{\mathrm{II}},\ldots)$, etc.—are completely determined by the charge conservation. Thus they may be found (at least parametrically) for the initial string solution of arbitrary complexity. Determining the evolution is much more complicated: one has to solve the string Eqs. (2.4) and (2.6) with the boundary conditions (2.18) and (2.19). At the moment, this is possible only in the simplest cases.

III. SPLITTING OF FOLDED STRINGS IN THE FLAT SPACE. A REVIEW

In this section we review splitting of the folded strings in flat Minkowski space [12]. The solution for the folded strings in flat Minkowski space reads

$$\begin{aligned} &X_0 = \ell\tau, \qquad X_1 = \rho\cos\phi = \ell\cos(\sigma)\cos(\tau), \\ &X_2 = \rho\sin\phi = \ell\cos(\sigma)\sin(\tau). \end{aligned} \tag{3.1}$$

The energy and spin

$$\mathcal{E} = \ell, \qquad \mathcal{J} = \frac{1}{2}\ell^2 \tag{3.2}$$

obey the standard Regge relation $\mathcal{E}^2 = 2\mathcal{J}$.

Any two points on the string parameterized by σ_1 and $\sigma_2 = 2\pi - \sigma_1$ coincide in the target space and their velocities agree at any given time. Let us assume that at $\tau_0 = 0$ the string splits into two pieces. The cut occurs at $X_1 = \ell\cos(a\pi)$, $X_2 = 0$, i.e. $\sigma_{\mathrm{cut}\,1} = a\pi$ and $\sigma_{\mathrm{cut}\,2} = 2\pi - a\pi$:

$$\begin{aligned} &\text{fragment I: } \sigma \in (0,a\pi)\cup(2\pi-a\pi,2\pi) \\ &\text{fragment II: } \sigma \in (a\pi,2\pi-a\pi), \\ &\qquad 0 < a < \frac{1}{2}. \end{aligned} \tag{3.3}$$

Here without loss of generality $0 < a < \frac{1}{2}$, i.e. the fragment I is always "smaller" than the fragment II (see schematic plot in Fig. 1).

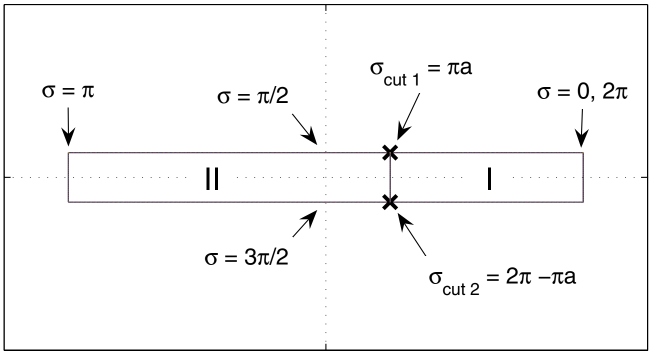

FIG. 1 (color online). Splitting of the folded string in the flat space.

Quantum numbers of the fragment I are the energy ($\mathcal{E}_{\rm I}$), linear momentum ($P_{{\rm I}i} = \sqrt{\lambda}\mathcal{P}_{{\rm I}i}$) and angular momentum ($\mathcal{J}_{\rm I}$):

$$\mathcal{E}_{\rm I} = \mathcal{P}_{\rm I0} = 2\int_0^{\pi a}\frac{d\sigma}{2\pi}\dot{X}_{0\rm I} = \ell a, \tag{3.4}$$

$$\mathcal{P}_{\rm I1} = 0, \tag{3.5}$$

$$\mathcal{P}_{\rm I2} = 2\int_0^{\pi a}\frac{d\sigma}{2\pi}\dot{X}_{\rm I2} = 2\int_0^{\pi a}\frac{d\sigma}{2\pi}\cos(\sigma) = \frac{\ell\sin(\pi a)}{\pi}, \tag{3.6}$$

$$\begin{aligned}\mathcal{J}_{\rm I} = \mathcal{L}_{\rm I} + \mathcal{S}_{\rm I} &= 2\int_0^{\pi a}\frac{d\sigma}{2\pi}(X_{\rm I1}\dot{X}_{\rm I2} - \dot{X}_{\rm I1}X_{\rm I2})\\ &= \ell^2 a\left(\frac{\sin(2\pi a)}{\pi a} - 1\right).\end{aligned} \tag{3.7}$$

Here the orbital momentum ($L_{\rm I} = \sqrt{\lambda}\mathcal{L}_{\rm I}$) and spin ($S_{\rm I} = \sqrt{\lambda}\mathcal{S}_{\rm I}$) are[1]

$$\mathcal{L}_{\rm I} = \ell^2 a\frac{\sin^2(\pi a)}{(\pi a)^2}, \quad \mathcal{S}_{\rm I} = \ell^2 a\left(\frac{\sin(2\pi a)}{\pi a} - \frac{\sin^2(\pi a)}{(\pi a)^2} - 1\right). \tag{3.8}$$

The mass of the fragment I, i.e. its energy in the center-of-mass system read

$$M_{\rm I}^2 = \mathcal{E}_{\rm I}^2 - \mathcal{P}_{\rm I}^2 = \ell^2\left(a^2 - \frac{\sin^2(\pi a)}{\pi^2}\right). \tag{3.9}$$

The conserved charges for the fragment II may be found similarly:

$$\begin{aligned}&\mathcal{E}_{\rm II} = \ell(1-a), \quad \mathcal{P}_{\rm II} = -\frac{\ell\sin(\pi a)}{\pi}, \quad \mathcal{J}_{\rm II} = \mathcal{L}_{\rm II} + \mathcal{S}_{\rm II},\\ &M_{\rm II}^2 = \mathcal{E}_{\rm II}^2 - \mathcal{P}_{\rm II}^2 = \ell^2\left((1-a)^2 - \frac{\sin^2(\pi a)}{\pi^2}\right),\\ &\mathcal{L}_{\rm II} = \ell^2(1-a)\frac{\sin^2(\pi a)}{(\pi(1-a))^2},\\ &\mathcal{S}_{\rm II} = -\ell^2(1-a)\left(1 + \frac{\sin^2(\pi a)}{(\pi(1-a))^2} + \frac{\sin(2\pi a)}{\pi(1-a)}\right).\end{aligned} \tag{3.10}$$

The energy, linear momentum, and angular momentum are conserved in the process of splitting:

$$\mathcal{E}_{\rm I} + \mathcal{E}_{\rm II} = \mathcal{E}, \quad \mathcal{P}_{\rm I2} + \mathcal{P}_{\rm II2} = 0, \quad \mathcal{J}_{\rm I} + \mathcal{J}_{\rm II} = \mathcal{J}. \tag{3.11}$$

The string solution describing the evolution of the final states may be found using the general solution for a closed bosonic string in flat Minkowski space. Imposing the boundary and periodicity conditions [12] on it, one finds

$$\begin{aligned}X_{\rm I0} &= 2\ell\tau,\\ X_{\rm I1} &= \frac{\ell\sin(\pi a)}{\pi a}\left(1 + 2\sum_{n=1}^{\infty}\frac{(-1)^n}{1 - \frac{n^2}{a^2}}\cos\left(\frac{n\tau}{a}\right)\cos\left(\frac{n\sigma}{a}\right)\right),\\ X_{\rm I2} &= \frac{\ell\sin(\pi a)}{\pi a}\left(\tau + 2a\sum_{n=1}^{\infty}\frac{(-1)^n}{n(1-\frac{n^2}{a^2})}\right.\\ &\quad\left.\times\sin\left(\frac{2n\tau}{a}\right)\cos\left(\frac{2n\sigma}{a}\right)\right),\end{aligned} \tag{3.12}$$

where $-\pi a < \sigma < \pi a$, and

$$\begin{aligned}X_{\rm II0} &= 2\ell\tau,\\ X_{\rm II1} &= -\frac{\ell\sin(\pi a)}{\pi(1-a)}\left(1 + 2\sum_{n=1}^{\infty}\frac{(-1)^n}{1 - \frac{n^2}{(1-a)^2}}\right.\\ &\quad\left.\times\cos\left(\frac{n\tau}{(1-a)}\right)\cos\left(\frac{n\sigma}{(1-a)}\right)\right),\\ X_{\rm II2} &= -\frac{\ell\sin(\pi a)}{\pi(1-a)}\left(\tau + 2(1-a)\sum_{n=1}^{\infty}\frac{(-1)^n}{n(1-\frac{n^2}{(1-a)^2})}\right.\\ &\quad\left.\times\sin\left(\frac{n\tau}{(1-a)}\right)\cos\left(\frac{n\sigma}{(1-a)}\right)\right),\end{aligned} \tag{3.13}$$

where $-\pi(1-a) < \sigma < \pi(1-a)$.

Summing the series up, we obtain

$$X_{{\rm I,II}\mu}(\sigma,\tau) = X^{+}_{{\rm I,II}\mu}(\sigma^+) + X^{-}_{{\rm I,II}\mu}(\sigma^-), \qquad \sigma^{\pm} = \sigma \pm \tau, \tag{3.14}$$

where

[1] Orbital momentum is defined as $\mathcal{L}_{\rm I} = X_{cm\rm I}\mathcal{P}_{\rm I2}$, where $X_{cm\rm I}$ is the coordinate of the center of mass of the string

$$X_{cm\rm I} = \frac{1}{\pi a}\int_0^{\pi a} d\sigma X_{\rm I1} = \frac{\ell\sin(\pi a)}{\pi a}.$$

$$X^{\pm}_{\mathrm{I}0} = \pm\frac{\ell}{2}a\sigma^{\pm}, \quad X^{\pm}_{\mathrm{I}1} = \frac{\ell}{2}\mathbb{C}_{\mathrm{I}}(\sigma^{\pm}),$$
$$X^{\pm}_{\mathrm{I}2} = \pm\frac{\ell}{2}\Big[\frac{\sin(a\pi)}{\pi}\sigma^{\pm} + \mathbb{S}_{\mathrm{I}}(\sigma^{\pm})\Big],$$
$$\mathbb{C}_{\mathrm{I}}(\xi) = \cos(a\xi), \quad \mathbb{S}_{\mathrm{I}}(\xi) = \sin(a\xi) - \frac{\sin(a\pi)}{\pi}\xi \quad \text{for } 0 \le \xi < \pi,$$
$$\mathbb{C}_{\mathrm{I}}(\xi) = \cos(a\xi - 2a\pi),$$
$$\mathbb{S}_{\mathrm{I}}(\xi) = \sin(a\xi - 2a\pi) - \frac{\sin(a\pi)}{\pi}(\xi - \pi) \quad \text{for } \pi \le \xi < 2\pi \tag{3.15}$$

and

$$X^{\pm}_{\mathrm{II}0} = \pm\frac{\ell}{2}(1-a)\sigma^{\pm}, \qquad X^{\pm}_{\mathrm{II}1} = \frac{\ell}{2}\mathbb{C}_{\mathrm{II}}(\sigma^{\pm}),$$
$$X^{\pm}_{\mathrm{II}2} = \pm\frac{\ell}{2}\Big[-\frac{\sin(a\pi)}{\pi}\sigma^{\pm} + \mathbb{S}_{\mathrm{II}}(\sigma^{\pm})\Big],$$
$$\mathbb{C}_{\mathrm{II}}(\xi) = \cos((1-a)\xi + a\pi),$$
$$\mathbb{S}_{\mathrm{II}}(\xi) = \sin((1-a)\xi + a\pi) + \frac{\sin(a\pi)}{\pi}\xi \quad \text{for } 0 \le \xi < 2\pi. \tag{3.16}$$

In the expressions (3.15) and (3.16), the world-sheet parameters are rescaled as

$$\begin{aligned} &\text{fragment I: } \tau, \sigma \to a\tau, a\sigma \\ &\text{fragment II: } \tau, \sigma \to (1-a)\tau, (1-a)\sigma. \end{aligned} \tag{3.17}$$

The derivatives $X'_{\mathrm{I,II}i}$, $i = 1, 2$ have discontinuities at the points of splitting, i.e. at $\sigma^{\pm} = \pi$ for the fragment I and $\sigma^{\pm} = 0$ for the fragment II. These discontinuities show up as an angular bending on the folded shape of the strings moving along the strings as a function of τ (for more details see the original paper [12]). Eqs. (2.6) are satisfied at each point on the string, in spite of the discontinuity. The δ-functions arising from the second derivative $\partial_{\sigma,\sigma}X_{\mathrm{I,II}i}$ cancel with those coming from $\partial_{\tau,\tau}X_{\mathrm{I,II}i}$, due to the chiral properties of (3.14).

IV. SPLITTING OF FOLDED STRINGS IN AdS_3

In this section we discuss splitting of Gubser-Klebanov-Polyakov folded spinning strings in AdS_3.

A. Folded string in AdS_3

The folded string solution in the AdS_3 in the embedding coordinates read [17]

$$Y_{05} = \cosh\rho\, e^{i\kappa\tau}, \qquad Y_{12} = \sinh\rho\, e^{i\omega\tau}, \tag{4.1}$$

where

$$\sinh\rho = \ell\,\mathrm{sn}[\kappa\ell^{-1}\sigma, -\ell^2],$$
$$\cosh\rho = \mathrm{dn}[\kappa\ell^{-1}\sigma, -\ell^2], \tag{4.2}$$
$$\frac{w^2}{\kappa^2} = 1 + \frac{1}{\ell^2}.$$

Here $\mathrm{sn}[z, m]$ and $\mathrm{dn}[z, m]$ are the Jacobi elliptic functions, ℓ defines the length of the string: $\sinh\rho_{\max} = \ell$.

Expressions (4.2) are valid on the interval $0 \le \sigma < \frac{\pi}{2}$ only. To get the formal periodic solution on the interval $0 \le \sigma < 2\pi$ one has to combine four stretches of (4.2):

$$Y_{05} = \cosh\rho(\sigma)e^{i\kappa\tau} \quad Y_{12} = \sinh\rho(\sigma)e^{i\omega\tau} \quad \text{for } \sigma \in [0, \tfrac{\pi}{2})$$
$$Y_{05} = \cosh\rho(\pi - \sigma)e^{i\kappa\tau} \quad Y_{12} = \sinh\rho(\pi - \sigma)e^{i\omega\tau} \quad \text{for } \sigma \in [\tfrac{\pi}{2}, \pi)$$
$$Y_{05} = \cosh\rho(\sigma - \pi)e^{i\kappa\tau} \quad Y_{12} = -\sinh\rho(\sigma - \pi)e^{i\omega\tau} \quad \text{for } \sigma \in [\pi, \tfrac{3\pi}{2})$$
$$Y_{05} = \cosh\rho(2\pi - \sigma)e^{i\kappa\tau} \quad Y_{12} = -\sinh\rho(2\pi - \sigma)e^{i\omega\tau} \quad \text{for } \sigma \in [\tfrac{3\pi}{2}, 2\pi). \tag{4.3}$$

and impose

$$Y_P(\sigma + 2\pi) = Y_P(\sigma). \tag{4.4}$$

The closed string periodicity conditions require

$$\kappa = \frac{2}{\pi}\ell\mathbb{K}[-\ell^2]. \tag{4.5}$$

The energy and spin are

$$\mathcal{E} = \frac{2}{\pi}\ell\mathbb{E}[-\ell^2], \quad \mathcal{S} = \frac{2}{\pi}\sqrt{1+\ell^2}(\mathbb{E}[-\ell^2] - \mathbb{K}[-\ell^2]). \tag{4.6}$$

Here $\mathbb{K}[z]$ and $\mathbb{E}[z]$ are the complete elliptic integrals of the first and second kinds, respectively.

The classical energy of the string in the limit of large spin is [17,24][2]

$$E \simeq S + \frac{\sqrt{\lambda}}{\pi}\ln\frac{S}{\sqrt{\lambda}} + \ldots, \qquad \frac{S}{\sqrt{\lambda}} \gg 1. \tag{4.7}$$

B. Large-spin limit. Formal $\kappa = \omega$ approximation

There is a useful simplification of the solution (4.2), when the spin of the folded string is large:

$$\rho = \kappa\sigma, \qquad \kappa = \omega \gg 1. \tag{4.8}$$

This is a formal limit, as $\kappa \to \omega$ implies $\ell \to \infty$.

[2] There is an elegant method to obtain expansion for $\mathcal{E}(\mathcal{S})$ in large or small $\mathcal{S}$ with arbitrary accuracy [25].

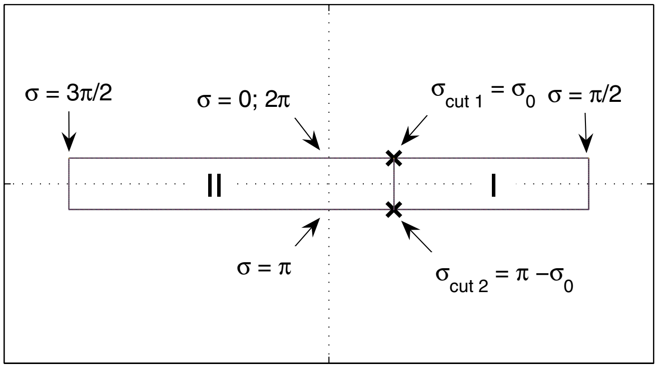

FIG. 2 (color online). Splitting of the folded string in AdS_3.

The energy and spin read

$$\mathcal{E} = \mathcal{S}_{05} \simeq \frac{\kappa}{2\pi}\pi + \frac{1}{4\pi}e^{\kappa\pi}, \quad \mathcal{S} = \mathcal{S}_{12} \simeq -\frac{\kappa}{2\pi}\pi + \frac{1}{4\pi}e^{\kappa\pi}. \tag{4.9}$$

Expansion of the classical energy in large $\mathcal{S}$ is consistent with the one coming from (4.6) in the first two orders[3]

$$E \simeq S + \frac{\sqrt{\lambda}}{\pi}\ln\frac{S}{\sqrt{\lambda}} + \ldots, \qquad \frac{S}{\sqrt{\lambda}} \gg 1. \tag{4.10}$$

Any two points on the string parameterized by σ_1 and $\sigma_2 = \pi - \sigma_1$ coincide in the target space and their velocities agree at any given time. Let us assume that at $\tau_0 = 0$ the string splits into two pieces. The cut occurs at $\rho = \kappa\sigma_0$, i.e. $\sigma_{\text{cut 1}} = \sigma_0$ and $\sigma_{\text{cut 2}} = \pi - \sigma_0$

$$\text{fragment I: } \sigma \in (\sigma_0, \pi - \sigma_0)$$
$$\text{fragment II: } \sigma \in (0, \sigma_0) \cup (\pi - \sigma_0, 2\pi), \quad 0 < \sigma_0 < \frac{1}{2}. \tag{4.11}$$

Here without loss of generality $0 < \sigma_0 < \frac{\pi}{2}$, i.e. the fragment I is always smaller than the fragment II (see schematic plot in Fig. 2).

Approximation (4.8) is invalid close to the string ends, thus we have to demand

$$\frac{\pi}{2} - \sigma_0 \gg \frac{1}{\kappa}. \tag{4.12}$$

The charges $(\mathcal{S}_{\text{I,II}})_{PQ}$ of the cut fragments read

$$\begin{aligned}
(\mathcal{S}_{\text{I,II}})_{05} &= \frac{\kappa}{2\pi}\left(\frac{\pi}{2} \mp \sigma_0\right) + \frac{\sinh(\kappa\pi) \mp \sinh(2\kappa\sigma_0)}{4\pi}, \\
(\mathcal{S}_{\text{I,II}})_{12} &= -\frac{\kappa}{2\pi}\left(\frac{\pi}{2} \mp \sigma_0\right) + \frac{\sinh(\kappa\pi) \mp \sinh(2\kappa\sigma_0)}{4\pi}, \\
(\mathcal{S}_{\text{I,II}})_{02} &= -(\mathcal{S}_{\text{I,II}})_{51} = \pm\frac{\cosh(\kappa\pi) - \cosh(2\kappa\sigma_0)}{4\pi}, \\
(\mathcal{S}_{\text{I,II}})_{01} &= 0, \qquad (\mathcal{S}_{\text{I}})_{52} = 0.
\end{aligned} \tag{4.13}$$

They are conserved in the process of splitting

$$\mathcal{E} = \mathcal{S}_{05} = (\mathcal{S}_{\text{I}})_{05} + (\mathcal{S}_{\text{II}})_{05}, \quad \mathcal{S} = (\mathcal{S}_{\text{I}})_{12} + (\mathcal{S}_{\text{II}})_{12},$$
$$(\mathcal{S}_{\text{in}})_{02} = (\mathcal{S}_{\text{I}})_{02} + (\mathcal{S}_{\text{II}})_{02}, \quad (\mathcal{S})_{51} = (\mathcal{S}_{\text{I}})_{51} + (\mathcal{S}_{\text{II}})_{51}. \tag{4.14}$$

Spins of the fragments I and II given in (4.13) have non-Cartan components, as they are written in the "center-of-mass system" of the initial string (the coordinate system where $\mathcal{S}_{PQ}$ of the string has Cartan components only). It is more natural to analyze the fragments in their own center-of-mass systems. Let us diagonalize $(\mathcal{S}_{\text{I,II}})_{PQ}$.

Performing the boost rotations independently for each string[4]

$$\begin{pmatrix} \tilde{Y}_{\text{I,II}0} \\ \tilde{Y}_{\text{I,II}1} \end{pmatrix} = \begin{pmatrix} \cosh\alpha_{\text{I,II}} & \sinh\alpha_{\text{I,II}} \\ \sinh\alpha_{\text{I,II}} & \cosh\alpha_{\text{I,II}} \end{pmatrix} \begin{pmatrix} Y_{\text{I,II}0} \\ Y_{\text{I,II}1} \end{pmatrix} \tag{4.15}$$

$$\begin{pmatrix} \tilde{Y}_{\text{I,II}5} \\ \tilde{Y}_{\text{I,II}2} \end{pmatrix} = \begin{pmatrix} \cosh\beta_{\text{I,II}} & \sinh\beta_{\text{I,II}} \\ \sinh\beta_{\text{I,II}} & \cosh\beta_{\text{I,II}} \end{pmatrix} \begin{pmatrix} Y_{\text{I,II}5} \\ Y_{\text{I,II}2} \end{pmatrix} \tag{4.16}$$

with the parameters (α_{I}, β_{I} for the fragment I and α_{II}, β_{II} for the fragment II)

$$\alpha_{\text{I,II}} = \beta_{\text{I,II}} = \mp\frac{\kappa}{2}\left(\frac{\pi}{2} \pm \sigma_0\right), \tag{4.17}$$

we find the energies and spins of the cut fragments in their own center-of-mass systems[5]

$$\mathcal{E}_{\text{I,II}} \simeq \frac{\kappa}{2\pi}\left(\frac{\pi}{2} \mp \sigma_0\right) + \frac{e^{\kappa((\pi/2)\mp\sigma_0)}}{4\pi}, \quad \mathcal{S}_{\text{I,II}} \simeq -\frac{\kappa}{2\pi}\left(\frac{\pi}{2} \mp \sigma_0\right) + \frac{e^{\kappa((\pi/2)\mp\sigma_0)}}{4\pi}. \tag{4.18}$$

These expressions coincide with (4.9) up to parameter definitions. The expansions of the classical energies $E_{\text{I,II}}(S_{\text{I,II}})$ in large spins obviously agree with (4.10):

[3]One has to be careful using (4.8) for computing charges. It is easy to see, that the absolute values of $\mathcal{E}$ and $\mathcal{S}$ in (4.9) approximately twice exceed those of (4.6) taken at equal κ. This inconsistency comes from the fact, that approximation (4.8) is invalid close to the string ends [24], while the largest contribution to the charges comes exactly from them.

[4]Any rotation in the Y_0Y_5, Y_1Y_2, Y_0Y_2 or Y_5Y_1 would result in $(\mathcal{S}_{\text{I,II}})_{01}$ and $(\mathcal{S}_{\text{I,II}})_{52}$ gaining nonzero values. We are left only with boosts in Y_0Y_1 and Y_5Y_2 planes.

[5]Making use of (4.12), we set $\sinh(\kappa(\frac{\pi}{2} \mp \sigma_0)) \sim \frac{1}{2}e^{\kappa((\pi/2)\mp\sigma_0)}$.

$$E_{\mathrm{I,II}} \simeq S_{\mathrm{I,II}} + \frac{\sqrt{\lambda}}{\pi}\ln\frac{S_{\mathrm{I,II}}}{\sqrt{\lambda}} + \ldots, \qquad \frac{S_{\mathrm{I,II}}}{\sqrt{\lambda}} \gg 1. \tag{4.19}$$

Let us find the string solutions describing the evolution of the cut fragments.

The evolution of the fragment I is governed by the string Eqs. (2.4) and (2.6) with the initial conditions at $\tau_0 = 0$ (written in the center-of-mass of the fragment):

$$\begin{aligned}
&\tilde{Y}_{\mathrm{I}0} = \cosh\left[\kappa\left(\sigma - \frac{\pi}{4} - \frac{\sigma_0}{2}\right)\right],\\
&\tilde{Y}_{\mathrm{I}1} = \sinh\left[\kappa\left(\sigma - \frac{\pi}{4} - \frac{\sigma_0}{2}\right)\right],\\
&\tilde{Y}_{\mathrm{I}5} = 0, \qquad \tilde{Y}_{I2} = 0,\\
&\frac{\partial}{\partial\tau}\tilde{Y}_{\mathrm{I}0} = 0, \qquad \frac{\partial}{\partial\tau}\tilde{Y}_{I1} = 0,\\
&\frac{\partial}{\partial\tau}\tilde{Y}_{\mathrm{I}5} = \kappa\cosh\left[\kappa\left(\sigma - \frac{\pi}{4} - \frac{\sigma_0}{2}\right)\right],\\
&\frac{\partial}{\partial\tau}\tilde{Y}_{\mathrm{I}2} = \kappa\sinh\left[\kappa\left(\sigma - \frac{\pi}{4} - \frac{\sigma_0}{2}\right)\right]
\end{aligned} \tag{4.20}$$

for the interval $\sigma_0 < \sigma < \frac{\pi}{2}$ and the same expressions with $\sigma \to \pi - \sigma$ for the interval $\frac{\pi}{2} < \sigma < \pi - \sigma_0$.

After rescaling of the world-sheet parameters σ to ξ in such a way that $\sigma_0 < \sigma < \frac{\pi}{2} \to -\frac{\pi}{2} < \xi < \frac{\pi}{2}$:

$$\sigma = \frac{\pi - 2\sigma_0}{2\pi}\xi + \frac{\pi}{4} + \frac{\sigma_0}{2} \quad \text{and} \quad \tau = \frac{\pi - 2\sigma_0}{2\pi\eta}, \tag{4.21}$$

we rewrite (4.20) in the following form

$$\begin{aligned}
&\tilde{Y}_{\mathrm{I}0} = \cosh(\kappa_{\mathrm{I}}\xi), \quad \tilde{Y}_{\mathrm{I}1} = \sinh(\kappa_{\mathrm{I}}\xi), \quad \tilde{Y}_{\mathrm{I}5} = 0, \quad \tilde{Y}_{\mathrm{I}2} = 0,\\
&\frac{\partial}{\partial\eta}\tilde{Y}_{\mathrm{I}0} = 0, \quad \frac{\partial}{\partial\eta}\tilde{Y}_{\mathrm{I}1} = 0,\\
&\frac{\partial}{\partial\eta}\tilde{Y}_{\mathrm{I}5} = \kappa_{\mathrm{I}}\cosh(\kappa_{\mathrm{I}}\xi), \quad \frac{\partial}{\partial\eta}\tilde{Y}_{\mathrm{I}2} = \kappa_{\mathrm{I}}\sinh(\kappa_{\mathrm{I}}\xi),\\
&\kappa_{\mathrm{I}} = \kappa\frac{\pi - 2\sigma_0}{2\pi}.
\end{aligned} \tag{4.22}$$

Such boundary conditions are satisfied by

$$\tilde{Y}_{\mathrm{I}05} = \cosh(\kappa_{\mathrm{I}}\xi)e^{i\kappa_{\mathrm{I}}\eta}, \quad \tilde{Y}_{\mathrm{I}12} = \sinh(\kappa_{\mathrm{I}}\xi)e^{i\kappa_{\mathrm{I}}\eta}. \tag{4.23}$$

That is the same as (4.8) up to parameter definitions.

For the fragment II we get similar result

$$\begin{aligned}
&\tilde{Y}_{\mathrm{II}05} = \cosh(\kappa_{\mathrm{II}}\xi)e^{i\kappa_{\mathrm{II}}\eta},\\
&\tilde{Y}_{\mathrm{II}12} = \sinh(\kappa_{\mathrm{II}}\xi)e^{i\kappa_{\mathrm{II}}\eta},\\
&\kappa_{\mathrm{II}} = \kappa\frac{\pi + 2\sigma_0}{2\pi}.
\end{aligned} \tag{4.24}$$

Making use of (4.9) and (4.18) the following conservation laws of the splitting may be derived[6]

$$\begin{aligned}
\mathcal{E}^{1-2/\mathcal{E}} &= 4\pi\mathcal{E}_{\mathrm{I}}^{1-2/\mathcal{E}_{\mathrm{I}}}\mathcal{E}_{\mathrm{II}}^{1-2/\mathcal{E}_{\mathrm{II}}},\\
\mathcal{S}^{1+2/\mathcal{S}} &= 4\pi\mathcal{S}_{\mathrm{I}}^{1+2/\mathcal{S}_{\mathrm{I}}}\mathcal{S}_{\mathrm{II}}^{1+2/\mathcal{S}_{\mathrm{II}}}.
\end{aligned} \tag{4.25}$$

The boost parameters (4.17) may be expressed as

$$\alpha_{\mathrm{I,II}} = \beta_{\mathrm{I,II}} \simeq \mp\ln\frac{\mathcal{E}^{1-2/\mathcal{E}}}{\mathcal{E}_{I,\mathrm{II}}^{1-2/\mathcal{E}_{\mathrm{I,II}}}} \simeq \mp\ln\frac{\mathcal{S}^{1+2/\mathcal{S}}}{\mathcal{S}_{I,\mathrm{II}}^{1+2/\mathcal{S}_{\mathrm{I,II}}}}. \tag{4.26}$$

Given (4.8), (4.23), (4.24), and (4.17) we see that when the initial string described by the formal $\kappa = \omega$ limit of the folded string (4.8) splits into two pieces the world surface does not change and the fragments are described by the solutions of the same type as the parent string.

It is interesting to point out that (4.8) is not just a formal approximation of the folded string profile (in the limit $\kappa = \omega \to \infty$), but a true solution of the string Eqs. (2.4) and (2.6) (with arbitrary values of $\kappa = \omega$). Strings of this type have a peculiar property. They may be divided into an arbitrary number of fragments, each of which is an independent solution of the same type as (4.8), simply boosted from its center of mass. However, its stretches may not be consistently glued to form a closed string. Such glued string would have jumps of the first derivatives at the string ends $\rho'(\sigma_{\mathrm{ends}} + 0) \neq \rho'(\sigma_{\mathrm{ends}} - 0)$, resulting in $\rho''(\sigma_{\mathrm{ends}}) \sim \delta(\sigma \pm \sigma_{\mathrm{ends}})$, and, consequently, would not satisfy (2.6).[7] The δ-functions arising on the right-hand side of Eq. (2.4) may be interpreted as point masses attached to the string ends [26].

C. String with an arbitrary spin. The general case

In this section we discuss the most general case of splitting of folded strings in AdS_3. Starting with the folded string solution in its exact form (4.1), (4.2), and (4.5) and, following the approach of Sec. IV B, we assume that the string splits into two fragments (I and II) defined in (4.11) and in Fig. 2. Their charges $(\mathcal{S}_{\mathrm{I,II}})_{PQ}$ read

[6]Here we used the relations

$$\mathcal{E} = \frac{\kappa}{2\pi}\pi + \frac{1}{4\pi}e^{\kappa\pi} \Rightarrow \ln\mathcal{E} = \kappa\pi - \ln 4\pi + 2\pi\kappa e^{-\kappa\pi}$$
$$\Rightarrow \kappa\pi = \ln\mathcal{E} + \ln 4\pi - \frac{2}{\mathcal{E}}\ln\mathcal{E}.$$

[7]In the flat space this inconsistency is avoided due to the chiral properties of the solutions for the final fragments (see above).

$$
\begin{aligned}
(\mathcal{S}_{\rm I,II})_{05} &= \kappa\mathbb{C}_{\rm I,II} = \frac{\ell}{\pi}(\mathbb{E}[-\ell^2] \mp \mathbb{E}[\mathrm{am}[\kappa\ell^{-1}\sigma_0, -\ell^2], -\ell^2]),\\
(\mathcal{S}_{\rm I,II})_{12} &= \omega\mathbb{S}_{\rm I,II} = -\frac{\sqrt{1+\ell^2}}{\pi^2}(\pi \mp 2\sigma_0)\mathbb{K}[-\ell^2]\\
&\quad + \frac{\sqrt{1+\ell^2}}{\pi^2}(\mathbb{E}[-\ell^2] \mp \mathbb{E}[\mathrm{am}[\kappa\ell^{-1}\sigma_0, -\ell^2], -\ell^2]),\\
(\mathcal{S}_{\rm I,II})_{02} &= \omega\mathbb{M}_{\rm I,II} = \pm\frac{1}{\pi}\ell\sqrt{1+\ell^2}\,\mathrm{cn}[\kappa\ell^{-1}\sigma_0, -\ell^2],\\
(\mathcal{S}_{\rm I,II})_{15} &= \kappa\mathbb{M}_{\rm I,II} = \pm\frac{1}{\pi}\ell^2\mathrm{cn}[\kappa\ell^{-1}\sigma_0, -\ell^2],\\
(\mathcal{S}_{\rm I,II})_{01} &= (\mathcal{S}_{\rm I,II})_{52} = 0,
\end{aligned}
\tag{4.27}
$$

where $\mathbb{E}[z, m]$ is the incomplete elliptic integral of the second kind. We want to find a coordinate systems where the non-Cartan components of the spins vanish and find the energies and spins of the final fragments. $(\mathcal{S}_{\rm I,II})_{PQ}$ may be diagonalized by boosts in the Y_0Y_1 and Y_5Y_2 planes (4.15) and (4.16) with parameters[8]

$$
\begin{aligned}
\sinh(\alpha_{\rm I,II} + \beta_{\rm I,II}) &= -\frac{\mathbb{M}_{\rm I,II}(\omega+\kappa)}{\sqrt{(\kappa\mathbb{C}_{\rm I,II} + \omega\mathbb{S}_{\rm I,II})^2 - \mathbb{M}_{\rm I,II}^2(\omega+\kappa)^2}}\\
\sinh(\alpha_{\rm I,II} - \beta_{\rm I,II}) &= \frac{\mathbb{M}_{\rm I,II}(\omega-\kappa)}{\sqrt{(\kappa\mathbb{C}_{\rm I,II} - \omega\mathbb{S}_{\rm I,II})^2 - \mathbb{M}_{\rm I,II}^2(\omega-\kappa)^2}},
\end{aligned}
\tag{4.28}
$$

where

$$
\begin{aligned}
\mathbb{C}_{\rm I,II} &= \frac{\ell}{\kappa\pi}(\mathbb{E}[-\ell^2] \mp \mathbb{E}[\mathrm{am}[\kappa\ell^{-1}\sigma_0, -\ell^2], -\ell^2]),\\
\mathbb{S}_{\rm I,II} &= -\frac{\pi \mp 2\sigma_0}{2} + \mathbb{C}_{\rm I,II}\\
\mathbb{M}_{\rm I,II} &= \pm\frac{\ell^2}{\kappa\pi}\mathrm{cn}[\kappa\ell^{-1}\sigma_0, -\ell^2],\\
\kappa &= \frac{2}{\pi}\ell\mathbb{K}[-\ell^2].
\end{aligned}
\tag{4.29}
$$

Then the energies and spins of the cut fragments read

[8] Vanishing of the non-Cartan components of the spins implies

$$
\begin{aligned}
&\mathbb{M}_{\rm I,II}(\kappa+\omega)\cosh(\alpha_{\rm I,II}+\beta_{\rm I,II}) + (\kappa\mathbb{C}_{\rm I,II} + \omega\mathbb{S}_{\rm I,II})\\
&\quad\times\sinh(\alpha_{\rm I,II}+\beta_{\rm I,II}) = 0\\
&\mathbb{M}_{\rm I,II}(\kappa-\omega)\cosh(\alpha_{\rm I,II}-\beta_{\rm I,II}) + (\kappa\mathbb{C}_{\rm I,II} - \omega\mathbb{S}_{\rm I,II})\\
&\quad\times\sinh(\alpha_{\rm I,II}-\beta_{\rm I,II}) = 0.
\end{aligned}
$$

That leads to (4.28).

$$
\begin{aligned}
\mathcal{E}_{\rm I,II} &= \frac{1}{2}\sqrt{(\kappa\mathbb{C}_{\rm I,II} + \omega\mathbb{S}_{\rm I,II})^2 - \mathbb{M}_{\rm I,II}^2(\omega+\kappa)^2}\\
&\quad + \frac{1}{2}\sqrt{(\kappa\mathbb{C}_{\rm I,II} - \omega\mathbb{S}_{\rm I,II})^2 - \mathbb{M}_{\rm I,II}^2(\omega-\kappa)^2}\\
\mathcal{S}_{\rm I,II} &= \frac{1}{2}\sqrt{(\kappa\mathbb{C}_{\rm I,II} + \omega\mathbb{S}_{\rm I,II})^2 - \mathbb{M}_{\rm I,II}^2(\omega+\kappa)^2}\\
&\quad - \frac{1}{2}\sqrt{(\kappa\mathbb{C}_{\rm I,II} - \omega\mathbb{S}_{\rm I,II})^2 - \mathbb{M}_{\rm I,II}^2(\omega-\kappa)^2}.
\end{aligned}
\tag{4.30}
$$

These relations parametrically encode the conservation laws of splitting, e.g. $E(E_{\rm I}, E_{\rm II})$, $S(S_{\rm I}, S_{\rm II})$, etc.

The evolution of the fragments I and II is governed by the string Eqs. (2.4) and (2.6) with the boundary conditions given by the initial string (4.1), (4.2), and (4.5) on the intervals (4.11) at $\tau_0 = 0$. Because of the complexity of the folded string profile (4.2), we are unable to find solutions to these equations. However, we could describe the evolution qualitatively based on the result of Sec. IV B and III, in the limit of large—but not infinitely large (as in (4.8))—spin, so long as the cut occurs far enough from the string ends for σ_0 to satisfy $\kappa(\pi/2 - \sigma_0) \gg 1$. In this case, one should expect the final pieces have almost the standard folded shape (4.1), (4.2), and (4.5), which is disturbed by a kink moving along the string, similar to that observed in flat Minkowski space; see Sec. III and [12]. The kink is a correction to the leading folded shape of the cut fragments, thus the angle of bending has to depend on the position of the kink. It may be substantial at the string ends but must be small close to the center.

V. SPLITTING OF STRINGS IN $\mathrm{AdS}_3 \times \mathrm{S}^5$

In this section we generalize the results for the splitting of the folded string in AdS_3 to $\mathrm{AdS}_3 \times \mathrm{S}^5$, including into consideration circular rotations and windings in S^5.

Let us consider the string solution having the folded shape in the AdS_3 and the circular one with windings in S^3:

$$
\begin{aligned}
&Y_{05} = \cosh\rho\, e^{i\kappa\tau}, \quad Y_{12} = \sinh\rho\, e^{i\omega\tau},\\
&X_{12} = a e^{i(\nu\tau + m\sigma)}, \quad X_{34} = b e^{i(\nu\tau - m\sigma)},\\
&a^2 + b^2 = 1, \quad m \in N,
\end{aligned}
\tag{5.1}
$$

where

$$
\begin{aligned}
&\sinh\rho = \tilde{\ell}\,\mathrm{sn}[\tilde{\kappa}\tilde{\ell}^{-1}\sigma, -\tilde{\ell}^2], \quad \tilde{\kappa} = \frac{2}{\pi}\tilde{\ell}\mathbb{K}[-\tilde{\ell}^2], \quad \frac{\tilde{\omega}^2}{\tilde{\kappa}^2} = 1 + \frac{1}{\tilde{\ell}^2},\\
&\tilde{\omega}^2 = \omega^2 - (\nu^2 + m^2), \quad \tilde{\kappa}^2 = \kappa^2 - (\nu^2 + m^2).
\end{aligned}
\tag{5.2}
$$

Comparing that with (4.1), (4.2), and (4.5), we see that the only result of accounting for the S^3 part is redefinition of κ and $\omega \rightarrow \tilde{k}$ and $\tilde{\omega}$. That is also true if one adds other spins and windings in S^5.

Combining together four stretches of (5.1), each of which is valid on the interval $0 \le \sigma < \frac{\pi}{2}$, we obtain a periodic solution on the interval $0 \le \sigma < 2\pi$. Its classical energy and spins read

$$\mathcal{E}_{\mathcal{J}} = \frac{2}{\pi}\frac{\kappa}{\tilde{\kappa}}\tilde{\ell}\mathbb{E}[-\tilde{\ell}^2] = \sqrt{\frac{4}{\pi^2}\tilde{\ell}^2 + \frac{m^2 + (\mathcal{J}_1+\mathcal{J}_2))^2}{\mathbb{K}^2[-\tilde{\ell}^2]}}\mathbb{E}[-\tilde{\ell}^2],$$
$$\mathcal{S}_{\mathcal{J}} = \frac{2}{\pi}\frac{\omega}{\tilde{\omega}}\sqrt{1+\tilde{\ell}^2}(\mathbb{E}[-\tilde{\ell}^2] - \mathbb{K}[-\tilde{\ell}^2])$$
$$= \sqrt{\frac{4}{\pi^2}(1+\tilde{\ell}^2) + \frac{m^2 + (\mathcal{J}_1+\mathcal{J}_2))^2}{\mathbb{K}^2[-\tilde{\ell}^2]}}(\mathbb{E}[-\tilde{\ell}^2] - \mathbb{K}[-\tilde{\ell}^2]),$$
$$\mathcal{J}_1 = a^2\nu, \quad \mathcal{J}_2 = b^2\nu, \quad \nu = \mathcal{J}_1 + \mathcal{J}_2. \tag{5.3}$$

Following the approach of Sec. IV, first, we consider the limit of the string with large spin in AdS_3. Then the AdS-part of the solution (5.2) may be approximated by

$$\rho = \sqrt{\kappa^2 - (\nu^2 + m^2)}\sigma = \tilde{\kappa}\sigma, \quad \kappa = \omega, \quad \tilde{\kappa} \gg 1. \tag{5.4}$$

This is a formal limit as $\kappa \to \omega$ implies $\tilde{\ell} \to \infty$.

The energy and AdS-spin of the string read

$$\mathcal{E}_{\mathcal{J}} = \mathcal{S}_{05} \simeq \frac{\kappa}{2\pi}\pi + \frac{\kappa}{4\pi\tilde{\kappa}}e^{\tilde{\kappa}\pi},$$
$$\mathcal{S}_{\mathcal{J}} = \mathcal{S}_{12} \simeq -\frac{\kappa}{2\pi}\pi + \frac{\kappa}{4\pi\tilde{\kappa}}e^{\tilde{\kappa}\pi}. \tag{5.5}$$

Spins in S^3 are unaffected by the limit.

Two points on the string parameterized by σ_1 and σ_2 coincide in the target space and their velocities agree, if $\sigma_1 = \sigma_0$, $\sigma_2 = \pi - \sigma_0$, and

$$\sigma_0 = \left(\frac{1}{2} - \frac{n}{m}\right)\pi, \qquad n \in N, \qquad \text{if } m \neq 0 \tag{5.6}$$

or for arbitrary σ_0 if $m = 0$. The string is not folded in $AdS_3 \times S^3$, when $m \neq 0$.

Approximation (5.4) is invalid close to the string ends, thus we have to demand

$$\frac{\pi}{2} - \sigma_0 \gg \frac{1}{\tilde{\kappa}} \tag{5.7}$$

for the coordinates of the cut ($\sigma_{\text{cut}1} = \sigma_0$ and $\sigma_{\text{cut}2} = \pi - \sigma_0$).

The charges $(\mathcal{S}^{\mathcal{J}}_{\text{I,II}})_{PQ}$ of the cut fragments read

$$(\mathcal{S}^{\mathcal{J}}_{\text{I,II}})_{05} = \frac{\kappa}{2\pi}\left(\frac{\pi}{2} \mp \sigma_0\right) + \frac{\kappa}{\tilde{\kappa}}\frac{\sinh(\tilde{\kappa}\pi) \mp \sinh(2\tilde{\kappa}\sigma_0)}{4\pi},$$
$$(\mathcal{S}^{\mathcal{J}}_{\text{I,II}})_{12} = -\frac{\kappa}{2\pi}\left(\frac{\pi}{2} \mp \sigma_0\right) + \frac{\kappa}{\tilde{\kappa}}\frac{\sinh(\tilde{\kappa}\pi) \mp \sinh(2\tilde{\kappa}\sigma_0)}{4\pi},$$
$$(\mathcal{S}^{\mathcal{J}}_{\text{I,II}})_{02} = -(\mathcal{S}^{\mathcal{J}}_{\text{I,II}})_{51} = \pm\frac{\kappa}{\tilde{\kappa}}\frac{\cosh(\tilde{\kappa}\pi) - \cosh(2\tilde{\kappa}\sigma_0)}{4\pi}, \tag{5.8}$$
$$(\mathcal{S}^{\mathcal{J}}_{\text{I,II}})_{01} = 0, \quad (\mathcal{S}^{\mathcal{J}}_{\text{I}})_{52} = 0,$$
$$(\mathcal{J}_{\text{I,II}})_1 = a^2\frac{\nu}{2\pi}(\pi \mp 2\sigma_0), \quad (\mathcal{J}_{\text{I,II}})_2 = b^2\frac{\nu}{2\pi}(\pi \mp 2\sigma_0).$$

They are conserved in the process of splitting

$$\mathcal{E}^{\mathcal{J}} = (\mathcal{S}^{\mathcal{J}}_{\text{I}})_{05} + (\mathcal{S}^{\mathcal{J}}_{\text{II}})_{05}, \quad \mathcal{S}^{\mathcal{J}} = (\mathcal{S}^{\mathcal{J}}_{\text{I}})_{12} + (\mathcal{S}^{\mathcal{J}}_{\text{II}})_{12},$$
$$\mathcal{S}^{\mathcal{J}}_{02} = (\mathcal{S}^{\mathcal{J}}_{\text{I}})_{02} + (\mathcal{S}^{\mathcal{J}}_{\text{II}})_{02}, \quad \mathcal{S}^{\mathcal{J}}_{51} = (\mathcal{S}^{\mathcal{J}}_{\text{I}})_{51} + (\mathcal{S}^{\mathcal{J}}_{\text{II}})_{51}, \tag{5.9}$$
$$\mathcal{J}_1 = (\mathcal{J}_{\text{I}})_1 + (\mathcal{J}_{\text{II}})_1, \quad \mathcal{J}_2 = (\mathcal{J}_{\text{I}})_2 + (\mathcal{J}_{\text{II}})_2.$$

It is natural to transform (5.8) to the center-of-mass systems of the final strings and explicitly find their energies and spins. $(\mathcal{S}^{\mathcal{J}}_{\text{I,II}})_{PQ}$ may be diagonalized by boosts in the Y_0Y_1 and Y_5Y_2 planes (4.15) and (4.16) with parameters

$$\alpha^{\mathcal{J}}_{\text{I,II}} = \beta^{\mathcal{J}}_{\text{I,II}} = \mp\frac{\tilde{\kappa}}{2}\left(\frac{\pi}{2} \pm \sigma_0\right). \tag{5.10}$$

We obtain the energies and AdS-spins of the fragments in the form[9]

$$\mathcal{E}^{\mathcal{J}}_{\text{I,II}} = \frac{\kappa}{2\pi}\left(\frac{\pi}{2} \mp \sigma_0\right) + \frac{\kappa}{\tilde{\kappa}}\frac{e^{\tilde{\kappa}((\pi/2)\mp\sigma_0)}}{4\pi},$$
$$\mathcal{S}^{\mathcal{J}}_{\text{I,II}} = -\frac{\kappa}{2\pi}\left(\frac{\pi}{2} \mp \sigma_0\right) + \frac{\kappa}{\tilde{\kappa}}\frac{e^{\tilde{\kappa}((\pi/2)\mp\sigma_0)}}{4\pi}. \tag{5.11}$$

The evolution of the fragments (in the own center-of-mass system for each fragment) is described by

$$(\tilde{Y}_{\text{I,II}})_{05} = \cosh(\tilde{\kappa}_{\text{I,II}}\xi)e^{i\kappa_{\text{I,II}}\eta} \quad (\tilde{Y}_{\text{I,II}})_{12} = \sinh(\tilde{\kappa}_{\text{I,II}}\xi)e^{i\kappa_{\text{I,II}}\eta}$$
$$(\tilde{X}_{\text{I,II}})_{12} = ae^{i(\nu_{\text{I,II}}\eta + m_{\text{I,II}}\xi)} \quad (\tilde{X}_{\text{I,II}})_{34} = be^{i(\nu_{\text{I,II}}\eta - m_{\text{I,II}}\xi)}, \tag{5.12}$$

where

$$\kappa_{\text{I,II}} = \kappa\frac{\pi_0 \mp 2\sigma_0}{2\pi}, \qquad \tilde{\kappa}_{\text{I,II}} = \tilde{\kappa}\frac{\pi_0 \mp 2\sigma_0}{2\pi},$$
$$\nu_{\text{I,II}} = \nu\frac{\pi \mp 2\sigma_0}{2\pi}, \qquad m_{\text{I,II}} = m\frac{\pi \mp 2\sigma_0}{2\pi} \tag{5.13}$$

and σ_0 satisfy (5.6) if $m \neq 0$. Note, that while the AdS-part of (5.12) is just a large-spin approximation, the solution for the S_3-part is exact.

Given (5.4), (5.12), and (5.10) we see that, when the initial string, described by the formal $\kappa = \omega$ limit of the string (5.4) in $AdS_3 \times S^3$, splits into two pieces, the world surface does not change and the fragments are described by the solutions of the same type as the parent string.

In the general case, starting from the exact solution in $AdS_3 \times S^3$ in the form (5.1) and (5.2), we obtain the following expressions for the charges of the cut fragments (in the center of mass of the initial string):

[9]Making use of (5.7), we set $\sinh(\tilde{\kappa}(\frac{\pi}{2} \mp \sigma_0)) \to \frac{1}{2}e^{\tilde{\kappa}((\pi/2)\mp\sigma_0)}$.

$$
\begin{aligned}
(\mathcal{S}^{\mathcal{J}}_{\mathrm{I,II}})_{05} &= \frac{1}{2}\mathcal{E}_{\mathcal{J}} \mp \sqrt{\frac{1}{\pi^2}\tilde{\ell}^2 + \frac{\mathcal{J}^2}{4\mathbb{K}^2[-\tilde{\ell}^2]}} \\
&\quad \times \mathbb{E}[\mathrm{am}[\tilde{\kappa}\tilde{\ell}^{-1}\sigma_0, -\tilde{\ell}^2], -\tilde{\ell}^2], \\
(\mathcal{S}^{\mathcal{J}}_{\mathrm{I,II}})_{12} &= \frac{1}{2}\mathcal{S}_{\mathcal{J}} \mp \sqrt{\frac{1}{\pi^2}(1+\tilde{\ell}^2) + \frac{\mathcal{J}^2}{4\mathbb{K}^2[-\tilde{\ell}^2]}} \\
&\quad \times (\mathbb{E}[\mathrm{am}[\tilde{\kappa}\tilde{\ell}^{-1}\sigma_0, -\tilde{\ell}^2], -\tilde{\ell}^2] - \frac{2}{\pi}\sigma_0\mathbb{K}[-\tilde{\ell}^2]), \\
(\mathcal{S}^{\mathcal{J}}_{\mathrm{I,II}})_{02} &= \pm\omega\frac{\tilde{\ell}^2\mathrm{cn}[\tilde{\kappa}\tilde{\ell}^{-1}\sigma_0, -\tilde{\ell}^2]}{\kappa\pi}, \\
(\mathcal{S}^{\mathcal{J}}_{\mathrm{I,II}})_{15} &= \pm\frac{\tilde{\ell}^2\mathrm{cn}[\tilde{\kappa}\tilde{\ell}^{-1}\sigma_0, -\tilde{\ell}^2]}{\pi}, \\
(\mathcal{S}^{\mathcal{J}}_{\mathrm{I,II}})_{01} &= (\mathcal{S}^{\mathcal{J}}_{\mathrm{I,II}})_{52} = 0, \\
(\mathcal{J}_{\mathrm{I,II}})_1 &= a^2\frac{\nu}{2\pi}(\pi \mp 2\sigma_0), \quad (\mathcal{J}_{\mathrm{I,II}})_2 = b^2\frac{\nu}{2\pi}(\pi \mp 2\sigma_0),
\end{aligned} \tag{5.14}
$$

where $\mathcal{E}_{\mathcal{J}}$ and $\mathcal{S}_{\mathcal{J}}$ are defined in (5.3). That may be transformed to the center-of-mass systems of the final states by the boosts in the Y_0Y_1 and Y_5Y_2 planes (4.15) and (4.16) with parameters

$$
\begin{aligned}
\sinh(\alpha^{\mathcal{J}}_{\mathrm{I,II}} + \beta^{\mathcal{J}}_{\mathrm{I,II}}) &= -\frac{\tilde{\mathbb{M}}_{\mathrm{I,II}}(\omega+\kappa)}{\sqrt{(\kappa\tilde{\mathbb{C}}_{\mathrm{I,II}} + \omega\tilde{\mathbb{S}}_{\mathrm{I,II}})^2 - \tilde{\mathbb{M}}^2_{\mathrm{I,II}}(\omega+\kappa)^2}} \\
\sinh(\alpha^{\mathcal{J}}_{\mathrm{I,II}} - \beta^{\mathcal{J}}_{\mathrm{I,II}}) &= \frac{\tilde{\mathbb{M}}_{\mathrm{I,II}}(\omega-\kappa)}{\sqrt{(\kappa\tilde{\mathbb{C}}_{\mathrm{I,II}} - \omega\tilde{\mathbb{S}}_{\mathrm{I,II}})^2 - \tilde{\mathbb{M}}^2_{\mathrm{I,II}}(\omega-\kappa)^2}},
\end{aligned} \tag{5.15}
$$

where $\tilde{\mathbb{C}}_{\mathrm{I,II}}$, $\tilde{\mathbb{S}}_{\mathrm{I,II}}$ and $\tilde{\mathbb{M}}_{\mathrm{I,II}}$ are given by (4.29) with ℓ replaced for $\tilde{\ell}$.

The general expressions for the energies and spins of the fragments read

$$
\begin{aligned}
\mathcal{E}^{\mathcal{J}}_{\mathrm{I,II}} &= \frac{1}{2}\sqrt{(\kappa\tilde{\mathbb{C}}_{\mathrm{I,II}} + \omega\tilde{\mathbb{S}}_{\mathrm{I,II}})^2 - \tilde{\mathbb{M}}^2_{\mathrm{I,II}}(\omega+\kappa)^2} \\
&\quad + \frac{1}{2}\sqrt{(\kappa\tilde{\mathbb{C}}_{\mathrm{I,II}} - \omega\tilde{\mathbb{S}}_{\mathrm{I,II}})^2 - \tilde{\mathbb{M}}^2_{\mathrm{I,II}}(\omega-\kappa)^2} \\
\mathcal{S}^{\mathcal{J}}_{\mathrm{I,II}} &= \frac{1}{2}\sqrt{(\kappa\tilde{\mathbb{C}}_{\mathrm{I,II}} + \omega\tilde{\mathbb{S}}_{\mathrm{I,II}})^2 - \tilde{\mathbb{M}}^2_{\mathrm{I,II}}(\omega+\kappa)^2} \\
&\quad - \frac{1}{2}\sqrt{(\kappa\tilde{\mathbb{C}}_{\mathrm{I,II}} - \omega\tilde{\mathbb{S}}_{\mathrm{I,II}})^2 - \tilde{\mathbb{M}}^2_{\mathrm{I,II}}(\omega-\kappa)^2} \\
(\mathcal{J}_{\mathrm{I,II}})_1 &= a^2\frac{\nu}{2\pi}(\pi \mp 2\sigma_0), \quad (\mathcal{J}_{\mathrm{I,II}})_2 = b^2\frac{\nu}{2\pi}(\pi \mp 2\sigma_0).
\end{aligned}
$$

These relations parametrically encode the conservation laws of splitting.

The evolution of the fragments I and II is governed by the string Eqs. (2.4) and (2.6) with the boundary conditions given by the initial string (5.1) and (5.2) on the intervals (4.11) at $\tau_0 = 0$ with σ_0 satisfying (5.6). The solutions describing the profiles of the fragments consist of AdS- and S^3-parts. The expressions for the S^3-parts presented in (5.12), but we are unable to find the exact expressions for the AdS-parts, due to the complexity of (5.2). Up to parameter definitions, the AdS-parts coincide with the solutions describing the splitted fragments of the folded string in pure AdS_3. This is based on the fact that the only result of accounting for the S^5 is redefinition of κ, $\omega \to \tilde{k}$, $\tilde{w}$ and discretizing of σ_0, if any.

VI. CONCLUDING REMARKS

In this paper we have investigated splitting of folded spinning strings in AdS_3 and its generalization to include circular rotations and windings in S^5. We computed the energies and spins of products of splitting and showed that in the case of splitting of strings with large AdS-spins (which is of greatest interest in the context of AdS/CFT duality), the cut fragments are described by the solutions very similar to the initial string. The complexity of the exact folded string profile prevents us from finding the evolution of the final fragments by solving the string equations with boundary conditions given by the initial string. However, one hopes that this might be reachable ‘‘indirectly’’ by applying the finite gap technique (see [27,28] for reviews). The profiles of the cut fragments are known at the moment of splitting, thus we can find the full set of the conserved charges for them, including the higher ones. This uniquely determines the algebraic surface which, being explicitly constructed, would allow the determination of the string profiles. Implementation of such an approach is promising, but quite complicated. It requires detailed investigation.

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