PHYSICAL REVIEW D **84,** 025005 (2011)

Stochastic analysis of an accelerated charged particle: Transverse fluctuations

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(Received 3 December 2010; revised manuscript received 2 June 2011; published 5 July 2011)

An accelerated particle sees the Minkowski vacuum as thermally excited, and the particle moves stochastically due to an interaction with the thermal bath. This interaction fluctuates the particle's transverse momenta like the Brownian motion in a heat bath. Because of this fluctuating motion, it has been discussed that the accelerated charged particle emits extra radiation (the Unruh radiation [P. Chen and T. Tajima, Phys. Rev. Lett. **83**, 256 (1999).]) in addition to the classical Larmor radiation, and experiments are under planning to detect such radiation by using ultrahigh intensity lasers constructed in near future [P. G. Thirolf, D. Habs, A. Henig, D. Jung, D. Kiefer, C. Lang, J. Schreiber1, C. Maia, G. Schaller, R. Schutzhold, and T. Tajima, Eur. Phys. J. D **55**, 379 (2009).][http://www.extreme-light-infrastructure.eu/.]. There are, however, counterarguments that the radiation is canceled by an interference effect between the vacuum fluctuation and the fluctuating motion. In fact, in the case of an internal detector where the Heisenberg equation of motion can be solved exactly, there is no additional radiation after the thermalization is completed [D. J. Raine, D. W. Sciama, and P. G. Grove, Proc. R. Soc. Lond. A **435**, 205 (1991).][A. Raval, B. L. Hu, and J. Anglin, Phys. Rev. D **53**, 7003 (1996).]. In this paper, we revisit the issue in the case of an accelerated charged particle in the scalar-field analog of QED. We prove the equipartition theorem of transverse momenta by investigating a stochastic motion of the particle, and show that the Unruh radiation is partially canceled by an interference effect.

DOI: 10.1103/PhysRevD.84.025005 PACS numbers: 03.70.+k, 04.62.+v, 05.40.−a

I. INTRODUCTION

Quantum field theories in the space-time with horizons exhibit interesting thermodynamic behavior. The most prominent phenomenon is the Hawking radiation [1] and the fundamental laws of thermodynamics hold in the black hole background [2]. This indicates an underlying microscopic description of the space-time and there are varieties of proposals including those based on D-branes [3] or quantum spin foam [4]. For a black hole with surface gravity κ at the horizon, the temperature of the Hawking radiation is given by

$$T_H = \frac{\hbar\kappa}{2\pi c k_B} = 6\times 10^{-8}\left(\frac{M_\odot}{M}\right)[\mathrm{K}], \tag{1.1}$$

where we have used $\kappa = 1/4M$ for the Schwarzschild black hole with mass M. The temperature is too small to be observed for astrophysical black holes. A similar phenomenon occurs for a uniformly accelerated observer in the Minkowski vacuum [5,6]. The equivalence principle of the general relativity relates acceleration with gravity. If a particle is uniformly accelerated with an acceleration a, there appears a causal horizon, the Rindler horizon, and no information can be transmitted from the other side of the horizon. Because of the existence of the Rindler horizon, the accelerated observer sees the Minkowski vacuum as thermally excited with the Unruh temperature

$$T_U = \frac{\hbar a}{2\pi c k_B} = 4\times 10^{-23}\left(\frac{a}{1\ \mathrm{cm/s^2}}\right)[\mathrm{K}]. \tag{1.2}$$

Furthermore it is discussed [7] that, if we assign entropy to the Rindler horizon and assume thermodynamic relations, the Einstein equation can be derived.

The Unruh temperature is very small for low acceleration, but the recent development of ultra-high intensity lasers makes the Unruh effect experimentally accessible. In the electromagnetic field of a laser with intensity $I[\mathrm{W/cm^2}]$, an electron can be accelerated to

$$a = 2\times 10^{12}\sqrt{\frac{I}{1\,\mathrm{W/cm^2}}}\,[\mathrm{cm/s^2}] \tag{1.3}$$

and the Unruh temperature is given by

$$T_U = 8\times 10^{-11}\sqrt{\frac{I}{1\ \mathrm{W/cm^2}}}\,[\mathrm{K}]. \tag{1.4}$$

The ELI (Extreme Light Infrastructure) project [8] recently approved is planning to construct Peta Watt lasers with an intensity as high as $5\times 10^{26}[\mathrm{W/cm^2}]$. Then the expected Unruh temperature becomes more than 10^3 K which is much higher than the room temperature. Can we experimentally observe such high Unruh temperature of an accelerated electron in the laser field? This is an interesting issue and worth being investigated [9].

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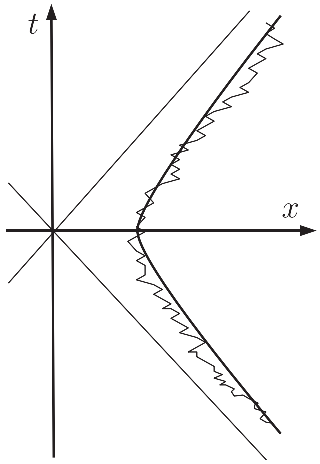

FIG. 1. Stochastic trajectories induced by quantum field fluctuations.

One such proposal was given by P. Chen and T. Tajima [10]. Their basic idea is the following. An electron is accelerated in the oscillating electromagnetic field of lasers. It is not a uniform acceleration, but they approximated the electron's motion around the turning points by a uniform acceleration. Since the electron feels the vacuum as thermally excited with the Unruh temperature, the motion of the electron will be thermalized and fluctuate around the classical trajectory (Fig. 1).

Because of this fluctuating motion of an electron, they conjectured that additional radiation, apart from the classical Larmor radiation, will emanate. Using an intuitive argument, they estimated the additional radiation and called it Unruh radiation. Though the estimated amount of radiation is much smaller than the classical one by 10^{-5}, they argued that the angular dependence is different. Especially there is a blind spot for the Larmor radiation in the direction along the acceleration while the Unruh radiation is expected to be radiated more spherically. Hence they proposed to detect the additional radiation in this direction.

The above heuristic argument sounds correct, but it has been known in a simpler situation that such radiation is canceled by an interference effect between the radiation field emanated from the fluctuating motion and the vacuum fluctuation of the radiation field [11,12]. The cancellation was shown to occur[1] for an internal detector in $1+1$ dimensions.[2] In order to see the importance of the interference effect, we briefly sketch the calculation of radiation from the internal detector. In these papers [11,12], the authors analyzed a uniformly accelerated internal detector Q coupled with a scalar field ϕ in $1+1$ dimensions. The action is given by

$$S = S(Q) + S(\phi) + e\int d\tau \frac{dQ}{d\tau}\phi(z(\tau)) \tag{1.5}$$

where $z(\tau) = (t(\tau), x(\tau))$ represents a classical trajectory of the detector. $S(Q)$ and $S(\phi)$ are quadratic actions of the internal degrees of freedom of the detector (i.e., a harmonic oscillator) and the scalar field in $1+1$ dimensions respectively;

$$S(Q) = \int d\tau\left(\frac{1}{2}\dot{Q}(\tau)^2 - \frac{\omega_0^2}{2}Q^2(\tau)\right) \tag{1.6}$$

$$S(\phi) = \int d^2x \frac{1}{2}(\partial\phi(x))^2. \tag{1.7}$$

Since the coupling term is linear both in Q and ϕ, the Heisenberg equations of motion can be exactly solved. A classical solution of the scalar field is written as a sum of the vacuum fluctuation $\phi_h(x)$, which is a solution to the homogeneous equation in the absence of Q, and an inhomogeneous term $\phi_{\rm inh}(x)$ as

$$\phi(x) = \phi_h(x) + \phi_{\rm inh}(x). \tag{1.8}$$

The inhomogeneous part $\phi_{\rm inh}$ is given by

$$\phi_{\rm inh}(x) = \int d\tau G_R(x, z(\tau))\frac{dQ}{d\tau} \tag{1.9}$$

where G_R is the retarded Green function of the scalar field. The equation of motion of Q becomes

$$\ddot{Q} + \omega_0^2 Q = -e\frac{d\phi_h}{d\tau} - e\frac{d\phi_{\rm inh}}{d\tau}. \tag{1.10}$$

Since $\phi_{\rm inh}$ is solved linearly in $Q(\tau)$,the second term of the r.h.s. of (1.10) is linear in Q. It can be shown that it becomes a dissipative term $\gamma\dot{Q}$ where $\gamma = e/2\pi$. Hence $Q(\tau)$ can be solved in terms of the homogeneous solution ϕ_h as

$$\tilde{Q}(\omega) = eh(\omega)\varphi(\omega) \tag{1.11}$$

where $\tilde{Q}(\omega)$ and $\varphi(\omega)$ are the Fourier modes of $Q(\tau)$ and $\phi_h(z(\tau))$ with respect to τ, and $h(\omega) = i\omega/(\omega^2 - \omega_0^2 - i\omega\gamma)$. By inserting (1.11) to (1.9), the inhomogeneous solution $\phi_{\rm inh}$ is solved in terms of the vacuum fluctuation $\phi_h(x)$.

Then it is straightforward to calculate the energy-momentum tensor. Since the energy flux is written in terms of the 2-point function, they first calculated the 2-point function

$$G(x, x') = \langle\phi(x)\phi(x')\rangle. \tag{1.12}$$

It is written as a sum of the following terms,

[1]We note that, though the radiation vanishes in generic points, a singular behavior of the radiation is pointed out to exist on the past horizon [13].

[2]We call a detector with internal degrees of freedom an internal detector in order to distinguish it from an accelerated charged particle. In the case of a charged particle, the position of the particle reacts to the thermal effect of acceleration. On the contrary, in the case of an internal detector, only its internal degree of freedom is excited by the thermal effect.

$$G(x,x') - G_0(x,x') = \langle\phi_{\rm inh}(x)\phi_{\rm inh}(x')\rangle + \langle\phi_{\rm inh}(x)\phi_h(x')\rangle + \langle\phi_h(x)\phi_{\rm inh}(x')\rangle \quad (1.13)$$

where vacuum fluctuation $G_0(x,x') = \langle\phi_h(x)\phi_h(x')\rangle$ is subtracted. The first term $\langle\phi_{\rm inh}(x)\phi_{\rm inh}(x')\rangle$ can be considered as an analog of the Unruh radiation proposed in [10]. It is nonzero because the detector is thermally excited from the classical ground state $Q = 0$. However, Sciama *et. al.* [11] and Hu. *et. al.* [12] have shown that in $(1+1)$-dimensional case, the contributions from the interference terms $\langle\phi_{\rm inh}(x)\phi_h(x')\rangle + \langle\phi_h(x)\phi_{\rm inh}(x')\rangle$ cancel the radiation $\langle\phi_{\rm inh}(x)\phi_{\rm inh}(x')\rangle$, except for the polarization cloud. The polarization cloud is the cloud of radiation field localized near the accelerated particle, and does not contribute to the energy-momentum tensor. Since we are interested in the energy flux far from the particle in this paper, we do not consider it in the following. Hu and Lin [14] extended the calculation to a detector with internal degrees of freedom in $3+1$ dimensions. The calculation in the $3+1$ dimensional case is reviewed in Appendix A.

The physical reason of the cancellation is discussed in [15,16] (see also [17]). The key point of their arguments is the invariance of the Minkowski vacuum state under boosts with the conservation of energy-momentum tensor. The metric of the Minkowski space is invariant under the translation of τ. So, after the thermalization occurs, the system becomes stationary and the energy-momentum tensor will not depend on τ explicitly. In [16], the following three conditions are assumed:

(i) Poincare invariance;
(ii) A detector follows a uniformly accelerated trajectory in flat space;
(iii) A τ-independent coupling.

With these conditions, the energy-momentum tensor does not depend explicitly on τ. They have argued that, if the energy conservation holds for the total system, there is no outgoing flux. Namely the cancellation of the radiation $\langle\phi_{\rm inh}(x)\phi_{\rm inh}(x')\rangle$ by the interference terms is not accidental but a consequence of the boost symmetry. In the case of an accelerated charged particle, however, the second condition does not hold since the particle's trajectory is a dynamical variable and fluctuates. Then the boost invariance of the trajectory is lost. Furthermore, the charged particle is accelerated by injecting energy from outside. It is therefore not obvious whether extra radiation vanishes or not.

The purpose of the paper is to investigate a stochastic motion of a uniformly accelerated charged particle and to study whether there is additional radiation, Unruh radiation associated with the stochastic motion of the particle. The situation becomes much more complicated than the internal detector case because the equations of motion are highly nonlinear. When the particle's motion $z(\tau)$ is affected by the vacuum fluctuation, the Green function $G_R(x, z(\tau))$ is also changed accordingly unlike the internal detector case. Hence in order to calculate the radiation from an accelerated charged particle, we need to approximate the fluctuating motion near the classical trajectory and assume that fluctuations are small. We first study a stochastic equation for a charged particle coupled with the scalar field [18]. This gives a simplified model of the real QED. A self-interaction with the scalar field created by the particle itself gives a backreaction to the particle's motion, and it gives a radiation damping term of the Abraham-Lorentz-Dirac equation [19]. If we further regard the vacuum fluctuation as stochastic noise, the particle's motion obeys a generalized Langevin equation. By solving the Langevin equation, we can obtain stochastic fluctuations of the particle's momenta. In this paper, we mainly focus on fluctuations in the transverse directions and leave analysis of longitudinal fluctuations for future investigation. Some comments are given in Discussions and in Appendix B.

The organization of the paper is the following. In Sec. II, we summarize the basic framework of our system, and obtain a generalized Langevin equation for a charged particle coupled with a scalar field. In Sec. III, we consider small fluctuations in the transverse directions. Then the stochastic equation can be solved and we can prove the equipartition theorem for the transverse momenta, i.e., a stochastic average of a square of momentum fluctuations in the transverse directions is shown to be proportional to the Unruh temperature. We also discuss the relaxation time of the thermalization process. Section IV is the main part of the paper. We calculate radiation emitted by a charged particle in the scalar-field analog of QED. The interference terms partially cancel the radiation coming from the contribution $\langle\phi_{\rm inh}(x)\phi_{\rm inh}(x')\rangle$, but unlike the case of an internal detector, they do not cancel exactly. In Sec. V, we obtain a similar stochastic equation for an accelerated charged particle in the real QED, and show the equipartition theorem for transverse momenta. Section VI is devoted to conclusions and discussions. In Appendix A, we review the calculation of radiation for a uniformly accelerated internal detector in $(3+1)$ dimensions [14]. In Appendix B, we consider fluctuations in the longitudinal and temporal directions. We use Planck units $c = \hbar = k_B = 1$ in most of the paper.

II. STOCHASTIC EQUATION OF AN ACCELERATED PARTICLE

We consider the scalar-field analog of QED. The model is analyzed in [18] and here we briefly review the settings and the derivation of the stochastic Abraham-Lorentz-Dirac (ALD) equations. In [18], the authors used the influence functional approach, but here we take a simplified method. The system composes of a relativistic particle $z^\mu(\tau)$ and the scalar field $\phi(x)$. The action is given by

$$S[z,\phi,h] = S[z,h] + S[\phi] + S[z,\phi], \quad (2.1)$$

with

$$S[z,h] = -m\int d\tau\sqrt{\dot{z}^\mu \dot{z}_\mu}, \tag{2.2}$$

$$S[\phi] = \int d^4x \frac{1}{2}(\partial_\mu \phi(x))^2, \tag{2.3}$$

$$S[z,\phi] = \int d^4x\, j(x;z)\phi(x). \tag{2.4}$$

The scalar current $j(x;z)$ is defined as

$$j(x;z) = e\int d\tau\sqrt{\dot{z}^\mu \dot{z}_\mu}\,\delta^4(x - z(\tau)), \tag{2.5}$$

where e is negative for an electron. We can parametrize the particle's path satisfying $\dot{z}^2 = 1$ by taking τ properly.

The equation of motion of the particle is given by

$$m\ddot{z}^\mu = F^\mu - \int d^4x \frac{\delta j(x;z)}{\delta z_\mu(\tau)}\phi(x) \tag{2.6}$$

where we have added the external force F^μ so as to accelerate the particle uniformly:

$$F^\mu = ma(\dot{z}^1, \dot{z}^0, 0, 0). \tag{2.7}$$

Then a classical solution of the particle (in the absence of the coupling to ϕ) is given by

$$z^\mu_{\rm cl} = \left(\frac{1}{a}\sinh a\tau, \frac{1}{a}\cosh a\tau, 0, 0\right). \tag{2.8}$$

Note that the external force satisfies $F^\mu \dot{z}_\mu = 0$ and therefore the classical equation of motion preserves the gauge condition $\dot{z}^2 = 1$. From the definition of the current (2.5), it is easy to prove the identity,

$$\int d^4x \frac{\delta j(x;z)}{\delta z^\mu(\tau)} f(x) = e\vec{\omega}_\mu f(x)|_{x=z(\tau)} \tag{2.9}$$

where $\vec{\omega}^\mu$ is given by

$$\vec{\omega}_\mu = \dot{z}^\nu \dot{z}_{[\nu}\partial_{\mu]} - \ddot{z}_\mu. \tag{2.10}$$

Here we have used the gauge condition $\dot{z}^2 = 1$ and $\ddot{z}\cdot\dot{z} = 0$. Hence the equation of motion (2.6) becomes

$$m\ddot{z}^\mu = F^\mu - e\vec{\omega}^\mu \phi(z(\tau)). \tag{2.11}$$

Since the differential operator $\vec{\omega}^\mu$ satisfies $\dot{z}^\mu\vec{\omega}_\mu = 0$ for a classical path satisfying the gauge condition, the stochastic Eq. (2.11) continues to preserve the condition $\dot{z}^2 = 1$. The second term of (2.11) represents a self-interaction of the particle with the radiation emitted by the particle itself.

The equation of motion of the radiation field $\partial^\mu\partial_\mu\phi(x) = j(x)$ is solved by using the retarded Green function G_R as

$$\phi(x) = \phi_h(x) + \phi_{\rm inh}, \qquad \phi_{\rm inh} = \int d^4x' G_R(x,x') j(x';z) \tag{2.12}$$

where ϕ_h is the homogeneous solution of the equation of motion and represents the vacuum fluctuation. The retarded Green function satisfies

$$\partial^\mu\partial_\mu G_R(x,x') = \delta^{(4)}(x - x') \tag{2.13}$$

and is given by

$$G_R(x,x') = i\langle[\phi(x),\phi(x')]\rangle\theta(t - t') \tag{2.14}$$

$$= \frac{\theta(t-t')\delta((x-x')^2)}{2\pi} = \frac{\delta((t-t')-r)}{4\pi r} \tag{2.15}$$

where $r^2 = |\mathbf{x} - \mathbf{x}'|^2$. Inserting the solution (2.12) into (2.11), we have the following stochastic equation for the particle

$$m\ddot{z}^\mu(\tau) = F^\mu(z(\tau)) - e\vec{\omega}^\mu\left(\phi_h(z(\tau)) + e\int d\tau' G_R(z(\tau), z(\tau'))\right). \tag{2.16}$$

Here we have used the gauge condition $\dot{z}^2 = 1$. The operator $\vec{\omega}_\mu$ acts on $z(\tau)$. The homogeneous part $\phi_h(z(\tau))$ of the scalar field describes Gaussian fluctuations of the vacuum, and hence the first term in the parenthesis can be interpreted as random noise to the particle's motion. Expanding ϕ_h as

$$\phi_h(x) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}(a_k e^{-ik^\mu x_\mu} + a_k^\dagger e^{ik^\mu x_\mu}), \tag{2.17}$$

the vacuum fluctuation is given by

$$\langle\phi_h(x)\phi_h(x')\rangle = -\frac{1}{4\pi^2}\frac{1}{(t - t' - i\epsilon)^2 - r^2}. \tag{2.18}$$

It is essentially quantum mechanical, but if it is evaluated on a world line of a uniformly accelerated particle $x = z(\tau)$, $x' = z(\tau')$; it behaves as the ordinary finite temperature noise.

The second term in the parenthesis of (2.16) is a functional of the total history of the particle's motion $z(\tau')$ for $\tau' \le \tau$, but it can be reduced to the so-called radiation damping term of a charged particle coupled with a radiation field. It is generally nonlocal, but since the Green function damps rapidly as a function of the distance r, the term is approximated by local derivative terms. First define $y^\mu(s) = z^\mu(\tau) - z^\mu(\tau')$ where $\tau' = \tau - s$ with τ kept fixed. Then it can be expanded as

$$y^\mu(s) = s\dot{z}^\mu(\tau) - \frac{s^2}{2}\ddot{z}^\mu(\tau) + \frac{s^3}{6}\dddot{z}^\mu(\tau) + \cdots. \qquad (2.19)$$

A square of the space-time distance σ is given by

$$\sigma(s) \equiv y^\mu y_\mu = s^2\left(1 - \frac{s^2}{12}(\ddot{z})^2 + \cdots\right) \qquad (2.20)$$

and

$$\frac{d\sigma(s)}{ds} = 2y^\mu \dot{y}_\mu = 2s\left(1 - \frac{s^2}{6}(\ddot{z})^2 + \cdots\right). \qquad (2.21)$$

In deriving them, we have used the gauge condition $(\dot{z})^2 = 1$, $\dot{z}\cdot\ddot{z} = 0$ and $\dot{z}\cdot\dddot{z} = -(\ddot{z})^2$. The derivative ∂_μ appearing in the operator $\vec{\omega}_\mu$ can be written in terms of $\frac{d}{ds}$, when it acts on a function of σ, as

$$\partial_\mu = \frac{\partial\sigma}{\partial z^\mu}\frac{d}{d\sigma} = 2y_\mu\left(\frac{d\sigma}{ds}\right)^{-1}\frac{d}{ds} = \frac{y_\mu}{y^\nu \dot{y}_\nu}\frac{d}{ds} \qquad (2.22)$$

$$= \left(\dot{z}_\mu - \frac{s}{2}\ddot{z}_\mu + \frac{s^2}{6}(\dddot{z}_\mu + \dot{z}_\mu(\ddot{z})^2) + \cdots\right)\frac{d}{ds}. \qquad (2.23)$$

Hence the second term in the parenthesis of (2.16) can be simplified as

$$e^2\vec{\omega}_\mu \int_{-\infty}^{\tau} d\tau' G_R(z(\tau), z(\tau')) \qquad (2.24)$$

$$= e^2\int_0^\infty ds\vec{\omega}_\mu G_R(s) \qquad (2.25)$$

$$= e^2\int_0^\infty ds\Big(a_\mu(\tau)G^R(s) + a_\mu(\tau)\frac{s}{2}\frac{d}{ds}G^R(s) + (\dot{z}_\mu\ddot{z}^2 + \dddot{z}_\mu)\frac{s^2}{6}\frac{d}{ds}G^R(s) + \mathcal{O}(s^3)\Big). \qquad (2.26)$$

In the first equality, we have neglected the singular term proportional to $\delta(\sigma)$. The first two terms can be absorbed by mass renormalization. The s integrals of them are divergent, so we assume that the renormalized mass becomes finite after the mass renormalization. The last one is the radiation reaction term and can be evaluated by using the identity

$$\int_0^\infty dss^2\frac{d}{ds}G^R(s) = \int_0^\infty dss^2\frac{d}{ds}\frac{\delta(s)}{4\pi s} = -\frac{1}{2\pi}. \qquad (2.27)$$

After the mass renormalization, we get the following generalized Langevin equation for the charged particle,

$$m\ddot{z}^\mu - F^\mu - \frac{e^2}{12\pi}(\dot{z}^\mu\ddot{z}^2 + \dddot{z}^\mu) = -e\vec{\omega}^\mu\phi_h(z). \qquad (2.28)$$

This is an analog of the Abraham-Lorentz-Dirac equation for a charged particle interacting with the electromagnetic field. The dissipation term is induced through an effect of backreaction of the particle's radiation to the particle's motion. Note that, if the noise term, the r.h.s., is absent, the classical solution (2.8) is still a solution to Eq. (2.28).

III. THERMALIZATION OF TRANSVERSE MOMENTUM FLUCTUATIONS

The stochastic Eq. (2.28) is nonlinear and difficult to solve. Here we consider small fluctuations around the classical trajectory induced by the vacuum fluctuation ϕ_h. Especially, we consider fluctuations in the transverse directions perpendicular to the direction of the acceleration. The classical trajectory of the particle does not move to the transverse directions, and we can easily treat fluctuations in these directions. In contrast, the particle is accelerated strongly to the longitudinal direction and it is more difficult to separate fluctuations from the classical solution. We shortly discuss longitudinal fluctuations in Appendix B.

First we expand the particle's motion around the classical trajectory z_0^μ as

$$z^\mu(\tau) = z_0^\mu + \delta z^\mu. \qquad (3.1)$$

The particle is accelerated along the x direction. Now we consider small fluctuation in transverse directions z^i. By expanding the stochastic Eq. (2.28), we can obtain a linearized stochastic equation for the transverse velocity fluctuation $\delta v^i \equiv \delta\dot{z}^i$ as,

$$m\delta\dot{v}^i = e\partial_i\phi_h + \frac{e^2}{12\pi}(\delta\ddot{v}^i - a^2\delta v^i). \qquad (3.2)$$

Performing Fourier transformation with respect to the trajectory's parameter τ

$$\delta v^i(\tau) = \int\frac{d\omega}{2\pi}\delta\tilde{v}^i(\omega)e^{-i\omega\tau},$$
$$\partial_i\phi_h(\tau) = \int\frac{d\omega}{2\pi}\partial_i\varphi(\omega)e^{-i\omega\tau}, \qquad (3.3)$$

the stochastic equation can be solved as

$$\delta\tilde{v}^i(\omega) = eh(\omega)\partial_i\varphi(\omega) \qquad (3.4)$$

where

$$h(\omega) = \frac{1}{-im\omega + \frac{e^2(\omega^2+a^2)}{12\pi}}. \qquad (3.5)$$

The vacuum 2-point function on the classical trajectory can be evaluated from (2.18) as

$$\langle\partial_i\phi_h(x)\partial_j\phi_h(x')\rangle|_{x=z(\tau),x'=z(\tau')} = \frac{1}{2\pi^2}\frac{\delta_{ij}}{((t-t'-i\epsilon)^2 - r^2)^2} \qquad (3.6)$$

$$= \frac{a^4}{32\pi^2} \frac{\delta_{ij}}{\sinh^4(\frac{a(\tau-\tau'-i\epsilon)}{2})}. \tag{3.7}$$

It has originated from the quantum fluctuations of the vacuum, but it can be interpreted as finite temperature noise if it is evaluated on the accelerated particle's trajectory [5].[3] By making the Fourier transformation with respect to τ, we have

$$\langle \partial_i \varphi(\omega) \partial_j \varphi(\omega') \rangle = 2\pi\delta(\omega + \omega')\delta_{ij} I(\omega) \tag{3.8}$$

where

$$I(\omega) = \frac{a^4}{32\pi^2} \int_{-\infty}^{\infty} d\tau \frac{e^{i\omega\tau}}{\sinh^4(\frac{a(\tau-i\epsilon)}{2})} = \frac{1}{6\pi} \frac{\omega^3 + \omega a^2}{1 - e^{-2\pi\omega/a}}. \tag{3.9}$$

For small ω, this is expanded as

$$I(\omega) = \frac{a}{12\pi^2}(a^2 + a\pi\omega + \cdots). \tag{3.10}$$

The expansion corresponds to the derivative expansion

$$\langle \partial_i \phi_h(x) \partial_j \phi_h(x') \rangle|_{x=z(\tau), x'=z(\tau')} = \frac{a^3}{12\pi^2} \delta_{ij} \delta(\tau - \tau') - i \frac{a^2}{12\pi} \delta_{ij} \delta'(\tau - \tau') + \cdots. \tag{3.11}$$

If we approximate the 2-point function by the first term, the noise correlation becomes white noise. The coefficient determines the strength of the noise. We show that it is consistent with the fluctuation-dissipation theorem at the Unruh temperature.

By symmetrizing it, i.e., $\langle \partial\phi(x)\partial\phi(x')\rangle_S = \langle\{\partial\phi(x), \partial\phi(x')\}\rangle/2$, the correlation function becomes

$$I_S(\omega) = \frac{\omega(\omega^2 + a^2)}{12\pi} \coth\left(\frac{\pi\omega}{a}\right). \tag{3.12}$$

It is an even function of ω. The correlators $I(\omega)$ and $I_S(\omega)$ should be regularized for large ω where quantum field theoretic effects of electrons become important.

The expectation value of a square of the velocity fluctuations in the transverse directions can be evaluated as

$$\langle \delta v^i(\tau) \delta v^j(\tau') \rangle_S = e^2 \int \frac{d\omega d\omega'}{(2\pi)^2} \langle \partial_i \varphi(\omega) \partial_j \varphi(\omega') \rangle_S h(\omega) h(\omega') e^{-i(\omega\tau + \omega'\tau')} \tag{3.13}$$

$$= e^2 \delta_{ij} \int \frac{d\omega}{2\pi} I_S(\omega) |h(\omega)|^2 e^{-i\omega(\tau-\tau')} \tag{3.14}$$

$$\sim e^2 \delta_{ij} \int \frac{d\omega}{24\pi^3} \frac{a^3}{(m\omega)^2 + (\frac{e^2}{12\pi})(\omega^2 + a^2)^2} e^{-i\omega(\tau-\tau')}. \tag{3.15}$$

The denominator has four poles at $\omega = \pm i\Omega_\pm$ where

$$\Omega_+ = \frac{12\pi m}{e^2}(1 + \mathcal{O}(a^2/m^2)), \tag{3.16}$$

$$\Omega_- = \frac{a^2 e^2}{12\pi m}(1 + \mathcal{O}(a^2/m^2)). \tag{3.17}$$

The acceleration of an electron in high-intensity laser fields in the near future can be at most 0.1 eV and much smaller than the electron mass 0.5 MeV. Hence, the values of these poles satisfy the following inequalities,

$$\Omega_+ \gg a \gg \Omega_-. \tag{3.18}$$

Since the energy scale of the dynamics of the accelerated particle is much smaller than the electron mass, the poles at $\pm i\Omega_+$ should be considered spurious and we should not take the contributions of the residues at $\pm i\Omega_+$.[4] By taking the residue at $\pm\omega = i\Omega_-$, we can evaluate the integral and get the following result,

$$\frac{m}{2} \langle \delta v^i(\tau) \delta v^j(\tau) \rangle = \frac{1}{2} \frac{a\hbar}{2\pi c} \delta_{ij}(1 + \mathcal{O}(a^2/m^2)). \tag{3.19}$$

Here we have recovered c and $\hbar$. This gives the equipartition relation for the transverse momentum fluctuations at the Unruh temperature $T_U = a\hbar/2\pi c$. The typical energy scale of the fluctuating motion is given by the value of the pole Ω_-. Since it is much smaller than the acceleration, the derivative expansion with respect to ω/a is justified for the transverse fluctuations.

The thermalization process of the stochastic Eq. (3.2) can be also discussed. For simplicity, we approximate the stochastic equation by dropping the second derivative term. This corresponds to the derivative expansion with respect to ω/a. Then it is solved as

$$\delta v^i(\tau) = e^{-\Omega_-\tau} \delta v^i(0) + \frac{e}{m} \int_0^\tau d\tau' \partial_i \phi(z(\tau')) e^{-\Omega_-(\tau-\tau')}. \tag{3.20}$$

[3] The finite temperature (Unruh) effect is caused by the appearance of the horizon for a uniformly accelerated observer in the Minkowski space-time and analogous to the Hawking radiation of the black hole, but you should not confuse the radiation we are discussing in this paper with the Hawking radiation. The accelerated observer sees the Minkowski vacuum as thermally excited, but it is excited from the Rindler vacuum (not from the Minkowski vacuum) and the energy-momentum tensor remains zero as ever. The radiation discussed in the paper is, if it exists, produced by an interaction with the vacuum and the accelerated charged particle.

[4] Or we can simply approximate the denominator by dropping the ω^4 term. Then only the poles at $\pm i\Omega_-$ survive.

The relaxation time is given by $\tau_R = 1/\Omega_-$. The momentum square can be calculated as

$$
\begin{aligned}
&\langle \delta v^i(\tau)\delta v^j(\tau)\rangle \\
&= e^{-2\Omega_-\tau}\delta v^i(0)\delta v^j(0) + e^2\int_0^\tau d\tau' \int_0^\tau d\tau'' e^{-\Omega_-(\tau-\tau')} \\
&\quad \times e^{-\Omega_-(\tau-\tau'')}\langle \partial_i\phi(z(\tau'))\partial_j\phi(z(\tau''))\rangle \\
&= e^{-2\Omega_-\tau}\delta v^i(0)\delta v^j(0) + \frac{a\delta_{ij}}{2\pi m}(1-e^{-2\Omega_-\tau}). \qquad (3.21)
\end{aligned}
$$

For $\tau \to \infty$, it approaches the thermalized average (3.19). The relaxation time in the proper time can be estimated, for $a = 0.1$ eV and $m = 0.5$ MeV, to be

$$\tau_R = \frac{12\pi m}{a^2 e^2} = 1.4 \times 10^{-5} \text{ sec}. \qquad (3.22)$$

This relaxation time should be compared with the laser frequency. The oscillation period of the laser field at ELI is about 3×10^{-15} seconds and much shorter than the above relaxation time. Hence an accelerated electron by the ELI laser does not thermalize during each oscillation and the uniform acceleration is not a good approximation. Even in such a situation, if an electron is accelerated in the laser field for a long time, it may feel averaged temperature. In order to fully understand the Unruh effect of nonuniform acceleration, we need to investigate the transient phenomena. It is beyond the present calculation, and we leave it for future investigations. See e.g. [20] for the Unruh effect of the circular motion.

The position of the particle in the transverse directions fluctuates like the ordinary Brownian motion in a heat bath. A mean square of the fluctuations is given by

$$
\begin{aligned}
R^2(\tau) &= \sum_{i=y,z}\langle (z^i(\tau) - z^i(0))^2\rangle \\
&= 2D\left(\tau - \frac{3 - 4e^{-\Omega_-\tau} + e^{-2\Omega_-\tau}}{2\Omega_-}\right). \qquad (3.23)
\end{aligned}
$$

The diffusion constant D is given by

$$D = \frac{2T_U}{\Omega_- m} = \frac{12}{ae^2}, \qquad (3.24)$$

which is estimated for the above parameters as $D \sim 7.8 \times 10^4\ \text{m}^2/\text{s}$. In the Ballistic region where $\tau < \tau_R$, the mean square becomes

$$R^2(\tau) = \frac{2T_U}{m}\tau^2 \qquad (3.25)$$

while in the diffusive region ($\tau > \tau_R$), it is proportional to the proper time as

$$R^2(\tau) = 2D\tau. \qquad (3.26)$$

As the ordinary Brownian motion, the mean square of the particle's transverse position grows linearly with time. If it becomes possible to accelerate the particle for a sufficiently long period, it may be possible to detect such a Brownian motion in future laser experiments.

IV. QUANTUM RADIATION BY TRANSVERSE FLUCTUATION

Once we obtain the stochastic motion of the accelerated particle, it is straightforward to calculate the energy flux of the radiation field emitted by this particle. In this section, we calculate the radiation induced by the fluctuation in the transverse directions.

A. 2-point function

First we evaluate the 2-point function

$$
\begin{aligned}
G(x,x') - G_0(x,x') &= \langle \phi_{\text{inh}}(x)\phi_{\text{inh}}(x')\rangle + \langle \phi_{\text{inh}}(x)\phi_h(x')\rangle \\
&\quad + \langle \phi_h(x)\phi_{\text{inh}}(x')\rangle. \qquad (4.1)
\end{aligned}
$$

The inhomogeneous solution ϕ_{inh} is produced by the accelerated charged particle while the homogeneous solution ϕ_h represents the vacuum fluctuation of the quantum field ϕ. The Unruh radiation estimated in [10] corresponds to calculating 2-point correlation function of the inhomogeneous terms $\langle \phi_{\text{inh}}(x)\phi_{\text{inh}}(x')\rangle$. As we will see later, this term also contains the classical Larmor radiation. However, this is not the end of the story. As it has been discussed in [11], the interference terms $\langle \phi_{\text{inh}}\phi_h\rangle + \langle \phi_h\phi_{\text{inh}}\rangle$ cannot be neglected and may possibly cancel the Unruh radiation in $\langle \phi_{\text{inh}}\phi_{\text{inh}}\rangle$ after the thermalization occurs. This is shown for an internal detector in (1 + 1) dimensions, but it is not obvious whether the same cancellation occurs for the case of a charged particle we are considering.

The inhomogeneous solution of the scalar field is written in terms of the position of the accelerated particle $z^\mu(\tau)$ as

$$\phi_{\text{inh}}(x) = e\int d\tau G_R(x - z(\tau)) \qquad (4.2)$$

$$= e\int d\tau \frac{\theta(t - z^0(\tau))\delta((x - z(\tau))^2)}{2\pi} = \frac{e}{4\pi\rho}, \qquad (4.3)$$

where ρ is defined by

$$\rho = \dot{z}(\tau_-^x)\cdot(x - z(\tau_-^x)). \qquad (4.4)$$

Because of the step and the delta functions in the integrand of (4.3), τ_-^x satisfies

$$(x - z(\tau_-^x))^2 = 0, \qquad x^0 > z^0(\tau_-^x), \qquad (4.5)$$

which is the proper time of the particle whose radiation travels to the space-time point x. Hence $z(\tau_-^x)$ lies on an intersection between the particle's world line and the light-cone extending from the observer's position x (see Fig. 2). We write the superscript x to make the x dependence of τ explicitly. The meaning of the subscript $(-)$ will be made clear later. By using the light-cone condition, ρ can be rewritten as

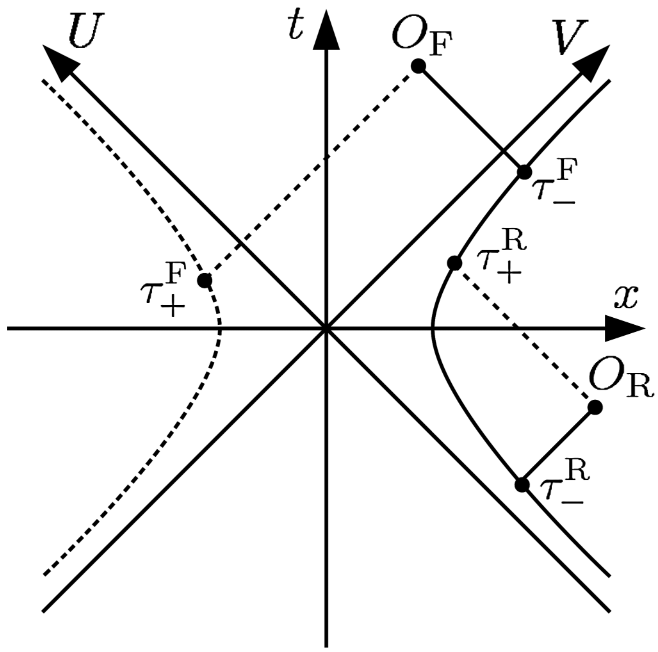

FIG. 2. The hyperbolic line in the right wedge denotes the world line of the particle. The points O_F and O_R are observers in the future and right wedges, respectively. For an observer in the right wedge, the light-cone of the observer has two intersections with the world line, and the proper time of the intersections is given by $\tau_\pm^R$. For an observer in the future wedge, there is only one intersection on the particle's real trajectory which corresponds to τ_-^F. The other solution $T_+^F = \tau_+^F + i\pi/a$ is complex. One may interpret this complex proper time as the intersection between the light-cone of the observer and the world line of a virtual particle with a real proper time τ_+^F in the left wedge. The superscript letters R or F are used to distinguish two different observers, but we do not use them in the body of the paper to leave the space for the observer's position x.

$$\rho(x) = \frac{dz^0(\tau_-^x)}{d\tau} r(\tau_-^x)\left(1 - \frac{\mathbf{v}\cdot\mathbf{r}}{r}\right) \tag{4.6}$$

where $\mathbf{v} = \frac{d\mathbf{z}}{dz^0}$, $\mathbf{r}(\tau_-^x) = \mathbf{x} - \mathbf{z}(\tau_-^x)$ and $r = |\mathbf{r}|$. It is the spatial distance for the observer moving with the particle.

The particle's trajectory is fluctuating around the classical trajectory and can be expressed as $z = z_0 + \delta z + \delta^2 z + \cdots$ where the expansion is given with respect to e. In terms of the expansion of the spatial distance $\rho = \rho_0 + \delta\rho + \delta^2\rho + \cdots$, the inhomogeneous solution (4.3) becomes

$$\phi_{\rm inh} = \frac{e}{4\pi\rho_0}\left(1 - \frac{\delta\rho}{\rho_0} + \left(\frac{\delta\rho}{\rho_0}\right)^2 - \frac{\delta^2\rho}{\rho_0} + \cdots\right). \tag{4.7}$$

The first term is the classical potential, but since the particle's trajectory deviates from the classical one, the potential also receives corrections. Here ρ_0, $\delta\rho$ and $\delta^2\rho$ are given by

$$\rho_0 = \dot{z}_0(\tau_-^x)\cdot(x - z_0(\tau_-^x)) \tag{4.8}$$

$$\delta\rho = \delta\dot{z}(\tau_-^x)\cdot(x - z_0(\tau_-^x)) - \dot{z}_0(\tau_-^x)\cdot\delta z(\tau_-^x), \tag{4.9}$$

$$\delta^2\rho = \delta^2\dot{z}(\tau_-^x)\cdot(x - z_0(\tau_-^x)) - \delta\dot{z}(\tau_-^x)\cdot\delta z(\tau_-^x) - \dot{z}_0(\tau_-^x)\cdot\delta^2 z(\tau_-^x). \tag{4.10}$$

From now on we only consider the transverse fluctuations. Then $\dot{z}_0\cdot\delta z = 0$ is satisfied. As seen from (4.6), ρ is proportional to the spatial distance from the particle to the observer. The variation of ρ becomes negligible for large distance r if we take a variation of $(x - z_0(\tau))$ in ρ. On the contrary, if we take a variation of $\dot{z}_0$, $\delta\rho$ or $\delta^2\rho$ is still proportional to the spatial distance r. Hence for large r, we can approximate the variations by

$$\delta\rho \sim \delta\dot{z}(\tau_-^x)\cdot(x - z_0(\tau_-^x)), \tag{4.11}$$

$$\delta^2\rho \sim \delta^2\dot{z}(\tau_-^x)\cdot(x - z_0(\tau_-^x)). \tag{4.12}$$

Note also $\langle\delta^2\dot{z}^i\rangle = 0$ since the velocity in the transverse directions fluctuates uniformly and its expectation value vanishes.

B. Correlations of the inhomogeneous terms

Now we calculate the 2-point function explicitly. If we take the classical part without the fluctuation of ρ, the 2-point function becomes

$$G(x,x') - G_0(x,x') \to \langle\phi_{\rm inh}(x)\phi_{\rm inh}(x')\rangle = \left(\frac{e}{4\pi}\right)^2\frac{1}{\rho_0(x)\rho_0(x')}. \tag{4.13}$$

This gives the classical radiation corresponding to the Larmor radiation. The interference term vanishes because the 1-point function vanishes identically $\langle\phi_h\rangle = 0$.

Corrections to the above classical Larmor radiation are induced by the transverse fluctuating motion $\delta\rho$. First we consider the 2-point correlation function between the inhomogeneous terms up to the second order of the transverse fluctuations. Since $\langle\delta^2\rho\rangle = 0$, we have

$$\langle\phi_{\rm inh}(x)\phi_{\rm inh}(y)\rangle = \left(\frac{e}{4\pi}\right)^2\left\langle\frac{1}{\rho(x)\rho(y)}\right\rangle = \left(\frac{e}{4\pi}\right)^2\frac{1}{\rho_0(x)\rho_0(y)}\left(1 + \frac{\langle\delta\rho(x)\delta\rho(y)\rangle}{\rho_0(x)\rho_0(y)} + \frac{\langle(\delta\rho(x))^2\rangle}{\rho_0^2(x)} + \frac{\langle(\delta\rho(y))^2\rangle}{\rho_0^2(y)}\right). \tag{4.14}$$

Note that all the terms in the parenthesis behave constantly as the distance r between the observer and the particle becomes large. The first term gives the Larmor radiation mentioned above. The other terms correspond to the radiation induced by the fluctuations. Its calculation is easy, because one can write $\langle\delta\rho\delta\rho\rangle$ in terms of $\langle\delta\dot{z}^i\delta\dot{z}^i\rangle = \langle\delta v^i\delta v^i\rangle$ which we have already evaluated in the previous section. With the expression (3.15), it becomes

$$\langle\phi_{\rm inh}(x)\phi_{\rm inh}(y)\rangle = \left(\frac{e}{4\pi}\right)^2 \frac{1}{\rho_0(x)\rho_0(y)}\left[1 + e^2\int\frac{d\omega}{2\pi}|h(\omega)|^2 I(\omega) \times\left(\frac{x^i y^i e^{-i\omega(\tau_-^x-\tau_-^y)}}{\rho_0(x)\rho_0(y)} + \frac{x^i x^i}{\rho_0(x)\rho_0(x)} + \frac{y^i y^i}{\rho_0(y)\rho_0(y)}\right)\right]. \tag{4.15}$$

As before, since we are considering the fluctuating motion whose frequency is smaller than the acceleration, we may as well approximate $I(\omega)$ by $a^3/12\pi^2$. In order to calculate the symmetrized correlation function between x and y, $I(\omega)$ is replaced by $I_S(\omega)$.

C. Interference terms

Next let us calculate the interference terms. They are rewritten as

$$\langle\phi_{\rm inh}(x)\phi_h(y)\rangle + \langle\phi_h(x)\phi_{\rm inh}(y)\rangle = -\frac{e}{4\pi}\left(\frac{\langle\delta\rho(x)\phi_h(y)\rangle}{\rho_0^2(x)} + \frac{\langle\phi_h(x)\delta\rho(y)\rangle}{\rho_0^2(y)}\right). \tag{4.16}$$

Calculation of the interference terms is more complicated since we need to evaluate the following correlation functions:

$$\langle\delta\rho(x)\phi_h(y)\rangle = -x^i\langle\delta\dot z^i(\tau_-^x)\phi_h(y)\rangle \tag{4.17}$$

$$= -ex^i\int\frac{d\omega}{2\pi}e^{-i\omega\tau_-^x}h(\omega)\langle\partial_i\varphi(\omega)\phi_h(y)\rangle \tag{4.18}$$

$$\langle\phi_h(x)\delta\rho(y)\rangle = -y^i\langle\phi_h(x)\delta\dot z^i(\tau_-^y)\rangle \tag{4.19}$$

$$= -ey^i\int\frac{d\omega}{2\pi}e^{-i\omega\tau_-^y}h(\omega)\langle\phi_h(x)\partial_i\varphi(\omega)\rangle. \tag{4.20}$$

Since two terms $\langle\partial_i\varphi(\omega)\phi_h(y)\rangle$ and $\langle\phi_h(x)\partial_i\varphi(\omega)\rangle$ are related by

$$\langle\partial_i\varphi(\omega)\phi_h(y)\rangle = (\langle\phi_h(y)\partial_i\varphi(-\omega)\rangle)^*, \tag{4.21}$$

it is sufficient to calculate one of them. From the definition of φ in (3.3), the interference term $\langle\phi_h(x)\partial_i\varphi(\omega)\rangle$ is written as

$$\langle\phi_h(x)\partial_i\varphi(\omega)\rangle = \int d\tau e^{i\omega\tau}\left(\frac{\partial}{\partial y^i}\langle\phi_h(x)\phi_h(y)\rangle\right)_{y=z(\tau)} \tag{4.22}$$

$$= -\frac{1}{4\pi^2}\int d\tau e^{i\omega\tau}\left(\frac{\partial}{\partial y^i}\frac{1}{(x^0-y^0-i\epsilon)^2-(\vec x-\vec y)^2}\right)_{y=z(\tau)} \tag{4.23}$$

$$= \frac{1}{4\pi^2}\frac{\partial}{\partial x^i}\int d\tau\frac{e^{i\omega\tau}}{(x^0-z^0(\tau)-i\epsilon)^2-(x^1-z^1(\tau))^2-x_\perp^2}, \tag{4.24}$$

where $x_\perp^2 = (x^2)^2 + (x^3)^2$ is the transverse distance. We first evaluate the integral and then take a derivative. The integral

$$P(x,\omega) \equiv \int d\tau\frac{e^{i\omega\tau}}{(x^0-z^0(\tau)-i\epsilon)^2-(x^1-z^1(\tau))^2-x_\perp^2}, \tag{4.25}$$

can be evaluated by the contour integral in the complex τ plane. The positions of the poles are given by a series of points

$$\tau_\pm^n = T_\pm + \frac{2n\pi i}{a} - i\epsilon, \tag{4.26}$$

where n is an integer. $T_\pm$ are complex numbers whose imaginary parts are 0 or π/a and satisfy

$$e^{aT_\pm} = \frac{a}{2u}\left(-L^2 \mp\sqrt{L^4 + \frac{4}{a^2}uv}\right). \tag{4.27}$$

Here we have defined

$$L^2 = -x^\mu x_\mu + \frac{1}{a^2}, \tag{4.28}$$

$$u = x^0 - x^1, \qquad v = x^0 + x^1. \tag{4.29}$$

Note the relation $e^{aT_+}e^{aT_-} = -v/u$. The positions of the poles reflect the finite temperature property of the uniformly accelerated observer. In the following we will consider two different types of observers as shown in Fig. 2. The first observer is to observe the radiation in the right wedge ($O_{\rm R}$) while the second one is in the future wedge ($O_{\rm F}$). For both cases, $v > 0$ is satisfied and the radiation can travel causally from the particle to the observers. There are two different types of poles $\tau_\pm$. A pole at $\tau_- = T_-$, which is real, is located at a classically acceptable point. Namely, τ_- is the proper time of the particle whose radiation travels to the observer in a causal way. The other pole at T_+ is more subtle.

For $u < 0$ (in the right wedge), $T_+ = \tau_+^R$ is real and corresponds to the advanced causal proper time. For $u > 0$ (in the future wedge), $T_+ = \tau_+^F + i\pi/a$ has an imaginary part and one can interpret it as the proper time of a trajectory of a virtual particle in the left wedge, as in Fig. 2. In the following, we drop the superscript F or R. In the region where $v < 0$, $\phi_{\rm inh}$ does not exist and no nontrivial correlation is observed there.

The residue of the pole at $\tau_\pm^n$ is given by $-e^{i\omega\tau_\pm^n}/(2\rho(\tau_\pm^n))$ where

$$\rho(\tau_\pm^n) = \dot z(\tau_\pm^n)\cdot(x - z_0(\tau_\pm^n)) \tag{4.30}$$

$$= \frac{1}{2}(ue^{a\tau_\pm^n} + ve^{-a\tau_\pm^n}). \tag{4.31}$$

Because of the periodicity, $\rho(\tau_\pm^n)$ is independent of n. The integral is now given by

$$P(x,\omega)=\frac{-\pi i}{\rho_0}\frac{1}{e^{2\pi\omega/a}-1}(e^{i\omega\tau_-^x}-e^{i\omega\tau_+^x}Z_x(\omega)), \tag{4.32}$$

where

$$Z_x(\omega)=e^{\pi\omega/a}\theta(u)+\theta(-u) \tag{4.33}$$

$\rho_0=\rho(\tau_-^n)$ can be rewritten in terms of L^2 as

$$\rho_0=\frac{a}{2}\sqrt{L^4+\frac{4}{a^2}uv}. \tag{4.34}$$

Note that the relation $\rho(\tau_+^n)=-\rho_0$ follows the identity $e^{aT_+}e^{aT_-}=-v/u$.

The second term of the parenthesis in (4.32) depends on τ^+. With naive intuition based on classical causality, the term may be removed by hand, but the calculation of the interference terms is essentially quantum mechanical, and it should not be neglected. It is puzzling how we can physically interpret such τ^+ dependence of the integral.

Taking a derivation of $P(x,\omega)$, we obtain $\langle\phi_h(x)\partial_i\varphi(\omega)\rangle$ as

$$\langle\phi_h(x)\partial_i\varphi(\omega)\rangle=\frac{iax^i}{4\pi\rho_0^2}\frac{1}{e^{2\pi\omega/a}-1}\bigg(\bigg(\frac{aL^2}{2\rho_0}+\frac{i\omega}{a}\bigg)e^{i\omega\tau_-^x}+\bigg(-\frac{aL^2}{2\rho_0}+\frac{i\omega}{a}\bigg)e^{i\omega\tau_+^x}Z_x(\omega)\bigg). \tag{4.35}$$

Here we have used the following identities,

$$\frac{\partial\rho_0}{\partial x^i}=\frac{a^2L^2}{2\rho_0}x^i, \tag{4.36}$$

$$\frac{\partial\tau_\pm^x}{\partial x^i}=\pm\frac{1}{\rho_0}x^i, \tag{4.37}$$

where i is the transverse direction. The second identity can be obtained by differentiating $(x-z(\tau_\pm^x))^2=0$ with respect to x^i, see (4.49).

The whole interference terms are now given by

$$\begin{aligned}&\langle\phi_h(x)\phi_{\rm inh}(y)\rangle+\langle\phi_{\rm inh}(x)\phi_h(y)\rangle\\&=\frac{-iae^2x^iy^i}{(4\pi)^2\rho_0(x)^2\rho(y)^2}\int\frac{d\omega}{2\pi}\frac{1}{1-e^{-2\pi\omega/a}}\\&\times\bigg[e^{-i\omega(\tau_-^x-\tau_-^y)}\bigg(h(-\omega)\bigg(\frac{aL_x^2}{2\rho_0(x)}-\frac{i\omega}{a}\bigg)\\&-h(\omega)\bigg(\frac{aL_y^2}{2\rho_0(y)}+\frac{i\omega}{a}\bigg)\bigg)+e^{-i\omega(\tau_+^x-\tau_-^y)}h(-\omega)\\&\times\bigg(-\frac{aL_x^2}{2\rho_0(x)}-\frac{i\omega}{a}\bigg)Z_x(-\omega)-e^{-i\omega(\tau_-^x-\tau_+^y)}h(\omega)\\&\times\bigg(-\frac{aL_y^2}{2\rho_0(y)}+\frac{i\omega}{a}\bigg)Z_y(-\omega)\bigg].\end{aligned} \tag{4.38}$$

D. Partial cancellation

In the following, in order to see whether there is a cancellation between the interference terms and the correlation function of the inhomogeneous terms, we look more closely at the first term in the parenthesis of (4.38) which depends only on τ_-. Note that the correlation function of the inhomogeneous terms (4.15) depends only on τ_-, and the τ_+ depending terms in the interference terms cannot be canceled with the correlations of the inhomogeneous terms.

Note that, by using the relation

$$h(\omega)+h(-\omega)=\frac{e^2}{6\pi}(\omega^2+a^2)|h(\omega)|^2, \tag{4.39}$$

one can show that a part of the interference terms in (4.38)

$$\begin{aligned}&\frac{iae^2x^iy^i}{(4\pi)^2\rho_0(x)^2\rho_0(y)^2}\int\frac{d\omega}{2\pi}\frac{1}{1-e^{-2\pi\omega/a}}e^{-i\omega(\tau_-^x-\tau_-^y)}\\&\times\bigg(h(-\omega)\frac{i\omega}{a}+h(\omega)\frac{i\omega}{a}\bigg)\end{aligned} \tag{4.40}$$

can be rewritten as

$$\begin{aligned}&-\bigg(\frac{e}{4\pi}\bigg)^2\frac{x^iy^i}{\rho_0(x)^2\rho_0(y)^2}\int\frac{d\omega}{2\pi}e^{-i\omega(\tau_-^x-\tau_-^y)}|h(\omega)|^2I(\omega)\\&=-\bigg(\frac{e}{4\pi}\bigg)^2\frac{\langle\delta\rho(x)\delta\rho(y)\rangle}{\rho_0(x)\rho_0(y)}.\end{aligned} \tag{4.41}$$

This cancels the first correction term in the correlations of the inhomogeneous parts in (4.15). Note that this canceled term was obtained by taking a derivative of $e^{i\omega\tau_-^x}$ in $P(x,\omega)$.

We have seen that there is a partial cancellation between the interference term and the correlations of the inhomogeneous terms, but the other terms are not canceled each other. Then, summing up both contributions, (4.15) and (4.38), we get the following result of the 2-point function;

$$\langle\phi(x)\phi(y)\rangle-\langle\phi_h(x)\phi_h(y)\rangle=\frac{e^2}{(4\pi)^2\rho_0(x)\rho_0(y)}F(x,y) \tag{4.42}$$

where

$$F(x,y) = 1 + e^2 \int \frac{d\omega}{2\pi} \frac{|h(\omega)|^2}{6\pi} I(\omega)\left(\left(\frac{x^i}{\rho_0(x)}\right)^2 + \left(\frac{y^i}{\rho_0(y)}\right)^2\right) - \frac{ia^2 x^i y^i}{\rho_0(x)\rho_0(y)} \int \frac{d\omega}{4\pi} \frac{1}{1-e^{-2\pi\omega/a}} \times \left[e^{-i\omega(\tau_-^x - \tau_-^y)}\left(h(-\omega)\frac{L_x^2}{\rho_0(x)} - h(\omega)\frac{L_y^2}{\rho_0(y)}\right) - e^{-i\omega(\tau_+^x - \tau_-^y)} h(-\omega)\left(\frac{L_x^2}{\rho_0(x)} + i\frac{2\omega}{a^2}\right) Z_x(-\omega) - e^{-i\omega(\tau_-^x - \tau_+^y)} h(\omega)\left(-\frac{L_y^2}{\rho_0(y)} + i\frac{2\omega}{a^2}\right) Z_y(-\omega)\right]. \tag{4.43}$$

The first term in $F(x, y)$ is the classical effect of radiation corresponding to the Larmor radiation. The second term comes from the inhomogeneous term $\langle(\delta\rho(x)/\rho_0(x))^2\rangle + \langle(\delta\rho(y)/\rho_0(y))^2\rangle$. The third term comes from the interference term, which is obtained by taking a derivative of $\rho(x)$ in $P(x, \omega)$. The fourth term is also an interference effect and depends on τ_+.

Let us compare the above result with the calculation for an internal detector. In the case of an internal detector in $(1 + 1)$ dimensions, the radiation is canceled by the interference effect, and there are no terms depending on τ_+. In the case of an internal detector in $(3 + 1)$ dimensions, there are τ_+-dependent terms. But if we neglect these terms, it was shown [18] that the interference terms completely cancel the radiation. The calculation is reviewed in Appendix A. In the case of a charged particle, since the position of the particle is fluctuating, only a part of the terms is canceled. In the following we focus on the τ_+-independent terms because the rapidly oscillating function $e^{-i\omega\tau}$ remains even after setting $x = y$ and suppresses the ω integral of the τ_+-dependent terms.

For a symmetrized 2-point function, $F(x, y)$ is replaced by $F_S(x, y)$

$$F_S(x,y) = 1 + e^2 \int \frac{d\omega}{2\pi} \frac{|h(\omega)|^2}{6\pi} I_S(\omega)\left(\left(\frac{x^i}{\rho_0(x)}\right)^2 + \left(\frac{y^i}{\rho_0(y)}\right)^2\right) - \frac{ia^2 x^i y^i}{\rho_0(x)\rho_0(y)} \times \int \frac{d\omega}{8\pi} \coth\left(\frac{\pi\omega}{a}\right) e^{-i\omega(\tau_-^x - \tau_-^y)} \times \left(h(-\omega)\frac{L_x^2}{\rho_0(x)} - h(\omega)\frac{L_y^2}{\rho_0(y)}\right) + \tau_+\text{-dependent terms}. \tag{4.44}$$

5. Energy-momentum tensor

In the remainder of this section we consider the radiation emitted by the accelerated particle. The energy-momentum tensor of the scalar field is given by

$$\langle T_{\mu\nu}\rangle = \langle :\partial_\mu\phi\partial_\nu\phi - \frac{1}{2} g_{\mu\nu}\partial^\alpha\phi\partial_\alpha\phi:\rangle_S. \tag{4.45}$$

Hence we can evaluate it by taking a derivative of the 2-point function (4.44).

The following relations are useful in taking derivatives:

$$\partial_\mu\rho_0 = (\ddot{z}_0 \cdot (x - z_0) - 1)\partial_\mu\tau_- + \dot{z}_{0\mu} \tag{4.46}$$

$$= -\frac{a^2L^2}{2}\partial_\mu\tau_- + \dot{z}_{0\mu} \tag{4.47}$$

$$\partial_\mu\tau_- = \frac{x_\mu - z_{0\mu}}{\rho_0}. \tag{4.48}$$

In the last line of the first equation, we used the explicit form of the classical solution (2.8) and $\ddot{z}_0 \cdot x = -a^2L^2/2$. The derivative $\partial_\mu\tau_-$ was obtained by taking a variation of the light-cone condition $(x - z_0(\tau_-^x))^2 = 0$;

$$2(x_\nu - z_{0\nu})(\delta x^\nu - \dot{z}_0^\nu \delta\tau_-^x) = 0 \rightarrow \frac{\delta\tau_-^x}{\delta x^\nu} = \frac{x_\nu - z_{0\nu}}{\rho_0}. \tag{4.49}$$

In particular, u and v derivatives are given by

$$\partial_u\tau_- = \frac{v - v_z}{2\rho_0}, \qquad \partial_v\tau_- = \frac{u - u_z}{2\rho_0} \tag{4.50}$$

$$\partial_u\rho_0 = -\frac{a^2L^2}{2}\partial_u\tau_- + \frac{av_z}{2}, \qquad \partial_v\rho_0 = -\frac{a^2L^2}{2}\partial_v\tau_- + \frac{1}{2av_z} \tag{4.51}$$

where $u_z = -e^{-a\tau_-}/a$, $v_z = e^{a\tau_-}/a$. From (4.48), we have $(\partial\rho_0)^2 = a^2x^2$. Since $(x - z(\tau))^2 = 0$, $x^2 \sim \mathcal{O}(r)$ and $(\partial\rho_0)^2$ is approximately proportional to the spatial distance r, not r^2. On the other hand, since $L^2 = -x_\mu^2 + 1/a^2$ is $\mathcal{O}(r)$, $\partial_\mu\rho_0$ itself is growing as $\mathcal{O}(r)$.

First we calculate the classical part of the energy-momentum tensor. It becomes

$$T_{\mathrm{cl},\mu\nu} = \frac{e^2(\partial_\mu\rho_0\partial_\nu\rho_0 - \frac{g_{\mu\nu}}{2}\partial_\alpha\rho_0\partial^\alpha\rho_0)}{(4\pi)^2\rho_0^4} \tag{4.52}$$

$$\sim \frac{e^2\partial_\mu\rho_0\partial_\nu\rho_0}{(4\pi)^2\rho_0^4}. \tag{4.53}$$

Note that $\partial_\alpha\rho_0\partial^\alpha\rho_0$ does not make a contribution here, since it is of the order of ρ_0 at infinity while $\partial_\mu\rho_0\partial_\nu\rho_0$ is in general of order ρ_0^2. This part of the energy-momentum tensor corresponds to the classical Larmor radiation and behaves as $1/\rho_0^2 \sim 1/r^2$ at infinity. The term $\dot{z}_{0\mu}(\tau_-^x)$ in $\partial_\mu\rho_0$ seems to be negligible, since it is $\mathcal{O}(1)$ while $\partial_\mu\rho_0$ is $\mathcal{O}(r)$. However, care should be taken because $\dot{z}_{0\mu}(\tau_-^x) = (\cosh a\tau_-^x, \sinh a\tau_-^x, 0, 0)$ behaves singularly if the observer is near the horizon.

Next we evaluate the other parts of the energy-momentum tensor. We especially consider the (u, u) and (v, v)-components in the following. From (4.44), extra terms of the energy-momentum tensor besides the classical ones are given by

$$\begin{aligned} T_{\mathrm{fluc},\mu\nu} = \frac{(x^i)^2}{\rho_0^2}\bigg[\bigg(\frac{e^2}{\pi}I_m - \frac{6ma^2 I_1 L^2}{\rho_0}\bigg)T_{\mathrm{cl},\mu\nu} - \frac{e^2a^2L^2}{(4\pi)^2\rho_0^3} \\ \times (mI_3\partial_\mu\tau_-^x\partial_\nu\tau_-^x + \frac{2mI_1}{\rho_0L^2}(x_\mu\partial_\nu\rho_0 + x_\nu\partial_\mu\rho_0) \\ + \frac{e^2I_m}{12\pi L^2}(x_\mu\partial_\nu\tau_-^x + x_\nu\partial_\mu\tau_-^x) \\ - \frac{e^2I_m}{24\pi\rho_0}(\partial_\mu\tau_-^x\partial_\nu\rho_0 + \partial_\nu\tau_-^x\partial_\mu\rho_0))\bigg] \end{aligned} \tag{4.54}$$

where we have defined the following ω integrals

$$I_1 = \int\frac{d\omega}{4\pi}|h(\omega)|^2\coth\bigg(\frac{\pi\omega}{a}\bigg)\omega, \tag{4.55}$$

$$I_3 = \int\frac{d\omega}{4\pi}|h(\omega)|^2\coth\bigg(\frac{\pi\omega}{a}\bigg)\omega^3, \tag{4.56}$$

$$I_m = \int\frac{d\omega}{4\pi}|h(\omega)|^2\coth\bigg(\frac{\pi\omega}{a}\bigg)(\omega^3 + a^2\omega) \tag{4.57}$$

$$= I_3 + a^2I_1. \tag{4.58}$$

These integrals can be similarly evaluated as in Sec. III, and we have

$$I_1 = \frac{3}{2mae^2}, \tag{4.59}$$

$$I_m \sim a^2I_1. \tag{4.60}$$

Because of the inequality $\Omega_- \ll a$, terms containing I_3 are generally negligible compared to other terms; $I_3 \sim \Omega_-^2 I_1 \ll a^2I_1$.

Near the past horizon, the $v \to 0$, the u-derivatives of ρ_0 and τ_-^x become very small and negligible. On the other hand, the v-derivative of τ_- becomes potentially large. u-derivatives of them are approximately given by

$$\partial_v\tau_-^x \to \frac{1}{av}, \qquad \partial_v\rho_0 \to -\frac{au}{2}. \tag{4.61}$$

A singular term of $\partial_v\rho_0$ near $v \sim 0$ is canceled and it remains finite near the past horizon. Hence the second term in (4.54) proportional to $(\partial\tau_-)^2$ may becomes large there. However, there are two reasons that the term cannot grow so large. One is a suppression by the ω integral, which is proportional to a very small coefficient I_3. The other reason is the overall factor $(x^i)^2/\rho_0^2$. Since the observer is much further than the acceleration scale $1/a$ from the particle, L^2 is much larger than $1/a^2$. Then $\rho_0 = (a/2)\sqrt{L^4 + (4/a^2)uv}$ can be approximated by $\rho_0 \sim (a/2)|x_\mu|^2$ and $(x^i)^2/\rho_0^2$ is also suppressed. Because of these two reasons, the singular behavior near the past horizon seems to be difficult to be observed experimentally.

V. THERMALIZATION IN ELECTROMAGNETIC FIELD

In this section, we consider the thermalization of an accelerated charged particle in the electromagnetic field. Calculations of the energy-momentum tensors are more involved and left for a future investigation. We study the thermalization of the transverse momenta of a uniformly accelerated particle in an electromagnetic field. The calculation is almost the same, but due to the presence of the polarization, several quantities become twice as large as those in the scalar case.

The action is given by

$$\begin{aligned} S_{\mathrm{EM}} = -m\int d\tau\sqrt{\dot{z}^\mu\dot{z}_\mu} - \int d^4xj^\mu(x)A_\mu(x) \\ -\frac{1}{4}\int d^4xF^{\mu\nu}F_{\mu\nu}, \end{aligned} \tag{5.1}$$

where the current is defined as

$$j^\mu(x) = e\int d\tau\dot{z}^\mu(\tau)\delta^4(x - z(\tau)). \tag{5.2}$$

The equations of motion are

$$m\ddot{z}_\mu = eF_{\mu\nu}\dot{z}^\nu \tag{5.3}$$

$$\partial_\mu F^{\mu\nu}(x) = j^\nu. \tag{5.4}$$

Using the gauge

$$\partial^\mu A_\mu = 0, \tag{5.5}$$

the equation of motion for A_μ becomes

$$\partial^\mu\partial_\mu A^\nu = j^\nu. \tag{5.6}$$

One can solve this equation as

$$\begin{aligned} A_\mu &= A_{h\mu} + \int d^4yG_R(x, y)j_\mu(y) \\ &= A_{h\mu} + e\int d\tau G_R(x, z(\tau))\dot{z}_\mu(\tau), \end{aligned} \tag{5.7}$$

where $A_{h\mu}$ is the homogeneous solution of the equation of motion which satisfies $\partial^2A_h^\mu = 0$. $G_R(x - y)$ is the retarded Green function

$$\begin{aligned} G_R(x, y) &= \theta(x^0 - y^0)\frac{\delta((x - y)^2)}{2\pi}, \\ \partial^2G_R(x, y) &= \delta^4(x - y). \end{aligned} \tag{5.8}$$

Inserting the solution of $A_\mu(x)$ back to the equation of motion for z^μ, we obtain the following stochastic equation

$$m\ddot{z}_\mu(\tau) = F_\mu + e(\partial_\mu A_{h\nu}(z) - \partial_\nu A_{h\mu}(z))\dot{z}^\nu + e^2 \int d\tau' \dot{z}^\nu(\tau)(\dot{z}_\nu(\tau')\partial_\mu - \dot{z}_\mu(\tau')\partial_\nu)G_R(z(\tau), z(\tau')). \quad (5.9)$$

The second line is the radiation reaction which can be treated similarly to the scalar case. It becomes

$$e^2 \int d\tau' \Big(\dot{z}^\nu \dot{z}_{[\nu}\partial_{\mu]} - \frac{s^2}{2}(\ddot{z}^2\partial_\mu - \dddot{z}^2\dot{z}^\nu\partial_\nu)\Big) \times \frac{d}{ds} G_R(z(\tau), z(\tau')) = -e^2 \int_\infty^\infty ds \frac{s^2}{3}(\dddot{z}_\mu(\tau) + \dot{z}_\mu(\tau)\ddot{z}^2(\tau))\frac{d}{ds}\frac{\delta(s^2)}{2\pi} = \frac{e^2}{6\pi}(\dddot{z}_\mu + \dot{z}_\mu \ddot{z}^2), \quad (5.10)$$

which has exactly the same form as the scalar case, but the coefficient is twice as large since the gauge field has two different polarizations. This is the Abraham-Lorentz-Dirac self-radiation term.

For small transverse momentum fluctuations $\delta v^i \equiv \delta \dot{z}^i$, we can simplify the stochastic equation similarly to the scalar case in previous sections. It can be solved in terms of the homogeneous solution of the gauge field as

$$\delta\tilde{v}^i(\omega) = -eh(\omega)(v_{0\alpha}\partial_i + \delta^i_\alpha(v_0 \cdot \partial))A^\alpha_h, \quad (5.11)$$

where

$$h(\omega) = \frac{1}{-im\omega + \frac{e^2}{6\pi}(\omega^2 + a^2)}. \quad (5.12)$$

The noise correlation of A^μ_h in the r.h.s. of (5.11) can be evaluated as

$$(v_{0\alpha}\partial_i + \delta^i_\alpha(v_0 \cdot \partial))(v'_{0\beta}\partial'_j + \delta^j_\beta(v'_0 \cdot \partial'))\langle A^\alpha_h(z)A^\beta_h(z')\rangle = \frac{a^4}{16\pi^2}\frac{\delta_{ij}}{\sinh^4(\frac{a(\tau-\tau'-i\epsilon)}{2})}. \quad (5.13)$$

It is also twice as large as the scalar case. Note that the quantity is gauge invariant

$$(\dot{z}_\alpha k_\mu - \eta_{\alpha\mu}(\dot{z}\cdot k))(\dot{z}'_\beta k'_\nu - \eta_{\beta\nu}(\dot{z}'\cdot k'))k^\alpha k'^\beta = 0. \quad (5.14)$$

Hence performing similar calculations to the scalar case, the fluctuations of the transverse momenta become

$$\frac{m}{2}\langle \delta v^i(\tau)\delta v^j(\tau)\rangle = \frac{1}{2}\frac{a\hbar}{2\pi c}\delta_{ij}\Big(1 + \mathcal{O}\Big(\frac{a^2}{m^2}\Big)\Big). \quad (5.15)$$

Since the coefficient of the dissipative term is twice as large as the scalar case, the relaxation time becomes the half of it: $\tau_R = \frac{6\pi m}{a^2 e^2}$.

VI. CONCLUSIONS AND DISCUSSIONS

In this paper, we studied a stochastic motion of a uniformly accelerated charged particle in the scalar-field analog of QED. The particle's motion fluctuates because of the thermal behavior of the uniformly accelerated observer (the Unruh effect). Because of this fluctuating motion, Chen and Tajima [10] conjectured that there is additional radiation besides the classical Larmor radiation. On the other hand, it was argued [11,12] that interferences between the radiation field induced by the fluctuating motion and the quantum fluctuation of the vacuum may cancel the above additional radiation. The cancellation was shown in the case of an internal detector, but it was not yet settled whether the same kind of cancellation occurs in the case of a fluctuating charged particle in QED.

In the present paper, in order to investigate the above issue systematically, we first formulated a motion of a uniformly accelerated particle in terms of the stochastic (Langevin) equation. By using this formalism, we showed that the momenta in the transverse directions actually get thermalized so as to satisfy the equipartition relation with the Unruh temperature. Then we calculated correlation functions and energy flux from the accelerated particle. Partial cancellation is actually shown to occur, but some terms still remain. Hence there is still a possibility that, besides the classical Larmor radiation, we can detect additional radiation associated with the fluctuating motion caused by the Unruh effect.

There are several issues to be clarified. First in calculating the energy flux at infinity there appeared classically unacceptable contributions (i.e. those depend on τ_+). If the observer is in the right wedge, the contribution to the energy flux come from the particle in the future of the observer. In the case of the observer in the future wedge, this contribution comes from the virtual particle in the left wedge. Both of them are classically unacceptable, and we do not yet have physical understanding why these contributions appear in the calculation.

Another issue is the calculation of longitudinal fluctuations. Since the particle is accelerated in the longitudinal direction with very high acceleration, longitudinal fluctuations caused by the Unruh effect are technically difficult to evaluate (see Appendix B). Furthermore, it is not clear what kind of quantities are thermalized in the longitudinal fluctuations. The particle feels finite temperature noise in the accelerating frame and we may expect that the kinetic energy written in the Rindler coordinate ξ is thermalized. However, the particle is uniformly accelerated in constant external force. Then if a particle at $P1$ on a trajectory $T1$ in Fig. 3 is kicked in the longitudinal direction to $P2$, it follows another trajectory $T2$ with a different asymptote. The Rindler coordinate ξ, which is defined in the original accelerating frame, becomes divergent at $\tau = \infty$ on the new trajectory $T2$. In this sense, the longitudinal fluctuations look kinematically unstable in the original Rindler

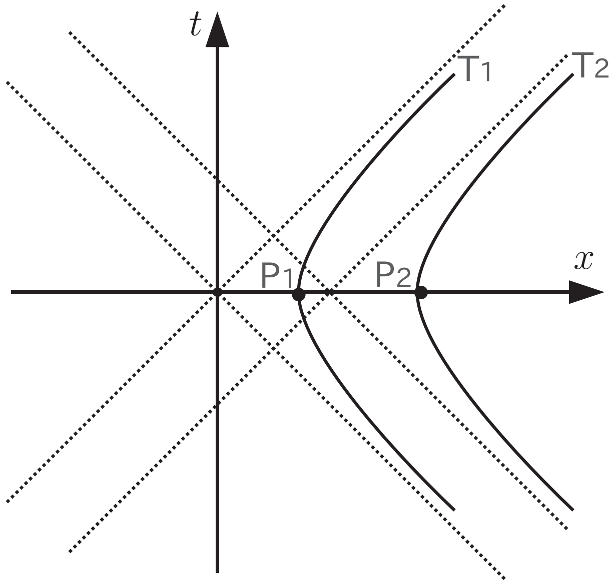

FIG. 3. $T1$ is a trajectory of the original accelerated particle. If the particle is kicked at $P1$ to a point $P2$, it follows a different trajectory $T2$ in the constant external force. It has a different asymptote from the original one, and the original Rindler coordinate ξ of the particle on $T2$ diverges at $\tau = \infty$.

coordinate and it is inappropriate to describe the thermalization of longitudinal fluctuations in the Rindler coordinate ξ. We give brief discussions on the calculations of longitudinal fluctuations in Appendix A.

Finally an interesting possibility is an effect of decoherence induced by interaction with environments. In this paper, we have treated the trajectory of the particle semiclassically in terms of the stochastic Langevin equation. If the initial state of the particle is a superposition of two localized wave-packets, we need to sum over the corresponding trajectories to obtain the transition amplitude [18]. Decoherence would suppress the correlations between these different worldlines. Furthermore the effect of decoherence might be important even for a single trajectory with the stochastic fluctuations. We have discussed the partial cancellation in the energy-momentum tensor between the inhomogeneous (fluctuation originated) terms and the interference terms. The fluctuating motions of the particle are quantum mechanically induced by the vacuum fluctuations of the radiation field. The inhomogeneous terms in energy-momentum tensor are evaluated at the same position on the trajectory, and robust against decoherence. Namely the variances of the trajectory never vanish. On the contrary, the interference terms are given by calculating the correlation function of the vacuum fluctuations at the position of the observer $\mathcal{O}_{\mathcal{F},\mathcal{R}}$ and at the particle's position on the trajectory, and can be easily killed by an additional interaction in between the trajectory and the observer. Then the (partial) cancellation is lost and only the correlation function between the inhomogeneous term $\langle \phi_{\rm inh}(x)\phi_{\rm inh}(y)\rangle$ (namely, the Unruh radiation) may survive.

ACKNOWLEDGMENTS

The work was started by discussions on physics of the high-intensity lasers with experimentalists K. Fujii, K. Homma, T. Saeki, T. Takahashi, T. Tauchi, and J. Urakawa. We would like to thank them for introducing us to this subject. We also acknowledge K. Itakura, Y. Kitazawa, H. Kodama and W. G. Unruh for useful discussions. A part of the work was presented at a workshop at LMU on November 24, 2009. We thank T. Tajima and D. Habs for organizing the workshop and the warm hospitality there. The works of Y. Y. and S. Z. are supported in part by Grant-in-Aid for JSPS Fellows, Japan Society for the Promotion of Science, No. 22-3834 and No. 21-1348, respectively. The research by S. I. is supported in part by the Grant-in-Aid for Scientific Research (19540316) from the Ministry of Education, Culture, Sports, Science and Technology, Japan. We are also supported in part by the Center for the Promotion of Integrated Sciences (CPIS) of Sokendai.

APPEMDIX A: INTERNAL DETECTOR IN 3 + 1 DIMENSIONS

In this appendix, we give a brief review of an internal detector in (3 + 1) dimensions to see how the Unruh radiation is canceled by the interference effects [14].

The action for an accelerated internal detector coupling with a massless scalar field is given by

$$S = \int d\tau \frac{m}{2}\Big((\partial_\tau Q(\tau))^2 - \Omega_0^2 Q^2\Big) + \int d^4x \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \lambda \int d^4x d\tau Q(\tau)\phi(x)\delta^4(x - z(\tau)\Big), \tag{A1}$$

where ∂_τ is used to denote a derivative with respect to the proper time τ. The equations of motion are given by

$$\partial^2 \phi(x) = \lambda \int d\tau Q(\tau)\delta^4(x - z(\tau)) \tag{A2}$$

$$(\partial_\tau^2 + \Omega_0^2)Q(\tau) = \frac{\lambda}{m}\phi(z(\tau)). \tag{A3}$$

Substituting the solution $\phi = \phi_h + \phi_{\rm inh}$,

$$\phi_{\rm inh}(x) = \lambda \int d\tau Q(\tau) G_R(x - z(\tau)), \tag{A4}$$

to the equation of the internal detector, we get the following equation,

$$(\partial_\tau^2 + \Omega_0^2)Q(\tau) - \frac{\lambda^2}{m}\int d\tau' Q(\tau') G_R(z(\tau) - z(\tau')) = \frac{\lambda}{m}\phi_h(z(\tau)). \tag{A5}$$

Here ϕ_h is the homogeneous solution representing the vacuum fluctuations. The inhomogeneous term is evaluated by expanding the Green function with respect to $(\tau - \tau')$ as we did in (2.27). Then after a renormalization of the mass term, we get the diffusive term of the radiation reaction,

$$\int d\tau' Q(\tau') G_R(z(\tau) - z(\tau')) \Rightarrow \frac{Q'(\tau)}{4\pi}. \quad \text{(A6)}$$

The stochastic equation can be solved by the Fourier transformation on the path as

$$\tilde{Q}(\tau) = \lambda h(\omega)\varphi(\omega), \quad \text{(A7)}$$

where

$$h(\omega)^{-1} = -m\omega^2 + m\Omega^2 - i\frac{\omega\lambda^2}{4\pi} \quad \text{(A8)}$$

and the Fourier transformations are defined as

$$\tilde{Q}(\omega) = \int d\tau e^{i\omega\tau} Q(\tau), \quad \text{(A9)}$$

$$\varphi(\omega) = \int d\tau e^{i\omega\tau} \phi_h(z(\tau)). \quad \text{(A10)}$$

Note that $G_R(z(\tau) - z(\tau'))$ is a function of $(\tau - \tau')$ if the classical solution $z(\tau)$ represents the accelerated path (2.8). The 2-point correlation function is decomposed into

$$\langle\phi(x)\phi(y)\rangle = \langle\phi_h(x)\phi_h(y)\rangle + \langle\phi_{\mathrm{inh}}(x)\phi_h(y)\rangle + \langle\phi_h(x)\phi_{\mathrm{inh}}(y)\rangle + \langle\phi_{\mathrm{inh}}(x)\phi_{\mathrm{inh}}(y)\rangle \quad \text{(A11)}$$

where

$$\langle\phi_{\mathrm{inh}}(x)\phi_h(y)\rangle + \langle\phi_h(x)\phi_{\mathrm{inh}}(y)\rangle = \int d\tau \frac{d\omega}{2\pi} e^{-i\omega\tau}\lambda^2 h(\omega)(G_R(y - z(\tau))\langle\phi_h(x)\varphi(\omega)\rangle + G_R(x - z(\tau))\langle\varphi(\omega)\phi_h(y)\rangle) \quad \text{(A12)}$$

$$\langle\phi_{\mathrm{inh}}(x)\phi_{\mathrm{inh}}(y)\rangle = \int d\tau d\tau' \frac{d\omega}{2\pi}\frac{d\omega'}{2\pi} e^{-i(\omega\tau + \omega'\tau')}\lambda^4 G_R(x - z(\tau)) \times G_R(y - z(\tau'))h(\omega)h(\omega')\langle\varphi(\omega)\varphi(\omega')\rangle. \quad \text{(A13)}$$

We first evaluate the interference term (A12);

$$\langle\phi_h(x)\varphi(\omega)\rangle = \int d\tau e^{i\omega\tau}\langle\phi_0(x)\phi_0(z(\tau))\rangle = -\frac{1}{4\pi^2}\int d\tau \frac{e^{i\omega\tau}}{(x^0 - z^0(\tau) - i\epsilon)^2 - (x^1 - z^1(\tau))^2 - \rho^2} = -\frac{1}{4\pi^2}P(x, \omega). \quad \text{(A14)}$$

Poles are given by solving the equation,

$$0 = \left(x^0 - \frac{\sinh a\tau}{a}\right)^2 - \left(x^1 - \frac{\cosh a\tau}{a}\right)^2 - \rho^2 \quad \text{(A15)}$$

$$= -u\frac{e^{a\tau}}{a} + u\frac{e^{-a\tau}}{a} + x^2 - \frac{1}{a^2}. \quad \text{(A16)}$$

The solutions of this equation are classified according to two different types of observers (see Fig. 2)

$\mathcal{O}_F$(in future wedge): $u > 0, v > 0$

$$\Rightarrow e^{a\tau_-^F} = \frac{a}{2u}\left(-L^2 + \sqrt{L^4 + \frac{4}{a^2}uv}\right) \quad \text{(A17)}$$

$$- e^{a\tau_+^F} = \frac{a}{2u}\left(-L^2 - \sqrt{L^4 + \frac{4}{a^2}uv}\right) \quad \text{(A18)}$$

$\mathcal{O}_R$(in right wedge): $u < 0, x^0 + x^1 > 0$

$$\Rightarrow e^{a\tau_-^R} = \frac{a}{2|u|}\left(L^2 - \sqrt{L^4 - \frac{4}{a^2}|uv|}\right) \quad \text{(A19)}$$

$$e^{a\tau_+^R} = \frac{a}{2|u|}\left(L^2 + \sqrt{L^4 - \frac{4}{a^2}|uv|}\right), \quad \text{(A20)}$$

where, $L^2 = -x^2 + 1/a^2$. The poles at $\tau_-^{F,R}$ correspond to the proper times at the intersections of the particle's world line and the past light-cone of the observer's position. Hence they are the physically acceptable poles. On the other hand, τ_+^F correspond to the proper time at a point on a "virtual path" in the left wedge. τ_+^R lies at an intersection of the world line and the future light-cone of the observer. Both of them are classically unacceptable.

Summing these contributions to the integral, we obtain

$$P(x, \omega) = \frac{-\pi i}{\rho_0}\frac{1}{e^{2\pi\omega/a} - 1}(e^{i\omega\tau_-^x} - e^{i\omega\tau_+^x}Z_x(\omega), \quad \text{(A21)}$$

where

$$Z_x = e^{\pi\omega/a}\theta(u) + \theta(-u), \quad \text{(A22)}$$

$$\rho_0 = \frac{a}{2}\sqrt{L^4 + \frac{4}{a^2}uv}. \quad \text{(A23)}$$

Using the following relation,

$$\int d\tau G_R(x - z(\tau))f(\tau) = \frac{1}{4\pi\rho_0}f(\tau_-), \quad \text{(A24)}$$

a part of the interference term depending on τ_-^R or τ_-^F can be written as

$$\langle\phi_h(x)\phi_{\rm inh}(y)\rangle \to i\lambda^2 \int d\tau d\tau' \frac{d\omega}{2\pi} G_R(x - z(\tau)) \times G_R(y - z(\tau'))e^{i\omega(\tau-\tau')}\frac{h(\omega)}{e^{2\pi\omega/a} - 1}. \tag{A25}$$

Similarly, we have

$$\langle\phi_{\rm inh}(x)\phi_h(y)\rangle \to i\lambda^2 \int d\tau d\tau' \frac{d\omega}{2\pi} G_R(x - z(\tau)) \times G_R(y - z(\tau'))e^{-i\omega(\tau-\tau')}\frac{h(\omega)}{1 - e^{-2\pi\omega/a}}, \tag{A26}$$

where we have used the identity

$$\langle\tilde{\varphi}(\omega)\phi_h(y)\rangle = (\langle\phi_h(y)\tilde{\varphi}(-\omega)\rangle)^*. \tag{A27}$$

The correlation function of inhomogeneous terms is given by

$$\begin{aligned}
&\langle\phi_{\rm inh}(x)\phi_{\rm inh}(y)\rangle \\
&= \lambda^4 \int d\tau d\tau' \frac{d\omega}{2\pi}\frac{d\omega'}{2\pi} e^{-i\omega\tau}e^{-i\omega'\tau'}G_R(x - z(\tau)) \\
&\quad \times G_R(y - z(\tau'))h(\omega)h(\omega')\langle\tilde{\varphi}(\omega)\tilde{\varphi}(\omega')\rangle \\
&= \lambda^4 \int d\tau d\tau' \frac{d\omega}{2\pi}\frac{d\omega'}{2\pi} e^{-i\omega(\tau-\tau')}G_R(x - z(\tau)) \\
&\quad \times G_R(y - z(\tau'))h(\omega)h(-\omega)\int d(\tau_a - \tau_b) \\
&\quad \times 2\pi\delta(\omega + \omega')e^{i\omega(\tau_a-\tau_b)}\langle\phi_0(z(\tau_a))\phi_0(z(\tau_b))\rangle \\
&= \lambda^4 \int d\tau d\tau' \frac{d\omega}{2\pi} e^{-i\omega(\tau-\tau')}G_R(x - z(\tau)) \\
&\quad \times G_R(y - z(\tau'))\frac{\omega}{2\pi}\frac{h(\omega)h(-\omega)}{1 - e^{-2\pi\omega/a}}.
\end{aligned} \tag{A28}$$

These three contributions (A25), (A26), and (A28) to the correlation function are shown to be canceled each other because of the relation

$$h(\omega) - h(-\omega) = \frac{i\omega\lambda^2}{2\pi}|h(\omega)|^2. \tag{A29}$$

Therefore if we neglect the contributions from the classically unacceptable poles at τ^+ the 2-point function vanishes, and therefore there are no energy-momentum flux after the thermalization occurs.

The remaining term in the 2-point function is the contributions of the τ_+ dependent terms to the interference term, and written as

$$\begin{aligned}
&\int \frac{d\omega}{2\pi}\frac{-ia^2\lambda^2}{8\pi\rho_0(x)\rho_0(y)}\frac{1}{1 - e^{-2\pi\omega/a}} \\
&\times (h(\omega)e^{-i\omega(\tau_-(x)-\tau_+(y))}Z_y(-\omega) \\
&- h(-\omega)e^{-i\omega(\tau_+(x)-\tau_-(y))}Z_x(-\omega)).
\end{aligned} \tag{A30}$$

It looks strange why we have such a (classically unacceptable) term in the final result.

APPENDIX B: LONGITUDINAL FLUCTUATIONS

In this appendix we briefly study fluctuations along the direction of the particle's classical motion. It is convenient to define the light-cone coordinates

$$z^\pm = z^t \pm z^x. \tag{B1}$$

The classical solution in the light-cone coordinates is given by

$$z^\pm_{cl} = \frac{\pm e^{\pm a\tau}}{a}. \tag{B2}$$

In order to describe the fluctuations around the classical solution, we define the Rindler coordinates $(\tilde{\tau}, \xi)$ by

$$z^\pm = \pm\frac{e^{\pm a(\tilde{\tau}\pm\xi)}}{a}. \tag{B3}$$

The classical solution corresponds to $\tilde{\tau}(\tau) = \tau$ and $\xi(\tau) = 0$.

Small fluctuations around the classical solution (B2) are written as

$$\delta z^\pm = (\delta\tilde{\tau} \pm \delta\xi)e^{\pm a\tau}. \tag{B4}$$

Writing the velocities as $\delta\dot{z}^\pm = \pm v^\pm e^{\pm a\tau}$, the stochastic equations for the longitudinal fluctuations become

$$m\dot{v}^\pm = \frac{e^2}{12\pi}(\ddot{v}^\pm \pm a\dot{v}^\pm - a^2v^\pm) + \frac{ae^2}{12\pi}(av^\mp \mp \dot{v}^\mp) + eQ\mathcal{O}\phi \tag{B5}$$

where

$$\mathcal{O} = a + e^{a\tau}\partial_+ - e^{-a\tau}\partial_-. \tag{B6}$$

Here $\mathcal{O}(\tau)$ acts on $\phi(z(\tau))$ and $\partial_\pm = \frac{1}{2}(\partial_0 \pm \partial_x)$.

The set of equations can be solved by using the Fourier transformations

$$v^\pm(\tau) = \int\frac{dw}{2\pi}e^{-i\omega\tau}\tilde{v}^\pm(\omega), \tag{B7}$$

$$\mathcal{O}\,\phi = \int\frac{dw}{2\pi}e^{-i\omega\tau}\mathcal{O}\varphi(\omega) \tag{B8}$$

as

$$\tilde{v}^{\,\pm}(\omega) = \frac{12\pi}{\omega(e^2\omega - 12i\pi m)}eQ\mathcal{O}\varphi(\omega). \tag{B9}$$

Since $(v^+ - v^-)$ is not affected by the quantum field φ, we can safely put it at zero. It is consistent with the gauge condition $\dot{z}\cdot\dot{z} = 1$, i.e. $\dot{z}_{cl}\cdot\delta\dot{z} = 0$.

By using the relation

$$v^\pm = \delta\dot{\xi} + a\delta\tilde{\tau} \pm (\delta\dot{\tilde{\tau}} + a\delta\xi) \tag{B10}$$

we can obtain fluctuations for $\tilde{\tau}$ and ξ as

$$\delta\xi(\omega) = \frac{i}{a^2 + \omega^2} \frac{12\pi}{(e^2\omega - 12i\pi m)} eQ\mathcal{O}\varphi(\omega) \quad \text{(B11)}$$

$$\delta\tilde{\tau}(\omega) = \frac{a}{a^2 + \omega^2} \frac{12\pi}{\omega(e^2\omega - 12i\pi m)} eQ\mathcal{O}\varphi(\omega). \quad \text{(B12)}$$

The vacuum noise fluctuations for $\mathcal{O}\varphi$ can be calculated as

$$\langle \mathcal{O}\phi(\tau)\mathcal{O}\phi(\tau')\rangle = \frac{a^4}{32\pi^2} \frac{1}{\sinh^4(\frac{a(\tau-\tau'-i\epsilon)}{2})}. \quad \text{(B13)}$$

It is interesting that this is exactly the same as the noise correlation (3.7) appearing in the stochastic equation for the transverse momenta. However the 2-point function of $\delta\dot{\xi}$ and $\delta\dot{\tilde{\tau}}$ behave differently from the 2-point function of the transverse momenta. After symmetrization, we have

$$\langle \delta\dot{\xi}(\tau)\delta\dot{\xi}(\tau')\rangle_S = 6e^2Q^2 \int d\omega \coth\left(\frac{\pi\omega}{a}\right)\left(\frac{\omega}{a^2 + \omega^2}\right)^2 \times \frac{\omega^3 + \omega a^2}{e^4\omega^2 + (12\pi m)^2}, \quad \text{(B14)}$$

$$\langle \delta\dot{\tilde{\tau}}(\tau)\delta\dot{\tilde{\tau}}(\tau')\rangle_S = 6e^2Q^2 \int d\omega \coth\left(\frac{\pi\omega}{a}\right)\left(\frac{a}{a^2 + \omega^2}\right)^2 \times \frac{\omega^3 + \omega a^2}{e^4\omega^2 + (12\pi m)^2}. \quad \text{(B15)}$$

The structure of the integral is quite different from that appeared in the transverse fluctuations. The poles at $\pm i\Omega_+$ are the same, but the poles at $\pm i\Omega_-$ in the transverse case are replaced by poles at $\pm ia$. Since $a \gg \Omega_-$, the longitudinal fluctuations are affected by higher frequency modes of the quantum vacuum fluctuations. Hence, the positions of the poles are of the same order as a and we cannot use the derivative expansion with respect to ω/a, which was used in the case of the transverse fluctuations. Because of this, we do not know yet the validity of the integrals and an appropriate way to evaluate them. This technical problem will be related to another problem mentioned in the discussions that it is not clear what kind of quantities are appropriate to describe the thermalization in the longitudinal direction. We leave further analysis for future investigations.

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