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Generalized virial theorem in warped DGP brane-world

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We generalize the virial theorem to the warped Dvali, Gabadadze, and Porrati brane-world scenario and consider its implications on the virail mass. In this theory the four-dimensional scalar curvature term is included in the bulk action and the resulting four-dimensional effective Einstein equation is augmented with extra terms which can be interpreted as geometrical mass, contributing to the gravitational energy. Estimating the geometrical mass $\mathcal{M}(r)$ using the observational data, we show that these geometric terms may account for the virial mass discrepancy in clusters of galaxies. Finally, we obtain the radial velocity dispersion of galaxy clusters $\sigma_r(r)$ and show that it is compatible with the radial velocity dispersion profile of such clusters.

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I. INTRODUCTION

In recent times, theories with higher dimensions have become popular in high energy physics, especially in the context of the hierarchy problem and cosmology [1]. In this scenario it is purported that our four-dimensional Universe is a subspace called the brane, embedded in a higher-dimensional space-time called the bulk. One of the most successful of such higher-dimensional models is that proposed by Randall and Sundrum whose bulk has the geometry of an anti–de Sitter space admitting Z_2 symmetry [2]. They were successful in explaining what is known as the hierarchy problem: the enormous disparity between the strength of the fundamental forces. The Randall-Sundrum (RS) scenario has greatly increased our understanding of the Universe and has brought higher-dimensional gravitational theories to the fore. In certain RS-type models, all matter and gauge interactions reside on the brane while gravity can propagate into the bulk. Using the Israel junction conditions [3] and the Gauss-Codazzi equations, one can obtain the field equations on the brane, as employed by Shiromizu, Maeda, and Sasaki [4]. There are two very important results that arise from the effective four-dimensional Einstein equations on the brane. The first one is the quadratic energy-momentum tensor, $\pi_{\mu\nu}$, which is relevant in high energy and the second one is the projected Weyl tensor, $\mathcal{E}_{\mu\nu}$, on the brane which is responsible for carrying on the brane the contribution of the bulk gravitational field. The cosmological evolution of such a brane universe has been extensively investigated and effects such as a quadratic density term in the Friedmann equations have been found [5–7].

An alternative scenario was subsequently proposed by Dvali, Gabadadze, and Porrati (DGP) [8]. The DGP proposal rests on the key assumption of the presence of a four-dimensional Ricci scalar in the bulk action. There are two main reasons that make this model phenomenologically appealing. First, it predicts that four-dimensional Newtonian gravity on a brane-world is regained at distances shorter than a given crossover scale r_c (high energy limit), whereas five-dimensional effects become manifest above that scale (low energy limit) [9]. Second, the model can explain late-time acceleration without having to invoke a cosmological constant or quintessential matter [10]. An extension of the DGP brane-world scenario have been constructed by Maeda, Mizuno, and Torii, which is the combination of the RS II model and DGP model [11]. In this combination, an induced curvature term appears on the brane in the RS II model. This model has been called the warped DGP brane-world in the literature [12]. In this paper, we consider the effective gravitational field equations within the context of the warped DGP brane-world model and obtain the spherically symmetric equations in this scenario. So much for the success of the DGP model, a word of caution is in order; the theory predicts the existence of ghostlike excitations. Many scenarios have been undertaken to explain away such ghosts, but as yet no satisfactory solution exists. The interested reader should consult [13] for further insight. We do not discuss such excitations since our aim lies in studying the virial mass discrepancy in warped DGP models.

Modern astrophysical and cosmological models are faced with two severe theoretical difficulties which can be summarized as dark energy and dark matter problems. The problem of dark matter is a longstanding problem in modern astrophysics. Two important observational issues, the behavior of the galactic rotation curves and the mass discrepancy in clusters of galaxies led to the necessity of considering the existence of dark matter at the galactic and extra-galactic scales [14]. The total mass of a cluster can be estimated in two ways. One can apply the virial theorem to estimate the total dynamic mass M_V of a rich galaxy cluster from measurements of the velocities of the member galaxies and the cluster radius from the volume they occupy.

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The second is obtained by separately estimating the mass of each individual members and summing them up to give a total baryonic mass M. It is always found that M_V is greater than M. This is known as the *missing mass problem*. Nevertheless, the existence of dark matter was not firmly established until the time when the measurement of the rotational velocity of stars and gas orbiting at a distance r from the galactic center was performed. Observations show that the rotational velocity increases near the center of galaxy and remain nearly constant. This discrepancy between the observed rotation velocity curves and the theoretical prediction from Newtonian mechanics is known as the *galactic rotation curves problem* (Fig. 1). These discrepancies are explained by postulating that every galaxy and cluster of galaxy is embedded in a halo made up of some dark matter [14]. To deal with the question of dark matter, a great number of efforts has been concentrated on various modifications to the Einstein and the Newtonian gravity [15–19]. Several theoretical models, based on a modification of Newton's law or of general relativity, have been proposed to explain the behavior of the galactic rotation curves. In the modified Newtonian dynamics theory which has been proposed by Milgrom [20], the Poisson equation for the gravitational potential, $\nabla^2\phi = 4\pi G\rho$, is replaced by an equation of the form $\nabla[\mu(x)(|\nabla\phi|/a_0)] = 4\pi G\rho$, where a_0 is a fixed constant and $\mu(x)$ a function satisfying the conditions $\mu(x) = x$ for $x \ll 1$ and $\mu(x) = 1$ for $x \gg 1$. The force law, giving the acceleration a of a test particle, becomes $a = a_N$ for $a_N \gg a_0$ and $a = \sqrt{a_N a_0}$ for $a_N \ll a_0$, where a_N is the usual Newtonian acceleration. The rotation curves of the galaxies are predicted to be flat, and they can be calculated once the distribution of the baryonic matter is known. A relativistic modified Newtonian dynamics inspired theory was developed by Bekenstein [21]. In this theory gravitation is mediated by a metric, a scalar field, and a four-vector field, all three dynamical. For alternative theoretical models to explain the galactic rotation curves, see [22]. One other such modification is that of the RS brane-world scenario [23]. It has been argued that a modified theory of gravity based on the RS brane-world scenario can explain the observations of the galactic rotation curve of spiral galaxies and the virial theorem mass discrepancy in clusters of galaxies without introducing any additional hypothesis [23].

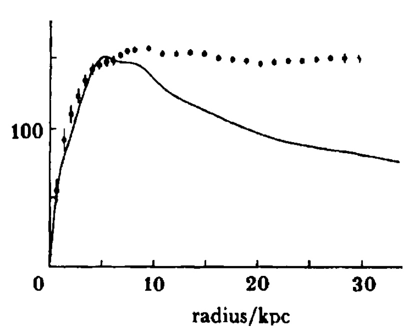

FIG. 1. Observed rotation velocity curve of the NGC3198 (dotted line) and the prediction from Newtonian theory (solid line).

Our main purpose in this paper is to obtain the generalized form of the virial theorem in the warped DGP brane-world model by using the collisionless Boltzmann equation. In what follows, we first give a brief review of the warped DGP brane-world model and the gravitational field equations that are derived in this model. Next, we use the relativistic Boltzmann equation to derive the virial theorem, which is modified by an extra term that may be used to explain the virial mass discrepancy in clusters of galaxies. Finally, we identify the geometrical mass of cluster in terms of the observable quantities and obtain the radial velocity dispersion of galaxy clusters.

II. EFFECTIVE FIELD EQUATIONS ON WARPED DGP BRANE

Let us start by presenting the model used in our calculation [11]. Consider a five-dimensional space-time with a four-dimensional brane that is located at $Y(X^A) = 0$, where X^A, $(A = 0, 1, 2, 3, 4)$ are five-dimensional coordinates. The effective action is given by

$$\mathcal{S} = \mathcal{S}_{\text{bulk}} + \mathcal{S}_{\text{brane}}, \tag{1}$$

where

$$\mathcal{S}_{\text{bulk}} = \int d^5X\sqrt{-\mathcal{G}}\left[\frac{1}{2\kappa_5^2}\mathcal{R} + \mathcal{L}_m^{(5)}\right], \tag{2}$$

and

$$\mathcal{S}_{\text{brane}} = \int_{Y=0} d^4x\sqrt{-g}\left[\frac{1}{\kappa_5^2}K^{\pm} + \mathcal{L}_{\text{brane}}(g_{\alpha\beta}, \psi)\right], \tag{3}$$

where $\kappa_5^2 = 8\pi G_5$ is the five-dimensional gravitational constant, and $\mathcal{R}$ and $\mathcal{L}_m^{(5)}$ are the five-dimensional scalar curvature and the matter Lagrangian in the bulk, respectively. Also, x^μ, $(\mu = 0, 1, 2, 3)$ are the induced four-dimensional coordinates on the brane, $K^{\pm}$ is the trace of extrinsic curvature on either side of the brane [24,25] and $\mathcal{L}_{\text{brane}}(g_{\alpha\beta}, \psi)$ is the effective four-dimensional Lagrangian, which is given by a generic functional of the brane metric $g_{\alpha\beta}$ and matter fields.

The five-dimensional Einstein field equations are given by

$$\mathcal{R}_{AB} - \tfrac{1}{2}\mathcal{R}\mathcal{G}_{AB} = \kappa_5^2[T_{AB}^{(5)} + \delta(Y)\tau_{AB}], \tag{4}$$

where

$$T_{AB}^{(5)} \equiv -2\frac{\delta\mathcal{L}_m^{(5)}}{\delta\mathcal{G}^{AB}} + \mathcal{G}_{AB}\mathcal{L}_m^{(5)}, \tag{5}$$

and

$$\tau_{\mu\nu} \equiv -2\frac{\delta \mathcal{L}_{\rm brane}}{\delta g^{\mu\nu}} + g_{\mu\nu}\mathcal{L}_{\rm brane}. \tag{6}$$

We study the case with an induced gravity on the brane due to quantum corrections [8]. The interaction between bulk gravity and the matter on the brane induces gravity on the brane through its quantum effects. If we take into account quantum effects of matter fields confined on the brane, the gravitational action on the brane is modified as

$$\mathcal{L}_{\rm brane}(g_{\alpha\beta}, \psi) = \frac{\mu^2}{2}R - \lambda_b + \mathcal{L}_m, \tag{7}$$

where μ is a mass scale which may correspond to the four-dimensional Planck mass, λ_b is the tension of the brane and $\mathcal{L}_m$ presents the Lagrangian of the matter fields on the brane. We note that for $\lambda_b = 0$ and $\Lambda^{(5)} = 0$ action (1) gives the DGP model and gives the RS II model if $\mu = 0$.

We obtain the gravitational field equations on the brane-world as [4]

$$G_{\mu\nu} = \frac{2\kappa_5^2}{3}\left[T^{(5)}_{AB}g^A_\mu g^B_\nu + g_{\mu\nu}\left(T^{(5)}_{AB}n^A n^B - \frac{1}{4}T^{(5)}\right)\right] + \kappa_5^2\pi_{\mu\nu} - \mathcal{E}_{\mu\nu}, \tag{8}$$

$$\nabla_\nu \tau^\nu_\mu = -2T^{(5)}_{AB}n^A g^B_\mu, \tag{9}$$

where ∇_ν is the covariant derivative with respect to $g_{\mu\nu}$ and the quadratic correction has the form

$$\pi_{\mu\nu} = -\tfrac{1}{4}\tau_{\mu\alpha}\tau^\alpha_\nu + \tfrac{1}{12}\tau\tau_{\mu\nu} + \tfrac{1}{8}g_{\mu\nu}\tau^{\alpha\beta}\tau_{\alpha\beta} - \tfrac{1}{24}g_{\mu\nu}\tau^2, \tag{10}$$

and the projection of the bulk Weyl tensor to the surface orthogonal to n^A is given by

$$\mathcal{E}_{\mu\nu} = C^{(5)}_{ABCD}n^A n^B g^C{}_\mu g^D{}_\nu. \tag{11}$$

The symmetry properties of $\mathcal{E}_{\mu\nu}$ imply that in general we can decompose it irreducibly with respect to a chosen 4-velocity field υ^μ as [6]

$$\mathcal{E}_{\mu\nu} = -\left(\frac{\kappa_5}{\kappa_4}\right)^4\left[U\left(\upsilon_\mu\upsilon_\nu + \frac{1}{3}h_{\mu\nu}\right) + 2Q_{(\mu}\upsilon_{\nu)} + P_{\mu\nu}\right], \tag{12}$$

where $h_{\mu\nu} = g_{\mu\nu} + \upsilon_\mu\upsilon_\nu$ projects orthogonal to υ_μ and the factor (κ_5/κ_4) is introduced for dimension reasons. Here

$$U = -\left(\frac{\kappa_4}{\kappa_5}\right)^4\mathcal{E}_{\mu\nu}\upsilon^\mu\upsilon^\nu,$$

is an effective nonlocal energy density or "dark radiation" term on the brane, arising from the free gravitational field in the bulk, $Q_\mu = (\kappa_4/\kappa_5)^4 h^\alpha_\mu \mathcal{E}_{\alpha\beta}\upsilon^\beta$ is an effective nonlocal energy flux, and

$$P_{\mu\nu} = -\left(\frac{\kappa_4}{\kappa_5}\right)^4\left[h_{(\mu}{}^\alpha h_{\nu)}{}^\beta - \frac{1}{3}h_{\mu\nu}h^{\alpha\beta}\right]\mathcal{E}_{\alpha\beta},$$

is a spatial, symmetric, and trace-free tensor. In what follows for the static spherically symmetric brane we have $Q_\mu = 0$, and we may choose $P_{\mu\nu} = P(r)\times(r_\mu r_\nu - \frac{1}{3}h_{\mu\nu})$, where the "dark pressure" $P(r)$ is a scalar function of the radial distance r, r_μ is a unit radial vector, and at any point on the brane in inertial frame $\upsilon^\mu = \delta^\mu_0$, $h_{\mu\nu} = \mathrm{diag}(0, 1, 1, 1)$ [26].

In order to find the basic field equations on the brane with induced gravity, we have to obtain the energy-momentum tensor of the brane $\tau_{\mu\nu}$, given by definition (6) from the Lagrangian (7), yielding

$$\tau^\mu_\nu = -\lambda_b\delta^\mu_\nu + T^\mu_\nu - \mu^2 G^\mu_\nu. \tag{13}$$

Assuming that the five-dimensional bulk space includes only a cosmological constant $\Lambda^{(5)}$ and inserting Eq. (13) into Eq. (8), we find the effective field equations for four-dimensional metric $g_{\mu\nu}$ as

$$\left(1 + \frac{\lambda_b}{6}\kappa_5^4\mu^2\right)G_{\mu\nu} = \frac{1}{6}\lambda_b\kappa_5^4 T_{\mu\nu} - \Lambda_4 g_{\mu\nu} - \kappa_5^4\mu^2 K_{\mu\nu\alpha\beta}G^{\alpha\beta} + \kappa_5^4[\pi^{(T)}_{\mu\nu} + \mu^4\pi^{(G)}_{\mu\nu}] - \mathcal{E}_{\mu\nu}, \tag{14}$$

where

$$K_{\mu\nu\rho\sigma} = \tfrac{1}{4}(g_{\mu\nu}T_{\rho\sigma} - g_{\mu\rho}T_{\nu\sigma} - g_{\nu\sigma}T_{\mu\rho}) + \tfrac{1}{12}[T_{\mu\nu}g_{\rho\sigma} + T(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\nu}g_{\rho\sigma})], \tag{15}$$

$$\pi^{(T)}_{\mu\nu} = -\tfrac{1}{4}T_{\mu\alpha}T^\alpha_\nu + \tfrac{1}{12}TT_{\mu\nu} + \tfrac{1}{8}g_{\mu\nu}T_{\alpha\beta}T^{\alpha\beta} - \tfrac{1}{24}g_{\mu\nu}T^2, \tag{16}$$

$$\pi^{(G)}_{\mu\nu} = -\tfrac{1}{4}G_{\mu\alpha}G^\alpha_\nu + \tfrac{1}{12}GG_{\mu\nu} + \tfrac{1}{8}g_{\mu\nu}G_{\alpha\beta}G^{\alpha\beta} - \tfrac{1}{24}g_{\mu\nu}G^2, \tag{17}$$

and the effective cosmological constant on the brane is given by

$$\Lambda_4 = \frac{\kappa_5^2}{2}\left[\Lambda^{(5)} + \frac{1}{6}\kappa_5^2\lambda_b^2\right]. \tag{18}$$

We note that for $\mu = 0$ these equations are exactly the same effective equations as in Ref. [4].

III. FIELD EQUATIONS FOR A CLUSTER INCLUDING IDENTICAL AND COLLISION-LESS POINT PARTICLES

Now we consider an isolated and spherically symmetric cluster being described by a static and spherically symmetric metric

$$ds^2 = -e^{\lambda(r)}dt^2 + e^{\nu(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{19}$$

Suppose that the clusters are constructed from identical and collisionless point particles (galaxies). This multiparticle system can be described by a continuous non-negative function $f_B(x^\mu, p^\mu)$, distribution function, which is defined over the phase space. In terms of the distribution function the energy-momentum tensor can be written as [27,28]

$$T_{\mu\nu} = \int f_B m \upsilon_\mu \upsilon_\nu d\upsilon, \tag{20}$$

where m is the mass of each galaxy, υ_μ is the four-velocity of the galaxy, and $d\upsilon = \frac{1}{\upsilon_t} d\upsilon_r d\upsilon_\theta d\upsilon_\varphi$ is the invariant volume element of the velocity space. We also assume that the matter content of the bulk is just a cosmological constant $\Lambda^{(5)}$ and the energy-momentum tensor of the matter in a cluster of galaxies can be represented in terms of spherically symmetric perfect fluid as

$$T_{\mu\nu} = (p_b + \rho_b)\upsilon_\mu \upsilon_\nu + p_b g_{\mu\nu}, \tag{21}$$

where $\upsilon_\mu \upsilon^\mu = -1$. Use of Eqs. (20) and (21) leads to the following relation for ρ_b and p_b:

$$\rho_b = \rho\langle \upsilon_t^2\rangle, \qquad p_b = \rho\langle \upsilon_r^2\rangle = \rho\langle \upsilon_\theta^2\rangle = \rho\langle \upsilon_\varphi^2\rangle; \tag{22}$$

here $\langle \upsilon_t^2\rangle$ represents the usual macroscopic averaging which is defined as $\langle \upsilon_t^2\rangle = \frac{1}{\rho}\int f_B \upsilon_t^2 m d\upsilon$, where ρ is the mass density and in terms of f_B is given by $\rho = \int f_B m d\upsilon$ [29,30].

Using Eq. (14), the gravitational field equations on the brane become

$$\left(1 + \frac{\lambda_b}{6}\kappa_5^4\mu^2\right)\frac{e^{-\nu}}{r^2}(-1 + r\nu' + e^\nu) = \frac{1}{6}\lambda_b\kappa_5^4\rho_b + \Lambda_4 + \kappa_5^4\mu^2\mathcal{K}_0^0 + \frac{\kappa_5^4}{12}\rho_b^2 - \kappa_5^4\mu^4\pi_0^{0(G)} + \frac{6}{\kappa_4^2\lambda_b}U(r), \tag{23}$$

$$\left(1 + \frac{\lambda_b}{6}\kappa_5^4\mu^2\right)\frac{e^{-\nu}}{r^2}(1 + r\lambda' - e^\nu) = \frac{1}{6}\lambda_b\kappa_5^4 p_b - \Lambda_4 - \kappa_5^4\mu^2\mathcal{K}_1^1 + \frac{\kappa_5^4}{12}(\rho_b^2 + 2\rho_b p_b) + \kappa_5^4\mu^4\pi_1^{1(G)} + \frac{2}{\kappa_4^2\lambda_b}[U(r) + 2P(r)], \tag{24}$$

$$\left(1 + \frac{\lambda_b}{6}\kappa_5^4\mu^2\right)\frac{e^{-\nu}}{2r}(2\lambda' - 2\nu' - \lambda'\nu' r + 2\lambda'' r + \lambda'^2 r) = \frac{1}{3}\lambda_b\kappa_5^4 p_b - 2\Lambda_4 - 2\kappa_5^4\mu^2\mathcal{K}_2^2 + \frac{\kappa_5^4}{6}(\rho_b^2 + 2\rho_b p_b) + 2\kappa_5^4\mu^4\pi_2^{2(G)} + \frac{4}{\kappa_4^2\lambda_b}[U(r) - P(r)], \tag{25}$$

where $U(r)$ and $P(r)$ are the dark radiation and dark pressure and the $K_{\mu\nu\alpha\beta}G^{\alpha\beta} \equiv \mathcal{K}_{\mu\nu}$ term is given by

$$\mathcal{K}_0^0 = -\frac{e^{-\nu}}{8}\left[\frac{2\nu'}{r}(\rho_b + p_b) - 4p_b\frac{\lambda'}{r} - \frac{2}{r^2}(\rho_b + p_b) + p_b\lambda'\nu' - 2p_b\lambda'' - p_b\lambda'^2 + \frac{2}{r^2}(\rho_b + p_b)e^\nu\right] - \frac{e^{-\nu}(\rho_b - 3p_b)}{24}\left[\frac{2e^\nu}{r^2} + \frac{2\nu'}{r} - \frac{2}{r^2} - \frac{4\lambda'}{r} + \lambda'\nu' - 2\lambda'' - \lambda'^2\right] + \frac{\rho_b e^{-\nu}}{24}\left[\frac{4\nu'}{r} - \frac{4\lambda'}{r} - \frac{4}{r^2} + \lambda'\nu' - 2\lambda'' - \lambda'^2 + \frac{4e^\nu}{r^2}\right], \tag{26}$$

$$\mathcal{K}_1^1 = -\frac{e^{-\nu}}{8}\left[p\lambda'\nu' - 2p\lambda'' - p\lambda'^2 - \frac{2\nu'}{r}(\rho_b - p_b) + \frac{2}{r^2}(\rho_b + p_b)e^\nu + \frac{2}{r^2}(\rho_b + p_b)\right] - \frac{e^{-\nu}(\rho_b - 3p_b)}{24}\left[\frac{4\nu'}{r} + \frac{2e^\nu}{r^2} - \frac{2}{r^2} - \frac{2\lambda'}{r} + \lambda'\nu' - 2\lambda'' - \lambda'^2\right] - \frac{p_b e^{-\nu}}{24}\left[\frac{4\nu'}{r} + \frac{4e^\nu}{r^2} - \frac{4}{r^2} - \frac{4\lambda'}{r} + \lambda'\nu' - 2\lambda'' - \lambda'^2\right], \tag{27}$$

$$\mathcal{K}_2^2 = \mathcal{K}_3^3 = \frac{e^{-\nu}}{4}\left[\frac{p_b\lambda'}{r} + \frac{\rho_b\nu'}{r} + \frac{(\rho_b - p_b)e^\nu}{r^2} - \frac{(\rho_b - p_b)}{r^2}\right] - \frac{e^{-\nu}(\rho_b - 3p_b)}{48}\left[\frac{6\nu'}{r} + \frac{8e^\nu}{r^2} - \frac{8}{r^2} - \frac{6\lambda'}{r} + \lambda'\nu' - 2\lambda'' - \lambda'^2\right] - \frac{p_b e^{-\nu}}{24}\left[\frac{4\nu'}{r} + \frac{4e^\nu}{r^2} - \frac{4}{r^2} - \frac{4\lambda'}{r} + \lambda'\nu' - 2\lambda'' - \lambda'^2\right], \tag{28}$$

and the use of Eq. (17) leads to the components of $\pi_\mu^{\nu(G)}$ as

$$\pi_0^{0(G)} = \frac{e^{-\nu}}{24}\left(\frac{-6\nu'}{r^3} - \frac{2\lambda'}{r^3} - \frac{\lambda'\nu'}{r^2} + \frac{2\lambda''}{r^2} + \frac{\lambda'^2}{r^2}\right) + \frac{e^{-2\nu}}{192}\left(\frac{48\nu'}{r^3} + \frac{16\lambda'}{r^3} - \frac{8\nu'\lambda''}{r} + \frac{16\lambda'\nu'}{r^2} - 4\lambda'\lambda''\nu' - \frac{8\lambda'\lambda''}{r} + \frac{4\lambda'\nu'^2}{r} + \lambda'^2\nu'^2 - 2\lambda'^3\nu' + 4\lambda''\lambda'^2 - \frac{12\nu'^2}{r^2} - \frac{4\lambda'^3}{r} - \frac{16\lambda''}{r^2} + 4\lambda''^2 + \lambda'^4 - \frac{4\lambda'^2}{r^2}\right), \tag{29}$$

$$\pi_1^{1(G)} = \frac{e^{-\nu}}{24}\left(\frac{2\nu'}{r^3} + \frac{6\lambda'}{r^3} - \frac{\lambda'\nu'}{r^2} + \frac{2\lambda''}{r^2} + \frac{\lambda'^2}{r^2}\right) + \frac{e^{-2\nu}}{192}\left(-\frac{16\nu'}{r^3} - \frac{48\lambda'}{r^3} + \frac{8\nu'\lambda''}{r} + \frac{16\lambda'\nu'}{r^2} - 4\lambda'\nu'\lambda'' + \frac{8\lambda'\lambda''}{r} - \frac{4\lambda'\nu'^2}{r} + \mu'^2\nu'^2 - 2\lambda'^3\nu' + 4\lambda''\lambda'^2 + \frac{4\nu'^2}{r^2} + \frac{4\lambda'^3}{r} - \frac{16\lambda''}{r^2} + 4\lambda''^2 + \lambda'^4 - \frac{20\lambda'^2}{r^2}\right), \tag{30}$$

$$\pi_2^{2(G)} = \pi_3^{3(G)} = -\frac{e^{-\nu}}{24}\left(\frac{4}{r^4} + \frac{\lambda'\nu'}{r^2} - \frac{2\lambda''}{r^2} - \frac{\lambda'^2}{r^2} - \frac{2e^{\nu}}{r^4}\right) - \frac{e^{-2\nu}}{48}\left(-\frac{2\lambda'^2\nu'}{r} - \frac{2\lambda''\nu'}{r} - \frac{4}{r^4} - \frac{10\lambda'\nu'}{r^2} + \frac{2\lambda'\lambda''}{r} + \frac{\lambda'\nu'^2}{r} - \frac{2\nu'^2}{r^2} + \frac{\lambda'^3}{r} + \frac{4\lambda''}{r^2}\right), \tag{31}$$

where a prime represents differentiation with respect to r. In the next section, we will investigate the influence of the bulk effects on the dynamics of the galaxies in warped DGP brane-world model.

IV. THE VIRIAL THEOREM IN WARPED DGP BRANE

In order to derive the virial theorem for galaxy clusters, we have to first write down the general relativistic Boltzmann equation governing the evolution of the distribution function f_B. The galaxies, which are treated as identical and collisionless point particles, are described by this distribution function. For the static spherically symmetric metric given by Eq. (19) we introduce the following frame of orthonormal vectors [27–29]:

$$e_\rho^{(0)} = e^{\lambda/2}\delta_\rho^0, \qquad e_\rho^{(1)} = e^{\nu/2}\delta_\rho^1, \qquad e_\rho^{(2)} = r\delta_\rho^2, \qquad e_\rho^{(3)} = r\sin\theta\,\delta_\rho^3, \tag{32}$$

where $g^{\mu\nu}e_\mu^{(a)}e_\nu^{(b)} = \eta^{(a)(b)}$. The four-velocity v^μ of a typical galaxy with $v^\mu v_\mu = -1$, in tetrad components is written as

$$v^{(a)} = v^\mu e_\mu^{(a)}, \qquad a = 0, 1, 2, 3. \tag{33}$$

The relativistic Boltzmann equation in tetrad components is given by

$$v^{(a)} e_{(a)}^\rho \frac{\partial f_B}{\partial x^\rho} + \gamma_{(b)(c)}^{(a)} v^{(b)} v^{(c)} \frac{\partial f_B}{\partial v^{(a)}} = 0, \tag{34}$$

where $f_B = f_B(x^\mu, v^{(a)})$ and $\gamma_{(b)(c)}^{(a)} = e_{\rho;\sigma}^{(a)} e_{(b)}^\rho e_{(c)}^\sigma$ are the distribution function and the Ricci rotation coefficients, respectively. Assuming that the distribution function is only a function of r, the relativistic Boltzmann equation becomes

$$v_r\frac{\partial f_B}{\partial r} - \left[\frac{v_t^2}{2}\frac{\partial\lambda}{\partial r} - \frac{(v_\theta^2 + v_\varphi^2)}{r}\right]\frac{\partial f_B}{\partial v_r} - \frac{v_r}{r}\left[v_\theta\frac{\partial f_B}{\partial v_\theta} + v_\varphi\frac{\partial f_B}{\partial v_\varphi}\right] - \frac{e^{\nu/2}v_\varphi}{r}\cot\theta\left[v_\theta\frac{\partial f_B}{\partial v_\varphi} - v_\varphi\frac{\partial f_B}{\partial v_\theta}\right] = 0, \tag{35}$$

where we have defined

$$v^{(0)} = v_t, \qquad v^{(1)} = v_r, \qquad v^{(2)} = v_\theta, \qquad v^{(3)} = v_\varphi. \tag{36}$$

Since we have assumed the system to be spherically symmetric, the term proportional to $\cot\theta$ must be zero. Multiplying Eq. (35) by $mv_r dv$ where $dv = \frac{1}{v_t}dv_r dv_\theta dv_\varphi$, and integrating over the velocity space and assuming that the distribution function vanishes rapidly as the velocities tend to $\pm\infty$, we obtain

$$r\frac{\partial}{\partial r}[\rho\langle v_r^2\rangle] + \frac{1}{2}\rho[\langle v_t^2\rangle + \langle v_r^2\rangle]r\frac{\partial\lambda}{\partial r} - \rho[\langle v_\theta^2\rangle + \langle v_\varphi^2\rangle - 2\langle v_r^2\rangle] = 0, \tag{37}$$

where ρ is the mass density and $\langle v_r^2\rangle$ represents the average value of v_r^2. Multiplying Eq. (37) by $4\pi r^2$ and integrating over the cluster leads to

$$\int_0^R 4\pi\rho[\langle v_r^2\rangle + \langle v_\theta^2\rangle + \langle v_\varphi^2\rangle]r^2 dr - \frac{1}{2}\int_0^R 4\pi r^3\rho[\langle v_t^2\rangle + \langle v_r^2\rangle]\frac{\partial\lambda}{\partial r}dr = 0. \tag{38}$$

This equation is reduced to

$$2K - \frac{1}{2}\int_0^R 4\pi r^3\rho[\langle v_t^2\rangle + \langle v_r^2\rangle]\frac{\partial\lambda}{\partial r}dr = 0, \tag{39}$$

where the total kinetic energy of the galaxies is defined as

$$K = \int_0^R 2\pi\rho[\langle v_r^2\rangle + \langle v_\theta^2\rangle + \langle v_\varphi^2\rangle]r^2 dr. \tag{40}$$

Now, using these relations and adding the gravitational field equations (23)–(25) and Eq. (22) we find

$$\left(1+\frac{\lambda_b}{6}\kappa_5^4\mu^2\right)e^{-\nu}\left(\frac{\lambda'}{r}-\frac{\lambda'\nu'}{4}+\frac{\lambda''}{2}+\frac{\lambda'^2}{4}\right)$$
$$=\frac{\kappa_4^2}{2}\rho[\langle v_t^2\rangle+\langle v_r^2\rangle+\langle v_\theta^2\rangle+\langle v_\varphi^2\rangle]+\frac{\kappa_4^2}{2\lambda_b}\rho^2[\langle v_t^2\rangle^2+\langle v_r^2\rangle^2+\langle v_\theta^2\rangle^2+\langle v_\varphi^2\rangle^2]-\Lambda_4$$
$$+\frac{\kappa_5^4\mu^2}{2}[\mathcal{K}_0^0-\mathcal{K}_1^1-2\mathcal{K}_2^2]-\frac{\kappa_5^4\mu^4}{2}[\pi_0^{0(G)}-\pi_1^{1(G)}-2\pi_2^{2(G)}]+\frac{6}{\kappa_4^2\lambda_b}U(r), \quad (41)$$

where $\frac{1}{6}\lambda_b\kappa_5^4=\kappa_4^2$. In order to obtain the generalized virial theorem we have to use some approximations. First, since the dispersion of the velocity of galaxies in the clusters is of the order 600–1000 km/s, i.e., $(\frac{v}{c})^2\approx 4\times 10^{-6}-1.11\times 10^{-5}\ll 1$, we can neglect the relativistic effects in the relativistic Boltzmann equation and use the small velocity limit approximation. In other words, $\langle v_r^2\rangle\approx\langle v_\theta^2\rangle\approx\langle v_\varphi^2\rangle\ll\langle v_t^2\rangle\approx 1$. Second, the intensity of the gravitational effects can be estimated from the ratio GM/R, which for typical clusters is of the order of $10^{-6}\ll 1$. Therefore, inside the galactic clusters the gravitational field is weak and we can use the weak gravitational field approximation. Then the term proportional to $\lambda'\nu'$ and λ'^2 in Eq. (41) may be ignored. Thus, assuming that $e^\lambda\approx e^\nu\approx 1$ inside the cluster [23], we can write Eqs. (41) as

$$\left(1+\frac{\lambda_b}{6}\kappa_5^4\mu^2\right)\left(\frac{\lambda'}{r}+\frac{\lambda''}{2}\right)=\frac{\kappa_4^2}{2}\rho[\langle v_t^2\rangle+\langle v_r^2\rangle+\langle v_\theta^2\rangle+\langle v_\varphi^2\rangle]+\frac{\kappa_4^2}{2\lambda_b}\rho^2[\langle v_t^2\rangle^2+\langle v_r^2\rangle^2+\langle v_\theta^2\rangle^2+\langle v_\varphi^2\rangle^2]$$
$$-\Lambda_4-\frac{\kappa_5^4\mu^2}{6r}\rho[3\nu'\langle v_t^2\rangle+2\lambda'\langle v_t^2\rangle+\lambda''r\langle v_t^2\rangle+3\nu'\langle v_r^2\rangle]+\frac{6}{\kappa_4^2\lambda_b}U(r). \quad (42)$$

On the other hand, for clusters of galaxies the ratio of the matter density and of the brane tension is much smaller than 1, $\rho/\lambda_b\ll 1$, so that one can neglect the quadratic term in the matter density in above equation. These conditions certainly apply to test particles in stable circular motion around galaxies, and to the galactic clusters. Thus, we can rewrite Eq. (42) as

$$\left(1+\frac{\lambda_b}{6}\kappa_5^4\mu^2\right)\frac{1}{2r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\lambda}{\partial r}\right)$$
$$=\frac{\kappa_4^2}{2}\rho-\Lambda_4+\kappa_5^4\mu^2[\mathcal{P}(r)+3\mathcal{U}(r)]+\frac{6}{\kappa_4^2\lambda_b}U(r), \quad (43)$$

where

$$\mathcal{U}(r)=-\frac{\rho\nu'}{6r}, \qquad \mathcal{P}(r)=-\frac{\rho}{6r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\lambda}{\partial r}\right).$$

Multiplying Eq. (43) by r^2 and integrating from 0 to r yields

$$\left(1+\frac{\lambda_b}{6}\kappa_5^4\mu^2\right)\frac{1}{2}\left(r^2\frac{\partial\lambda}{\partial r}\right)-\frac{\kappa_4^2}{8\pi}M(r)+\frac{1}{3}\Lambda_4r^3$$
$$-\frac{\kappa_4^2}{8\pi}M_{\rm DGP}(r)-\frac{\kappa_4^2}{8\pi}M_{\rm RS}(r)=0. \quad (44)$$

The total baryonic mass and the geometrical masses of the system are given by

$$M(r)=4\pi\int_0^r\rho(r')r'^2dr', \quad (45)$$

and

$$\kappa_4^2M_{\rm RS}(r)=\frac{48\pi}{\kappa_4^2\lambda_b}\int_0^rU(r')r'^2dr', \quad (46)$$

$$\kappa_4^2M_{\rm DGP}(r)=8\pi\kappa_5^4\mu^2\int_0^r[\mathcal{P}(r')+3\mathcal{U}(r')]r'^2dr', \quad (47)$$

where

$$\mathcal{M}(r)=M_{\rm RS}(r)+M_{\rm DGP}(r). \quad (48)$$

Multiplying Eq. (44) by $\frac{dM(r)}{r}$ and integrating from 0 to R, we finally obtain the generalized virial theorem in warped DGP scenario as

$$\left(1+\frac{\lambda_b}{6}\kappa_5^4\mu^2\right)2K+W+\frac{1}{3}\Lambda_4I+\mathcal{W}=0, \quad (49)$$

where

$$W=-\frac{\kappa_4^2}{8\pi}\int_0^R\frac{M(r)}{r}dM(r), \quad (50)$$

$$\mathcal{W}=-\frac{\kappa_4^2}{2}\int_0^R\mathcal{M}(r)\rho r dr, \quad (51)$$

and

$$I=\int_0^Rr^2dM(r), \quad (52)$$

where W is the gravitational potential energy of the system. At this point it is worth nothing that for $\mu=0$, we have $M_{\rm DGP}=0$ and the virial theorem in the warped DGP brane-world is reduced to the virial theorem in the RS brane scenario [23]

$$2K + W + \tfrac{1}{3}\Lambda_4 I + W_{\rm RS} = 0, \tag{53}$$

where

$$W_{\rm RS} = -\frac{\kappa_4^2}{2}\int_0^R M_{\rm RS}(r)\rho r dr. \tag{54}$$

As one can see the gravitational energy modified by $W_{\rm RS}$ which has its origin in the global bulk effect due to the $\mathcal{E}_{\mu\nu}$ term. We can also recover the virial theorm in the standard general relativity from Eq. (53) in the limit $\lambda_b^{-1} \rightarrow 0$. A alternative possibility in recovering the four-dimensional virial theorem is to take the limit $\kappa_5 \rightarrow 0$, while keeping the Newtonian gravitational constant κ_4^2 finite [11].

In the case $\lambda_b = \Lambda^{(5)} = 0$, the virial theorem in the warped DGP brane-world is reduced to the virial theorem in the DGP brane scenario [19]

$$2K + W + W_{\rm DGP} = 0, \tag{55}$$

where

$$W_{\rm DGP} = -\frac{\kappa_4^2}{2}\int_0^R M_{\rm DGP}(r)\rho r dr. \tag{56}$$

Note for a Minkowski DGP bulk space we have $\mathcal{E}_{\mu\nu} = 0$, thus, in the above equation $W_{\rm RS} = 0$. There is difference between our model and Refs. [19,23]. The virial theorem in the warped DGP brane-world is modified by both $W_{\rm RS}$ and $W_{\rm DGP}$, which the first is due to the global bulk effect whereas the second term has its origins in the induced gravity on the brane due to quantum correction.

Now, we introduce the radii R_V, R_I, and $\mathcal{R}$ as

$$R_V = \frac{M^2(r)}{\int_0^R \frac{M(r)}{r} dM(r)}, \tag{57}$$

$$R_I^2 = \frac{\int_0^R r^2 dM(r)}{M(r)}, \tag{58}$$

$$\mathcal{R} = -\frac{\kappa_4^2}{8\pi}\frac{\mathcal{M}^2(r)}{\mathcal{W}}, \tag{59}$$

where R_V is the virial radius and $\mathcal{R}$ is defined as the geometrical radius of the clusters of galaxies. Defining the virial mass as [29]

$$2K = -\frac{\kappa_4^2}{8\pi}\frac{M_V^2}{R_V}, \tag{60}$$

and using the following relations:

$$W = -\frac{\kappa_4^2}{8\pi}\frac{M^2}{R_V}, \qquad I = MR_I^2, \tag{61}$$

the generalized virial theorem (49) is simplified as

$$\left(1 + \frac{\lambda_b}{6}\kappa_5^4\mu^2\right)\left(\frac{M_V}{M}\right)^2 = 1 - \frac{8\pi\Lambda_4}{3\kappa_4^2}\frac{R_V R_I^2}{M} + \left(\frac{\mathcal{M}}{M}\right)^2\left(\frac{R_V}{\mathcal{R}}\right). \tag{62}$$

We have three types of mass in Eq. (62), namely, the total baryonic mass of the system represented by M (including the baryonic mass of the intracluster gas and of the stars, other particles like massive neutrinos), the virial mass represented by M_V, and finally, the geometrical mass represented by $\mathcal{M}$.

On large distance scales associated with galaxies, we can ignore the contribution of the effective cosmological constant to the mass energy of the galaxy. Also, it is found that M_V is considerably greater than M for most of the clusters and we can neglect the term unitary in Eq. (62). Therefore, the virial mass is given by

$$M_V(r) \simeq \mathcal{M}(r)\left(\frac{R_V}{\mathcal{R}}\right)^{1/2}. \tag{63}$$

This equation shows that the virial mass is proportional to the geometrical mass.

V. ASTROPHYSICAL APPLICATIONS

In this section, we emphasize that the astrophysical observations together with the cosmological simulations have shown that the virilized part of the cluster is a measure of a fixed density such as a critical density, $\rho_c(z)$ at a special red-shift, so that $\rho_V = 3M_V/4\pi R_V^3 = \delta\rho_c(z)$, where M_V and R_V are the virial mass and radius, respectively. As is well known, $\rho_c(z) = h^2(z)3H_0^2/8\pi G$, where the Hubble parameter is normalized to its local value, i.e., $h^2(z) = \Omega_m(1+z)^3 + \Omega_\Lambda$, where Ω_m and Ω_Λ are the mass density and dark energy-density parameters, respectively, [31]. By knowing the integrated mass of the galaxy cluster as a function of the radius, one can estimate the appropriate physical radius for the mass measurement. The radii commonly used are r_{200} or r_{500}. These radii lie within the radii corresponding to the mean gravitational mass density of the matter $\rho_{tot} = 200\rho_c$ or $500\rho_c$. A useful radius is r_{200} to find the virial mass. The numerical values of the radius r_{200} for the cluster NGC 4636 are in the ranges $r_{200} = 0.85$ Mpc and $r_{200} = 4.49$ Mpc for the cluster A2163, so one can deduce that a typical value for r_{200} is 2 Mpc. The masses corresponding to r_{200} and r_{500} are denoted by M_{200} and M_{500}, respectively, and it is usually assumed that $M_V = M_{200}$ and $R_V = r_{200}$ [32].

A. Geometrical mass estimated using the Jean's relation

Now, we are going to obtain $\mathcal{M}(r)$ as a function of r by comparing the virial theorem results with the observational data for galaxy cluster which can be obtained from the X-ray observation of the gas in the cluster. The most of the baryonic mass in clusters is in the gas form, therefore we assume that the energy density and pressure in $T_{\mu\nu}$ is that of a gas as

$$\rho = \rho_g(r), \qquad p = p_g(r). \tag{64}$$

In a majority of clusters most of the baryonic mass is in the form of the intracluster gas. The gas mass density $\rho_g(r)$ distribution can be fitted with the observational data by using the following expression for the radial baryonic mass distribution [32]:

$$\rho_g(r) = \rho_0\left(1 + \frac{r^2}{r_c^2}\right)^{-3\beta/2}, \tag{65}$$

where r_c is the core radius, and ρ_0 and β are cluster independent constants. A static spherically symmetric system of collisionless particles that is in equilibrium can be described by the Jean's equation [14]

$$\frac{d}{dr}[\rho_g(r)\sigma_r^2] + \frac{2\rho_g(r)}{r}(\sigma_r^2 - \sigma_{\theta,\varphi}^2) = -\rho_g(r)\frac{d\Phi(r)}{dr}, \tag{66}$$

where $\Phi(r)$ is the gravitational potential, and σ_r and $\sigma_{\theta,\varphi}^2$ are the mass-weighted velocity dispersions in the radial and tangential directions. We assume that the gas is isotropically distributed inside the cluster, so $\sigma_r = \sigma_{\theta,\varphi}$. The gas pressure is related to the velocity dispersion and gas profile density by $p_g = \rho_g \sigma_r^2$. By assuming that the gravitational field is weak so that it satisfies the usual Poisson equation $2\nabla^2\Phi \approx \kappa_4^2 \rho_{tot}$, where ρ_{tot} is the energy density including ρ_g and other forms of matter, like luminous matter and the geometrical matter, etc., the Jean's equation becomes

$$\frac{dp_g(r)}{dr} = -\rho_g(r)\frac{d\Phi(r)}{dr} = -\frac{\kappa_4^2 M_{\rm tot}}{8\pi r^2}\rho_g(r), \tag{67}$$

where $M_{\rm tot}(r)$ is the total mass inside the radius r. The observed X-ray emission from the hot ionized intracluster gas is usually interpreted by assuming that the gas is in isothermal equilibrium. Therefore, we assume that the gas is in equilibrium state having the equation of state $p_g(r) = \frac{k_B T_g}{\mu m_p}\rho_g(r)$, where k_B is a Boltzmann constant, T_g is the gas temperature, $\mu = 0.61$ is the mean atomic weight of the particles in the gas cluster, and m_p is the proton mass. Equation (67) then gives

$$M_{\rm tot}(r) = -\frac{8\pi k_B T_g}{\mu m_p \kappa_4^2} r^2 \frac{d}{dr}\ln\rho_g(r). \tag{68}$$

Now, use of the density profile of the gas given by Eq. (65) leads to the mass profile inside the cluster as

$$M_{\rm tot}(r) = \frac{24\pi k_B T_g \beta}{\mu m_p \kappa_4^2}\frac{r^3}{r^2 + r_c^2}. \tag{69}$$

On the other hand, using Eqs. (45) and (48) we can obtain another expression for the total mass

$$\frac{dM_{\rm tot}(r)}{dr} = 4\pi r^2\rho_g(r) + \frac{48\pi}{\kappa_4^2\lambda_b}U(r)r^2 + \frac{8\pi\kappa_5^4\mu^2}{\kappa_4^2}[\mathcal{P}(r) + 3\mathcal{U}(r)]r^2, \tag{70}$$

substituting Eqs. (69) and (65) into Eq. (70) we obtain the following expression:

$$\frac{12}{\kappa_4^2\lambda_b}U(r) + \frac{2\kappa_5^2\mu^2}{\kappa_4^4}[\mathcal{P}(r) + 3\mathcal{U}(r)] = \frac{6k_B T_g\beta}{\mu m_p\kappa_4^2}\frac{r^2 + 3r_c^2}{(r^2 + r_c^2)^2} - \rho_0\left(1 + \frac{r^2}{r_c^2}\right)^{-3\beta/2}. \tag{71}$$

Finally, substituting above equation into Eq. (48), in the limit $r \gg r_c$ considered here, we obtain the following geometrical mass:

$$\mathcal{M}(r) \simeq \left[\frac{24\pi k_B T_g\beta}{\mu m_p\kappa_4^2} - 4\pi\rho_0 r_c^{3\beta}\frac{r^{2-3\beta}}{3(1-\beta)}\right]r, \tag{72}$$

which includes both the local and nonlocal bulk effects. Observations show that the intracluster gas has a small contribution to the total mass [31–34], thus we can neglect the contribution of the gas to the geometrical mass and rewrite Eq. (72) as

$$\mathcal{M}(r) \simeq \left(\frac{24\pi k_B T_g\beta}{\mu m_p\kappa_4^2}\right)r. \tag{73}$$

Now, let us estimate the value of $\mathcal{M}(r)$. First, we note that $k_B T_g \approx 5$ KeV for most clusters. The virial radius of the clusters of galaxies is usually assumed to be r_{200}, indicating the radius for which the energy density of the cluster becomes $\rho_{200} = 200\rho_{cr}$, where $\rho_{cr} = 4.6975 \times 10^{-30} h_{50}^2$ gr/cm^3 [32]. Using Eq. (72) we find

$$r_{\rm cr} = 91.33\beta^{1/2}\left(\frac{k_B T_g}{5\ \text{KeV}}\right)^{1/2} h_{50}^{-1}\ \text{Mpc}. \tag{74}$$

The total geometrical mass corresponding to this value is

$$\mathcal{M}(r) = 4.83 \times 10^{16}\beta^{3/2}\left(\frac{k_B T_g}{5\ \text{KeV}}\right)^{1/2} h_{50}^{-1} M_\odot, \tag{75}$$

which is consistent with the observational values for the virial mass of clusters [32].

B. Radial velocity dispersion in galactic clusters

Radial velocity dispersion in galactic clusters plays an important role in estimating the virial mass of the clusters. It can be expressed in terms of the virial mass as [34]

$$M_V = \frac{3}{G}\sigma_1^2 R_V. \tag{76}$$

Assuming that the velocity distribution in the cluster is isotropic, we have $\langle v^2\rangle = \langle v_r^2\rangle + \langle v_\theta^2\rangle + \langle v_\varphi^2\rangle = 3\langle v_r^2\rangle = 3\sigma_r^2$, the radial velocity dispersion σ_r^2 for clusters in the warped DGP model can be obtained from Eq. (37) as

$$\frac{d}{dr}(\rho\sigma_r^2) + \frac{1}{2}\rho\lambda' = 0, \tag{77}$$

where σ_1 and σ_r are related by $3\sigma_1^2 = \sigma_r^2$. On the other hand, by neglecting the cosmological constant the Einstein field equation (43) becomes

$$\left(1 + \frac{\lambda_b}{6}\kappa_5^4\mu^2\right)\frac{1}{2r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\lambda}{\partial r}\right) = \frac{\kappa_4^2}{2}\rho + \kappa_5^4\mu^2[\mathcal{P}(r) + 3\mathcal{U}(r)] + \frac{6}{\kappa_4^2\lambda_b}U(r). \tag{78}$$

Integrating, we obtain

$$\left(1 + \frac{\lambda_b}{6}\kappa_5^4\mu^2\right)\frac{1}{2}\left(r^2\frac{\partial\lambda}{\partial r}\right) = \frac{\kappa_4^2}{8\pi}M(r) + \frac{\kappa_4^2}{8\pi}\mathcal{M}(r) + \mathcal{C}_1, \tag{79}$$

where $\mathcal{C}_1$ is an integration constant. By eliminating λ' from Eqs. (77) and (79), we obtain

$$\left(1 + \frac{\lambda_b}{6}\kappa_5^4\mu^2\right)\frac{d}{dr}(\rho\sigma_r^2) = -\frac{\kappa_4^2 M(r)}{8\pi r^2}\rho(r) - \frac{\kappa_4^2\mathcal{M}(r)}{8\pi r^2}\rho(r) - \frac{\mathcal{C}_1}{r^2}\rho(r). \tag{80}$$

Integration now gives the following solution:

$$\left(1 + \frac{\lambda_b}{6}\kappa_5^4\mu^2\right)\sigma_r^2 = -\frac{1}{\rho}\int\frac{\kappa_4^2 M(r)}{8\pi r^2}\rho(r)dr - \frac{1}{\rho}\int\frac{\kappa_4^2\mathcal{M}(r)}{8\pi r^2}\rho(r)dr - \frac{1}{\rho}\int\frac{\mathcal{C}_1}{r^2}\rho(r)dr - \frac{\mathcal{C}_2}{\rho}, \tag{81}$$

where $\mathcal{C}_2$ is an integration constant. For most clusters $\beta \geq \frac{2}{3}$ and therefore, in the limit $r \gg r_c$, the gas density profile (65) can be written as [23]

$$\rho_g(r) = \rho_0\left(\frac{r}{r_c}\right)^{-3\beta}, \qquad \beta \geq \frac{2}{3}. \tag{82}$$

Now, substituting Eqs. (45), (73), and (82) into Eq. (81), for $\beta \neq 1$ we obtain

$$\left(1 + \frac{\lambda_b}{6}\kappa_5^4\mu^2\right)\sigma_r^2 = -\frac{\rho_0\kappa_4^2}{12(1-\beta)(1-3\beta)}r^2\left(\frac{r}{r_c}\right)^{-3\beta} + \frac{k_B T_g}{\mu m_p} + \frac{\mathcal{C}_1}{(1+3\beta)}\frac{1}{r} - \frac{\mathcal{C}_2}{\rho_0}\left(\frac{r}{r_c}\right)^{3\beta}, \tag{83}$$

and for $\beta = 1$ we find

$$\left(1 + \frac{\lambda_b}{6}\kappa_5^4\mu^2\right)\sigma_r^2 = \frac{\rho_0\kappa_4^2}{8}r_c^3\left(\frac{1}{4r^4} + \frac{\ln r}{r^4}\right)\left(\frac{r}{r_c}\right)^3 + \frac{k_B T_g}{\mu m_p} + \frac{\mathcal{C}_1}{4}\frac{1}{r} - \frac{\mathcal{C}_2}{\rho_0}\left(\frac{r}{r_c}\right)^3. \tag{84}$$

As is well known, the simple form $\sigma_r^2(r) = B/(r+b)$ for the radial velocity dispersion and the relation $\rho(r) = A/r(r+a)^2$ for the density of the galaxies in cluster, with B, b, a and A constants, can be used to fit the observational data [34]. For $r \gg a$, $\rho(r) \simeq A/r$, while for $r \gg a$, $\rho(r)$ behaves like $\rho(r) \simeq A/r^3$. Here, our expression for σ_r^2 can be also used to fit the observational data. Therefore, the comparison of the observed velocity dispersion profiles of the galaxy clusters and the velocity dispersion profiles predicted by the warped DGP brane-world model may give a powerful method to discriminate between the different theoretical scenarios.

Finally, we compare the radial velocity dispersion in the warped DGP brane-world model with the radial velocity dispersion in the other theoretical models. As we noted before for $\Lambda^{(5)} = \lambda_b = 0$, the warped DGP model reduces to the DGP model and Eq. (84) for $\beta = 1$ reduces to

$$\sigma_r^2 = \frac{\rho_0\kappa_4^2}{8}r_c^3\left(\frac{1}{4r^4} + \frac{\ln r}{r^4}\right)\left(\frac{r}{r_c}\right)^3 + \frac{k_B T_g}{\mu m_p} + \frac{\mathcal{C}_1}{4}\frac{1}{r} - \frac{\mathcal{C}_2}{\rho_0}\left(\frac{r}{r_c}\right)^3, \tag{85}$$

which is the radial velocity dispersion in the DGP model, Eq. (65) in [19]. The radial velocity dispersion of galaxy clusters in Palatini $f(R)$ gravity for $\gamma = 3$, which is corresponding to $\beta = 1$, is also presented as [18]

$$\sigma_r^2 = -r^3\int\frac{F'}{2F}r^{-3}dr + \frac{k_B T_g}{\mu m_p} + \pi G\rho_0\left(\frac{1}{4r} + \frac{\ln r}{r}\right) + \frac{c}{4}\frac{1}{r} - \frac{c'}{\rho_0}r^3, \tag{86}$$

where $F(R) = \frac{df(R)}{dR}$. For $f(R) = R$ this relation reduces to

$$\sigma_r^2 = \frac{\kappa_4^2\rho_0}{8}\left(\frac{1}{4r} + \frac{\ln r}{r}\right) + \frac{k_B T_g}{\mu m_p} + \frac{c}{4}\frac{1}{r} - \frac{c'}{\rho_0}r^3, \tag{87}$$

which is Eq. (84) with $8\pi G = \kappa_4^2$ and $\Lambda^{(5)} = \lambda_b = 0$. The same relation has been also obtained in the Randall-Sundrum II model with this difference that the origin of the geometrical mass in $f(R)$ gravity is the extra terms in the Einstein-Hilbert action whereas in the latter it is the global bulk effect. In Fig. 2 we have plotted the radial velocity dispersion for the cluster NGC 5813. The numerical values of it are in the ranges $\beta = 0.766$, $r_c = 25$ Kpc,

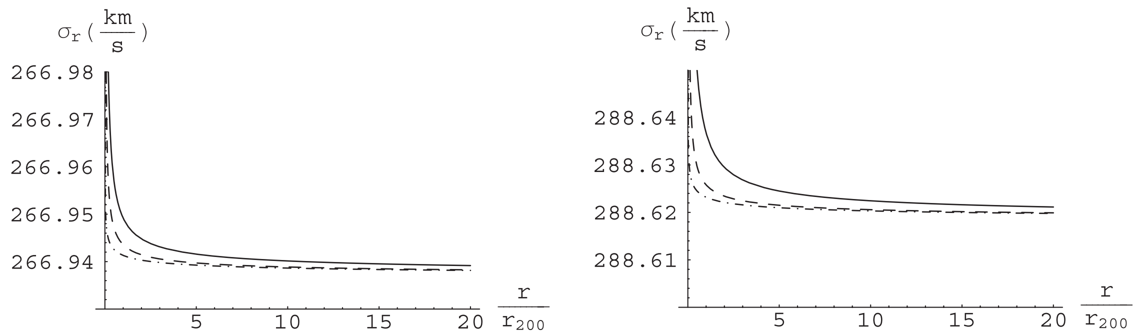

FIG. 2. Left panel: The radial velocity dispersion in warped DGP brane-world model. Right panel: The same parameter in the DGP brane-world scenario for the NGC 5813 cluster with $\beta = 0.766$, $r_{200} = 0.87$ Mpc, $r_c = 25$ Kpc, $k_B T_g = 0.52$ KeV and $\mathcal{C}_1 = 0.503$, 1.005×10^8, $2.011 \times 10^8 M_\odot$, $\mathcal{C}_2 = 0.02, 0.03, 0.04 M_\odot^2/\mathrm{Kpc}^4$ for solid, dashed, and dot-dashed curves, respectively.

$k_B T_g = 0.52$ Kev, $r_{200} = 0.087$ Mpc [32] and the radial velocity is about 240 km/s [35]. As one can see the radial velocity dispersion in the warped DGP brane-world is compatible with the observed profiles and for the same value of constants $\mathcal{C}_1$ and $\mathcal{C}_2$ is slower than the DGP brane-world model.

VI. CONCLUSIONS

The virial theorem plays an important role in astrophysics because of its generality and wide range of applications. One of the important results that can be obtained with the use of the virial theorem is to derive the mean density of astrophysical objects such as galaxy clusters and it can be used to predict the total mass of the clusters of galaxies. In the present paper, using the collisionless Boltzmann equation, we have obtained the generalized virial theorem within the context of the warped DGP brane-world model. The additional geometric terms due to the induced curvature term on the brane and nonlocal bulk effect in the modified gravitational field equations provide an effective contribution to the gravitational energy, Eq. (44), which may be used to explain the well-known virial theorem mass discrepancy in clusters of galaxies. Finally, we have compared the virial theorem results with the observational data for galaxy cluster which can be obtained from the X-ray observation of the gas in the cluster and expressed the geometrical mass in term of observational quantities, like the temperature and the gas profile density.

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