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Nordström gravity in $(1+1)$ dimensions coupled to matter

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We consider a (1 + 1)-dimensional model of general relativity that is based on a geometric theory of gravity due to Nordström. We show that the model can be reinterpreted as a scalar field theory in flat spacetime in which the scalar field couples to the trace of the total energy-momentum tensor, and we use the flat-spacetime interpretation to formulate the initial value problem for the model. We illustrate our results by using computer simulations of the model to obtain several example solutions.

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I. INTRODUCTION

The Einstein field equations do not yield a viable theory of gravity in (1 + 1) dimensions because in (1 + 1) dimensions the Riemann and Ricci tensors are given by

$$R^{\gamma}{}_{\nu\alpha\beta} = (R/2)g^{\gamma\mu}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}),$$
$$R_{\nu\beta} = R^{\gamma}{}_{\nu\gamma\beta} = (R/2)g_{\nu\beta}, \tag{1}$$

where $g_{\mu\nu}$ is the metric tensor and $R = g^{\nu\beta}R_{\nu\beta}$ is the curvature scalar [1]. These expressions imply that the Einstein tensor vanishes identically ($G_{\mu\nu} = R_{\mu\nu} - (R/2)g_{\mu\nu} = 0$), so the Einstein field equations reduce to $G_{\mu\nu} = 8\pi G T_{\mu\nu} = 0$, where G is the gravitational constant and $T_{\mu\nu}$ is the energy-momentum tensor for matter. Thus, in (1 + 1) dimensions the Einstein field equations simply state that the energy-momentum tensor vanishes.

As an alternative to the Einstein field equations, we consider a (1 + 1)-dimensional model of gravity that is based on the field equation

$$R = 4Gg_{\alpha\beta}T^{\alpha\beta}, \tag{2}$$

where the energy-momentum tensor $T^{\alpha\beta}$ is required to satisfy the conservation law

$$\nabla_\beta T^{\alpha\beta} = \partial_\beta T^{\alpha\beta} + \Gamma^{\alpha}{}_{\mu\beta}T^{\mu\beta} + \Gamma^{\beta}{}_{\mu\beta}T^{\alpha\mu} = 0. \tag{3}$$

This model is a useful theoretical laboratory for investigating conceptual problems in general relativity because it is generally covariant, has a sensible Newtonian limit, and accounts for gravitation in terms of spacetime curvature. The model has been discussed by a number of authors [2–5]. Topics that have been studied using the model include black hole solutions [6], point particle dynamics [7–9], and quantum gravity [10]. The model is the direct (1 + 1)-dimensional analog of a (3 + 1)-dimensional theory of gravity proposed by Nordström in which the field equations are taken to be

$$R = 24\pi G g_{\alpha\beta}T^{\alpha\beta}, \qquad C_{\alpha\beta\mu\nu} = 0, \tag{4}$$

where $C_{\alpha\beta\mu\nu}$ is the Weyl tensor [11,12]. In (1 + 1) dimensions the Weyl tensor vanishes identically, and the Nordström field equations reduce to Eq. (2) after a trivial rescaling of the gravitational constant.

In this paper, we show that Nordström gravity in (1 + 1) dimensions can be reinterpreted as a scalar field theory in flat spacetime in which the scalar field couples to the trace of the total energy-momentum tensor. Because of the general covariance of Nordström gravity, the flat-spacetime interpretation has a nontrivial spacetime symmetry in addition to Lorentz invariance. We demonstrate the invariance of the flat-spacetime interpretation under this new symmetry and explain its physical meaning. The flat-spacetime interpretation allows us to formulate the initial value problem for the model, leading to a simple scheme for simulating the model on a computer. We illustrate our scheme by performing computer simulations of the model for the case of a perfect fluid that obeys a simple equation of state.

The paper is organized as follows. In Sec. II, we show that Nordström gravity in (1 + 1) dimensions can be reinterpreted as a scalar field theory in flat spacetime in which the scalar field couples to the trace of the total energy-momentum tensor. In Sec. III, we discuss the symmetry properties of the flat-spacetime interpretation. In Sec. IV, we discuss the energy-momentum tensor for matter and consider in detail the special case of a perfect fluid that obeys a simple equation of state. In Sec. V, we use the flat-spacetime interpretation to formulate the initial value problem for Nordström gravity. In Sec. VI, we present several example solutions.

The following notation is used in this paper. The Minkowski metric tensor $\eta_{\mu\nu}$ is defined such that $\eta_{00} = -\eta_{11} = 1$, $\eta_{01} = \eta_{10} = 0$. The Levi-Civita tensor $\epsilon_{\mu\nu}$ is defined such that $\epsilon_{01} = -\epsilon_{10} = 1$, $\epsilon_{00} = \epsilon_{11} = 0$.

II. FLAT-SPACETIME INTERPRETATION

We will begin by showing that Nordström gravity in (1 + 1) dimensions can be reinterpreted as a scalar field theory in flat spacetime in which the scalar field couples to the trace of the total energy-momentum tensor. Consider a

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scalar field ϕ in a flat spacetime with metric tensor $\eta_{\mu\nu}$, together with some matter whose energy-momentum tensor is $\Theta_m^{\alpha\beta}$. We will assume that the total energy-momentum tensor for the system has the form $\Theta^{\alpha\beta} = \Theta_m^{\alpha\beta} + \Theta_f^{\alpha\beta} + \Theta_i^{\alpha\beta}$, where

$$\Theta_f^{\alpha\beta} = (1/2G)(\eta^{\alpha\mu}\eta^{\beta\nu} - (1/2)\eta^{\alpha\beta}\eta^{\mu\nu})(\partial_\mu\phi)(\partial_\nu\phi) \tag{5}$$

is the energy-momentum tensor for a free massless scalar field and $\Theta_i^{\alpha\beta}$ is an energy-momentum tensor associated with a coupling between the scalar field and the matter. We will require that the scalar field couple to the trace of the total energy-momentum tensor:

$$\Box\phi = -2G\eta_{\alpha\beta}\Theta^{\alpha\beta}. \tag{6}$$

Since $\Theta_f^{\alpha\beta}$ is traceless, it follows that

$$\Box\phi = -2G\eta_{\alpha\beta}\Theta_{mi}^{\alpha\beta}, \tag{7}$$

where $\Theta_{mi}^{\alpha\beta} \equiv \Theta_m^{\alpha\beta} + \Theta_i^{\alpha\beta}$. From Eqs. (5) and (7), it follows that

$$\partial_\beta\Theta_f^{\alpha\beta} = -\eta_{\mu\nu}\Theta_{mi}^{\mu\nu}\eta^{\alpha\beta}\partial_\beta\phi. \tag{8}$$

We will assume that the total energy momentum is conserved ($\partial_\beta\Theta^{\alpha\beta} = \partial_\beta\Theta_f^{\alpha\beta} + \partial_\beta\Theta_{mi}^{\alpha\beta} = 0$), so Eq. (8) implies that

$$\partial_\beta\Theta_{mi}^{\alpha\beta} = \eta_{\mu\nu}\Theta_{mi}^{\mu\nu}\eta^{\alpha\beta}\partial_\beta\phi. \tag{9}$$

We have obtained a theory describing a scalar field ϕ coupled to an energy-momentum tensor $\Theta_{mi}^{\alpha\beta}$ in flat spacetime. The field equation for ϕ is given by Eq. (7), and $\Theta_{mi}^{\alpha\beta}$ satisfies the conservation law given by Eq. (9).

We will now relate this theory to Nordström gravity. In $(1 + 1)$ dimensions, spacetime is conformally flat for all metrics; that is, one can always choose a coordinate system $x^\mu = (t, x)$ in which the metric tensor takes the form $g_{\mu\nu} = e^{2\phi}\eta_{\mu\nu}$ for some scalar field ϕ [13]. We will call such a coordinate system a conformal coordinate system. In a conformal coordinate system, the Christoffel symbols are given by

$$\Gamma^\mu{}_{\alpha\beta} = \delta^\mu{}_\beta\partial_\alpha\phi + \delta^\mu{}_\alpha\partial_\beta\phi - \eta_{\alpha\beta}\eta^{\mu\nu}\partial_\nu\phi, \tag{10}$$

and the curvature scalar is given by

$$R = -2e^{-2\phi}\Box\phi. \tag{11}$$

From Eqs. (10) and (11), it follows that in a conformal coordinate system the field equation (2) for Nordström gravity takes the form

$$\Box\phi = -2Ge^{4\phi}\eta_{\alpha\beta}T^{\alpha\beta}, \tag{12}$$

and the conservation law (3) for the energy-momentum tensor takes the form

$$\partial_\beta T^{\alpha\beta} + (4T^{\alpha\beta} - \eta_{\mu\nu}T^{\mu\nu}\eta^{\alpha\beta})\partial_\beta\phi = 0. \tag{13}$$

If we define $\Theta_{mi}^{\alpha\beta} \equiv e^{4\phi}T^{\alpha\beta}$, we find that Eqs. (12) and (13) can be identified with the field equation (7) and conservation law (9) for our scalar field theory in flat spacetime. The scalar field theory can thus be viewed as a flat-spacetime interpretation of Nordström gravity.

III. SYMMETRY PROPERTIES OF THE FLAT-SPACETIME INTERPRETATION

Let us now investigate the symmetry properties of the flat-spacetime interpretation. The field equation (7) and conservation law (9) are clearly invariant under the transformation

$$\begin{aligned} x^\mu &\to \bar{x}^\mu = A^\mu{}_\nu x^\nu + a^\mu, \\ \Theta_{mi}^{\alpha\beta} &\to \bar{\Theta}_{mi}^{\alpha\beta} = A^\alpha{}_\mu A^\beta{}_\nu \Theta_{mi}^{\mu\nu}, \\ \phi &\to \bar{\phi} = \phi, \end{aligned} \tag{14}$$

where $A^\mu{}_\nu$ is an arbitrary Lorentz boost and a^μ is an arbitrary spacetime translation. The theory also has a less obvious spacetime symmetry that follows from the general covariance of Nordström gravity. Let λ be an arbitrary solution to the homogeneous wave equation $\Box\lambda = 0$. We can express λ as $\lambda(t, x) = f_+(t + x) + f_-(t - x)$, where f_+ and f_- describe left-moving and right-moving waves. Define a coordinate transformation $x^\mu \to \bar{x}^\mu$ by

$$\begin{aligned} \bar{t}(t, x) &= (1/2)(I_+(t, x) + I_-(t, x)), \\ \bar{x}(t, x) &= (1/2)(I_+(t, x) - I_-(t, x)), \end{aligned} \tag{15}$$

where

$$I_\pm(t, x) = \int_0^{t\pm x} e^{2f_\pm(u)}du. \tag{16}$$

This coordinate transformation is illustrated in Fig. 1 for the case of a right-moving Gaussian wave packet. The theory is then invariant under the transformation

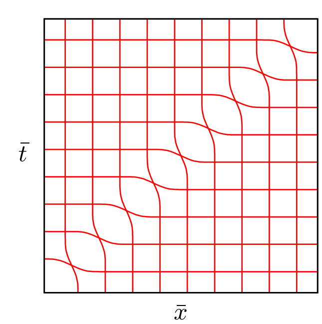

FIG. 1 (color online). Coordinate transformation $x^\mu \to \bar{x}^\mu$ described by Eq. (15). Shown are lines of constant x and lines of constant t in the $\bar{x} - \bar{t}$ plane for the case of a right-moving Gaussian wave packet λ.

$$
\begin{aligned}
x^\mu &\rightarrow \bar{x}^\mu, \\
\Theta^{\alpha\beta}_{mi} &\rightarrow \bar{\Theta}^{\alpha\beta}_{mi} = e^{-4\lambda}\Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu \Theta^{\mu\nu}_{mi}, \\
\phi &\rightarrow \bar{\phi} = \phi - \lambda,
\end{aligned} \tag{17}
$$

where

$$
\begin{aligned}
\Lambda^\alpha{}_\mu &= \partial_\mu \bar{x}^\alpha \\
&= (1/2)(e^{2f_+(t+x)} + e^{2f_-(t-x)})\delta^\alpha{}_\mu \\
&\quad + (1/2)(e^{2f_+(t+x)} - e^{2f_-(t-x)})\epsilon^\alpha{}_\mu.
\end{aligned} \tag{18}
$$

We can demonstrate this invariance by applying the coordinate transformation described by Eq. (15) to Nordström gravity in a conformal coordinate system. We first note that from Eq. (15) it follows that

$$
\begin{aligned}
\eta_{\alpha\beta}(\partial_\mu \bar{x}^\alpha)(\partial_\nu \bar{x}^\beta) &= e^{2\lambda}\eta_{\mu\nu}, \\
\eta_{\mu\nu}(\bar{\partial}_\alpha x^\mu)(\bar{\partial}_\beta x^\nu) &= e^{-2\lambda}\eta_{\alpha\beta}.
\end{aligned} \tag{19}
$$

Since the metric tensor in the original coordinate system is $g_{\mu\nu} = e^{2\phi}\eta_{\mu\nu}$, the metric tensor in the transformed coordinate system is

$$
\begin{aligned}
\bar{g}_{\alpha\beta} &= g_{\mu\nu}(\bar{\partial}_\alpha x^\mu)(\bar{\partial}_\beta x^\nu) = e^{2\phi}\eta_{\mu\nu}(\bar{\partial}_\alpha x^\mu)(\bar{\partial}_\beta x^\nu) \\
&= e^{2(\phi-\lambda)}\eta_{\alpha\beta} = e^{2\bar{\phi}}\eta_{\alpha\beta},
\end{aligned} \tag{20}
$$

where $\bar{\phi} = \phi - \lambda$. So the transformed coordinate system is also a conformal coordinate system, and Nordström gravity in the transformed coordinate system can also be identified with a scalar field theory in flat spacetime. The scalar field is given by $\bar{\phi}$, and the energy-momentum tensor is given by

$$
\begin{aligned}
\bar{\Theta}^{\alpha\beta}_{mi} &\equiv e^{4\bar{\phi}}\bar{T}^{\alpha\beta} = e^{4(\phi-\lambda)}(\partial_\mu \bar{x}^\alpha)(\partial_\nu \bar{x}^\beta)T^{\mu\nu} \\
&= e^{-4\lambda}\Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu \Theta^{\mu\nu}_{mi}.
\end{aligned} \tag{21}
$$

We have thus shown that the flat-spacetime interpretation is invariant under the transformation described by Eq. (17). One can also demonstrate this invariance by a direct calculation, using the fact that

$$
\partial_\mu \Lambda^\alpha{}_\nu = \Lambda^\alpha{}_\nu \partial_\mu \lambda + \Lambda^\alpha{}_\mu \partial_\nu \lambda - e^{2\lambda}\eta_{\mu\nu}\eta^{\alpha\beta}\bar{\partial}_\beta \lambda. \tag{22}
$$

We can understand the physical meaning of this symmetry from the following considerations. We first observe that for Nordström gravity in (1 + 1) dimensions the gravitational field is not dynamical; that is, there are no physical gravitational waves. This property follows directly from the field equation (2): note that if the energy-momentum tensor vanishes then the curvature scalar vanishes, and hence, by Eq. (1), the Riemann tensor vanishes. So in vacuum there is no curvature and thus no physical gravitational waves. In the flat-spacetime interpretation, however, the scalar field is dynamical and supports freely propagating waves. These waves cannot correspond to physical gravitational waves; rather, they correspond to gauge waves λ that can be eliminated via the transformation described by Eq. (17). Note that the scalar field ϕ is pure gauge only in vacuum; in the presence of matter, there is a component to ϕ that cannot be eliminated by this transformation, and it is this component that gives rise to the attractive gravitational force.

IV. ENERGY-MOMENTUM TENSOR FOR MATTER

In (1 + 1) dimensions, any energy-momentum tensor for which $2T_{\alpha\beta}T^{\alpha\beta} > (g_{\alpha\beta}T^{\alpha\beta})^2$ can be expressed in the form of an energy-momentum tensor for a perfect fluid:

$$
T^{\alpha\beta} = (\rho + p)u^\alpha u^\beta - p g^{\alpha\beta}, \tag{23}
$$

where p is the pressure, ρ is the proper energy density, and u^α is the fluid velocity. The pressure and proper energy density are given by

$$
\begin{aligned}
p &= (1/2)((2T_{\alpha\beta}T^{\alpha\beta} - T^2)^{1/2} - T), \\
\rho &= (1/2)((2T_{\alpha\beta}T^{\alpha\beta} - T^2)^{1/2} + T),
\end{aligned} \tag{24}
$$

where $T = g_{\alpha\beta}T^{\alpha\beta}$, and the fluid velocity is given by

$$
\begin{aligned}
u^0 &= (\rho + p)^{-1/2}(T^{00} + p g^{00})^{1/2}, \\
u^1 &= (\rho + p)^{-1/2}(T^{11} + p g^{11})^{1/2}.
\end{aligned} \tag{25}
$$

As a specific example, we will consider a perfect fluid with the simple equation of state $p = r\rho$, where r is a constant in the range $0 \le r \le 1$. If we substitute the expressions for p and ρ given by Eq. (24) into this equation of state, we find that

$$
T^2 = (1 - r)^2(1 + r^2)^{-1}T_{\alpha\beta}T^{\alpha\beta}. \tag{26}
$$

In what follows, it will be useful to solve Eq. (26) for T^{11} in terms of T^{00} and T^{01}:

$$
\begin{aligned}
T^{11} &= (1/2r)(1 + r^2)T^{00} - (1/2r)(1 - r)((1 + r)^2(T^{00})^2 \\
&\quad - 4r(T^{01})^2)^{1/2}.
\end{aligned} \tag{27}
$$

V. INITIAL VALUE PROBLEM

We can use the flat-spacetime interpretation described in Sec. II to formulate the initial value problem for Nordström gravity. We will take the dynamical variables for the system to be ϕ, $B \equiv \partial_t \phi$, $\Theta^{00}_{mi} \equiv e^{4\phi}T^{00}$, and $\Theta^{01}_{mi} \equiv e^{4\phi}T^{01}$. From the field equation (7) and the conservation equation (9), it follows that the equations of motion for these dynamical variables are

$$
\partial_t \phi = B, \tag{28}
$$

$$
\partial_t B = \partial_x^2 \phi - 2G(\Theta^{00}_{mi} - \Theta^{11}_{mi}), \tag{29}
$$

$$
\partial_t \Theta^{00}_{mi} = -\partial_x \Theta^{01}_{mi} + (\Theta^{00}_{mi} - \Theta^{11}_{mi})B, \tag{30}
$$

$$
\partial_t \Theta^{01}_{mi} = -\partial_x \Theta^{11}_{mi} - (\Theta^{00}_{mi} - \Theta^{11}_{mi})E, \tag{31}
$$

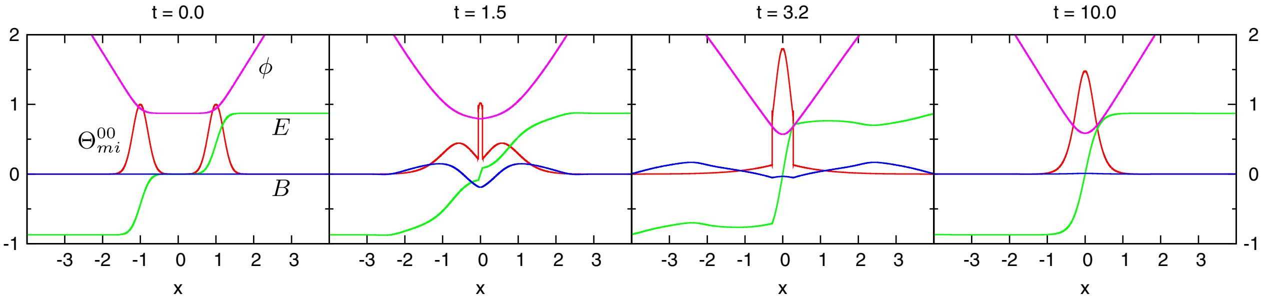

FIG. 2 (color online). Collision of two spatially separated mass distributions. Shown are the scalar field ϕ, the field derivatives $B = \partial_t\phi$ and $E = \partial_x\phi$, and the energy-momentum tensor component Θ^{00}_{mi} versus x at times $t = 0$, 1.5, 3.2, 10.0.

where $E \equiv \partial_x\phi$ and

$$\Theta^{11}_{mi} \equiv (1/2r)(1 + r^2)\Theta^{00}_{mi} - (1/2r)(1 - r)((1 + r)^2 \times (\Theta^{00}_{mi})^2 - 4r(\Theta^{01}_{mi})^2)^{1/2}. \tag{32}$$

To obtain Eq. (32), we have used the definition $\Theta^{11}_{mi} \equiv e^{4\phi}T^{11}$ and substituted for T^{11} using Eq. (27). We can evolve the system in time by numerically integrating Eqs. (28)–(31) subject to a set of initial conditions. To improve the numerical stability, we use a standard technique from computational fluid dynamics: we add artificial diffusion terms $\nu\partial_x^2\Theta^{00}$ and $\nu\partial_x^2\Theta^{01}$ to Eqs. (30) and (31), respectively, where the diffusion constant ν is chosen to be sufficiently small as to have negligible influence on the evolution of the system [14].

The dynamical variables we have chosen describe the system in the flat-spacetime interpretation and are thus dependent on the choice of coordinate system. We can obtain coordinate-independent quantities by translating back into the curved-spacetime interpretation. For example, the pressure and proper energy density are given by

$$\begin{aligned} p &= (1/2)((2\Theta_{mi}\cdot\Theta_{mi} - \Theta^2_{mi})^{1/2} - \Theta_{mi})e^{-2\phi}, \\ \rho &= (1/2)((2\Theta_{mi}\cdot\Theta_{mi} - \Theta^2_{mi})^{1/2} + \Theta_{mi})e^{-2\phi}, \end{aligned} \tag{33}$$

and the curvature scalar is given by

$$R = 4\Theta_{mi}e^{-2\phi}, \tag{34}$$

where $\Theta_{mi} \equiv \eta_{\alpha\beta}\Theta^{\alpha\beta}_{mi}$ and $\Theta_{mi}\cdot\Theta_{mi} \equiv \eta_{\alpha\mu}\eta_{\beta\nu}\Theta^{\alpha\beta}_{mi}\Theta^{\mu\nu}_{mi}$.

VI. EXAMPLE SOLUTIONS

We will now present several example solutions. For our first example, we simulate the collision of two spatially separated mass distributions. We take the initial conditions for Θ^{00}_{mi} and Θ^{01}_{mi} to be

$$\begin{aligned} \Theta^{00}_{mi}(0, x) &= ae^{-(x+\mu)^2/2\sigma^2} + ae^{-(x-\mu)^2/2\sigma^2}, \\ \Theta^{01}_{mi}(0, x) &= 0, \end{aligned} \tag{35}$$

and we take the initial conditions for ϕ and B to be the static fields generated by the matter:

$$\begin{aligned} \phi(0, x) &= G\int |x - x'|(\Theta^{00}_{mi}(0, x') - \Theta^{11}_{mi}(0, x'))dx', \\ B(0, x) &= 0. \end{aligned} \tag{36}$$

We integrate the equations of motion (28)–(31) subject to these initial conditions and plot the resulting evolution in Fig. 2, where the parameters are taken to be $G = 1$, $r = 0.13$, $a = 1$, $\mu = 1$, $\sigma = 0.2$, and $\nu = 10^{-4}$. The two mass distributions approach each other due to their mutual gravitational attraction, collide at $t \simeq 1$, and reach an equilibrium state at $t \simeq 8$. During the collision process, gauge waves are emitted that propagate outwards to the left and right.

For our second example, we demonstrate the invariance of the flat-spacetime interpretation under the transformation described in Sec. III by simulating the propagation of a gauge wave packet through a static mass distribution. From the equations of motion (28)–(31), it follows that for a static solution $B = \Theta^{01}_{mi} = 0$, and ϕ and Θ^{00}_{mi} satisfy the equations

$$\begin{aligned} \partial_x\phi &= E, \\ \partial_x E &= 2G(1 - r)\Theta^{00}_{mi}, \\ \partial_x\Theta^{00}_{mi} &= (1/r)(1 - r)\Theta^{00}_{mi}E. \end{aligned} \tag{37}$$

Here we have used the fact that $\Theta^{11}_{mi} = r\Theta^{00}_{mi}$ when $\Theta^{01}_{mi} = 0$. We obtain a static solution that is symmetric about $x = 0$ by integrating these equations subject to the boundary conditions

$$\phi(0, 0) = 0, \quad E(0, 0) = 0, \quad \Theta^{00}_{mi}(0, 0) = \rho_0, \tag{38}$$

where ρ_0 is the proper energy density of the fluid at $x = 0$. We obtain initial conditions by starting with a static solution and adding a right-moving Gaussian wave packet; that is, we make the replacements

$$\begin{aligned} \phi(0, x) &\to \phi(0, x) + \lambda(0, x), \\ B(0, x) &\to B(0, x) - \partial_x\lambda(0, x), \end{aligned} \tag{39}$$

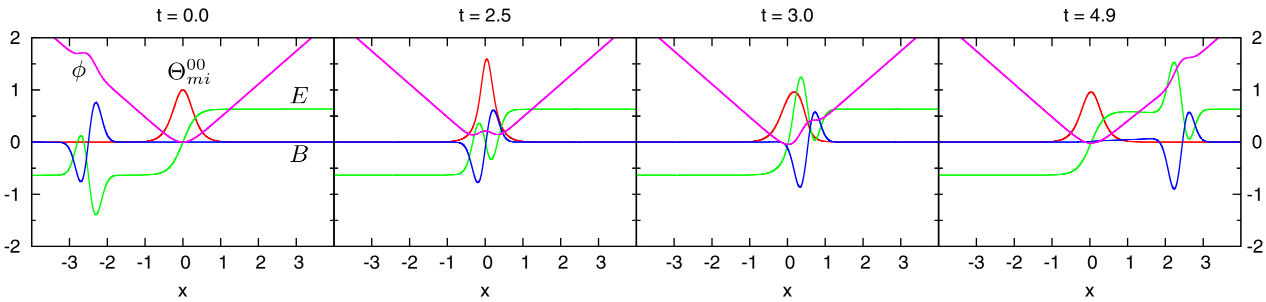

FIG. 3 (color online). Propagation of a gauge wave packet through a static mass distribution. Shown are the scalar field ϕ, the field derivatives $B = \partial_t \phi$ and $E = \partial_x \phi$, and the energy-momentum tensor component Θ^{00}_{mi} versus x at times $t = 0$, 2.5, 3.0, 4.9.

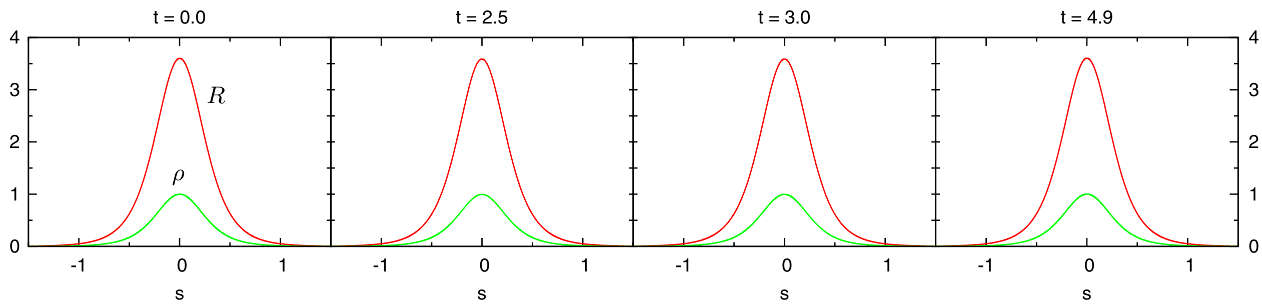

FIG. 4 (color online). Propagation of a gauge wave packet through a static mass distribution. Shown are the proper energy density ρ and the curvature scalar R versus proper distance s at times $t = 0$, 2.5, 3.0, 4.9.

where $\lambda(0, x) = ae^{-(x-\mu)^2/2\sigma^2}$. We integrate the equations of motion (28)–(31) subject to these initial conditions and plot the resulting evolution in Fig. 3, where the parameters are taken to be $G = 1$, $r = 0.1$, $\rho_0 = 1$, $a = 0.25$, $\mu = -2.5$, $\sigma = 0.2$, and $\nu = 10^{-4}$. As described in Sec. III, the gauge wave packet can be eliminated by a suitable transformation, and thus its presence should not alter the behavior of the system in the curved-spacetime interpretation. It follows that, although coordinate-dependent quantities like $\Theta^{\alpha\beta}_{mi}$ are allowed to vary in time, coordinate-independent quantities like the proper energy density ρ and the curvature scalar R should remain constant. We verify this claim by using Eqs. (33) and (34) to calculate ρ and R as a function of time. In Fig. 4, we plot these quantities against the proper distance s, which is given by

$$s(x) = \int_{x_0}^{x} e^{\phi(x')} dx', \tag{40}$$

where x_0 is defined to be the point of maximum curvature. As expected, the curves $\rho(s)$ and $R(s)$ do not change as the gauge wave packet propagates through the mass distribution.

VII. CONCLUSION

We have considered a model of general relativity in (1 + 1) dimensions that is the direct (1 + 1)-dimensional analog of a theory of gravity proposed by Nordström. We have shown that the model can be reinterpreted as a scalar field theory in flat spacetime in which the scalar field couples to the trace of the total energy-momentum tensor. In the model system, the gravitational field does not support physical gravitational waves, and as a consequence freely propagating waves in the flat-spacetime interpretation correspond to gauge waves that can be eliminated via a symmetry transformation. We have used the flat-spacetime interpretation to formulate the initial value problem for the model and have presented several example solutions. The model of gravity that we have considered is a useful theoretical laboratory for studying general relativity, and the results presented here provide new tools for understanding this system.

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