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Acceleration of particles by black holes: Kinematic explanation

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A new simple and general explanation of the effect of acceleration of particles by black holes to infinite energies in the center of mass frame is suggested. It is based on kinematics of particles moving near the horizon. This effect arises when particles of two kinds collide near the horizon. For massive particles, the first kind represents a particle with the generic energy and angular momentum (I call them ''usual''). Near the horizon, such a particle has a velocity almost equal to that of light in the frame that corotates with a black hole (the frame is static if a black hole is static). The second kind (called ''critical'') consists of particles with the velocity $v < c$ near the horizon due to special relationship between the energy and angular momentum (or charge). As a result, the relative velocity approaches the speed of light c, and the Lorentz factor grows unbound. This explanation applies both to generic rotating black holes and charged ones (even for radial motion of particles). If one of the colliding particles is massless (photon), the critical particle is distinguished by the fact that its frequency is finite near the horizon. The existence (or absence) of the effect is determined depending on competition of two factors—gravitational blue shift for a photon propagating towards a black hole and the Doppler effect due to transformation from the locally nonrotating frame to a comoving one. Classification of all possible types of collisions is suggested depending on whether massive or massless particle is critical or usual.

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I. INTRODUCTION

Recently, an interesting effect was discovered. It turned out that two particles can collide near the horizon of the Kerr black hole in such a way that the energy in the center of mass frame grows unbound [1] (we will call it the BSW effect). This provoked a series of consequent papers [2–19] in which details of collision were studied; the effect has been found in different and more general space-times, etc. The key role in the BSW effect is played by the fact that one of the two colliding particles should be ''critical.'' By definition, this means that its energy E and angular momentum L are connected by the relationship $E - \omega_H L = 0$ where ω_H is the angular velocity of the black hole. (Otherwise, I call a particle ''usual''). For the Kerr metric, this was observed in [1] and traced in detail in subsequent papers [6,14], including even nonequatorial motion [15]. For a generic rotating axially-symmetric dirty black hole (surrounded by matter) this was found in [11]. In [20] the most general geometric explanation was suggested that relies on the relative orientation of the particle's timelike four-velocity and the generator of a black hole horizon.

The aim of the present work is to give an alternative, purely kinematic explanation of the BSW effect with the emphasis on the role of critical particles in the terms of the particles' three-velocities. To the best of my knowledge, this has not been done yet. In Ref. [1] the remark has been made in passing that the effect is connected with a crucial difference between the kinematics of usual and critical particles. In the first case, a particle hits the horizon of a rotating black hole perpendicularly; in the second one it does it at some incident angle. This important observation does not give, however, the full explanation of the phenomenon. For example, the effect exists even for pure radial motion of charged particles in the Reissner-Nordström space-time [12] when, obviously, all particles approach the horizon perpendicularly. On the other hand, two different critical particles can collide at some nonzero angle if they have different momenta. However, this does not produce the infinite energy in the center of mass frame. Thus, an interesting property mentioned in [1] is neither necessary nor sufficient for the explanation of the effect under discussion.

Below, Secs. II, III, IV, V, and VI show that for the collision of massive particles, the crucial point is whether or not a particle has near the horizon the velocity approaching the speed of light. Only the collision of particles of both different kinds produces the effect. The frame in which the velocity under discussion is measured is either the static one (for charged static black holes) or the frame of the zero angular momentum observer (ZAMO) [21]. Then, as we will see, the essence of the effect can be understood in the terms of special relativity in combination with general consequences of geodesic motion near the horizon.

In Sec. VI, I consider separately the case of collision between massive and massless particles since explanation for the BSW effect and the definition and a role of the critical particle are somewhat different in both cases. Collisions of this type were considered in [15] but for the concrete case of the Kerr metric only.

In what follows we assume (as usual for the BSW effect) that both colliding particles are ingoing. (The case when

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one of two particles is outgoing is more simple and always leads to infinite energies simply due to blueshift near the horizon [18].) The effect under consideration is interesting both from the theoretical viewpoint (since it gives hope to probe Planck physics near the black hole horizon) and the astrophysical one (since collisions with divergent energies can, in principle, leave their imprint on the emergent flux of particles escaping from a black hole [16,17]).

II. BASIC EQUATIONS

Let us consider the space-time of a rotating black hole described by the metric

$$ds^2 = -N^2 dt^2 + g_{\phi\phi}(d\phi - \omega dt)^2 + dl^2 + g_{zz}dz^2. \quad (1)$$

Here, the metric coefficients do not depend on t and ϕ. On the horizon $N = 0$. Alternatively, one can use coordinates θ and r, similar to Boyer-Lindquist ones for the Kerr metric, instead of l and z. In (1) we assume that the metric coefficients are even functions of z, so the equatorial plane $\theta = \frac{\pi}{2}$ $(z = 0)$ is a symmetry one. The explicit form of the metric coefficients is not specified, so consideration applies to "dirty" black holes surrounded by matter in equilibrium with the horizon.

We consider the geodesic motion of massive particles in the equatorial plane $\theta = \frac{\pi}{2}$. The equations of motion have the form

$$\dot{t} = u^0 = \frac{E - \omega L}{N^2}, \quad (2)$$

$$\dot{\phi} = \frac{L}{g_{\phi\phi}} + \frac{\omega(E - \omega L)}{N^2}, \quad (3)$$

$$\dot{l}^2 = \frac{(E - \omega L)^2}{N^2} - 1 - \frac{L^2}{g_{\phi\phi}}, \quad (4)$$

where $E = -u_0$ and $L = u_\phi$ are conserved energy and angular momentum per unit mass, and u^μ is the four-velocity. In the present paper I use units in which the gravitational constant $G = 1$ and the speed of light $c = 1$.

We assume that $\dot{t} > 0$, so that $E - \omega L > 0$ (motion forward in time), except, possibly, on the horizon where we admit the equality $E - \omega_H L = 0$ (subscript "H" denotes quantities calculated on the horizon). By definition, if $E - \omega_H L > 0$ a particle is usual and if $E - \omega_H L = 0$ it is critical.

In what follows we will use the tetrad basis. Denoting coordinates x^μ as $x^0 = t$, $x^1 = l$, $x^2 = z$, $x^3 = \phi$, we choose the tetrad vectors $h_{(a)\mu}$ in the following way:

$$h_{(0)\mu} = -N(1, 0, 0, 0), \quad (5)$$

$$h_{(1)\mu} = (0, 1, 0, 0), \quad (6)$$

$$h_{(2)\mu} = \sqrt{g_{zz}}(0, 0, 0, 1), \quad (7)$$

$$h_{(3)\mu} = \sqrt{g_{\phi\phi}}(-\omega, 0, 0, 1). \quad (8)$$

If such a tetrad is attached to an observer moving in the metric (1), it has the meaning of zero angular momentum observer (ZAMO) [21]. They are "rotate with the geometry" in the sense that $\frac{d\phi}{dt} \equiv \omega$ for them. The advantage of using the tetrad components consists in that one can use the formulas of special relativity in the flat space-time tangent to any given point.

Then, we can introduce the three-velocity according to

$$v^{(i)} = v_{(i)} = \frac{u^\mu h_{\mu(i)}}{-u^\mu h_{\mu(0)}}. \quad (9)$$

One can check that

$$-u_\mu h^\mu_{(0)} = \frac{E - \omega L}{N}, \quad (10)$$

$$u_\mu h^\mu_{(3)} = \frac{L}{\sqrt{g_{\phi\phi}}}. \quad (11)$$

From equations of motion (2)–(4) and formulas for tetrad components, we obtain

$$v^{(3)} = \frac{LN}{\sqrt{g_{\phi\phi}}(E - \omega L)}, \quad (12)$$

$$v^{(1)} = \sqrt{1 - \frac{N^2}{(E - \omega L)^2}\left(1 + \frac{L^2}{g_{\phi\phi}}\right)}. \quad (13)$$

Then, introducing also the absolute value of the velocity v according to

$$v^2 = [v^{(1)}]^2 + [v^{(2)}]^2 \quad (14)$$

one can find that

$$E - \omega L = \frac{N}{\sqrt{1 - v^2}}, \quad (15)$$

$$v^2 = 1 - \left(\frac{N}{E - \omega L}\right)^2. \quad (16)$$

III. LIMITING TRANSITIONS FOR RELATIVE VELOCITY

The energy $E_{\rm c.m.}$ in the center of mass frame of two colliding particles can be defined as (see [1] and consequent papers)

$$E^2_{\rm c.m.} = -(p^\mu_1 + p^\mu_2)(p_{1\mu} + p_{2\mu}) = m_1^2 + m_2^2 - 2m_1 m_2 u^\mu_1 u_{2\mu}. \quad (17)$$

Here, $p^\mu_i = m_i u^\mu_i$ $(i = 1, 2)$ is the four-momentum of each particle; m_i are their rest masses. By definition, this is a scalar which can be calculated in any frame. It is convenient to use a frame comoving with respect to one of colliding particles (say, particle 2). If one uses tetrad representation, one can exploit formulas known in a flat space-time. Then, the quantity of interest is

$$\gamma = -u_1^{\mu} u_{2\mu} = \frac{1}{\sqrt{1-w^2}} \tag{18}$$

where w is, by definition, their relative velocity (which in this frame coincides with the velocity of particle 1); γ has the meaning of the Lorentz factor.

The effect of unbound energies occurs if $w \to 1$, so $\gamma \to \infty$.

Now, let me remind some simple formulas from special relativity. If in the laboratory frame particle 1 has the velocity $\vec{v}_1 = v_1 \vec{n}_1$ and particle 2 has the velocity $\vec{v}_2 = v_2 \vec{n}_2$, the value of the relative velocity is equal to

$$w^2 = 1 - \frac{(1-v_1^2)(1-v_2^2)}{[1 - v_1 v_2 (\vec{n}_1 \vec{n}_2)]^2}. \tag{19}$$

This formula can be found in textbooks (see. e.g., problem 1.3. in [22]). Now, we enumerate different limiting transitions for this quantity relevant in our context.

(a) $v_1 \to 1$, $v_2 < 1$, $(\vec{n}_1 \vec{n}_2)$ is arbitrary.
It is obvious from (19) that in this case $w \to 1$ independent of the quantity $(\vec{n}_1 \vec{n}_2)$. This corresponds to the well-known fact that the velocity of light c is always equal to 1 (in geometrical units) in any frame.

(b) $v_1 \to 1$, $v_2 \to 1$ in such a way that $v_i = 1 - A_i \delta$ where A_i $(i = 1, 2)$ are constants, $\delta \ll 1$.
(b1). If $(\vec{n}_1 \vec{n}_2) \neq 1$, it is seen from (19) that

$$w^2 \approx 1 - \frac{4A_1 A_2 \delta^2}{[1 - (\vec{n}_1 \vec{n}_2)]^2}, \tag{20}$$

so we have $v \to 1$ again.
(b2) If $(\vec{n}_1 \vec{n}_2) = 1$, the situation changes radically. Then,

$$w \approx \frac{|A_1 - A_2|}{A_1 + A_2} < 1. \tag{21}$$

(c) $v_1 < 1$, $v_2 < 1$, $(\vec{n}_1 \vec{n}_2)$ is arbitrary. Then, it is obvious that $w < 1$. By itself, this case is trivial. However, it plays a nontrivial role in the context under consideration (see below).

IV. ASYMPTOTICS NEAR HORIZON

Let us now look at what happens to particles' velocities near the horizon. For an usual particle, $E - \omega_H L \neq 0$, and it follows from (15) that in the horizon limit $N \to 0$, $v \to 1$. Apart from this, it follows from (12) and (13) that in this limit $v^{(3)} \to 0$, $v^{(1)} \to 1$. Therefore, the unit vector $\vec{n}$ is pointed along the l direction, so for any two such particles $(\vec{n}_1 \vec{n}_2) = 1$.

However, for a critical particle, the situation is different. At first, consider the extremal horizon. Then, near it, we have an expansion

$$\omega = \omega_H - B_1 N + B_2 N^2 + \ldots. \tag{22}$$

For example, for the Kerr metric $B_1 = M^{-1}$ where B is the black hole mass [11]. We obtain from (15) that

$$v^2 = 1 - \frac{1}{L^2 B_1^2} < 1. \tag{23}$$

Apart from this, in the critical case the quantities $v^{(1)}$ and $v^{(3)}$ have the same order, so a particle hits the horizon at some nonzero angle with respect to the normal direction in accordance with the remark made in [1]. Correspondingly, $(\vec{n}_1 \vec{n}_2) \neq 1$. Now, using the above properties, we can enumerate different types of collisions near the horizon.

A. Collision between two usual particles

This situation corresponds to case (b2). Then, it follows from (21) that $w < 1$, the Lorentz factor γ is finite, so the effect of infinite energies is absent.

B. Collision between two critical particles

This situation corresponds to case (c). Then, we have that $w < 1$, so the effect under discussion is also absent.

C. Collision between an usual (1) and critical (2) particles

This type of collision falls into the class (a) described above. As a result, we have $w \to 1$, $\gamma \to \infty$ and the effect of infinite acceleration is present. The fact that $v_2 < 1$ explains why the critical particle cannot reach the extremal horizon for a finite proper time [6,11]. Indeed, the proper distance is infinite, so the proper time for a particle 2 having $v_2 < 1$ everywhere on its trajectory is certainly infinite.

In the nonextremal case a near-critical particle cannot reach the horizon since $\omega - \omega_H \sim N^2$ when $N \to 0$ [11,23], so the right-hand side of (4) cannot be positive. However, it can approach the horizon as nearly as one likes. Let $E = \omega_H L(1 + \delta)$, $\delta \ll 1$. Then, we must keep δ such that $\delta \gtrsim N$ to ensure the positivity of $\dot{l}^2$ in (4). Let $\delta = AN(P)$ where A is some finite coefficient, P is the point of collision. Then, $1 - v^2 = (\frac{N}{E - \omega_H L})^2 \approx \frac{1}{(\omega_H L A)^2} \neq 0$. Thus, taking the point of collision closer and closer to the horizon and simultaneously taking the energy closer and closer to the critical value, we can gain $v < 1$ and, thus, the effect of infinite acceleration for the energy in the center of mass for collision between an usual and the critical particles. However, this requires multiple scattering since, say, for the Kerr metric, such a particle cannot come from infinity. Apart from this, the collision should occur in a narrow strip near the horizon (see [6,11] for details).

V. CHARGED STATIC BLACK HOLES

All the above consideration applies also to charged static with minimum changes. For simplicity, let us consider the spherically-symmetric black holes. Then, equations of motion give us

$$\dot{t} = u^0 = \frac{E - \varphi q}{N^2}, \tag{24}$$

$$\dot{\phi} = \frac{L}{g_{\phi\phi}}, \tag{25}$$

$$\dot{l}^2 = \frac{(E - \varphi q)^2}{N^2} - 1 - \frac{L^2}{g_{\phi\phi}} \tag{26}$$

where φ is the electric potential with respect to infinity. The tetrad basis can be obtained by putting $\omega = 0$ in (5)–(8). One can find easily that

$$v^{(3)} = \frac{LN}{\sqrt{g_{\phi\phi}}(E - \varphi q)}, \tag{27}$$

$$v^{(1)} = -\sqrt{1 - \frac{N^2}{(E - \varphi q)^2}\left(1 + \frac{L^2}{g_{\phi\phi}}\right)} \tag{28}$$

where $v^{(1)} < 0$ since a particle is moving towards the horizon.

Now, instead of (15), we have

$$E - \varphi q = \frac{mN}{\sqrt{1 - v^2}} \tag{29}$$

where we restored explicitly in (29) the particle's rest mass m.

The condition of criticality is now $E - \varphi_+ q = 0$ where φ_+ is the potential of the black hole. Then, for the extremal case, $N \sim r - r_+ \sim \varphi_+ - \varphi$ where r is the standard curvature coordinate, r_+ is the horizon radius. As a result, $v \neq 1$.

If $L \neq 0$, the previous consideration applies and we again obtain that the effect under consideration is possible only when collision occurs between an usual (1) and the critical (2) particles: $v_1 \to 1$, $v_2 < 1$, so $w \to 1$, $\gamma \to \infty$. In doing so, an usual particle hits the horizon perpendicularly whereas the critical one does it at some incident angle, $(\vec{n}_1 \vec{n}_2) \neq 1$.

A new situation having no analog for rotating case, arises if $L = 0$ [12]. Then, for all colliding particles $(\vec{n}_1 \vec{n}_2) = 1$. Nonetheless, the main conclusion about the effect produced by collision between an usual and the critical particles is still valid.

In a similar way, for the nonextremal horizon the energy is finite but can be made as large as one like if one uses near-critical particles with $E = \varphi_+ q(1 + \delta)$, where $\delta \sim N \ll 1$.

VI. COLLISION BETWEEN MASSIVE AND MASSLESS PARTICLES

If one of particles is massless, the above explanation is not valid since (i) there is no comoving frame for a massless particle, (ii) in any frame, such a particle moves always with the velocity of light. Therefore, kinematic explanation should be somewhat changed. For brevity, we call a massive particle "electron" and a massless one "photon," although consideration applies to any kinds of such particles.

We do not consider the case when both particles are massless. Classical electrodynamics is linear theory, so interaction between photons could occur due to weak quantum-electrodynamic effects only which are neglected in the present work.

We again consider the geodesic motion of particles in the equatorial plane $\theta = \frac{\pi}{2}$. For photons, the equations of motion have the form

$$\frac{dt}{d\lambda} = k^0 = \frac{\nu_0 - \omega L_2}{N^2}, \tag{30}$$

$$\frac{d\phi}{d\lambda} = \frac{L_2}{g_{\phi\phi}} + \frac{\omega(\nu_0 - \omega L_2)}{N^2}, \tag{31}$$

$$\left(\frac{dl}{d\lambda}\right)^2 = \frac{(\nu_0 - CL_2)^2}{N^2} - \frac{L_2^2}{g_{\phi\phi}}, \tag{32}$$

where $\nu_0 = -k_0$, and $L_2 = k_\phi$ are conserved frequency and angular momentum, k^μ is the wave vector, and λ is the affine parameter. The quantity ν_0 has a meaning of frequency measured by a remote observer at infinity where we assume that $\omega \to 0$, $N \to 1$.

Thus, the only difference in the form of equations between the massive (2)–(4) and massless cases reveals itself is in Eqs. (32) and (4). We assume that $\frac{dt}{d\lambda} > 0$, so that $\nu_0 - \omega L > 0$ (motion forward in time), except, possibly on the horizon where we admit the equality $\nu_0 - \omega_H L_1 = 0$ (critical photon).

Now, the energy $E_{\text{c.m.}}$ in the center of mass frame is given by the expression

$$E_{\text{c.m.}}^2 = -(p^\mu + k^\mu)^2 \tag{33}$$

where the Planck constant $\hbar = 1$, $p^\mu = mu^\mu$, m is the electron rest mass. Then,

$$E_{\text{c.m.}}^2 = m^2 - 2m(uk), \qquad (uk) \equiv u^\mu l_\mu. \tag{34}$$

It follows from (30) and (4) that

$$-(uk) = \frac{X_1 X_2 - Z_1 Z_2}{N^2} - \frac{L_1 L_2}{g_{\phi\phi}}, \tag{35}$$

where $X_1 \equiv E_1 - CL_1$, $X_2 = \nu_0 - \omega L_2$,

$$Z_i = \sqrt{X_i^2 - N^2 b_i}, \qquad b_1 = 1 + \frac{L_i^2}{g_{\phi\phi}}, \qquad b_2 = \frac{L_2^2}{g_{\phi\phi}}. \tag{36}$$

When, repeating the straightforward calculations along the lines of [11] step by step, one can arrive at the conclusions that unbound growth of $E_{\text{c.m.}}^2$ is indeed possible if the electron is critical, photon is usual or vice versa. Meanwhile, it is more important to obtain qualitative explanation of infinite growth of $E^2_{\text{c.m.}}$ without explicit calculation of (35). To this end, we use again the ZAMO frame (5)–(8). We can obtain formulas for the photon. In contrast to (9), now formulas for the k^μ do not contain denominator:

$$k^{(i)} = k_{(i)} = k^\mu h_{\mu(i)}, \qquad k^{(0)} = k^\mu h^{(0)}_\mu = -k^\mu h_{\mu(0)}. \quad (37)$$

This is due to the fact that instead of the proper time τ the parameter λ along the geodesics is used, the vector k^μ being lightlike.

From equations of motion (2)–(4) and formulas for tetrad components, we have

$$k^{(1)} = -\sqrt{\nu^2 - \frac{L^2}{g_{\phi\phi}}}, \quad (38)$$

$$k^{(3)} = \frac{L}{\sqrt{g_{\phi\phi}}}, \quad (39)$$

where we took sign "-" in (38) since we consider an ingoing photon. The analog of Eq. (15) reads

$$\nu = \frac{\nu_0 - \omega N}{N}. \quad (40)$$

It can be also obtained writing the scalar (uk) in two frames—the original system (1) and the ZAMO one.

Defining $k^2 = [k^{(1)}]^2 + [k^{(2)}]^2$, it is seen that

$$k^2 = \frac{(\nu_0 - \omega L)^2}{N^2} = \nu^2 \quad (41)$$

$$k^{(0)} = -k_\mu h^\mu_{(0)} = \frac{\nu_0 - \omega L}{N} = \nu \quad (42)$$

as it should be for the lightlike vector since $k^2 - (k^{(0)})^2 = 0$.

In the horizon limit $N \to 0$, the component $v^{(3)} \to 0$, $v^{(1)} \to 1$ for an usual electron. Therefore, the unit vector $\vec{n}_1 = \frac{\vec{v}}{v}$ is pointed along l direction, perpendicularly to the horizon. For the critical particle this is not so [1] since $v^{(1)} \sim v^{(3)}$ have the same order. The similar properties hold in the case of a photon for the vector $\vec{n}_2 = \frac{\vec{k}}{k}$. Thus, in the horizon limit $(\vec{n}_1\vec{n}_2) = 1$ when both particles are usual and $(\vec{n}_1\vec{n}_2) \neq 1$ in other cases.

A. Different types of collisions

Now, we consider separately different cases depending on which particle (if any) is critical.

1. Case 1: Electron is critical, photon is usual

Let us pass to the frame which is comoving with respect to the electron. Then, the frequency ν' measured in this frame is related to the frequency ν in the ZAMO frame by the standard relativistic formula

$$\nu' = \gamma(\nu - \vec{k}\,\vec{v}) = \nu\gamma[1 - v(\vec{n}_1\vec{n}_2)]. \quad (43)$$

For a critical particle, as is explained above, $v \neq 1$, so the Lorentz factor γ is finite. The scalar product $(\vec{n}_1\vec{n}_2) \neq 1$, the quantity ν' has the order ν. But, as a photon is usual, $\nu \to \infty$. Thus, $\nu' \to \infty$ as well, so the effect reveals itself.

The resulting effect can be interpreted as a consequence of two factors. On one hand, there is an infinite blueshift of radiation due to strong gravitating field near a black hole. On the other hand, there is redshift due to the Doppler effect since in the laboratory frame a receiver of radiation is moving apart from a photon (both $v^{(1)} < 0$ and $k^{(1)} < 0$). It turned out that in the case under discussion the first factor is infinite whereas the second one is finite, so the net outcome is due to blueshift.

2. Case 2: Electron is usual, photon is critical

As the photon is critical, ν is finite. But, as the electron is usual, $v \to 1$, $\gamma \to \infty$. The quantity $(\vec{n}_1\vec{n}_2) \neq 1$. Thus, as a result, $\nu' \to \infty$ and we again obtain the effect under discussion.

Interpretation again involves the Doppler effect but the concrete details change. Let in a flat space-time a photon with the frequency ν propagate in the laboratory frame and some observer moves with the velocity v with respect to this frame. Then, in its own frame, the observer measures the frequency of the process which is equal to ν'. In the case under discussion, $(\vec{n}_1\vec{n}_2) \neq 1$. For simplicity, we can take $(\vec{n}_1\vec{n}_2) = 0$. Then, the frequency measured in the frame of a receiver $\nu' = \nu\gamma > \nu$ due to the transverse Doppler effect. In the limit $v \to 1$, the Lorentz factor $\gamma \to 1$ and the frequency $\nu' \to \infty$. In other words, even despite a moderate gravitational blueshift that resulted in a finite ν, the net outcome is infinite due to the Doppler effect.

3. Case 3: Both particles are critical

Then, $(\vec{n}_1\vec{n}_2) = 1$ but $v < 1$, ν is finite. It follows from (43) that ν' is also finite, so there is no effect under discussion. In other words, both factors—gravitational blueshifting and the Doppler effect are restricted and cannot give rise to infinite energies.

4. Case 4: Both particles are usual

Here, an accurate estimate of different terms in the horizon limit is required. In the limit $N \to 0$ the quantities $\gamma \sim \frac{1}{N}$, $\nu \sim \frac{1}{N}$ as it is seen from (15) and (40). It follows also from (13), (12), (38), and (39) that

$$1 - (\vec{n}_1\vec{n}_2) \sim N^2. \quad (44)$$

As a result, the factors N^2 in the numerator and denominator compensate each other, ν' remains finite, the effect of infinite acceleration is absent. One can say that the effect of infinite redshift due to the Doppler effect for a receiver moving apart from the photon is completely compensated by an infinite blueshifting of the photon frequency.

VII. CONCLUSION

We gave a simple and general explanation of the effect of infinite energy in the center of mass of particles colliding near the horizon of a black hole. It is based on kinematics of particles in a flat space-time plus properties of the horizon. It is given for massive and massless particles separately.

For massive ones, it is essential that in the ZAMO frame (or static one in the case of static charged black holes) (i) usual particles have the velocities approaching the speed of light near the horizon, (ii) for a special class of critical particles this limit differs from the speed of light. Then, collision between an usual and critical particles produces the effect under discussion. Thus, critical particles play a distinguished role in the kinematics of the process. These particles have also some other properties that distinguish them from usual ones: they hit the horizon nonperpendicularly (in the case of rotating black holes), and the proper time required to reach the extremal horizon is infinite. The distinction between usual and critical particles is also seen from the simple formulas for the particle's energy in the stationary gravitational field (15) and (29), which shows what happens to the velocity when a particle approaches the horizon.

For massless particles, we showed that, again, the distinguished role is played by critical particles although their definition and properties are somewhat different. Now, interpretation in terms of the velocity is not valid. Instead, it is done in terms of the frequency: for photons their frequency in the same frame remains finite notwithstanding the vanishing of the lapse function near the horizon. The crucial point is that the BSW effect is possible only for the case when one and only one of the colliding particles is critical. The role of critical particles gave rise to natural classification taking into account two factors—gravitational blue shift (GB) and the Doppler effect (DE). Namely, we have four cases: (1) critical electron, usual photon: infinite GB, finite DE, $E_{\rm c.m.}$ is infinite, (2) critical photon, usual electron: finite GB, infinite DE, $E_{\rm c.m.}$ is infinite, (3) both particles are critical: finite GB, finite DE, $E_{\rm c.m.}$ is finite, and (4) both particles are usual: infinite GB, infinite DE, $E_{\rm c.m.}$ is finite due to their compensation. The corresponding results can be used for investigation of the Compton effect near black holes. Meanwhile, the possibility of infinite $E_{\rm c.m.}$ means that, apart from mutual scattering of electrons and photons, qualitatively new reactions can occur with creation of new kinds of high-energy particles.

The above consideration is based on test particle approximation, with backreaction, gravitation, and electromagnetic radiation neglected. Whether and how these results can be changed if these factors are taken into account remains an interesting task for further studies.

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