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Casimir force for cosmological domain walls

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We calculate the vacuum fluctuations that may affect the evolution of cosmological domain walls. Considering domain walls, which are classically stable and have interaction with a scalar field, we show that explicit symmetry violation in the interaction may cause quantum bias that can solve the cosmological domain-wall problem.

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I. INTRODUCTION

The Casimir effect suggested originally in 1948 has been used to understand the contribution from the vacuum fluctuations of quantum fields [1]. The variation in the vacuum fluctuations, which appears in the excitations as the consequence of the nontrivial boundary conditions or the topology of the space, causes a shift of the vacuum energy. The original model, which appears as the two conducting parallel plates in the free R^3 space, the attractive force is confirmed experimentally by Sparnaay [2] and a more precise result have been given more recently in Ref. [3].

For the simplest one-dimensional model, the sum of excitations in the $(1 + 1)$-dimensional spacetime, where the boundaries are separated by the distance L, is given by

$$E_L \equiv \sum_{n=1}^{\infty} \frac{1}{2} \hbar \omega_n \equiv \frac{\pi\hbar}{2L} \sum_{n=1}^{\infty} n, \tag{1.1}$$

where $\hbar$ is the reduced Planck constant. We define the averaged energy density by

$$\rho_L \equiv \frac{E_L}{L} = \frac{\pi\hbar}{2L^2} \sum_{n=1}^{\infty} n. \tag{1.2}$$

Hereafter we set $\hbar = c = 1$. In the limit of $L \to \infty$, this gives for the massless field

$$\rho_\infty \equiv \frac{E_\infty}{L} = \int_0^\infty \frac{dk}{2\pi} k, \tag{1.3}$$

where k denotes the continuous $(Ł \to \infty)$ limit of $k_n \equiv \frac{\pi}{L} n$. The Casimir energy is defined (regularized) by $\Delta E \equiv E_L - E_\infty$. To obtain a finite result, consider the regularization [4]

$$\hat{\rho}_L \equiv \frac{1}{2L} \sum_{n=1}^{\infty} \omega_n e^{-\omega_n/\Lambda} = \frac{\Lambda^2}{2\pi} - \frac{\pi}{24L^2} + \mathcal{O}\left(\frac{1}{\Lambda}\right) \tag{1.4}$$

and

$$\hat{\rho}_\infty \equiv \frac{1}{2\pi} \int_0^\infty d\omega w e^{-\omega/\Lambda} = \frac{\Lambda^2}{2\pi}, \tag{1.5}$$

where Λ is introduced as the manifestation of the cutoff scale. Here $e^{-\omega_n/\Lambda} \to 0$ is assumed for $n \to \infty$. Regularization using the ζ-function is also possible. Considering the regularization, the energy shift caused by the boundary is estimated as

$$\rho^R \equiv \rho_L - \rho_\infty \simeq -\frac{\pi}{24L^2}. \tag{1.6}$$

Domain Walls in cosmology

In the context of the hot big bang theory, the fundamental theory of unification predicts a sequence of phase transitions during the cosmological evolution of the Universe. These phase transitions can be accompanied by the formation of domain structures that is determined by the symmetry breaking at the phase transition. Wall domination, which always leads to a serious problem if the energy scale of the domain wall exceeds 1 MeV, can be avoided if a small bias $\delta\rho \equiv \epsilon \neq 0$ appears. The bias (here we use this word specifically for the energy difference between the false and the true vacua) becomes important when the force per unit area on the walls becomes comparable to the tension of the wall. Then, the condition for the successful decay of the cosmological domain walls is satisfied when [5]

$$\epsilon > G\sigma^2, \tag{1.7}$$

where σ denotes the tension of the wall.

The condition shows that the global discrete symmetry, which leads to the domain-wall formation, must be broken explicitly. In that case, the magnitude of the (explicit) breaking parameter must explain the bias $\epsilon > G\sigma^2$. This idea is useful in supersymmetric theory, in which the supergravity potential breaks discrete symmetry with the required magnitude [6]. Usually, the origin of the bias is considered for the explicit symmetry breaking in the potential. Walls that are formed after brane inflation are

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realized by the deformation of the brane configuration in the compactified space [7]. Wall-like structure observed on a cosmic string may appear as a monopole connected by the strings, which turns out to be a so-called cosmic necklace [5,8]

In this paper, we consider a simple model in which the mass of an additional field χ is induced by the interaction

$$\mathcal{L}_{\rm int} = \frac{1}{2} g^2 \phi^2 \chi^2, \tag{1.8}$$

where ϕ is the field that forms the wall configuration. It would be easy to find that an explicit symmetry breaking in the interaction causes a small mass difference in m_χ. The small mass difference between the adjacent vacua, which may be very small compared with the energy scale of the domain wall, may cause a bias when the vacuum fluctuations are considered. In addition to the conventional vacuum fluctuations, which may be the dominant contribution, the mass difference causes a boundary for the excitations of the χ-field, which leads to another source of the bias. Calculation of the vacuum fluctuations (i.e., the quantum bias caused by the Casimir effect) shows that the Casimir force may have a significant impact on the evolution of the cosmological domain walls.

Although the Casimir force for the massless field seems to be consistent with the experiments, it may have serious drawbacks[1] when it is applied to the issue of the cosmological constant. Since we are considering quantum bias between adjacent vacua, we cannot be free from the peculiar assumptions that are needed to understand or explain the (almost) vanishing cosmological constant. In this paper we are making best effort to find a sensible result for the quantum bias, but it should be noted that our results are based on these assumptions, which will be further explained in Sec. III.

II. CASIMIR ENERGY FOR THE DOMAIN WALLS WITH A SMALL MASS-GAP (1 + 1 DIMENSIONAL TOY MODEL)

In the four-dimensional model, it is possible to consider a double-well potential $V(\phi) = \frac{1}{4}\lambda(\phi^2 - v^2)^2$, which has a Z_2 symmetry ($\phi = +v \leftrightarrow \phi = -v$). This symmetry is broken spontaneously in either vacuum. Interaction with a real scalar field χ can be given by $\mathcal{L}_{\rm int} = \frac{1}{2} g \phi^2 \chi^2$, which gives a mass to the field χ. The domain walls are kinks of a real scalar field ϕ, which mediate between the two vacua $\phi = v$ and $\phi = -v$. As we mentioned, we do not consider explicit breaking of the discrete symmetry in the potential $V(\phi)$. Instead, we add a small but explicit symmetry breaking to the interaction, so that it leads to a small mass-gap for the χ-field. Obviously, there is no bias in the classical vacua. The source of the quantum bias can be explained by a small breaking of the Z_2 symmetry in the interaction, which can be expressed as

$$\mathcal{L}_{\rm int} = \frac{1}{2} g^2 (\phi - \epsilon_z)^2 \chi^2, \tag{2.1}$$

where ϵ_z measures the explicit symmetry breaking in the interaction.[2] Denoting the mass of χ in each domain by $m_\chi^{lo} \equiv g(v - \epsilon_z)$ and $m_\chi^{hi} \equiv g(v + \epsilon_z)$, and placing the lower-mass domain in the area sandwiched by two domain walls, the χ-field excitations that are discretized (trapped) by the boundary have $\omega_n^2 = k_n^2 + (m^{lo})^2 < (m^{hi})^2$. Obviously, the excitations with $\omega_n < m_\chi^{hi}$ can exist only in the domain sandwiched by the walls, while other (higher) excitations are continuous in both domains.[3]

Let us first consider a simple model in one-dimensional space (x), and place domain walls at $x = 0$ and $x = L$. The sum of the excitations discretized by the domain walls leads to the energy density

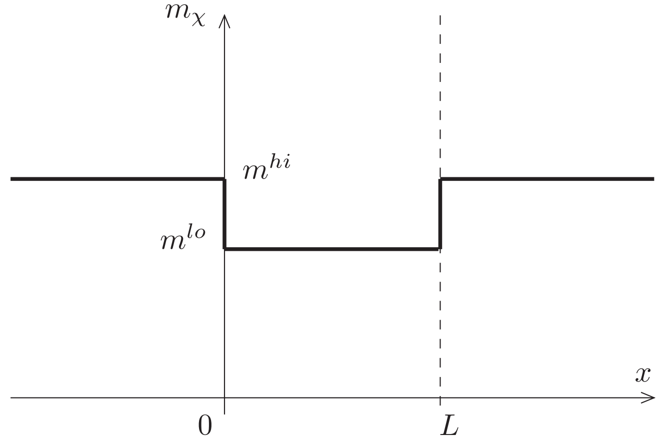

FIG. 1. The field χ feels a mass-gap at the domain walls, which are placed at $x = 0$ and $x = L$. The excitations that are trapped inside $0 < x < L$ are discrete.

[1]The regularization of the Casimir energy is under control. On the other hand, the root of the cosmological constant problem is not quite obvious. At this moment, it is not obvious whether an improvement is required in the regularization in solving the cosmological constant problem. The word ‘‘drawbacks’’ has been used to mention the specific situation in relation to the cosmological constant.

[2]We pointed out that the bias introduced by the Casimir effect may be important for the evolution of the cosmological domain walls. On the other hand, if one diagonalizes the whole Lagrangian, although the calculation is highly model-dependent and is not suitable for our argument, these domain walls may be unstable classically. In that case the bias introduced by the Casimir effect may be smaller than the classical bias. Note that we are not arguing that the classical bias is always smaller than the Casimir effect.

[3]See Fig. 1.

$$\rho_{\rm trap} \equiv \frac{1}{2L}\sum_{n=1}^{n_{\rm Max}} \omega_n, \tag{2.2}$$

where the integer $n_{\rm Max}$ is approximately given by $n_{\rm Max} \sim \frac{L}{\pi}\sqrt{(m_\chi^{hi})^2 - (m_\chi^{lo})^2} = \frac{2Lg}{\pi}\sqrt{v\epsilon_z}$. For the estimation of the Casimir effect, we set $n_{\rm Max} = \frac{2Lg}{\pi}\sqrt{v\epsilon_z}$ hereafter. With regard to the wavelength of these excitations, the effective length $L^{\rm eff}$ may depend on ω. The difference $L^{\rm eff}(\omega) - L \neq 0$ may be significant near $n_{\rm Max}$, where the excitations penetrate into the higher-mass domains. However, for the simple estimation of the Casimir effect, we choose the approximation $L^{\rm eff}(\omega) = L$ in this paper.

Despite the simplicity of the scenario, regularization of the vacuum fluctuations requires nontrivial assumptions that are far from obvious. In order to compare our result with the usual Casimir effect, it would be useful to start with a massless field. Therefore, we first consider a model with $m^{lo} = 0$ and $m^{hi} \neq 0$, so that we can calculate the Casimir energy using the conventional assumptions. The Casimir energy density in the constrained massless domain, which is sandwiched by the walls at $x = 0$ and $x = L$, is given by

$$\rho^R \equiv \rho_L - \rho_\infty = \frac{1}{2L}\sum_{n=1}^{n_{\rm Max}} \frac{\pi}{L} n - \int_0^{k_{\rm Max}} \frac{dk}{2\pi} k, \tag{2.3}$$

where the cancellation occurs in the continuous part $k > k_{\rm Max}$. Here we set $n_{\rm Max} = \frac{Lm^{hi}}{\pi}$ and $k_{\rm Max} = m^{hi}$. Executing the finite sum, it leads to

$$\rho^R = \frac{\pi}{2L^2}\frac{n_{\rm Max}(n_{\rm Max}-1)}{2} - \frac{1}{4\pi}(m^{hi})^2 = -\frac{1}{4L}m^{hi}, \tag{2.4}$$

where $n_{\rm Max} = \frac{Lm^{hi}}{\pi} > 1$ is required to obtain a nontrivial result.

In the above calculation, the origin of the Casimir energy is the discretization of the excitations, which is obviously finite. We subtracted the "common" part, which is continuous and divergent. In doing this, we assumed that the discretization of the excitations does not affect the regularization of the continuous part above $k > k_{\rm Max}$.

In the above calculation, we defined the Casimir energy in the massless domain by $\rho_L - \rho_\infty$ [4]. However, the bias between the adjacent vacua may "not" be measured by the Casimir effect in the massless domain. Namely, there is the possibility that $\rho_\infty^{lo} - \rho_\infty^{hi}$ may become the dominant part of the bias. In such calculation we have to reconsider regularization in the massive domain.

To understand the problem, consider two domains denoted by "A" and "B," which are separated by a wall. Then the vacuum fluctuations are (naively) given by $\rho_\infty^A \equiv \int_0^\infty \frac{dk}{2\pi}\sqrt{k^2 + m_A^2}$ and $\rho_\infty^B \equiv \int_0^\infty \frac{dk}{2\pi}\sqrt{k^2 + m_B^2}$, respectively. Without additional principle for the regularization, $m_A \neq m_B$ leads to $|\rho_\infty^A - \rho_\infty^B| \sim m_A^2 - m_B^2$. If the above calculation is true, the total vacuum fluctuations may depend explicitly on the particle content of the vacuum. The simplest way to avoid this problem is to assume some (unknown) regularization scheme that explains vanishing vacuum fluctuations in the $L \to \infty$ limit, which works for each field. If this assumption is true, one always finds $\rho_\infty^i = 0$ for the entire field labeled by i. Instead, one may consider a delicate cancellation between fields with different masses and spins, which eventually leads to the effective cosmological constant $\Lambda_c^{\rm eff} \equiv \Lambda_0 + \sum \rho_\infty^i \simeq 0$, where Λ_0 denotes contributions from other effects. Obviously, in the latter case the delicate cancellation is crucial for the bias calculation. Namely, if the delicate cancellation is violated in a false-vacuum domain, where the mass distribution is different from the true vacuum, the quantum bias is more significant compared with the Casimir energy calculated above.

In the next section, we consider a realistic four-dimensional model of cosmological domain walls and examine the cosmological domain-wall problem with the use of the quantum bias, which is caused by the symmetry breaking in the interaction.

III. CASIMIR ENERGY FOR THE DOMAIN WALLS WITH A SMALL MASS-GAP

We first consider a massless domain sandwiched by massive domains. For this thought experiment, a specific form of the potential can be expressed by $V(\phi) \sim \phi^2(\phi - v)^2$ (in this case there is no Z_2 symmetry but the degeneracy of the classical vacua still remains), or alternatively a fine-tuning can be considered in the interaction term (i.e., $v = \epsilon_z$ makes $m^{lo} = 0$). In order to realize a small mass-gap compared with the domain-wall tension ($m^{hi} \ll v$), we consider $g \ll 1$. Consider the domain sandwiched by two infinite and flat walls in the free R^3 space. The walls are placed at distance L apart and lie in the xy-plane. The standing waves are

$$\chi_n(x, y, x, t) = e^{-i\omega_n t}[e^{ik_x x}e^{ik_y y}]\sin k_n z, \tag{3.1}$$

where k_x and k_y are the wave vectors in the direction parallel to the walls, which are continuous, while the discretized wave vector

$$k_n = \frac{\pi}{L} n \tag{3.2}$$

is perpendicular to the walls. Because of the "shallow" potential, discretization occurs for the low-energy excitations $k \leq k_{\rm Max}$. The vacuum fluctuations in the massless domain are given by

$$\rho_L = \int \frac{dk_x dk_y}{(2\pi)^2}\left[\frac{1}{2L}\sum_{n=1}^{n_{\rm max}} \omega_n + \int_{k_{\rm max}}^\infty \frac{dk_z}{(2\pi)}\omega\right], \tag{3.3}$$

where the discretized wave (ω_n) is given by

$$\omega_n \equiv \left[k_x^2 + k_y^2 + \left(\frac{n\pi}{L}\right)^2\right]^{1/2}, \tag{3.4}$$

while for the continuous wave, it is given by

$$\omega \equiv [k_x^2 + k_y^2 + k_z^2]^{1/2}. \tag{3.5}$$

After integration and subtraction of ρ_∞, it leads to the regularized vacuum energy [4]

$$\begin{aligned}\rho^R &\equiv \rho_L - \rho_\infty \\ &= \frac{1}{12\pi}\left[-\frac{\pi^3}{2L^4}\sum_{n=1}^{n_{\max}} n^3 + \int_0^{k_{\max}} \frac{dk_z}{2\pi} k_z^3,\right] \\ &= \frac{1}{12\pi}\left[-\frac{\pi^3}{2L^4}\frac{n_{\max}^2(n_{\max}+1)^2}{4} + \frac{k_{\max}^4}{2\pi},\right] \\ &= -\frac{1}{96}\left[\frac{2m^3}{L} + \frac{\pi m^2}{L^2}\right],\end{aligned} \tag{3.6}$$

where the last equation is derived for $k_{\max} = \frac{\pi}{L} n_{\max} \equiv m$. Here m denotes the mass of the field χ in the massive domain.

In the above calculation, we considered a massless domain sandwiched by massive domains with a mass-gap $\delta m_\chi = m$ at the boundary (domain wall). Unlike the conventional Casimir effect, however, the discretization occurs only for a finite number of the excitations.

The above calculation for the "massless" domain is straight and very instructive, but here we must remember that the most important situation in our model is that the "massive" field feels a small mass-gap at a domain wall. In that case, the Casimir energy may cause the energy difference that depends on the mass distribution, as we already mentioned in the previous section for the domain walls. The situation seems unfavorable for the cosmological constant problem. Since the root of the cosmological constant problem is not obvious yet, and this is not the topic I am discussing in this paper, we follow the conventional calculation and try to find sensible consequences that may have phenomenological interest.

For instance, let us consider a bias ϵ for the cosmological domain walls with the tension $\sigma \sim v^3$. If the (almost) vanishing cosmological constant is explained by the delicate cancellation ($\Lambda_c^{\rm eff} \equiv \Lambda_0 + \sum \rho_\infty^i \simeq 0$), the bias between the domains with $m_A = m + \delta m$ and $m_B = m$ is calculated as

$$\epsilon \sim (m + \delta m)^4 - m^4 \sim 4m^3 \delta m. \tag{3.7}$$

For the interaction that leads to $m = gv$, the required condition for the safe decay is

$$\delta m > \delta_c \equiv \frac{v^3}{4M_p^2 g^3} = \frac{1}{4g^3}\left(\frac{v}{M_p}\right)^2 v, \tag{3.8}$$

which is about $g^{-3}(v/M_p)^2 \ll 1$ times smaller than v.[4] In this case, we may conclude that a small mass-gap that is caused by the symmetry breaking in the interaction term can be used to solve the cosmological domain walls problem. We find, therefore, a very useful solution to the cosmological domain-wall problem when the cosmological constant is tuned by the delicate cancellation.

If one cannot agree with such cancellation, an alternative assumption would be that the Casimir energy vanishes for each massive field in the infinite-volume limit (i.e., $\rho_\infty^i = 0$ for any i). Our purpose in this paper is not to argue which assumption is plausible, but to address the consequences that result from these assumptions. If the result obtained from our calculation turns out to be false in some experiment, one needs to introduce an additional principle for the regularization, so that one can calculate correctly the Casimir effect caused by the mass-gap. In any case, we believe that studying cosmological domain walls in terms of the Casimir effect can make a difference to the usual approaches to these problems.

Note however that, as far as the "Casimir energy" is defined using $\rho_L - \rho_\infty$, it is always possible to calculate the Casimir energy in an automatic manner. Therefore, in this paragraph, we are not going to argue the authenticity of the Casimir effect for a massive field, which may need improvement if it is responsible for the cosmological constant, but to calculate the Casimir effect in the automatic way. Consider two domains in which the masses of the field χ are given by $(m^{hi})^2 = m^2 + \delta m^2$ in the high-mass domain and $m^{lo} = m$ in the low-mass domain.[5] The vacuum fluctuations in the low-mass domain, which is sandwiched by the two walls and the high-mass domains outside, is given by

$$\rho_L = \frac{1}{L}\int \frac{dk_x dk_y}{(2\pi)^2}\left[\frac{1}{2}\sum_{n=1}^{n_{\max}} \omega_n + \int_{k_{\max}}^{\infty} \frac{dk_z}{(2\pi)}\omega\right]. \tag{3.9}$$

Here, for the discretized wave, ω_n is given by

$$\omega_n \equiv \left[m^2 + k_x^2 + k_y^2 + \left(\frac{n\pi}{L}\right)^2\right]^{1/2}, \tag{3.10}$$

while for the continuous wave, ω is given by

$$\omega \equiv [m^2 + k_x^2 + k_y^2 + k_z^2]^{1/2}. \tag{3.11}$$

After integration and subtraction of ρ_∞, it leads to the regularized vacuum energy[6]

[4]Even if g is very small, the condition is conceivable for $v \sim$ TeV $\ll M_p$ domain walls.

[5]In this notation δm^2 is not identical to $(\delta m)^2$.

[6]Here the word "regularization" means specifically the subtraction of ρ_∞. See also Ref. [4] in which "normal ordering" has been discussed in relation to the regularization.

$$\rho^R \equiv \rho_L - \rho_\infty$$
$$= \frac{1}{12\pi}\left[-\frac{1}{2L}\sum_{n=1}^{n_{\max}}\left[m^2 + \left(\frac{n\pi}{L}\right)^2\right]^{3/2} + \int_0^{k_{\max}} \frac{dk_z}{2\pi}(m^2 + k_z^2)^{3/2}\right], \tag{3.12}$$

where we set $k_{\max} \simeq \frac{\pi}{L} n_{\max} \equiv \sqrt{\delta m^2}$. In the limit of $\delta m^2/m^2 \ll 1$ we consider the approximation

$$(m^2 + k_z^2)^{3/2} \simeq m^3 + \frac{3}{2} m k_z^2, \tag{3.13}$$

which leads to

$$\rho^R \simeq -\frac{1}{24\pi L}\left[n_{\mathrm{Max}} m^3 + \frac{\pi^2 m}{4L^2} n_{\mathrm{Max}}(n_{\mathrm{Max}} + 1)(2n_{\mathrm{Max}} + 1)\right] + \frac{1}{24\pi^2} m^3 k_{\mathrm{Max}} + \frac{1}{48\pi^2} m k_{\mathrm{Max}}^3$$
$$= -\frac{1}{96}\left[\frac{3m\delta m^2}{\pi L} + \frac{m}{L^2}\sqrt{\delta m^2}\right]. \tag{3.14}$$

Going back to cosmological domain walls, we find immediately that the walls may appear equally in the x and y directions. Also, the shape of the domain may affect the calculation. However, a simple estimation of the vacuum fluctuation is not difficult, which leads to

$$\rho^R \sim c\frac{m\delta m^2}{\xi(t)}, \tag{3.15}$$

where $c \sim 10^{-2}$ is a numerical constant and $\xi(t)$ denotes the distance between walls. Since the Hubble parameter at the beginning of the wall domination is $H_d \simeq \sigma/M_p^2$ [5], the Casimir energy at the wall domination is expressed as

$$\rho^R \sim c\frac{\sigma}{M_p^2} m\delta m^2, \tag{3.16}$$

where the typical scale of the wall structure is assumed to be $\xi_d \sim H_d^{-1}$. Considering $\delta m^2 \equiv (m + \delta m)^2 - m^2 = 2m\delta m + (\delta m)^2$ in our notation, the Casimir force may satisfy the bias condition $\epsilon_z > \sigma^2/M_p^2$ when

$$\delta m > \frac{v}{cg^2}. \tag{3.17}$$

Therefore, the "Casimir force" calculated above seems to be unimportant for the domain-wall problem. On the other hand, structures like wiggles or foldings may typically have much smaller scale compared with H^{-1}. The evolution of these small-scale structures of the cosmological domain walls may be affected by the Casimir force.[7]

[7] See Fig. 2

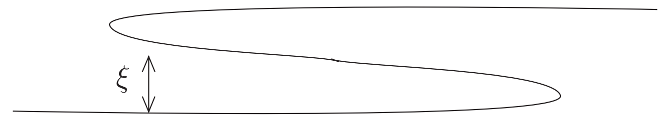

FIG. 2. Wiggles or foldings may appear in the small-scale structure of the cosmological domain wall.

Can the multiplicity enhance the Casimir effect?

If the field χ has the origin in some higher dimensional theory, one cannot neglect the multiplicity of the field. Namely, if the explicit symmetry breaking in the interaction term causes a mass-gap $\sim\delta m$ to the lowest state, it may also cause the same mass-gap to the Kaluza-Klein states. Then, the Casimir force calculated above can be enhanced by the number of the Kaluza-Klein states. For the specific scenario, we consider the mass for the k-th Kaluza-Klein state;

$$m_k^2 = m^2 + \frac{k^2}{R_5^2}, \tag{3.18}$$

where k is an integer. Then the Casimir energy is calculated as

$$\rho^R \equiv \rho_L - \rho_\infty$$
$$= \frac{1}{12\pi}\sum_k\left[-\frac{1}{2L}\sum_{n=1}^{n_{\max}}\left[m_k^2 + \left(\frac{n\pi}{L}\right)^2\right]^{3/2} + \int_0^{k_{\max}}\frac{dk_z}{2\pi}(m_k^2 + k_z^2)^{3/2}\right]$$
$$\simeq -\frac{1}{96}\left[\frac{3(\delta m^2)}{\pi L} + \frac{\delta m}{L^2}\right]\sum_k \sqrt{m^2 + \frac{k^2}{R_5^2}}, \tag{3.19}$$

where the result can be expressed formally in terms of Epstein ζ-function.[8]

IV. CONCLUSIONS

In this paper we considered two types of vacuum fluctuations for the evolution of the cosmological domain walls. We considered a potential which does not break explicitly the Z_2 symmetry. Instead, we added the interaction that breaks explicitly the symmetry. This term does not cause any bias in the classical vacua, but may source the

[8] The Casimir energy in relation to extra dimensions has been studied by many authors [9,10]. With regard to brane models, the finite temperature Casimir force due to a massless scalar in the bulk of a brane model has been calculated in Ref. [11]. The Casimir energy of massless and massive bulk fields can generate a potential that stabilizes the radius of the compact direction while it may be driving the accelerated expansion in the noncompact directions [12]. The Casimir force acting on two parallel planes lying within the single brane of a Randall-Sundrum scenario has been discussed in Ref. [13].

quantum bias. Then the two vacua at $\phi = \pm v$, which are classically degenerate, can be split by the Casimir effect. In the first example, in which we compared the vacuum fluctuations in different domains assuming that the vanishing cosmological constant is explained by the delicate cancellation, we find that domain walls can decay safely due to the quantum bias. In the second example, in which we considered the effect of the boundaries that are formed by the domain walls, the mass gap leads to a discretization of the excitations. The latter effect may be important for the small-scale structures of the domain walls, while it may be unimportant for the safe decay.

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