

Coupled dark energy and dark matter from dilatation anomalyJ. Beyer,¹ S. Nurmi,^{1,2} and C. Wetterich¹¹*Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany*²*NORDITA, Roslagstullsbacken 23, SE-106 91 Stockholm, Sweden*

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Cosmological runaway solutions may exhibit an exact dilatation symmetry in the asymptotic limit of infinite time. In this limit, the massless dilaton or cosmon could be accompanied by another massless scalar field—the bolon. At finite time, small time-dependent masses for both the cosmon and the bolon are still present due to imperfect dilatation symmetry. For a sufficiently large mass, the bolon will start oscillating and play the role of dark matter, while the cosmon is responsible for dark energy. The common origin of the mass of both fields leads to an effective interaction between dark matter and dark energy. Realistic cosmologies are possible for a simple form of the effective cosmon-bolon potential. We find an inverse bolon mass of a size where it could reduce subgalactic structure formation.

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I. INTRODUCTION

Dilatation symmetry and its anomaly could play an important role for cosmology [1]. Models with a dilatation-symmetric fixed point could provide a dynamical solution for the cosmological constant problem [1–3]. As one of the most characteristic features, such models have predicted the presence of a homogeneous dark energy component [1], long before its observational discovery. Recent investigations of higher-dimensional settings have shed new light on such theories [4]. It has been shown that dimensional reduction of a dilatation-symmetric quantum effective action in higher dimensions leads to four-dimensional models with a vanishing cosmological constant. If the cosmological solution approaches the dilatation-symmetric fixed point in the limit $t \rightarrow \infty$, such models can naturally give rise to an asymptotically vanishing cosmological constant. In such a scenario the observed particle masses are due to the spontaneous breaking of dilatation symmetry by the cosmological or vacuum solution. At the fixed point one will therefore encounter a massless Goldstone boson—the dilaton. For finite time, the dilatation symmetry is broken by anomalous terms which generate a small time-dependent mass for the dilaton. This picture can give rise to a quintessence cosmology where the dilaton rolling toward the fixed point plays the role of the scalar “cosmon” field with a slowly decreasing mass [1,2]. This higher-dimensional dilatation-symmetric setup naturally leads to scaling solutions where the cosmon mass tracks the Hubble parameter.

First investigations of the manifold of extrema of a dilatation-symmetric higher-dimensional quantum effective action often reveal the presence of additional massless scalar fields besides the dilaton [4]. These may correspond to a change of the characteristic length scale of internal space—the radion—or other changes in geometry, similar to the moduli fields in string theory. In this work we will concentrate on one such field and name it the “bolon,”

since it will ultimately be responsible for dark matter and therefore for the emergence of structure (i.e. “lumps” or “bola”) in the cosmos.

We start with a quick revision of the key concept of dilatation symmetry. Then we will investigate the coupled system of cosmon and bolon and show that it can reproduce the standard cosmological evolution at the background level for rather simple potentials. After a radiation-dominated period during which the cosmon and bolon act as early dark energy and play a subdominant role, the bolon starts to oscillate around a partial potential minimum. Its oscillation energy (potential and kinetic) is diluted as nonrelativistic matter and thus it will eventually become dominant, enforcing a transition to a matter-dominated period. After the transition the fluid equations for the energy density of bolon fluctuations obey the standard form for cold dark matter, with a small coupling to the cosmon. Thus the simple model with two scalars describes a cosmology with coupled dark matter and dark energy.

We further proceed to slight modifications of the simplest potential for which dark energy finally dominates the energy density of the Universe. This can be achieved by an effective stop in the cosmon evolution, induced either by a characteristic change in the scalar potential or a leaping kinetic term, or if the scaling behavior of the cosmon gets terminated by a cosmic trigger event. An example of the last scenario is given by the growing neutrino quintessence model [5].

II. DILATATION SYMMETRY

Dilatations correspond to a rescaling of each dynamical field of a given theory corresponding to an appropriate scaling dimension. For a theory involving a metric $g_{\mu\nu}$ and a scalar field ξ in d dimensions this means

$$g_{\mu\nu} \rightarrow \alpha^2 g_{\mu\nu}, \quad \xi \rightarrow \alpha^{-((d-2)/2)} \xi. \quad (1)$$

Any dilatation-symmetric quantum effective action Γ is then given by an action

$$\Gamma = \int d^4x g^{1/2} \mathcal{L} \quad (2)$$

with

$$\mathcal{L} \rightarrow \alpha^{-d} \mathcal{L} \quad (3)$$

under dilatations. This puts severe restrictions on the allowed interactions in \mathcal{L} . Dilatation symmetry ensures the absence of any explicit mass scales of the model. These restrictions have particularly interesting consequences in a higher-dimensional setting. For $d > 6$ no polynomial potential for the scalar field is allowed, and dimensional reduction of such a higher-dimensional dilatation-symmetric theory gives rise to a rather generic class of effective four-dimensional theories with a vanishing cosmological constant [4]. This finding has been generalized to arbitrary dilatation-symmetric Γ , for example, based only on the metric without a scalar field [4].

In our scenario we consider theories with a dilatation-symmetric fixed point, which corresponds to field configurations reached in the limit of cosmic time going to infinity. One possibility is to start with a generic action Γ , not necessarily dilatation symmetric, and to consider a family Γ_κ obtained by a rescaling

$$\Gamma_\kappa[g_{\mu\nu}, \xi] = \Gamma[\kappa^{-2}g_{\mu\nu}, \kappa^{d(d-2)/2}\xi]. \quad (4)$$

This becomes dilatation symmetric in the limit $\kappa \rightarrow \infty$. One may imagine that a dynamical runaway solution drives κ effectively to infinity for $t \rightarrow \infty$, for example, by a monotonic increase of the cosmological value of the scalar ξ .

Dimensional reduction of such an action leads to infinitely many fields in four dimensions. We consider only massless or light fields which play a role in cosmology after a possible inflationary period. They are described by an effective theory, encoded in the four-dimensional, dimensionally reduced action, plus the metric and the particles of the standard model of particle physics. The latter will dominate the radiation-dominated era, while in the matter-dominated era the small component of baryons is subleading and can be neglected for the overall picture of dark matter. Our starting point for the effective action of two scalars φ and χ is a standard kinetic term and a potential $V(\varphi, \chi)$. Dilatation symmetry at the fixed point enforces the common potential of the two fields to vanish for $\varphi \rightarrow \infty$ [4].

III. THE MODEL

Away from the dilatation-symmetric fixed point the four-dimensional quantum effective action will contain an effective potential $V(\varphi, \chi)$ for the cosmon field φ and

the bolon field χ , generated by dilatation anomalies. The formulation in terms of the quantum effective action means that no further quantum corrections to the potential are present. If we were to start at the classical theory, various quantum corrections would appear in the process of quantization. They may in fact be responsible for the existence of a fixed point. In this case the effective dilatation symmetry at the fixed point is due to quantum fluctuations (rather than being destroyed by them) and may be realized even for a classical action without dilatation symmetry. In the vicinity of a fixed point, asymptotic dilatation symmetry guarantees that the potential vanishes as the fixed point is approached for $\varphi \rightarrow \infty$. It also ensures a vanishing mass at the fixed point. The bolon mass also vanishes at the fixed point.

We work within the effective four-dimensional theory (obtained after dimensional reduction) and use the Einstein frame (with fixed Planck mass). Furthermore we assume first a normalization of φ and χ such that both have standard kinetic terms. In this frame the φ -dependence of the potential is typically of exponential shape [1], and we start with a simple model where

$$V = M^4 \left[\left(\frac{\mu}{M} \right)^A e^{-\alpha\varphi/M} + \left(\frac{\mu}{M} \right)^B e^{-2\beta\varphi/M} \left(\frac{\chi}{M} \right)^2 \right]. \quad (5)$$

Here M is the four-dimensional effective Planck mass, and $\mu \ll M$ is the scale of anomalous dilatation symmetry breaking. We assume the dimensionless constants A , B , α and β to be all positive. The ‘‘anomalous dimensions’’ A , B characterize the impact of explicit mass scales of the model (violation of dilatation symmetry) on the four-dimensional world. The asymptotic solution will correspond to $\varphi \rightarrow \infty$ where the potential V vanishes and dilatation symmetry becomes exact. Then also the mass matrix for the cosmon and bolon, given by the second derivatives of V , approaches zero. The simple quadratic form for χ is assumed to be a good approximation for χ close to a partial minimum of the potential in this direction. Small deviations of χ from the partial extremum will be sufficient for the understanding of late cosmology. For very early cosmology the quadratic approximation for χ may become insufficient, and we discuss generalizations below. There we also briefly address the possibility that the φ -dependence of the partial minimum for χ cannot be neglected [as for the potential (5)].

An important ingredient for our setting concerns the observation that the approach to the fixed point at $\varphi \rightarrow \infty$, $\chi \rightarrow 0$ can be characterized by a different scaling for the cosmon and the bolon directions. For χ at its minimum, the size of the ‘‘anomalous potential’’ is dictated by the parameters A and α . Different parameters B and β may describe the scaling of V away from the partial minimum, e.g. for $\chi \neq 0$. The characteristic scaling behavior and its motivation by the properties of solutions in the vicinity of a higher-dimensional dilatation-symmetric

fixed point distinguishes our approach from other settings where dark energy and dark matter are described by two scalar fields, as realized, for example, in a model for the axion and the cosmon or in the work of Refs. [6–11]. Because of the factor $\exp(-2\beta\varphi/M)$, the mass of the bolon field will depend on the value of the cosmon field. If the bolon plays the role of dark matter, we expect a coupling between dark energy and dark matter $\sim\beta$, as discussed previously in models of coupled quintessence [2,12]. We emphasize that a cosmon-bolon coupling is necessary in our setting since the bolon mass is assumed to vanish for the asymptotic solution.

IV. COSMIC EVOLUTION

To simplify the analysis of the dynamics, we perform a shift $\varphi \rightarrow \varphi - (MA/\alpha)\ln(\mu/M)$ which corresponds to $A \rightarrow 0$ and $B \rightarrow \tilde{B} = B - \beta A/\alpha$ in Eq. (5). This rescaling shows that our model depends effectively on three parameters, namely, the dimensionless couplings α and β and the effective scale for the bolon mass, which involves the parameter $(\mu/M)^{\tilde{B}}$. We define the energy densities

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + M^4 e^{-\alpha\varphi/M}, \quad (6)$$

$$\rho_\chi = \frac{1}{2}\dot{\chi}^2 + M^4 \left(\frac{\mu}{M}\right)^{\tilde{B}} e^{-2\beta\varphi/M} \left(\frac{\chi}{M}\right)^2, \quad (7)$$

which will be associated with dark energy and dark matter components, respectively. In addition to ρ_χ and ρ_φ , we assume that the Universe also contains a homogeneous radiation component ρ_r which dominates at early times.

The φ -dependent mass of the bolon, reflected by the factor $\exp(-2\beta\varphi/M)$ in Eq. (7), will lead to a coupling between dark matter and dark energy of strength β , similar to generic models of coupled quintessence [2,12]. We would like to point out that in our model the coupling between the bolon and the cosmon does not arise from matter particles coupling to a metric related to the Einstein frame metric by a conformal transformation as e.g. in generalized Jordan-Brans-Dicke theories [13]. Rather, it emerges as a feature of the common cosmon-bolon potential from the breaking of dilatation symmetry and is crucial to restore dilatation symmetry at the fixed point. Possible couplings of the cosmon to other forms of matter as baryons or neutrinos are not directly related to the coupling β appearing in Eq. (5). In particular, there is no reason to expect a baryon-cosmon coupling of equal strength as the coupling between dark matter and dark energy. Limits on the time variation of couplings and violations of the weak equivalence principle imply for most models that the baryon-cosmon coupling must be much smaller than the value of β discussed in this paper [1,2,14]. Nevertheless it is possible to add a baryonic fluid which can be either uncoupled, very weakly coupled or coupled with gravitational strength to the dilaton without qualitatively changing

our conclusions. In the last case, however, one would need to invoke some kind of mechanism to avoid constraints from local tests of gravity, like the Damour-Polyakov effect [15,16] or the chameleon mechanism [17,18].

For $\alpha \neq 2\beta$, the two terms in (5) decay at different rates as $\varphi \rightarrow \infty$. The qualitative features of the dynamics in the early radiation-dominated epoch depend on the ratio of α and β and the magnitude of the prefactor $(\mu/M)^{\tilde{B}}$. We start by discussing solutions for which the potential (5) is dominated by the first term in the early radiation-dominated Universe and the second term becomes dominant at late times, causing a transition into a matter-dominated epoch. The growth of the second term in (5) relative to the first one requires $2\beta < \alpha$, but we will actually impose a stronger condition, $\beta \ll \alpha$. This will also ensure that the resulting dark matter–dark energy interaction remains compatible with cosmological observations. The dominance of the first term in (5) (small positive values of φ) requires $(\mu/M)^{\tilde{B}}\chi^2 \ll M^2$.

In the early radiation-dominated Universe, we then have $V \simeq M^4 e^{-\alpha\varphi/M}$, and the system rapidly approaches the well-known attractor solution [1,2]

$$\varphi = -\frac{M}{\alpha} \ln\left(\frac{4H^2}{M^2\alpha^2}\right), \quad \rho_\varphi = \frac{12}{\alpha^2} H^2 M^2. \quad (8)$$

The behavior of χ depends on the magnitude of its effective mass which obeys for the scaling solution (8)

$$m_\chi^2 = 2M^2 \left(\frac{\mu}{M}\right)^{\tilde{B}} \Omega_\varphi^{2\beta/\alpha} \left(\frac{H}{M}\right)^{4\beta/\alpha}, \quad (9)$$

where $\Omega_\varphi = \rho_\varphi/(3M^2H^2) = 4/\alpha^2$. For $2\beta < \alpha$, the mass grows relative to the Hubble rate, $m_\chi/H \propto t^{1-2\beta/\alpha}$. For early cosmology one has $H \gg m_\chi$ and the field χ , starting from some initial configuration χ_{in} , $\dot{\chi}_{\text{in}}$, settles rapidly to a constant value χ_0 . Subsequently, it remains nearly frozen at χ_0 until $m_\chi \sim H$, when χ starts to oscillate.

Assuming ρ_χ to be still subdominant compared to ρ_φ at this point, we can use the scaling solution (8) to obtain $\dot{m}_\chi/m_\chi^2 = -4(\beta/\alpha)H/m_\chi$. For $\beta \ll \alpha$, m_χ soon becomes nearly constant in a time scale of one oscillation cycle $1/m_\chi$. As a consequence, the bolon χ oscillates in an effectively quadratic potential with a slowly decreasing mass term. Its energy density behaves almost as that of a nonrelativistic matter component [19] and eventually comes to dominate over ρ_φ and over the radiation component.

For an estimate of the onset of oscillations we define t_{osc} by the condition $H(t_{\text{osc}}) = m_\chi(t_{\text{osc}})$. In the absence of a coupling between χ and φ , $\beta \rightarrow 0$, we obtain $H(t_{\text{osc}}) = \sqrt{2}M(\mu/M)^{\tilde{B}/2}$. The time of matter-radiation equality is approximately given by

$$\frac{H_{\text{eq}}}{M} \sim \left(\frac{\mu}{M}\right)^{\tilde{B}/2} \left(\frac{\chi_{\text{eq}}}{M}\right). \quad (10)$$

Here χ_{eq} denotes the bolon amplitude at t_{eq} and obeys

$$\frac{\chi_{\text{eq}}}{M} \sim \left(\frac{\chi_0}{M}\right)^4 \quad (11)$$

if $\chi_0 \ll M$. For a weakly coupled system with $\beta/\alpha \ll 1$, the energy density stored in the oscillations $\rho_\chi \sim (1/2)m_\chi^2 \chi^2$ dilutes slightly faster than a^{-3} due to the decrease of the mass $m_\chi^2(\varphi)$. Our numerical analysis reveals that Eq. (10) gives a fairly accurate estimate in the weakly coupled case as well. Realistic cosmologies require matter-radiation equality to happen at $H_{\text{eq}}/M \approx 8 \times 10^{-56}$. For initial conditions leading to χ_0 of the order $0.1M$, this requires a small value of $(\mu/M)^{\tilde{B}/2}$, say, $\mu \approx 10^8$ GeV for $\tilde{B} = 10$. (Still, the value of μ is much larger than all the energy scales of the standard model of particle physics—smaller μ can be obtained for smaller \tilde{B} .)

After the onset of the bolon oscillations, the evolution equations for the system can be expressed in the form

$$\ddot{\phi} + 3H\dot{\phi} - \alpha M^3 e^{-\alpha\varphi/M} = \frac{\beta}{M} \rho_\chi, \quad (12)$$

$$\dot{\rho}_\chi + 3H\rho_\chi = -\frac{\beta}{M} \rho_\chi \dot{\phi}, \quad (13)$$

$$\dot{\rho}_\gamma + 4H\rho_\gamma = 0, \quad (14)$$

$$3M^2 H^2 = (\rho_r + \rho_\chi + \rho_\varphi), \quad (15)$$

where we have averaged over an oscillation cycle and used the results $\langle \dot{\chi}^2 \rangle = 2\langle M^2(\mu/M)^{\tilde{B}} e^{-2\beta\varphi/M} \chi^2 \rangle = \rho_\chi$, valid for $H \ll m_\chi$ and $(\dot{m}_\chi/m_\chi) \ll H$. For these “late” times the cosmon-bolon system bears much resemblance to other theories where a quintessence field couples to dark matter through the trace of the energy-momentum tensor, e.g. chameleon models [17] or the model investigated in [20]. (The main difference between the cosmic evolution described in [20] and our scenario—besides the obvious differences for early cosmology—is that the authors of [20] describe a “freezing” solution. There the universal attractor is characterized by an accelerated expansion and the cosmon field remains frozen until very recently and then starts to dominate the cosmic evolution. While this type of solution is also present in our scenario for a suitable choice of parameters and initial values, it is very sensitive to the choice of initial conditions. For this reason we will focus here on scaling solutions for the cosmon field during both the matter- and the radiation-dominated eras. We will see in Sec. V how this can ensure insensitivity to initial conditions. There we also discuss possibilities of how to break this behavior and reach cosmic acceleration.)

The dynamics of the system described by Eqs. (12)–(15) was analyzed in [2,12]. For $\beta/\alpha \ll 1$, it admits an effectively matter-dominated attractor solution with

$$a \propto t^{1-\beta/\alpha}, \quad \rho_\varphi = 3M^2 H^2 f(\alpha, \beta), \quad (16)$$

where $f = (18 + 6\beta^2 - 6\beta\alpha)/(6(\alpha - \beta)^2)$. For $\alpha \gg 1$, as required by $\rho_\varphi \ll \rho_r$ in the early scaling epoch, stability and existence of the solution is guaranteed for $4\beta < \alpha < \beta + 3/\beta$. Expanding $f(\alpha, \beta)$ around $\beta/\alpha = 0$, we find

$$\Omega_\varphi = \frac{3}{\alpha^2} - \frac{\beta}{\alpha} \left(1 - \frac{6}{\alpha^2}\right) + \mathcal{O}(\beta^2/\alpha^2), \quad (17)$$

which shows that Ω_φ decreases slightly as compared to the early radiation-dominated epoch. For $\alpha \sim \mathcal{O}(10)$, ρ_φ constitutes a few percent of the total energy density in the matter-dominated epoch.

At this stage the coupled cosmon-bolon system describes a cosmology with dark matter, radiation and a small constant fraction of early dark energy. A realistic cosmology requires a dark matter–dark energy crossover where, after redshift $z \approx 5$, dark energy increases from a few percent to its present fraction $\Omega_\varphi > 0.7$. This can be achieved by an effective stop or strong slowing down of the evolution of φ . Such a stop may be induced by a cosmological trigger event—such as neutrinos becoming nonrelativistic [5] or scalar backreaction due to structure formation [21]—or by a qualitative change of the cosmon potential or the kinetic term for φ exceeding a characteristic value [22].

V. MODIFICATIONS OF THE SCALAR POTENTIAL

A. Bolon scaling in the early Universe

For the potential (5) quadratic in χ , the time of matter-radiation equality t_{eq} depends on both the model parameter $(\mu/M)^{\tilde{B}}$ and the initial conditions for χ ; see Eq. (10). This is illustrated in Fig. 1. The memory of the initial conditions for φ disappears as the cosmon hits the attractor solution. For a very wide range of initial conditions, this happens long before the transition to matter domination, as demonstrated in Fig. 1. A dependence of t_{eq} (and the final

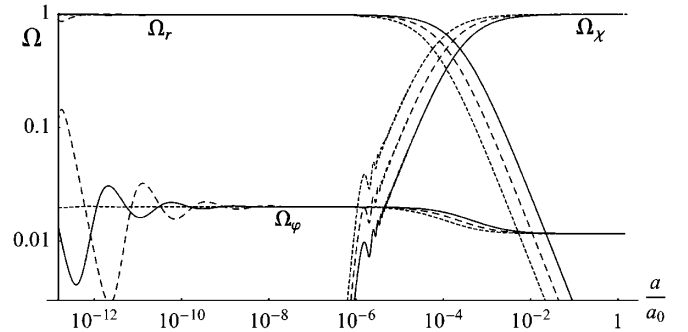


FIG. 1. Evolution of the density parameters Ω_r , Ω_χ and Ω_φ for $\alpha = \sqrt{2} \times 10$, $\beta = 0.05$, $(\mu/M)^{\tilde{B}} = 10^{-100}$ and $\dot{\chi}_{\text{in}} = 0$. We show three different values for χ_{in}/M : 0.1 (solid line), $\sqrt{2} \times 0.1$ (dashed line) and 0.2 (dotted line). We also vary Ω_φ ($z = 10^{13}$).

value of $\rho_\chi a^3$) on the initial conditions for χ is common to other dark matter models like the axion. While realistic cosmology can be obtained for rather natural-looking initial conditions, e.g. for $\chi_0 \sim M$ with $(\mu/M)^\beta \sim 10^{-100}$, it is noteworthy that the initial energy density ρ_χ needs to be considerably smaller than ρ_φ (and ρ_r) for the potential (5) to work. One could presume that the fields φ and χ have been excited at some point after inflation in a similar process, possibly connected to the production of baryonic matter and radiation. In this case, initial energy densities of the same order of magnitude would seem more natural. Furthermore, assuming the quadratic dependence on χ in Eq. (5) to remain valid all the way to the inflationary epoch, one would run into problems with the excessive isocurvature perturbations arising from the fluctuations of the light bolon.

We therefore consider Eq. (5) to be a valid approximation only for late cosmology (i.e. field configurations close to the fixed point). Indeed, there is no reason to assume a quadratic dependence on χ for large values, as relevant for early cosmology. In this regime the potential may be much steeper, typically involving χ exponentially. Steep potentials typically admit tracker solutions for which ρ_χ will either scale like the dominant background fluid (for an exponential χ -dependence) or slowly catch up to it. This could lead to a setting where the time of matter-radiation equality depends only on model parameters and not any longer on the initial conditions for χ . In a first period, χ will then rapidly evolve until it settles at a value $\bar{\chi}_0$. There it sits until ρ_χ has caught up with the energy density of the tracker solution, which it will follow from this point on. Once χ has reached values sufficiently close to its minimum, the quadratic approximation for the χ -dependence may be used. This behavior occurs for a large range of initial conditions and guarantees that the bolon enters the flat part of the potential with predetermined values of χ and $\dot{\chi}$, which now depend only on model parameters.

A simple example for such a mechanism can be obtained by replacing χ^2/M^2 in Eq. (5) by $(\cosh(\lambda\chi/M) - 1)$, a form considered in the context of uncoupled scalar dark matter in Refs. [6,7]. This potential has an asymptotically exponential shape for large χ -values, allowing for an attractive scaling solution in the uncoupled case $\beta = 0$. Our numerics reveal that the qualitative behavior is maintained in the weakly coupled case with small β . The range of initial conditions resulting in realistic cosmologies for this modification is rather large, including equipartition of the energy densities $\Omega_\chi(t_{\text{in}}) \approx \Omega_r(t_{\text{in}})$. Figure 2 demonstrates that this wide range of initial conditions influences only early cosmology. Close to matter-radiation equality, essentially no memory of the initial conditions survives. For late cosmology the behavior is very similar to the potential (5), but now with an effective value χ_0 that is determined by model parameters and no longer by initial conditions. We would like to point out that the price we pay

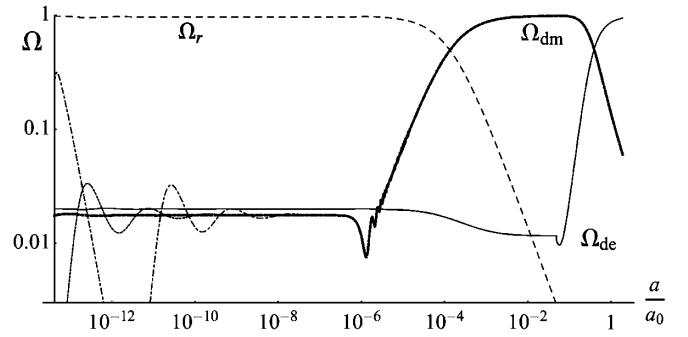


FIG. 2. Dark matter–dark energy crossover for parameters $\gamma = 0$, $\alpha = \sqrt{2} \times 10$, $\beta = 0.05$, $(\mu/M)^\beta = 10^{-96}$, $c = 10$ and $\varphi_0 = 8.7M$. The potential for large χ values was modified to have exponential shape by replacing χ^2 by $(\cosh(\lambda\chi) - 1)$, $\lambda = 15$. The bold curve for Ω_{dm} reflects a large range of initial conditions after inflation. We demonstrate the insensitivity to initial values by setting new “initial conditions” at $z = 13$ with χ between $4M$ and $5M$, $\dot{\chi} = 0$ (dashed curves).

for the independence of initial conditions is, as usual, the introduction of an additional parameter (in this simple model, λ). It determines at which field value the bolon potential makes a transition from a “steep” to a “flat” regime. The parameter needs to be adjusted to give a realistic cosmic evolution with the correct “timing” of matter-radiation equality.

B. Breaking the scaling behavior with a cosmon-dependent minimum

A further interesting generalization of our model potentially concerns a φ -dependence of the partial minimum for χ according to substituting χ^2 in Eq. (5) by $e^{-2\gamma\chi/M}(\chi - g(\varphi))^2$. This could lead to a late time accelerated expansion if $g(\varphi)$ has a characteristic shape where it changes its form abruptly once φ passes a characteristic value φ_0 . We demonstrate this in Fig. 2 for a simple toy model where $g(\varphi) = c(\varphi - \varphi_0)\Theta(\varphi - \varphi_0)$, Θ being the Heaviside function. One observes a crossover from matter domination to dark energy domination in the present cosmological epoch.

If the partial minimum for the bolon depends on φ , the separation of the combined cosmon-bolon energy density into dark matter and dark energy is less straightforward. A reasonable approximation for $\varphi > \varphi_0$ is given by

$$\begin{aligned} \rho_{\text{dm}} &= \frac{1}{1 + c^2} (\dot{\chi} - c\dot{\varphi})^2 + V(\varphi, \chi) - V(\varphi, g(\varphi)), \\ \rho_{\text{de}} &= \frac{1}{1 + c^2} (c\dot{\chi} + \dot{\varphi})^2 + V(\varphi, g(\varphi)). \end{aligned} \quad (18)$$

The time-averaged evolution equations for these quantities are given by Eqs. (12) and (13), but with ρ_χ , ρ_φ , α , β and φ replaced by ρ_{dm} , ρ_{de} , $\alpha_c = \alpha/\sqrt{1 + c^2}$, $\beta_c = (\beta + c\gamma)/\sqrt{1 + c^2}$ and the approximation for the cosmon, $\phi = (c\chi + \varphi)/\sqrt{1 + c^2}$, respectively. For $\varphi > \varphi_0$, the scaling solution (17) remains no longer valid—the

asymptotic behavior is now governed by α_c and the new coupling β_c . For an appropriate choice of parameters, the only stable fixed point of this system is an accelerated expansion [12]. We show an example in Fig. 2. Such a setting resembles certain cases discussed in Ref. [23].

We expect that similar features can also arise from a qualitative change of the scalar kinetic terms for $\varphi > \varphi_0$. Let us recall, however, that the dark matter–dark energy crossover may also be induced by a cosmological trigger event [5,21]. In this case, no particular feature for the scalar potential or the kinetic terms at $\varphi = \varphi_0$ is needed, as for the leaping kinetic term of Ref. [22].

VI. DISCUSSION

In summary, we have found rather simple models for coupled cosmon and bolon scalar fields which realize a consistent cosmology without the need of other dark matter particles. The essential ingredients are the presence of a period where the bolon energy density is much smaller than radiation, and the increase of the bolon mass relative to the Hubble parameter due to $\beta \ll \alpha$.

Remarkably, the inverse bolon mass in the present cosmological period,

$$m_\chi^{-1} = \sqrt{\frac{1}{3}} \frac{\chi_{\text{eq}}}{M} H_{\text{eq}}^{-1} e^{\beta \Delta\varphi/M} \approx \left(\frac{10\chi_0}{M}\right)^4 \text{pc}, \quad (19)$$

is typically found at subgalactic scales and could reduce the clustering of dark matter on these and smaller scales

[6,7]. Here, $\Delta\varphi = \varphi_{\text{today}} - \varphi_{\text{eq}}$, and the last factor is neglected in the weak-coupling approximation. We found that for the type of models discussed in Sec. V, values of $\chi_0/M \approx 1$ or somewhat larger are naturally realized, independent of the precise initial conditions. (A bolon mass of a galactic scale of 10 kpc is realized for $\chi_0/M \approx 1$.)

Since a coupling of the bolon to ordinary matter has presumably at most gravitational strength, a direct detection by the searches for WIMP-like particles or axions seems excluded. On the other hand, a coupling β between dark energy and dark matter is a generic feature of our setting. It reflects the common origin of the cosmon-bolon potential and the cosmon and bolon masses from the deviations from dilatation symmetry. Interestingly, if cosmological measurements should indicate an equation of state $w_{\text{de}} < -1$ for uncoupled dark energy, the coupling β can be used to explain such observations [24]. Furthermore β influences both the behavior of the cosmological solution and the properties of dark matter on smaller-length scales. A test of the interesting observational consequences can constrain β or give hints in the direction of the coupled dark energy and dark matter of our model.

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