Testing and selecting dark energy models with lens redshift data

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In this paper, we compare seven popular dark energy models under the assumption of a flat universe by using the latest observational data of gravitationally-lensed image separations observed in the Cosmic Lens All-Sky Survey (CLASS), the PMN-NVSS Extragalactic Lens Survey (PANELS), the Sloan Digital Sky Survey (SDSS) and other surveys, which are (nearly) complete for the image separation range $0''.3 \leq \Delta\theta \leq 7''$. We combine the 29 lens redshift data with the cosmic microwave background (CMB) observation from the Wilkinson Microwave Anisotropy Probe (WMAP7) results, the baryonic acoustic oscillation (BAO) observation from the spectroscopic Sloan Digital Sky Survey (SDSS) Data Release. The model comparison statistic, the Bayesian information criterion is also applied to assess the worth of the models. This statistic favors models that give a good fit with fewer parameters. Based on this analysis, we find that the simplest cosmological constant model that has only one free parameter is still preferred by the current data. For the other dynamical dark energy models, we find that some of them, such as the Ricci dark energy model, the Affine equation-of-state dark energy, and the generalized Chaplygin gas, can provide good fits to the current data. The Dvali-Gabadadze-Porrati model is the only one-parameter model that can give a rather good fit but also nest Λ while the three-parameter model, namely, the interactive dark energy, is clearly disfavored by the data, as it is unable to provide a good fit.

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I. INTRODUCTION

Dark energy has become one of the most important issues of modern cosmology ever since the observations of type Ia supernovae (SNe Ia) first indicated that the Universe is undergoing an accelerated expansion at the present stage [1,2]. It is known that the ordinary matter and fields of the standard model are not sufficient to accommodate the present phase of acceleration. Moreover, recent observational data when interpreting the Big Bang model have provided some interesting information about another composition of the Universe called dark energy. It has been predicted that this quaint composition must have negative pressure and is uniformly distributed (i.e., unclustered), and thus may be responsible for the present acceleration of our Universe. In the past decades various models concerning dark energy have been put forward, the most simple candidate for which is considered to be in the form of vacuum energy density or cosmological constant (Λ). However, the cosmological constant is always entangled with two fatal problems: (i) fine tuning problem (present amount of the dark energy is so small compared with fundamental scale) and (ii) coincidence problem (dark energy density is comparable to critical density today). Alternatively there exist other choices such as (i) an X-matter component, which is characterized by an equation of state $p = w\rho$, where $-1 \leq w < 0$ [3,4], (ii) an exotic fluid, Chaplygin gas [5], (iii) a result of gravity leaking out into the bulk at large scales [6], etc.

In the face of so many competing dark energy candidates, it is important to find other effective methods to decide which one is right, or at least, which one is most favored by the observational data, although the accumulation of the current observational data including the Constitution SNIa data [7,8] and other cosmological probes such as the distance information measured by the Wilkinson Microwave Anisotropy Probe (WMAP) [9] and Baryon Acoustic Oscillations (BAO) [10] have opened a robust window for constraining the parameter space of dark energy models. In the meantime, strong lensing has become an important astrophysical tool for probing both cosmology [11–20] and galaxies (their structures, formations, and evolutions [21–31]). At the time of this writing we have collected 29 galactic-scale strong lenses with measured source redshift, lens redshift and image separation, which form a well-defined sample useful for statistical analysis [32]. They are from the Cosmic Lens ALL-Sky Survey (CLASS; [33,34]), the Sloan Digital Sky Survey (SDSS), the PMN-NVSS Extragalactic Lens Survey (PANELS; [35]), and other large systematic surveys of gravitationally-lensed quasars. We note that these well-defined samples are particularly useful not only for constraining the statistical properties of galaxies such as optical region velocity dispersions (e.g., [27,29]) and galaxy evolutions (e.g., [25,26]), but also for constraining cosmological parameters such as the present-day matter density Ω_m, dark energy density Ω_x and its equation of state w [15,16,18,32,36].

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In this paper, we pay attention to assessing several popular dark energy models with this observational lens redshift data. The goal of this work is to use the lens redshift data concerning strong lensing statistics with the SDSS DR5 velocity dispersion function (VDF) of early-type galaxies and the newly measured late-type galaxies VDF, together with the BAO, CMB data to constrain seven popular cosmological models assuming a flat cosmology. Moreover, in order to make a comparison for various dark energy models with different numbers of parameters and decide on the model preferred by the current data, following Davis *et al.* [37], we apply a model comparison statistic, i.e., the Bayesian information criterion (BIC) [38] in our analysis.

This paper is organized as follows. In Sec. II, we briefly introduce the statistical method applied in the paper, discuss the information criterion in the context of dark energy model selection, and give details of the lens redshift data. Section III describes a certain number of popular dark energy models, illustrates the constraint results, and assesses which one is preferred by the observational data. Finally the results are discussed in Sec. IV.

II. DATA AND METHOD

The number density of galaxies as a function of luminosity is described by the Schechter luminosity function (LF) $\phi_{\rm L}$ given by

$$dn = \phi_{\rm L}(L)dL = \phi_*\left(\frac{L}{L_*}\right)^{\alpha_{\rm L}}\exp\left(-\frac{L}{L_*}\right)\frac{dL}{L_*}. \qquad (1)$$

We note that the velocity dispersion σ may also relate to various observable qualities such as the Einstein radius (θ_E) and the luminosity (L). For instance, Lee and Ng (2007) [20] used the Einstein radius (θ_E), which is proportional to both D_{ds}/D_s and velocity dispersion squared, σ^2, to investigate the property of dark energy, and concluded that a single strong gravitational lensing by use of the Einstein radius may not be a proper method to constrain cosmological parameters. In our analysis, we assume an effective power-law relation between the luminosity (L) and the velocity dispersion (σ)

$$\frac{L}{L_*} = \left(\frac{\sigma}{\sigma_*}\right)^{\beta}, \qquad (2)$$

then the number density of galaxies as a function of velocity dispersion can be described by the VDF $\phi_{\rm VD}$ given by

$$dn = \phi_*\left(\frac{\sigma}{\sigma_*}\right)^{\alpha}\exp\left[-\left(\frac{\sigma}{\sigma_*}\right)^{\beta}\right]\frac{\beta}{\Gamma(\alpha/\beta)}\frac{d\sigma}{\sigma}, \qquad (3)$$

where $\alpha = (\alpha_{\rm L}+1)\beta$.

As for the parameters in the VDF for the early-type and late-type galaxies, we use the latest much larger SDSS DR5 [39] and the result from Ref. [36] using the Tully-Fisher relation and SIE galaxy model:

$$(\sigma_*, \alpha, \beta)_{\rm DR5} = [161\ {\rm km\,s^{-1}}, 2.32, 2.67];$$
$$(\sigma_*, \alpha, \beta)_{\rm late} = [133\ {\rm km\,s^{-1}}, 0.30, 2.91], \qquad (4)$$

with the evolutions of the characteristic velocity dispersion σ_*: $\sigma_*(z) = \sigma_{*,0}(1+z)^{\nu_v}$, where the best-fit parameters are [40] $\nu_v = -0.01$ for early-type galaxies and $\nu_v = -0.186$ for late-type.

The particular differential probability that a source at redshift z_s be multiply-imaged with image separation $\Delta\theta$ by a distribution of galaxies at redshift z_l following Eq. (3) can be written as

$$\delta p_{\rm IS} \equiv \frac{d^2p}{dzd(\Delta\theta)}\Big/\frac{dp}{dz} = \frac{1}{2}\frac{\beta}{\Gamma[(\alpha+4)/\beta]}\frac{1}{\Delta\theta_*}\left(\frac{\Delta\theta}{\Delta\theta_*}\right)^{\alpha/2+1}\exp\left[-\left(\frac{\Delta\theta}{\Delta\theta_*}\right)^{\beta/2}\right], \qquad (5)$$

where $\Delta\theta_*$ is the characteristic image separation given by

$$\Delta\theta_* = \lambda 8\pi\frac{D(z,z_s)}{D(0,z_s)}\left(\frac{\sigma_*}{c}\right)^2, \qquad (6)$$

where $\lambda \approx 1$ is a dynamical normalization factor [16,27] and $D(z_1, z_2)$ is the angular-diameter distance between redshifts z_1 and z_2.

To fit the DE model parameters we use a maximum likelihood method based on those recently used by Refs. [15,16,27,36]. The likelihood for lensing is defined by

$$\ln\mathcal{L} = \sum_{j=1}^{N_{\rm IS}} w_j \ln\delta p_{\rm IS}(j), \qquad (7)$$

which is due to the relative image separation probabilities of lensed sources as defined by Eq. (5). Parameters w_j are the weight factors for different types of galaxies defined by Ref. [32]. Using Eq. (7), we define a "χ^2" as follows:

$$\chi^2_{lens} = -2\ln\mathcal{L}. \qquad (8)$$

In this paper, we use the lens redshift data of a well-defined statistical sample including 29 lenses from the CLASS, the PANELS, the SDSS, the JVAS and the Snapshot survey. The sample is summarized in Table I of Ref. [32], where

TABLE I. Summary of models.

Model	Abbreviation	Parameters	k
Cosmological constant	Λ	Ω_Λ	1
Constant w	w	Ω_x, w	2
Interacting dark energy	IDE	Ω_x, w, ξ	3
Dvali-Gabadadze-Porrati	DGP	Ω_m	1
Generalized Chaplygin gas	GCG	A_s, γ	2
Affine equation-of-state	AEOS	Ω_x, α	2
Ricci dark energy	RDE	Ω_m, β	2

one can find the source and lens redshifts, the largest image separations and galaxy types. Here we just list the names of lensing systems considered. From the CLASS survey: B0414 + 054, B0712 + 472†, B1933 + 503†, B1938 + 666, 0712 + 472, B1359 + 154, B2045 + 265, B1608 + 656, B0128 + 437, B1152 + 199†, B0218 + 357, B1600 + 434; from the PANELS survey: J1632 − 0033, J1838 − 3427, J0134.0931; from the SDSS survey: J0246 − 0825, SBS0909 + 523, J0924 + 0219, J1226 − 0006, J1335 + 0118, Q0957 + 561, J1332 + 0347, J1524 + 4409, J1620 + 1203; from the JVAS survey: B1030 + 074, B1422 + 231† and from the Snapshot survey: Q0142 − 100, PG1115 + 080, Q1208 + 1011.

However, given the fact that the lens redshift data only give a weak constraint on the model parameters [32], there are some other observational data relevant to this work, such as the observations of CMB anisotropy [9] and large-scale structure (LSS) [41,42]. We use the shift parameter R from the CMB, and the distance parameter A of the measurement of the baryon acoustic oscillation (BAO) peak in the distribution of SDSS luminous red galaxies, which are argued as model-independent [43]. As is well known, the shift parameter R of the CMB is defined as [43,44]

$$R \equiv \Omega_m^{1/2} \int_0^{z_*} \frac{d\tilde{z}}{E(\tilde{z})}, \tag{9}$$

where Ω_m is the present fractional density of pressureless matter and $z_* = 1091.3$ is the redshift of recombination, which has been updated in the WMAP7 data [9]. The shift parameter R relates the angular-diameter distance to the last scattering surface, the comoving size of the sound horizon at z_*, and the angular scale of the first acoustic peak in the CMB power spectrum of temperature fluctuations [43,44]. The value of R has been updated to 1.725 ± 0.018 from the WMAP7 data [9]. On the other hand, the distance parameter A of the measurement of the BAO peak in the distribution of SDSS luminous red galaxies [42] is given by

$$A \equiv \Omega_m^{1/2} E(z_b)^{-1/3} \left[\frac{1}{z_b} \int_0^{z_b} \frac{d\tilde{z}}{E(\tilde{z})} \right]^{2/3}, \tag{10}$$

where $z_b = 0.35$. Eisenstein *et al.* has determined the value of A as $0.469(n_s/0.98)^{-0.35} \pm 0.017$ [41]. Here the scalar spectral index n_s is taken to be 0.963, which has been updated from the WMAP7 data [9]. So, the total χ^2 is given by

$$\chi^2 = \chi^2_{\rm lens} + \chi^2_{\rm BAO} + \chi^2_{\rm CMB}, \tag{11}$$

where $\chi^2_{\rm CMB} = (R - R_{\rm obs})^2/\sigma_R^2$ and $\chi^2_{\rm BAO} = (A - A_{\rm obs})^2/\sigma_A^2$. The best-fit model parameters are determined by minimizing the total χ^2. The 68.3% confidence level is determined by $\Delta\chi^2 \equiv \chi^2 - \chi^2_{\rm min} \le 1.0$, 2.3 and 3.53 for $k = 1$, 2 and 3, respectively, where k is the number of free model parameters. On the other hand, the 95.4% confidence level is determined by $\Delta\chi^2 \equiv \chi^2 - \chi^2_{\rm min} \le 4.0$, 6.17 and 8.02 for $k = 1$, 2 and 3, respectively.

However, the χ^2 statistic alone is not sufficient enough to provide an effective way to make a comparison between different models since this method is based on the assumption that the underlying model is the correct one. Since in general a model with more parameters tends to give a lower $\chi^2_{\rm min}$, it is unwise to compare different models by simply considering $\chi^2_{\rm min}$ with likelihood contours or best-fit parameters. Instead, one may employ the information criteria (IC) to assess different models. In this paper, we use the BIC, also known as the Schwarz information criterion [38] as a model selection criterion. The BIC is given by

$$\mathrm{BIC} = -2\ln\mathcal{L}_{\rm max} + k\ln N, \tag{12}$$

where $\mathcal{L}_{\rm max}$ is the maximum likelihood, k is the number of parameters, and N is the number of data points used in the fit. Note that for Gaussian errors, $\chi^2_{\rm min} = -2\ln\mathcal{L}_{\rm max}$, and the difference in BIC can be simplified to $\Delta\mathrm{BIC} = \Delta\chi^2_{\rm min} + \Delta k \ln N$. A difference in BIC ($\Delta$BIC) of 2 is considered positive evidence against the model with the higher BIC, while a ΔBIC of 6 is considered strong evidence.

III. DE MODELS AND CONSTRAINT RESULTS

All of the models considered in this section are currently possible candidates to explain the cosmic acceleration, i.e. they have not been completely falsified by available tests of the background cosmology. Considering the current cosmological observations, there is no strong reason to go beyond the simple, standard cosmological model with zero curvature and a cosmological constant Λ; however, when one attempts to reconcile its observed value with some estimates derived from fundamental arguments [45], a vast difference is arising. Therefore, it is still interesting to explore as many cosmological models as possible. Unless stated otherwise, throughout our paper we calculate the best-fit values found in that work, and vary the parameters within their 2σ uncertainties for each class of model, on an assumption of a flat universe.

In what follows, we choose several popular dark energy models and examine whether they are consistent with the lens redshift data available to us. We divide these models into seven classes:

(1) Cosmological constant model;
(2) Dark energy with constant equation-of-state;
(3) Interacting dark energy;
(4) Dvali-Gabadadze-Porrati model;
(5) Chaplygin gas models;
(6) Affine equation of state;
(7) Ricci dark energy model.

The models discussed and the parameters that describe each model are summarized in Table I. The fit and information criterion results are summarized in Table II.

TABLE II. Summary of the information criterion. The ΔBIC values for all other models in the table are measured with respect to the cosmological constant model.

Model	Abbreviation	ΔBIC
Cosmological constant	Λ	0
Constant w	w	2.34
Interacting dark energy	IDE	5.21
Dvali-Gabadadze-Porrati	DGP	0.40
Generalized Chaplygin gas	GCG	1.88
Affine equation of state	AEOS	1.86
Ricci dark energy	RDE	0.86

Next, we shall outline the basic equations describing the evolution of the cosmic expansion in each of the dark energy models, calculate the best-fit values of their parameters, and find their corresponding ΔBIC values.

A. The standard cosmological model

The current standard cosmological model, also known as the simplest scenario is the case where the dark energy is simply a cosmological constant, Λ, i.e., a component with constant equation of state $w = p/\rho = -1$. If flatness of the FRW metric is assumed, the Hubble parameter evolves according to the Friedmann equation, which, for this model, is

$$E^2(z) = \frac{\Omega_m}{a^3} + \Omega_\Lambda, \tag{13}$$

where Ω_m and Ω_Λ quantify the density of matter and cosmological constant, respectively. When flatness is assumed, $\Omega = \Omega_m + \Omega_\Lambda = 1$, this model has only one free parameter Ω_Λ.

The best-fit value of the parameter is

$$\Omega_\Lambda = 0.75^{+0.02}_{-0.02}. \tag{14}$$

We plot the 68.3% and 95.4% confidence limits for this model in the $\Omega_\Lambda - \Delta\chi^2$ plane in Fig. 1. This model has the lowest value of BIC in all the tested models, so ΔBIC is measured with respect to this model (see Table II). Meanwhile, for comparison, we also show the constraining result with BAO+CMB to demonstrate the non-negligible effect of the lens redshift data on the parameter constraints.

Note that a similar analysis for this concordance model was previously done in Ref. [36], which improved the constraints on cosmological parameters with the CLASS statistical sample and the image separation distribution of the CLASS and the PANELS radio-selected lenses, and found that the cosmological matter density $\Omega_m = 0.25^{+0.12}_{-0.08}$ (68.3% CL) assuming evolutions of galaxies predicted by a semianalytical model of galaxy formation and $\Omega_m = 0.26^{+0.12}_{-0.08}$ assuming no evolution of galaxies for the ΛCDM model. Moreover, the current best-fit value from cosmological observations is $\Omega_\Lambda = 0.73 \pm 0.04$ in the flat case [37]. Komatsu *et al.* (2009) [46] also gave the best-fit parameter: $\Omega_m = 0.274$ for the flat ΛCDM model from WMAP 5-year data combined with BAO, CMB and the SNe Ia Union data. We find that our result from the lens redshift data is consistent with the previous works.

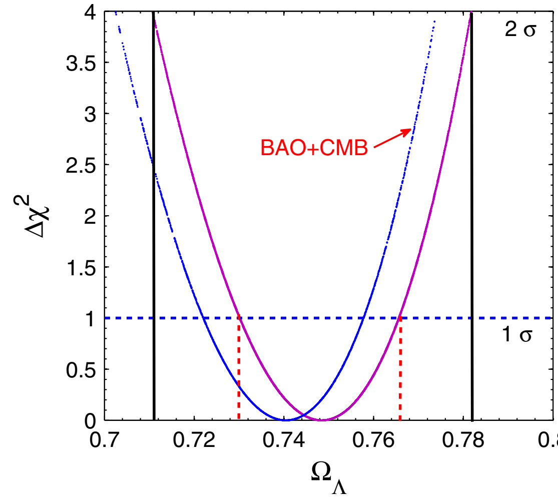

FIG. 1 (color online). The ΛCDM model constraint by the joint analysis (purple dots) as well as BAO + CMB (blue dots): 68.3% and and 95.4% confidence limits.

B. Dark energy with constant equation of state

In this case, the equation-of-state parameter of dark energy is assumed to be a constant. In a zero-curvature universe, the Hubble parameter for this generic dark energy component with density Ω_x then becomes

$$E^2(z) = \frac{\Omega_m}{a^3} + \frac{\Omega_x}{a^{3(1+w)}}. \tag{15}$$

When flatness is assumed, $\Omega = \Omega_m + \Omega_\Lambda = 1$. It is apparent that this is a two-parameter model with model parameters: $\theta = \{\Omega_x, w\}$.

We plot the likelihood contours for the this model in the $\Omega_x - w$ plane in Fig. 2 with the best fit (see Ref. [32]):

$$\Omega_x = 0.71^{+0.07}_{-0.07}, \qquad w = -0.78^{+0.22}_{-0.34}. \tag{16}$$

For comparison, we also present the constraining results only with BAO and CMB. Obviously, the combined data contours including the lens redshift data are shifted compared to those with BAO + CMB, which also demonstrates the non-negligible effect of the strong lensing data on model constraints. We also find that this constraint result is much more stringent than that of Ref. [36], which obtained $w < -1.2$ at the 68.3% C.L. Meanwhile, compared to the cosmological constant model, this flat cosmology with constant EoS dark energy gives a lower $\chi^2_{\rm min}$, but due to the one extra parameter it has, it is punished by the information criterion: ΔBIC = 2.34.

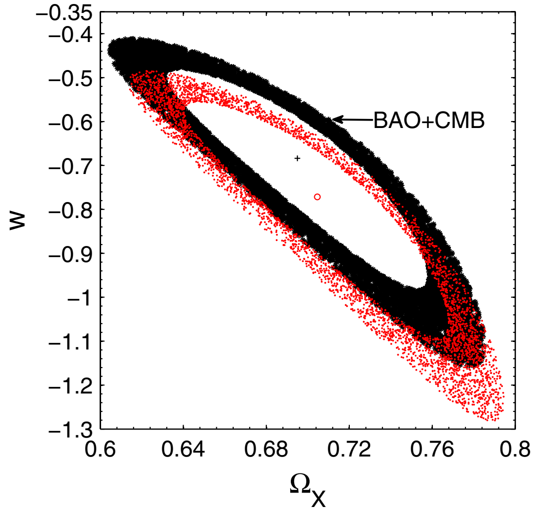

FIG. 2 (color online). The w model constraint by the joint analysis (red dots) as well as BAO + CMB (black dots): likelihood contours at 68.3% and 95.4% confidence levels in the $\Omega_x - w$. The unfilled circle and the cross here and in other figures correspond to the best-fit parameters for the two data sets, respectively.

C. Interacting dark energy

In the interacting dark energy model, the dark energy interacts with other components including baryonic and dark matter through an energy exchange term. The conservation equations for matter and dark energy can be written in a very general way as

$$\dot{\rho_m} + 3H\rho_m = 3QH\rho_m, \tag{17}$$

$$\dot{\rho}_x + 3H\rho_x(1 + w) = -3QH\rho_m, \tag{18}$$

which preserves the total energy conservation equation $\dot{\rho}_{\rm tot} + 3H(\rho_{\rm tot} + p_{\rm tot}) = 0$. We assume that the EoS of dark energy $w \equiv p/\rho$ is a constant in spatially flat FRW. If Q is a nonzero function of the scale factor, the interaction makes ρ_m and ρ_x to deviate from the standard scaling. One Q with only one parameter comes from the assumption [47,48]:

$$\frac{\rho_X}{\rho_m} = \frac{\rho_{X0}}{\rho_{m0}} a^{\xi}, \tag{19}$$

where ξ is a constant parameter, which quantifies the severity of the coincidence problem. The Hubble parameter or this model then reads

$$E^2(z) = a^{-3}(1 - \Omega_x(1 - a^{\xi}))^{-3(w/\xi)}, \tag{20}$$

which reduces to the uncoupled case for $\xi = -3w$. Evidently, there are three independent model parameters in this model: $\theta = \{\Omega_x, w, \xi\}$. According to the current data analysis, we find the best-fit parameters:

$$\Omega_x = 0.76, \qquad w = -1.30, \qquad \xi = 4.77. \tag{21}$$

We plot the likelihood contours for the IDE model in the $\Omega_x - \xi$ plane in Fig. 3. Note that the best-fit parameters of the ΛCDM model ($\Omega_x = 0.73$, $w = -1$ and $\xi = 3$) still lie in the 1σ regions of the IDE model, indicating that the ΛCDM model is fairly in concordance with the current observational data. However, compared with Ω_x, the EoS parameter w and the interaction term ξ are weakly constrained with $\xi = 4.77^{+5.23}_{-4.75}$ at 68.3% confidence level. Meanwhile, since the IDE model has three free model parameters, it should have made considerable improvement in the fit, however, it gives a slightly lower $\chi^2_{\rm min}$ contrasting to the constant w model (only smaller by 1.31). The difference in the information criterion with respect to the ΛCDM model is ΔBIC $= 5.21$. Such a poor information criterion result implies that the IDE model is too complex to be necessary in explaining the strong lensing data, compared with the simpler models

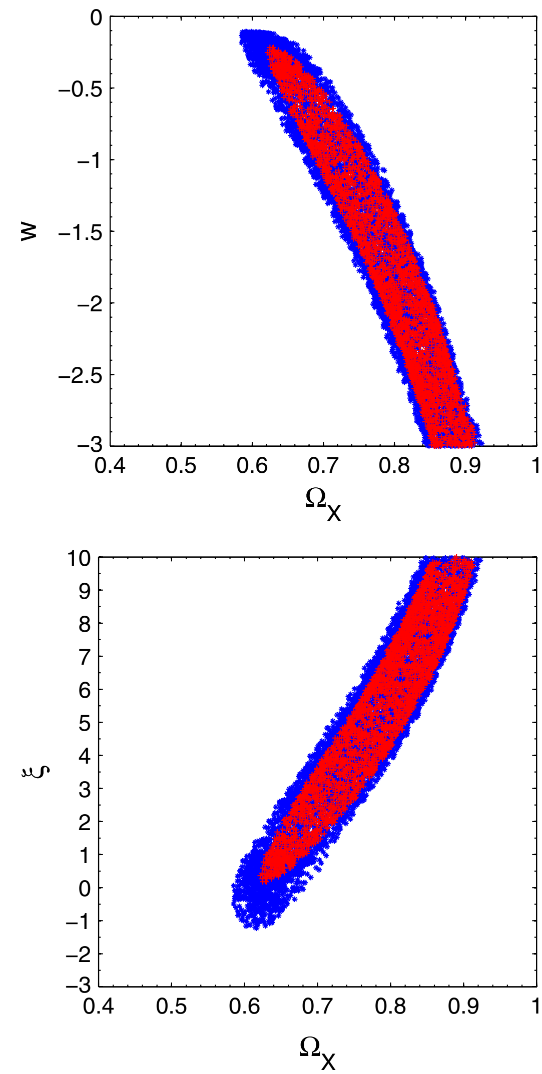

FIG. 3 (color online). The IDE model constraint by the joint analysis: likelihood contours at 68.3% and 95.4% confidence levels in the $\Omega_x - w$ and $\Omega_x - \xi$ planes.

such as the Λ model and the constant w model. Moreover, in respect to the relatively large range of w and ξ in the 1σ region, both of the cosmological parameters are not really well constrained by the lens redshift data compared with the SNe Ia Union data [49].

D. Dvali-Gabadadze-Porrati model

The DGP model arises from the brane world theory in which gravity leaks out into the bulk at large scales, giving birth to the possibility of an accelerated expansion of the Universe [6]. In this model, the Friedmann equation is modified as

$$3M_{\rm Pl}^2\left(H^2 - \frac{H}{r_c}\right) = \rho_m(1+z)^3, \tag{22}$$

where $r_c = (H_0(1-\Omega_m))^{-1}$ is the crossover scale. In this model, $E(z)$ is given by

$$E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_{r_c}} + \sqrt{\Omega_{r_c}}, \tag{23}$$

where $\Omega_{rc} = 1/(4r_c^2H_0^2)$ is a constant. The flat DGP model only contains one free model parameter, $\theta = \{\Omega_m\}$.

For the DGP model, the best-fit parameter is

$$\Omega_m = 0.32^{+0.02}_{-0.02}. \tag{24}$$

The 68.3% and 95.4% confidence limits for this model in the $\Omega_\Lambda - \Delta\chi^2$ plane are shown in Fig. 4. We find that the DGP model, as a single-parameter model, is only a bit worse than the ΛCDM model under the observational test. Its $\chi^2_{\rm min}$ is smaller than that of the ΛCDM model by about 0.4, and it yields ΔBIC $= 0.40$. So, the fitting result shows that the DGP model fares the best, except for the ΛCDM model, under the current observational tests (see also Ref. [37]), which is in good agreement with the results from gravitational lensing statistics using the Cosmic Lens All-Sky Survey (CLASS) lensing sample, $\Omega_m = 0.30^{+0.19}_{-0.11}$ [50].

E. Generalized Chaplygin gas model

Bento *et al.* (2008) [5] proposed an interesting model of dark energy, named the generalized Chaplygin gas (GCG) model, which has an exotic equation of state:

$$p = -\frac{A}{\rho^\gamma}, \tag{25}$$

where A is a positive constant. Combining the equation of state and the energy conservation equation for the GCG, we gain the energy density of the GCG:

$$\rho(a) = \rho(0)\left(A_s + \frac{1-A_s}{a^{3(1+\gamma)}}\right)^{1/1+\gamma}, \tag{26}$$

where $A_s \equiv A/\rho^{1+\gamma}(0)$. Apparently, for $\alpha = 0$, the GCG model behaves like the cold dark matter plus a cosmological constant. When $A_s = 0$ the GCG behaves always like matter and when $A_s = 1$ the GCG behaves like a cosmological constant. So, the GCG model is considered to be able to unify dark energy and dark matter. In a flat universe, we have

$$E^2(z) = \Omega_b a^{-3} + (1-\Omega_b)(A_s + (1-A_s)a^{-3(1+\gamma)})^{1/1+\gamma}, \tag{27}$$

where Ω_b is the present dimensionless density parameter of the baryonic matter. We consider $\Omega_b h^2 = 0.0233 \pm 0.0008$ and the dimensionless Hubble constant $h = 0.72$ given by the WMAP5 observations. This model has two independent model parameters: $\theta = \{A_s, \alpha\}$. The cosmological constant model is recovered for $\alpha = 0$ and $\Omega_m = 1 - A_s(1-\Omega_b)$.

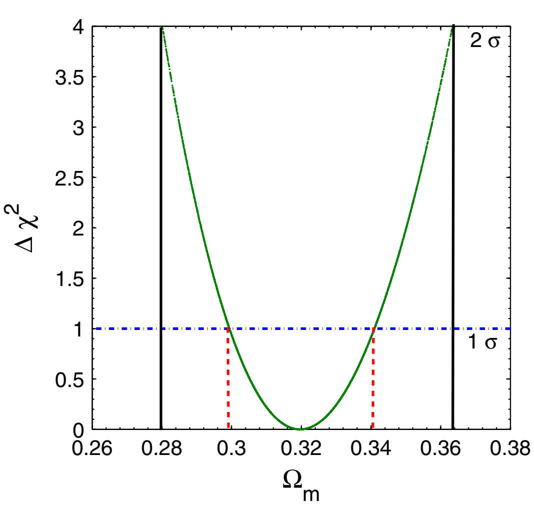

FIG. 4 (color online). The DGP model constraint by the joint analysis: 68.3% and 95.4% confidence limits.

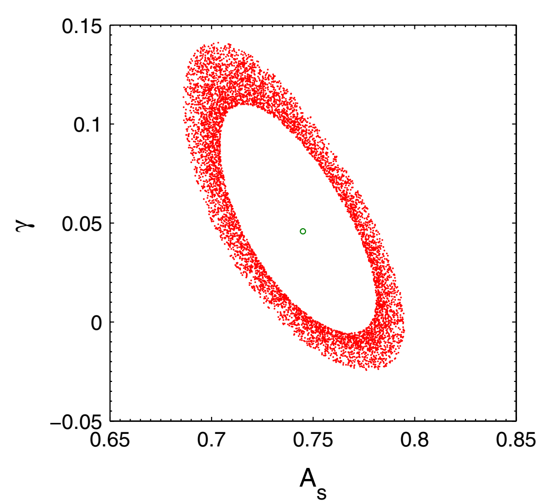

FIG. 5 (color online). The GCG model constraint by the joint analysis: likelihood contours at 68.3% and 95.4% confidence levels in the $A_s - \gamma$ plane.

The best-fit parameters are

$$A_s = 0.75^{+0.04}_{-0.04}, \qquad \gamma = 0.046^{+0.064}_{-0.052}. \tag{28}$$

The best-fit value of γ is approaching zero, implying that the ΛCDM limit of this model is still favored. We plot the likelihood contours for the GCG model in the $A_s - \gamma$ in Fig. 5. However, we can see that compared with the cosmological constant model, this model gives a lower $\chi^2_{\rm min}$, with the information criterion: ΔBIC $= 1.88$, which means the GCG model, as a two-parameter model, performs very well under the information criterion test.

F. Affine equation of state

Considering the equation of state for the dark energy, it is possibly modeled by a generic expression $p = p(\rho)$, as is extensively discussed by Refs. [51–53]. In particular, when the Taylor expansion of an arbitrary equation of state is truncated to the first order, e.g., $p = p_0 + \alpha\rho$ [54], it is interesting to find that such an affine equation of state can be used to describe a simple unified dark matter model. In this model, the time evolution of the background density reads $\rho(a) = \rho_\Lambda + (\rho_o - \rho_\Lambda)a^{-3(1+\alpha)}$, where $\rho_\Lambda \equiv -p_0/(1+\alpha)$ and ρ_0 is the dark energy fluid energy density at present. The Hubble parameter is given by

$$E^2(z) = \frac{\tilde{\Omega}_m}{a^{3(1+\alpha)}} + \Omega_\Lambda, \tag{29}$$

where $\tilde{\Omega}_m \equiv (\rho_o - \rho_x)/\rho_c$. With $\alpha = 0$ this model recovers the standard ΛCDM case. In a flat universe, $\tilde{\Omega}_m + \Omega_x = 1$, and the model has two independent parameters: $\theta = \{\Omega_\Lambda, \alpha\}$.

The best-fit parameters are

$$\Omega_\Lambda = 0.73^{+0.04}_{-0.04}, \qquad \alpha = 0.007^{+0.008}_{-0.008}. \tag{30}$$

We plot the likelihood contours for this model in the $\Omega_x - \alpha$ plane in Fig. 6. The currently preferred values of α in this model still include the cosmological constant case: $\alpha = 0$. Compared with the cosmological constant model, this model gives a lower $\chi^2_{\rm min}$ with the information criterion: ΔBIC $= 1.86$, which means the AEOS model, as a two-parameter model, is in concordance with the ΛCDM model in the 1σ region under this observational test. Meanwhile, considering the small value of α, the fitting result shows that the AEOS model is generally consistent with the cosmological constant case.

G. Ricci dark energy model

There exists a possibility that the average radius of the Ricci scalar curvature $|\mathcal{R}|^{-1/2}$ may provide us with an IR cutoff length scale. In a flat universe, the Ricci scalar is $\mathcal{R} = -6(\dot{H} + 2H^2)$ [55–57], and therefore the energy density of RDE reads

$$\rho_{de} = 3\beta M_{Pl}^2(\dot{H} + 2H^2), \tag{31}$$

where α is a positive constant. The Hubble parameter could be derived rom the Friedmann equation:

$$E^2(z) = \frac{2\Omega_m}{2-\beta}(1+z)^3 + \left(1 - \frac{2\Omega_m}{2-\beta}\right)(1+z)^{(4-(2/\beta))}. \tag{32}$$

This is a two-parameter model, and its free model parameters are $\theta = \{\Omega_m, \beta\}$.

For the RDE model, the best-fit parameters and the corresponding $\chi^2_{\rm min}$ are

$$\Omega_m = 0.14^{+0.08}_{-0.07}, \; \beta = 0.23^{+0.06}_{-0.06}. \tag{33}$$

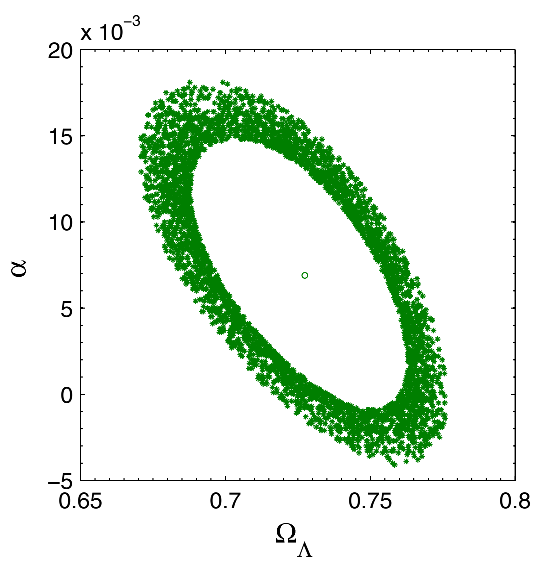

FIG. 6 (color online). The AEOS model constraint by the joint analysis: likelihood contours at 68.3% and 95.4% confidence levels in the $\Omega_\Lambda - \alpha$ plane.

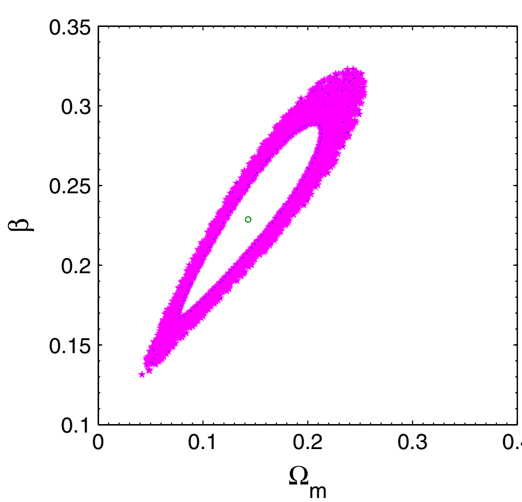

FIG. 7 (color online). The Ricci model constraint by the joint analysis: likelihood contours at 68.3% and 95.4% confidence levels in the $\Omega_m - \beta$ plane.

We plot the likelihood contours for the RDE model in the $\Omega_m - \beta$ plane in Fig. 7. Out of all the two-parameter cosmological models we consider, the RDE model performs the best, which gives the smallest $\chi^2_{\min}$ and smallest information criterion result: $\Delta\mathrm{BIC} = 0.86$.

IV. DISCUSSION

In this work, we use the distribution of gravitationally-lensed image separations observed in the Cosmic Lens All-Sky Survey (CLASS), the PMN-NVSS Extragalactic Lens Survey (PANELS), the Sloan Digital Sky Survey (SDSS), and other large systematic surveys of gravitationally-lensed quasars, combined with the BAO and CMB data, to constrain several popular dark energy models with the new measurements of the velocity dispersion function of early-type and late-type galaxies based on the SDSS DR5 data. We have explored seven popular dark energy models in the context of a flat universe assumption. To assess various competing dark energy models and make a comparison, the information criterion, the BIC, is also applied in this analysis.

For each model, we have calculated the best-fit values of its parameters and found its ΔBIC value. Table I summarizes all the models in consideration and the parameters describing each model. We have plotted the likelihood contours of parameters for all the models. The fit and information criteria results have been summarized in Table II. Note that the cosmological constant model has the lowest value of BIC and the value of ΔBIC is measured with respect to this model.

From Table I, we see that according to the number of parameters, these models can be divided into three classes: the one-parameter models, including the Λ and DGP models; the two-parameter models, including the constant w, GCG, AEOS and RDE models; and the three-parameter model, namely, the IDE model. If only comparing $\chi^2_{\min}$, we find that the Λ model is not the best one, and there are five models, namely, the constant w, GCG, AEOS, IDE, and RDE models, a little bit better than the Λ model according to this criterion. However, if the economical efficiency is considered, we also will find that the Λ model is the best one to explain the current lens redshift data together with BAO and CMB, since the BIC value it yields is the smallest. For the two-parameter models, the RDE model provides the smallest $\chi^2_{\min}$ and smallest information criterion result among all of the two-parameter cosmological models. Meanwhile, the AEOS model performs best in explaining the current data. As for the three-parameter model, although the IDE model can fit the current data well with lower $\chi^2_{\min}$, it is too complex (it has three free model parameters) to be necessary, yet.

We also provide a graphical representation of the IC results in Fig. 8 which directly shows the results in the BIC test for each model. Out of all the candidate models, it is obvious that the simplest Λ model remains the best one.

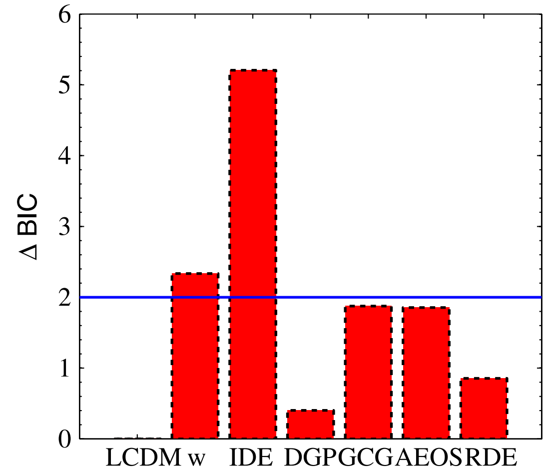

FIG. 8 (color online). Graphical representation of the results in Table II: the values of ΔBIC for each model. The order of models from left to right is the same as that in Table II.

Following it are the RDE model and two models that give comparably good fits but have one more free parameter, the AEOS and GCG models, which can also reduce to the Λ model and their best-fit parameters indeed do so (within 1σ range). The DGP model is the only one-parameter model that can give a rather good fit but also nest Λ. The three-parameter model, IDE, is clearly disfavored as it is unable to provide a good fit to the data.

In conclusion, given the current observational lens redshift data combined with the BAO and CMB data, under the assumption of a flat universe, the information criterion indicates that the cosmological constant model is still the best one and there is no reason to prefer more complex models. This conclusion is in accordance with the previous results [37]. Finally, as mentioned by Ref. [32], we pin our hope on the future observational lens redshift data of high accuracy and more precise CMB data from the ESA Planck satellite [58], as well as other complementary data such as Gamma Ray Bursts data [59–61], the data of X-ray gas mass fraction in cluster [62–64] and gravitational lensing data (see Ref. [65] and corresponding references therein).

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