PHYSICAL REVIEW D **84,** 016007 (2011)

Triangle relation of dark matter, electric dipole moment, and *CP* violation in B^0 mixing in a supersymmetric Q_6 model

Yoshiyuki Kaburaki,[*] Kazuhiro Konya,[†] and Jisuke Kubo[‡]
Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan

Alexander Lenz[§]
Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany
(Received 13 April 2011; published 25 July 2011)

We consider a recently proposed supersymmetric model based on the discrete Q_6 family group. Because of the family symmetry and spontaneous *CP* violation, the electric dipole moment, the *CP* violation in the mixing of the neutral mesons, and the dark matter mass $m_{\rm DM}$ are closely related. This triangle relation is controlled by the size of the μ parameters. Loop effects can give rise to large contributions to the soft mass insertions, and we find that the model allows a large *CP* violation in the B^0 system. Its size is comparable with the recent experimental observations by D0 and CDF, and it could be observed at the LHCb in the first years. If the parameter space is constrained by the neutron electric dipole moment, flavor changing neutral currents, and *CP* violations in K^0 as well as B^0 mixing, the triangle relation yields the following bound on the dark matter candidate: 0.12 TeV $< m_{\rm DM} <$ 0.33 TeV, which is directly observable at the LHC. We also compute $a_{sl}^s - a_{sl}^d$, which is observable at the LHCb, where $a_{sl}^{s(d)}$ is the semileptonic *CP* asymmetry for the $B_{s(d)}$ system.

DOI: 10.1103/PhysRevD.84.016007 PACS numbers: 11.30.Hv, 12.15.Ff, 13.20.He

I. INTRODUCTION

Family symmetry is a useful tool [1–7] to suppress flavor changing neutral currents (FCNCs) in supersymmetric (SUSY) extensions of the standard model (SM).[1] If it is combined with spontaneous violation of *CP* in SUSY models, *CP* violation in these models can be suppressed, too [4,5,7]. However, this theoretical idea may be in conflict with the recent measurement of the *CP* violating dimuon asymmetry A_{sl}^b by the D0 Collaboration [9]. Its measured value $A_{sl}^b = -(9.57 \pm 2.51 \pm 1.46) \times 10^{-3}$ disagrees by 3.2 standard deviations [9] with the SM prediction $A_{sl}^b = -(2.3^{+0.5}_{-0.6}) \times 10^{-4}$ [10], which has stimulated a number of papers [11,12] dealing with a large *CP* violation in B^0 mixing.[2] Moreover, the CKMfitter Group [14,15] also obtained, from a global fit to flavor observables, a large value for the dimuon asymmetry: $A_{sl}^b = -(4.2^{+1.9}_{-1.8}) \times 10^{-3}$ [15].[3] If the size of *CP* violation in a symmetry-based mechanism to suppress *CP* violation turns out to be of the same order of the SM value, we may be running into a dilemma between suppressed and large *CP* violation. In any case, the mechanism has to take care of small *CP* violation in K^0 mixing and, at the same time, allow large *CP* violation in B^0 mixing. See [15] for a large list of references in which diverse theoretical possibilities for large *CP* violation in B^0 mixing have been proposed.

Recently, two of us [12] considered a supersymmetric extension of the SM based on the discrete Q_6 family symmetry [4–7].[4] Because of the family symmetry, this model contains three pairs of $SU(2)_L$ doublet Higgs supermultiplets. We found that the one-loop effects of the extra Higgs multiplets on the soft mass insertions can generically give rise to large contributions to the soft mass insertions and that the model allows values for A_{sl}^b that touch the 1σ range of the fit result of [15]. In this paper we will continue with our investigation of this model. In this model the size of the μ parameters plays an important role: It enters directly into the above-mentioned one-loop corrections to the soft mass insertions and into the electric dipole moment (EDM) [7]. If the neutralino lightest supersymmetric particle (LSP) should be a dark matter candidate, then its mass also depends on the μ parameters. We are thus particularly interested in the triangle relationship between *CP* violation in B^0 mixing, the EDM, and the mass of the dark matter candidate.

II. THE MODEL

We start by considering the superpotential

$$W = Y_{ij}^{uI} Q_i U_j^c H_I^u + Y_{ij}^{dI} Q_i D_j^c H_I^d + \mu^{IJ} H_I^u H_J^d, \quad (1)$$

[*]kab@hep.s.kanazawa-u.ac.jp
[†]konya@hep.s.kanazawa-u.ac.jp
[‡]jik@hep.s.kanazawa-u.ac.jp
[§]alexander.lenz@physik.uni-regensburg.de

[1]For a recent review on family symmetry, see [8] for instance.
[2]For earlier works see e.g. [13].
[3]This value is for the new physics scenario I of [15]. The UTfit Group [16] and Lunghi and Soni [17] also reported large *CP* violating effects in B^0 mixing.
[4]Q_6 was considered in the past in [18].

TABLE I. The Q_6 assignment of the chiral matter supermultiplets. The irreducible representations of Q_6 fall into two doublets ($2, 2'$) and four singlets ($\mathbf{1}_{+,0}$, $\mathbf{1}_{+,2}$, $\mathbf{1}_{-,1}$, $\mathbf{1}_{-,3}$). The 2 is complex valued but pseudoreal, while the $2'$ is real valued. The singlets can be classified according to the subgroup $Z_2 \times Z_4$, and the subscripts denote its quantum number. The $\mathbf{1}_{+,0}$ and $\mathbf{1}_{+,2}$ are real, while the $\mathbf{1}_{-,1}$ and $\mathbf{1}_{-,3}$ are complex conjugates of each other. More details can be found in Ref. [4]. For completeness we include leptons, L, E^c, and N^c. R parity is also imposed.

	Q	Q_3	U^c, D^c	U_3^c, D_3^c	L	L_3	E^c, N^c	E_3^c	N_3^c	H^u, H^d	H_3^u, H_3^d
Q_6	$\mathbf{2}$	$\mathbf{1}_{+,2}$	$\mathbf{2}'$	$\mathbf{1}_{-,1}$	$\mathbf{2}'$	$\mathbf{1}_{+,0}$	$\mathbf{2}'$	$\mathbf{1}_{+,0}$	$\mathbf{1}_{-,3}$	$\mathbf{2}'$	$\mathbf{1}_{-,1}$

where we have restricted ourselves to the quark sector and the Higgs sector. Here Q, H^u, and H^d stand for $SU(2)_L$ doublets of the quark and Higgs supermultiplets, respectively. The indices I and J indicate different kinds of Higgs $SU(2)_L$ doublets. Similarly, U^c and D^c stand for $SU(2)_L$ singlets of the quark supermultiplets. The structure of the Y and μ terms in the Yukawa matrices are fixed by the Q_6 family symmetry.[5] The Q_6 assignment is shown in Table I, and the Q_6 invariance yields [4]

$$\mathbf{Y}^{u1(d1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & Y_b^{u(d)} \\ 0 & Y_{b'}^{u(d)} & 0 \end{pmatrix}, \quad \mathbf{Y}^{u2(d2)} = \begin{pmatrix} 0 & 0 & Y_b^{u(d)} \\ 0 & 0 & 0 \\ -Y_{b'}^{u(d)} & 0 & 0 \end{pmatrix}, \quad \mathbf{Y}^{u3(d3)} = \begin{pmatrix} 0 & Y_c^{u(d)} & 0 \\ Y_c^{u(d)} & 0 & 0 \\ 0 & 0 & Y_a^{u(d)} \end{pmatrix}. \tag{2}$$

The only Q_6 invariant μ term is $(H_1^u H_1^d + H_2^u H_2^d)$, and no $H_3^u H_3^d$ and no mixing between the Q_6 doublet and singlet Higgs multiplets is allowed. Therefore, there is an accidental global $SU(2)$, implying the existence of Nambu-Goldstone modes. In [19] the Higgs sector is extended to include a certain set of SM singlet Higgs multiplets to avoid this problem. With this extended Higgs sector one can break the flavor symmetry Q_6 and CP invariance spontaneously. Moreover, the scalar potential of the original theory turns out to have an accidental Z_2 invariance,

$$h_+^{u,d} = \frac{1}{\sqrt{2}}(h_1^{u,d} + h_2^{u,d}) \rightarrow h_+^{u,d}, \quad h_-^{u,d} = \frac{1}{\sqrt{2}}(h_1^{u,d} - h_2^{u,d}) \rightarrow -h_-^{u,d}, \tag{3}$$

where h's are scalar components of H's, and $H_\pm^{u,d} = (H_1^{u,d} \pm H_2^{u,d})/\sqrt{2}$. After the singlet sector has been integrated out, we obtain an effective μ term

$$W^{\rm eff} = \mu^{++}(H_+^u H_+^d + H_-^u H_-^d) + \mu^{+3} H_+^u H_3^d + \mu^{3+} H_3^u H_+^d, \tag{4}$$

and the soft-supersymmetry-breaking Lagrangian

$$\mathcal{L}_{\rm soft}^{\rm eff} = m_{H^u}^2(|h_+^u|^2 + |h_-^u|^2) + m_{H_3^u}^2|h_3^u|^2 + m_{H^d}^2(|h_+^d|^2 + |h_-^d|^2) + m_{H_3^d}^2|h_3^d|^2 + [B^{++}(h_+^u h_+^d + h_-^u h_-^d) + B^{+3} h_+^u h_3^d + B^{3+} h_3^u h_+^d + \text{H.c.}]. \tag{5}$$

(The A terms are suppressed.) The parameters μ and B are complex: They originate from the complex vacuum expectation values (VEVs) of the SM singlet Higgs fields of the original theory [19]. But because of the CP invariance of the original theory the Yukawa matrices and soft scalar masses are real. So, the effective superpotential (4) and the effective soft-supersymmetry-breaking Lagrangian (5) break Q_6 and CP softly. However, thanks to (3), the VEVs of the form

$$\langle h_-^{u,d0}\rangle = 0, \quad \langle h_+^{u,d0}\rangle = \frac{v_+^{u,d}}{\sqrt{2}} \exp i\theta_+^{u,d}, \quad \langle h_3^{u,d0}\rangle = \frac{v_3^{u,d}}{\sqrt{2}} \exp i\theta_3^{u,d} \tag{6}$$

can be realized, where the $SU(2)$ components of the Higgs fields are defined as

$$h_I^u = (h_I^{u+}, h_I^{u0}), \qquad h_I^d = (h_I^{d0}, h_I^{d-})(I = 3, +, -). \tag{7}$$

To proceed with our discussion we make a phase rotation of the Higgs superfields so that their VEVs become real: $\tilde{H}_\pm^{u,d} = H_\pm^{u,d} e^{-i\theta_+^{u,d}}$, $\tilde{H}_3^{u,d} = H_3^{u,d} e^{-i\theta_3^{u,d}}$. Then we define

$$\begin{pmatrix} \Phi_L^{u,d} \\ \Phi_H^{u,d} \\ \Phi_-^{u,d} \end{pmatrix} := \begin{pmatrix} \cos\gamma^{u,d} & \sin\gamma^{u,d} & 0 \\ -\sin\gamma^{u,d} & \cos\gamma^{u,d} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{H}_3^{u,d} \\ \tilde{H}_+^{u,d} \\ \tilde{H}_-^{u,d} \end{pmatrix}, \tag{8}$$

where

$$\cos\gamma^u = c_{\gamma^u} = v_3^u/v_u, \quad \sin\gamma^u = s_{\gamma^u} = v_+^u/v_u, \quad v_{u,d} = \sqrt{(v_3^u)^2 + (v_+^u)^2}, \tag{9}$$

and similarly for the down sector. We further define the components of the $SU(2)$ doublet Higgs superfields as

$$\Phi_I^u = \begin{pmatrix} \Phi_I^{u+} \\ \Phi_I^{u0} \end{pmatrix}, \qquad \Phi_I^d = \begin{pmatrix} \Phi_I^{d0} \\ \Phi_I^{d-} \end{pmatrix} (I = L, H, -). \tag{10}$$

[5] More details of the model can be found in [5,7].

The light and heavy MSSM-like Higgs scalars are then given by

$$\begin{aligned}(v+h-iX)/\sqrt{2} &= (\phi_L^{d0})^* \cos\beta + (\phi_L^{u0})\sin\beta,\\ (H+iA)/\sqrt{2} &= -(\phi_L^{d0})^* \sin\beta + (\phi_L^{u0})\cos\beta,\\ G^+ &= -(\phi_L^{d-})^* \cos\beta + (\phi_L^{u+})\sin\beta,\\ H^+ &= (\phi_L^{d-})^* \sin\beta + \phi_L^{u+}\cos\beta,\end{aligned} \tag{11}$$

where X and G^+ are the Nambu-Goldstone fields, ϕ's are scalar components of Φ's of (10), $v = \sqrt{v_u^2 + v_d^2}\times$ ($\simeq 246$ GeV) and $\tan\beta = v_u/v_d$. In terms of Φ's of (10) the superpotential (4) becomes

$$\begin{aligned}W'^{\text{eff}} = {}& \mu_L \Phi_L^u \Phi_L^d + \mu_{LH}\Phi_L^u \Phi_H^d + \mu_{HL}\Phi_H^u \Phi_L^d\\ & + \mu_H \Phi_H^u \Phi_H^d + \mu_- \Phi_-^u \Phi_-^d,\end{aligned} \tag{12}$$

which we shall use when calculating the dark matter mass, EDM, and CP violation later on. The μ parameters in W'^{eff} are related to the original ones according to

$$\begin{aligned}\mu_L &= c_{\gamma^u} s_{\gamma^d} \mu^{3+} e^{i(\theta_3^u+\theta_+^d)} + s_{\gamma^u} c_{\gamma^d} \mu^{+3} e^{i(\theta_+^u+\theta_3^d)}\\ &\quad + s_{\gamma^u} s_{\gamma^d} \mu^{++} e^{i(\theta_+^u+\theta_+^d)},\\ \mu_{LH} &= c_{\gamma^u} c_{\gamma^d} \mu^{3+} e^{i(\theta_3^u+\theta_+^d)} - s_{\gamma^u} s_{\gamma^d} \mu^{+3} e^{i(\theta_+^u+\theta_3^d)}\\ &\quad + s_{\gamma^u} c_{\gamma^d} \mu^{++} e^{i(\theta_+^u+\theta_+^d)},\\ \mu_{HL} &= -s_{\gamma^u} s_{\gamma^d} \mu^{3+} e^{i(\theta_3^u+\theta_+^d)} + c_{\gamma^u} c_{\gamma^d} \mu^{+3} e^{i(\theta_+^u+\theta_3^d)}\\ &\quad + c_{\gamma^u} s_{\gamma^d} \mu^{++} e^{i(\theta_+^u+\theta_+^d)},\\ \mu_H &= -s_{\gamma^u} c_{\gamma^d} \mu^{3+} e^{i(\theta_3^u+\theta_+^d)} - c_{\gamma^u} s_{\gamma^d} \mu^{+3} e^{i(\theta_+^u+\theta_3^d)}\\ &\quad + c_{\gamma^u} c_{\gamma^d} \mu^{++} e^{i(\theta_+^u+\theta_+^d)},\\ \mu_- &= \mu^{++} e^{i(\theta_+^u+\theta_+^d)}.\end{aligned} \tag{13}$$

[θ's, $c_{\gamma^{u,d}}$, and $s_{\gamma^{u,d}}$ are defined in (6) and (9), respectively.]

III. THE YUKAWA SECTOR IN THE QUARK MASS EIGENSTATES

The Yukawa sector in terms of the quark mass eigenstates is needed to compute EDMs mediated by the exchange of Higgs fields. As we will see, the set of theory parameters in this sector is overconstrained, so satisfying the EDM constraint is a nontrivial matter. We shall briefly discuss below how the Yukawa sector is constrained by the family symmetry and derive a Lagrangian which we will use to compute the EDMs mediated by the exchange of Higgs fields.

The quark mass matrices $\mathbf{m}^u$ and $\mathbf{m}^d$ can be read off from the superpotential (1) along with (2) and (6). Then, using the rotation and phase matrices defined below,

$$\begin{aligned}R_L &= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & 0\\ -1 & 1 & 0\\ 0 & 0 & \sqrt{2}\end{pmatrix},\\ R_R &= \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & -1 & 0\\ -1 & 1 & 0\\ 0 & 0 & \sqrt{2}\end{pmatrix},\end{aligned} \tag{14}$$

$$\begin{aligned}P_L^q &= \text{diag}(1, \exp i2\Delta\theta^q, \exp i\Delta\theta^q),\\ P_R^u &= (-1)\exp i\theta_3^u \text{diag}(\exp i2\Delta\theta^u, 1, \exp i\Delta\theta^u),\\ P_R^d &= \exp i\theta_3^d \text{diag}(\exp i2\Delta\theta^d, 1, \exp i\Delta\theta^d),\\ \Delta\theta^q &= \theta_3^q - \theta_+^q \quad (q = u, d),\end{aligned} \tag{15}$$

we can bring $\mathbf{m}^q$ into a real form $\hat{\mathbf{m}}^q = P_L^{q\dagger} R_L^T \mathbf{m}^q R_R P_R^q$. The mass matrix $\hat{\mathbf{m}}^u$ can then be diagonalized as $O_L^{uT}\hat{\mathbf{m}}^u O_R^u = \text{diag}(m_u, m_c, m_t)$, and similarly for $\mathbf{m}^d$, where $O_{L,R}^{u,d}$ are orthogonal matrices. So, the mass eigenstates $u'_{iL} = (u'_L, c'_L, t'_L)$, etc. can be obtained from $q_L = U_L^q q'_L$, $q_R = U_R^q q'_R$, where $U_{L(R)}^q = R_{L(R)} P_{L(R)}^q O_{L(R)}^q$. Therefore, the Cabibbo-Kobayashi-Maskawa (CKM) matrix V_{CKM} is given by

$$V_{\text{CKM}} = O_L^{uT} P_L^{u\dagger} P_L^d O_L^d = O_L^{uT} P_q O_L^d, \tag{16}$$

where

$$\begin{aligned}P_q &= \text{diag}(1, \exp(i2\theta_q), \exp(i\theta_q)),\\ \theta_q &= \theta_+^u - \theta_+^d - \theta_3^u + \theta_3^d.\end{aligned} \tag{17}$$

There are nine independent theory parameters which describe the CKM parameters and the quark masses: $Y_a^{u,d} v_3^{u,d}$, $Y_c^{u,d} v_3^{u,d}$, $Y_b^{u,d} v_+^{u,d}$, $Y_{b'}^{u,d} v_+^{u,d}$, and θ_q, where $Y^{u,d}$'s are entries of the Yukawa matrices (2). The set of theory parameters is thus overconstrained. Therefore, there is not much freedom in the parameter space, and so it is sufficient to consider a single point in the space of the theory parameters of this sector:

$$\begin{aligned}&Y_a^u v_3^u = 1.409 m_t, \quad Y_c^u v_3^u = 2.135\times 10^{-4} m_t,\\ &Y_b^u v_+^u = 0.0847 m_t, \quad Y_{b'}^u v_+^u = 0.0879 m_t,\\ &Y_a^d v_3^d = 1.258 m_b, \quad Y_c^d v_3^d = -6.037\times 10^{-3} m_b,\\ &Y_b^d v_+^d = 0.0495 m_b, \quad Y_{b'}^d v_+^d = 0.6447 m_b,\\ &\theta_q = -0.7125.\end{aligned} \tag{18}$$

With these parameter values we obtain [20]

$$\begin{aligned}m_u/m_t &= 0.609\times 10^{-5},\\ m_c/m_t &= 3.73\times 10^{-3},\\ m_d/m_b &= 0.958\times 10^{-3},\end{aligned} \tag{19}$$

$$m_s/m_b = 1.69\times 10^{-2},$$
$$|V_{\rm CKM}| = \begin{pmatrix} 0.9740 & 0.2266 & 0.00361 \\ 0.2264 & 0.9731 & 0.0414 \\ 0.00858 & 0.0407 & 0.9991 \end{pmatrix}, \tag{20}$$

$$\begin{aligned} |V_{td}/V_{ts}| &= 0.211, \\ \sin 2\beta(\phi_1) &= 0.695, \\ \bar\rho &= 0.152, \\ \bar\eta &= 0.343. \end{aligned} \tag{21}$$

The mass ratio (20) is defined at M_Z and consistent with the recent updates of [21], and the CKM parameters above agree with those of the Particle Data Group [22] and the CKMfitter Group [14,16]. (See [23] for the predictions of the model in the lepton sector.)

In the basis of the fermion mass eigenstates the Yukawa couplings have the following form:

$$\begin{aligned} \mathcal{L}_Y = &-\sum_{I=L,H,-} Y^{u0I}_{ij}(\phi^{u0}_I)^* \bar u'_{iL} u'_{jR} \\ &+ \sum_{I=L,H,-} Y^{d-I}_{ij}(\phi^{d-}_I)^* \bar u'_{iL} d'_{jR} \\ &- \sum_{I=L,H,-} Y^{d0I}_{ij}(\phi^{d0}_I)^* \bar d'_{iL} d'_{jR} \\ &+ \sum_{I=L,H,-} Y^{u+I}_{ij}(\phi^{u+}_I)^* \bar d'_{iL} u'_{jR} + \text{H.c.}, \end{aligned} \tag{22}$$

where the Higgs fields are defined in (10) and (11), the Yukawa matrices $\mathbf{Y}^{u1}$, etc. are given in (2), and

$$\begin{aligned} \mathbf{Y}^{d0L} &= O^{dT}_L R^T_L \mathbf{Y}^{dL} R_R O^d_R \\ &= \sqrt{2}\,\mathrm{diag}(m_d, m_s, m_b)/v\cos\beta, \\ \mathbf{Y}^{d0H} &= O^{dT}_L R^T_L \mathbf{Y}^{dH} R_R O^d_R, \\ \mathbf{Y}^{d0-} &= \frac{1}{\sqrt{2}} O^{dT}_L R^T_L (\mathbf{Y}^{d1} - \mathbf{Y}^{d2}) R_R O^d_R e^{2i\Delta\theta^d}, \\ \mathbf{Y}^{d-L} &= O^{uT}_L P_q R^T_L \mathbf{Y}^{dL} R_R O^d_R, \\ \mathbf{Y}^{d-H} &= O^{uT}_L P_q R^T_L \mathbf{Y}^{dH} R_R O^d_R, \end{aligned} \tag{23}$$

$$\begin{aligned} \mathbf{Y}^{d--} &= \frac{1}{\sqrt{2}} O^{uT}_L P_q R^T_L (\mathbf{Y}^{d1} - \mathbf{Y}^{d2}) R_R O^d_R e^{2i\Delta\theta^d}, \\ \mathbf{Y}^{dL} &= \left[\frac{1}{\sqrt{2}}\sin\gamma^d(\mathbf{Y}^{d1} + \mathbf{Y}^{d2}) + \cos\gamma^d \mathbf{Y}^{d3}\right], \\ \mathbf{Y}^{dH} &= \left[\frac{1}{\sqrt{2}}\cos\gamma^d(\mathbf{Y}^{d1} + \mathbf{Y}^{d2}) - \sin\gamma^d \mathbf{Y}^{d3}\right], \end{aligned} \tag{24}$$

and similarly for the $\mathbf{Y}^u$'s, where the matrices other than the Yukawa matrices are defined in (14) and (17). One finds that $\mathbf{Y}^{d0L}$ and $\mathbf{Y}^{d0H}$ are real and that the only phase appearing in $\mathbf{Y}^{d-L}$ and $\mathbf{Y}^{d-H}$ is θ_q given in (17), which is the same phase entering into $V_{\rm CKM}$. As we can see from (23) and (24) the free parameters in (22) are only β, $\gamma^{u,d}$ and $\Delta\theta^d + \Delta\theta^u$ [because $\theta_q = \Delta\theta^d - \Delta\theta^u$ is fixed at (18)]. We will use the Lagrangian (22) to compute the EDM mediated by the exchange of the Higgs fields. The EDM depends, therefore, on the Higgs masses, which we will discuss when calculating the EDM in Sec. V.

IV. SOFT MASS INSERTIONS

We shall make use of the soft mass insertions [24,25] to calculate FCNC and CP violations coming from the SUSY breaking sector. Because of the family symmetry and CP invariance in this model, however, the soft-supersymmetry-breaking sector is strongly constrained. We will describe below how the soft mass insertions in this model are constrained and parametrized.

The A terms and soft scalar mass terms obey the Q_6 family symmetry in the effective theory. Therefore, the soft mass matrices have the form

$$\begin{aligned} \tilde{\mathbf{m}}^2{}_{aLL} &= m^2_{\tilde a}\mathrm{diag}(a^a_L, a^a_L, b^a_L) \qquad (a = q, l), \\ \tilde{\mathbf{m}}^2_{aRR} &= m^2_{\tilde a}\mathrm{diag}(a^a_R, a^a_R, b^a_R) \qquad (a = u, d, e), \\ (\tilde{\mathbf{m}}^2_{aLR})_{ij} &= \sum_{I=1,2,3} A^a_{ij}(\mathbf{Y}^{aI})_{ij}\frac{|v^a_I|}{\sqrt{2}} \\ &= A^a_{ij}(\mathbf{m}^a)_{ij} \qquad (a = u, d, e; v^e_I = v^d_I), \end{aligned} \tag{25}$$

where $m_{\tilde a}$ denote the average of the squark and slepton masses, respectively, $(a^a_{L(R)}, b^a_{L(R)})$ are dimensionless free real parameters, A^a_{ij} are free real parameters of dimension one, and $\mathbf{m}^a$ are the respective fermion mass matrices. According to [24,25] we define the tree-level supersymmetry-breaking soft mass insertions as

$$\delta^{a0}_{LL(RR)} = U^{a\dagger}_{L(R)}\tilde{\mathbf{m}}^2_{aLL(RR)}U^a_{L(R)}/m^2_{\tilde a}, \tag{26}$$

$$\delta^{u0}_{LR} = U^{u\dagger}_L(\tilde{\mathbf{m}}^2_{uLR} - \mu^{IJ}\langle h^{d0}_J\rangle \mathbf{Y}^{uJ})U^u_R/m^2_{\tilde u}, \tag{27}$$

$$\delta^{d0}_{LR} = U^{d\dagger}_L(\tilde{\mathbf{m}}^2_{dLR} + \mu^{JI}\langle h^{u0}_J\rangle \mathbf{Y}^{dJ})U^d_R/m^2_{\tilde d}, \tag{28}$$

in the super CKM basis, where U's are unitary matrices that diagonalize the quark mass matrices, and h's are the neutral Higgs fields defined in (7). (We restrict ourselves to the quark sector.) The μ-term and A-term contributions to $\delta^{u0(d0)}_{LR}$ are the first and second terms in (27) and (28), respectively. A phase alignment embedded in the model is working for the A-term contributions. That is, although $A_{ij} \neq 1$ in (25), the phases in the fermion mass matrices $(\mathbf{m}^a)_{ij}$ can be rotated away by the unitary matrices in (28). Therefore, only the μ terms contribute to EDMs.

For the input parameters given in (18) we obtain the following A-term contributions to the left-right insertions:

$$\begin{aligned}
(\delta^{d0}_{12})_{LR}(A) &\simeq 1.9(\tilde{A}^d_1 - \tilde{A}^d_2) \times 10^{-5},\\
(\delta^{d0}_{21})_{LR}(A) &\simeq (-2.2\tilde{A}^d_1 + 1.7\tilde{A}^d_2) \times 10^{-5},\\
(\delta^{d0}_{13})_{LR}(A) &\simeq (1.1\tilde{A}^d_1 - 1.0\tilde{A}^d_2 + 5.0\tilde{A}^d_3) \times 10^{-5},\\
(\delta^{d0}_{31})_{LR}(A) &\simeq 5.8\tilde{A}^d_2 \times 10^{-4},\\
(\delta^{d0}_{23})_{LR}(A) &\simeq 1.7\tilde{A}^d_3 \times 10^{-4},\\
(\delta^{d0}_{32})_{LR}(A) &\simeq -2.3\tilde{A}^d_2 \times 10^{-2},
\end{aligned} \tag{29}$$

where $\tilde{A}^d_i = [A^d_i/m_{\tilde{d}}][0.5\ \text{TeV}/m_{\tilde{d}}]$ $(i = 1, 2, 3)$ are dimensionless free parameters. The real parameters A^d_i are given by

$$A^d_1 = A^d_b - A^d_c, \qquad A^d_2 = A^d_a - A^d_{b'}, \qquad A^d_3 = A^d_b - A^d_a, \tag{30}$$

where $A^d_{a,b,b',c}$ are associated with the independent elements $Y^d_{a,b,b',c}$ of the Yukawa matrices (2) and defined such that the corresponding trilinear couplings of the squarks and Higgs fields have the form $A^d_b Y^d_b$, etc., as one can see from (25). The left-right mass insertions $(\delta_{ij})_{LR}(A)$ contribute to the radiative correction to the Yukawa couplings [26–29] and can also enhance FCNCs and EDMs if $A's/m_{\tilde{d}} \gg 1$ [30–32] (which means $\tilde{A}'s \gg 1$). Similar effects are present due to the μ-term contribution to $(\delta^d_{ij})_{LR}$ for a large $\tan\beta$ [26–32]. In this paper we use $\tan\beta = 3.18$ as a benchmark value (see Table II). As we can see from (29), the strong constraints on $(\delta^d_{ij})_{LR}$ given in [29,33] are satisfied for $\tilde{A}'s \lesssim O(1)$. Therefore, we assume in this paper that $\tilde{A}'s \lesssim O(1)$. The μ-term contributions to $(\delta^d_{ij})_{LR}$ can be obtained from the second terms of (27) and (28). They are complex and hence can contribute to EDMs at the one-loop level as we will see in Sec. V B.

The other insertions are found to be

$$\begin{aligned}
(\delta^{d0}_{12})_{LL} &= (\delta^{d0}_{21})^*_{LL} \simeq -2.6 \times 10^{-4}\Delta a^q_L,\\
(\delta^{d0}_{13})_{LL} &= (\delta^{d0}_{31})^*_{LL} \simeq -8.7 \times 10^{-3}\Delta a^q_L,\\
(\delta^{d0}_{23})_{LL} &= (\delta^{d0}_{32})^*_{LL} \simeq -3.0 \times 10^{-2}\Delta a^q_L,\\
(\delta^{d0}_{12})_{RR} &= (\delta^{d0}_{21})^*_{RR} \simeq 5.0 \times 10^{-2}\Delta a^d_R,\\
(\delta^{d0}_{13})_{RR} &= (\delta^{d0}_{31})^*_{RR} \simeq -0.10\Delta a^d_R,\\
(\delta^{d0}_{23})_{RR} &= (\delta^{d0}_{320})^*_{RR} \simeq 0.39\Delta a^d_R,
\end{aligned} \tag{31}$$

where $\Delta a^q_L = a^q_L - b^q_L$, $\Delta a^d_R = a^d_R - b^d_R$. These mass insertions are the tree-level ones. In [12] it has been shown that the one-loop corrections to them, especially to $(\delta^{d0}_{ij})_{LL}$, can be large in the presence of more than one pair of the Higgs $SU(2)_L$ doublets. Moreover, it has been found that in the present model the one-loop corrections are needed to obtain a large CP violation in B^0 mixing that is comparable with the observations at Tevatron. These one-loop corrections $(\Delta\delta_{ij})$ depend on the parameters in the Higgs sector. For the input parameters given in (18) and in Table II, we find the following corrections:

$$\begin{aligned}
(\Delta\delta^d_{12})_{LL} &= (\Delta\delta^d_{21})^*_{LL} \simeq (6.7 - i0.8) \times 10^{-4}\left[\frac{0.5\ \text{TeV}}{m_{\tilde{d}}}\right]^2,\\
(\Delta\delta^d_{13})_{LL} &= (\Delta\delta^d_{31})^*_{LL} \simeq (1.2 + i2.5) \times 10^{-2}\left[\frac{0.5\ \text{TeV}}{m_{\tilde{d}}}\right]^2,\\
(\Delta\delta^d_{23})_{LL} &= (\Delta\delta^d_{32})^*_{LL} \simeq -(8.5 + i6.8) \times 10^{-2}\left[\frac{0.5\ \text{TeV}}{m_{\tilde{d}}}\right]^2.
\end{aligned} \tag{32}$$

The one-loop corrections to $(\delta_{ij})_{RR,LR}$ are negligibly small.

TABLE II. A benchmark set of the parameter values. That B_L is real, and is a consequence of the minimization of the Higgs potential. With this set of parameters, we find that the eigenvalues of $\mathbf{M}^2_{CH\text{even}}$ are $(9.60^2, 5.01^2, 2.15^2)$ [TeV^2], while the eigenvalues in the Z_2 odd sector are $(9.49^2, 4.26^2)$ [TeV^2].

$s_{\gamma^u} = \sin\gamma^u$	-0.16	$s_{\gamma^d} = \sin\gamma^d$	-0.688
$c_\beta = \cos\beta$	0.3	$c_W = \cos\theta_W$	$\sqrt{0.77}$
	[TeV]		[TeV^2]
μ_L	$0.377 - I0.066$	$B_{++} = B_-$	$(5.992)^2 + I(1.232)^2$
μ_{LH}	$-0.754 + I0.011$	B_L	$(2.949)^2$
μ_{HL}	$-0.0058 - I0.0444$	B_H	$(4.257)^2 + I(1.035)^2$
μ_H	$2.201 + I0.405$	$m^2_{H^u}$	$-(7.234)^2$
$\mu_{++} = \mu_-$	$1.710 + I0.312$	$m^2_{H^u_3}$	$-(0.834)^2$
		$m^2_{H^d}$	$-(7.065)^2$
		$m^2_{H^d_3}$	$-(2.716)^2$

V. DARK MATTER, EDM, AND B^0 MIXING

A. LSP and dark matter

We assume that the LSP is a neutralino and is a dark matter candidate in this model. Because of Z_2 defined in (3) the Higgsinos can also be grouped into the Z_2-even and odd sectors.[6] The Higgsinos in the Z_2 odd sector have no mixing with the gauginos. If, therefore, the LSP belongs to the Z_2 odd sector, the LSP is a pure Higgsinos state with the mass μ^{++}. For this LSP to be a dark matter candidate, μ^{++} has to be larger than $O(1)$ TeV and, at the same time, smaller than the other μ's and gaugino masses. This parameter region cannot satisfy the EDM constraint without an extreme fine-tuning because we need relatively small μ's to satisfy the EDM constraint in the present model [7]. So, we may assume that the LSP belongs to the Z_2-even sector. The mass matrix of the neutralinos in the Z_2-even sector is

$$\mathbf{M}^F_{N\mathrm{even}} = \begin{pmatrix} M_1 & 0 & s_W s_\beta M_Z & -s_W c_\beta M_Z & 0 & 0 \\ 0 & M_2 & -c_W s_\beta M_Z & c_W c_\beta M_Z & 0 & 0 \\ s_W s_\beta M_Z & -c_W s_\beta M_Z & 0 & -\mu_L & 0 & -\mu_{LH} \\ -s_W c_\beta M_Z & c_W c_\beta M_Z & -\mu_L & 0 & -\mu_{HL} & 0 \\ 0 & 0 & 0 & -\mu_{HL} & 0 & -\mu_H \\ 0 & 0 & -\mu_{LH} & 0 & -\mu_H & 0 \end{pmatrix}, \tag{33}$$

where $c_\beta = \cos\beta$, $c_W = \cos\theta_W$, and similarly for s_β and s_W (θ_W is the Weinberg angle). Because of the EDM constraint, we expect that the mass of the LSP is relatively light, $O(\text{few } 100)$ GeV. Therefore, the LSP has to be a mixture of the Higgsinos and the gauginos to obtain a desirable relic density $\Omega h^2 \simeq 0.11$. So, we require that the gaugino fraction of the LSP is in a range between 65% and 95% (see, for instance, [34]), and assume that if this is satisfied, the neutralino LSP can be a dark matter candidate in the present model.

B. EDM

Our concern here is the neutron EDM, d_n, because the electron EDM in this model is extremely suppressed [7]. There are two sources for d_n: the Yukawa sector, because of the multi-Higgs structure, and the SUSY breaking sector.[7] Here we simply assume that d_n can be obtained from $d_n = \frac{1}{3}(4d_d - d_u)$, where $d_{u(d)}$ is the EDM of the u (d) quark. The experimental upper bound is given by [22]

$$d_n/e \lesssim 6.3 \times 10^{-26}\ \text{cm}. \tag{34}$$

1. Yukawa contribution

We start in the Yukawa sector. The one-loop diagrams can be divided as follows: the photon is attached to a quark or a charged Higgs, and the internal Higgs is neutral or charged [36]. The contribution to d_n/e with the neutral Higgs boson exchange [satisfying the constraint (45)] is less than $O(10^{-31})$ cm, as was previously found in [7]. The contribution with the Z_2-even charged Higgs boson exchange to d_u is $O(10^{-29})$ [e cm] and is 3 orders of magnitude smaller than that to d_d. This is because (i) d_n is proportional to the fermion mass in the internal loop, and (ii) the Yukawa couplings in the down-type quark sector are smaller than those of the up-type quark sector for a moderate $\tan\beta$ [that is, $Y^{u+}_{1,dt} \gg Y^{d-}_{1,ub}$, for instance, where $Y^{u+}_{i,qq'}$ and $Y^{d-}_{i,q'q}$ are defined in (36) below]. We first concentrate on the contribution to d_d from the Z_2-even charged Higgs boson, which takes the form [36]

$$d_d = \frac{e}{24\pi^2} \sum_{k,l,m=1,2,3} \sum_{q=u,c,t} \mathrm{Im}(U^C_{lk} U^{C*}_{mk} Y^{d-}_{l,qd} Y^{u+}_{m,dq}) \times \frac{m_q}{(M_C)^2_k}\left[\frac{3}{4} - \ln\left(\frac{(M_C)^2_k}{m_q^2}\right)\right]\xi. \tag{35}$$

The up-type quark mass in the internal line is denoted by m_q, and $\xi \simeq 0.12$ stands for the QCD correction [36,37]. The Yukawa matrices in (35) are given by

$$\begin{aligned} Y^{u+}_{1,qq'} &= \cos\beta Y^{u+L}_{qq'}, & Y^{u+}_{2,qq'} &= Y^{u+H}_{qq'}, \\ Y^{u+}_{3,qq'} &= 0, & Y^{d-}_{1,qq'} &= \sin\beta Y^{d-L}_{qq'}, \\ Y^{d-}_{2,qq'} &= 0, & Y^{d-}_{3,qq'} &= Y^{d-H}_{qq'}, \end{aligned} \tag{36}$$

where $\mathbf{Y}^{u+H,L}$, $\mathbf{Y}^{d-H,L}$ are given in (23) and (24). U^C is the unitary matrix that diagonalizes the squared mass matrix of the charged Higgs bosons of the Z_2-even sector, and M^2_C is the corresponding eigenvalue. The squared mass matrix of the charged Higgs bosons in the Z_2-even sector has the form

[6] Since Z_2 is not an exact symmetry of the theory, the even and odd states will mix with each other in higher orders in perturbation theory.

[7] See, for instance, [35].

$$\mathbf{M}^2_{CH\mathrm{even}} = \begin{pmatrix} \frac{2B_L}{s_{2\beta}} + c_W^2 M_Z^2 & -\hat{m}^2_{uLH}/c_\beta & -\hat{m}^{2*}_{dLH}/s_\beta \\ -\hat{m}^{2*}_{uLH}/c_\beta & -\hat{m}^2_{uH} + c_{2\beta}(c_W^2 - 1/2)M_Z^2 & B_H^* \\ -\hat{m}^2_{dLH}/s_\beta & B_H & -\hat{m}^2_{dH} - c_{2\beta}(c_W^2 - 1/2)M_Z^2 \end{pmatrix}, \tag{37}$$

in the basis $(H^+, \phi_H^{u+}, (\phi_H^{d-})^*)$, where

$$\begin{aligned} B_L &= c_{\gamma^u} s_{\gamma^d} B^{3+} e^{i(\theta_3^u + \theta_+^d)} + s_{\gamma^u} c_{\gamma^d} B^{+3} e^{i(\theta_+^u + \theta_3^d)} + s_{\gamma^u} s_{\gamma^d} B^{++} e^{i(\theta_+^u + \theta_+^d)}, \\ B_H &= -s_{\gamma^u} c_{\gamma^d} B^{3+} e^{i(\theta_3^u + \theta_+^d)} - c_{\gamma^u} s_{\gamma^d} B^{+3} e^{i(\theta_+^u + \theta_3^d)} + c_{\gamma^u} c_{\gamma^d} B^{++} e^{i(\theta_+^u + \theta_+^d)}, \\ \hat{m}^2_{uLH} &= c_{\gamma^u} s_{\gamma^u}(m^2_{H^u} - m^2_{H_3^u}) - \mu_H \mu^*_{LH} - \mu_{HL}\mu^*_L, \\ \hat{m}^2_{dLH} &= c_{\gamma^d} s_{\gamma^d}(m^2_{H^d} - m^2_{H_3^d}) - \mu_H \mu^*_{HL} - \mu_{LH}\mu^*_L, \\ \hat{m}^2_{uH} &= c^2_{\gamma^u} m^2_{H^u} + s^2_{\gamma^u} m^2_{H_3^u} - |\mu_H|^2 - |\mu_{HL}|^2 + c_{2\beta} M_Z^2/2, \\ \hat{m}^2_{dH} &= c^2_{\gamma^d} m^2_{H^d} + s^2_{\gamma^d} m^2_{H_3^d} - |\mu_H|^2 - |\mu_{LH}|^2 - c_{2\beta} M_Z^2/2. \end{aligned} \tag{38}$$

Therefore, $[(U^C)^\dagger \mathbf{M}^2_{CH\mathrm{even}} U^C]_{ik} = (M_C)^2_k \delta_{ik}$, and we see that d_n depends on many mass parameters of the model, even though the Lagrangian (22) has only a few parameters. Note that only $\theta_q = \Delta\theta^d - \Delta\theta^u$ given in (17) enters in the contribution coming from the exchange of the Z_2-even charged Higgs bosons. As a benchmark set of the parameters we consider the parameter values given in Table II. These parameters are chosen so that they do not cause problems with FCNC and CP violations other than EDMs. We then find

$$d_d(\text{even charged}) \simeq 3.9 \times 10^{-26}\ \text{e cm}, \qquad d_u(\text{even charged}) \simeq 2.5 \times 10^{-29}\ \text{e cm}, \tag{39}$$

where we have used $\theta_q = \Delta\theta^d - \Delta\theta^u = -0.7125$ [see (18)].

As for the contribution coming from the exchange of the Z_2-odd charged Higgs bosons, the phase structure is different from that in the case of the Z_2-even charged Higgs bosons. In fact, $2(\Delta\theta^u + \Delta\theta^d) \pm \theta_q$ enters in the dominant contribution. Using the squared mass matrix of the charged Higgs bosons in the Z_2 odd sector

$$\mathbf{M}^2_{C\mathrm{odd}} = \begin{pmatrix} -m^2_{H^u} + |\mu_{++}|^2 + c_{2\beta} c_W^2 M_Z^2 & B^*_{++} \\ B_{++} & -m^2_{H^d} + |\mu_{++}|^2 - c_{2\beta} c_W^2 M_Z^2 \end{pmatrix} \tag{40}$$

in the $(\phi_-^{u+}, (\phi_-^{d-})^*)$ basis, we perform similar calculations as in the case of the Z_2-even charged Higgs bosons and find

$$\begin{aligned} d_d(\text{odd charged}) &\simeq 2.7 \times 10^{-24} \sin[2(\Delta\theta^u + \Delta\theta^d) \\ &\quad + \theta_q - 0.042]\ \text{e cm}, \\ d_u(\text{odd charged}) &\simeq 4.3 \times 10^{-27} \sin[2(\Delta\theta^u + \Delta\theta^d) \\ &\quad - \theta_q - 3.099]\ \text{e cm}. \end{aligned} \tag{41}$$

Therefore, for a certain range of $\Delta\theta^u + \Delta\theta^d$ we can satisfy the constraint (34). We will discuss the SUSY breaking contribution to the EDMs below, while assuming that $\Delta\theta^u + \Delta\theta^d$ is so tuned that the contribution from the Yukawa sector discussed here satisfies the constraint (34).

2. SUSY breaking contribution

The second source is the SUSY breaking sector. To obtain d_n we use the approximate result of [33] which takes into account only the gluino contribution[8]

$$\begin{aligned} d_d/e &= -\frac{2\alpha_s}{9\pi}\xi\, \mathrm{Im}(\delta^{d0}_{11})_{LR}, \\ d_u/e &= \frac{4\alpha_s}{9\pi}\xi\, \mathrm{Im}(\delta^{u0}_{11})_{LR}, \end{aligned} \tag{42}$$

[8]There is a chromo EDM, which depends strongly on $\tan\beta$ (see [38] and the references therein). In our case A parameters are still free parameters, and so it is possible to cancel the chromo EDM. It is not a problematic fine-tuning, because our $\tan\beta \simeq 3.12$ is not large.

TABLE III. Parameter values used in the text. For the calculations in the text we use only the central values. f_K, $M_{K,d,s}$, $\Delta M^{\rm exp}_{K,d,s}$ are from [22]. f_{B_s} belongs to the conservative sets of [10] (see the references therein), and f_{B_d} is obtained from f_{B_s}/ξ with $\xi = 1.24$. $m_d(2\ \text{GeV})$ and $m_s(2\ \text{GeV})$ are from [22], while those at m_b are taken from [21].

Input		Input	
f_K	$(159.8 \pm 1.4 \pm 0.44) \times 10^{-3}$ GeV	f_{B_d}	0.194 ± 0.032 GeV
f_{B_s}	0.240 ± 0.040 GeV		
M_K	$0.497\,648 \pm 0.000\,022$ GeV	$\Delta M_K^{\rm exp}$	$(0.5292 \pm 0.0009) \times 10^{-2}\text{ps}^{-1}$
M_s	5.3661 ± 0.0006 GeV	$\Delta M_s^{\rm exp}$	$17.77 \pm 0.10 \pm 0.07\ \text{ps}^{-1}$
M_d	$5.279\,50 \pm 0.000\,33$ GeV	$\Delta M_d^{\rm exp}$	$0.507 \pm 0.005\ \text{ps}^{-1}$
$m_d(2\ \text{GeV})$	$(5.04^{+0.96}_{-1.54}) \times 10^{-3}$ GeV	$m_s(2\ \text{GeV})$	$0.105^{+0.025}_{-0.035}$ GeV
$m_d(m_b)$	$(4.23^{+1.74}_{-1.71}) \times 10^{-3}$ GeV	$m_s(m_b)$	0.080 ± 0.022 GeV
$m_b(m_b)$	4.20 ± 0.07 GeV		

where we have assumed that $m_{\tilde g} = m_{\tilde u} = m_{\tilde d} = m_{\tilde q}$, and $\xi \simeq 0.12$ is the QCD correction [36,37]. Since the A's are real, only the μ terms contribute to $\text{Im}(\delta_{11}^{u(d)0})_{LR}$, and therefore,

$$\text{Im}(\delta_{11}^{u0})_{LR} = \left[\frac{1}{\tan\beta}\text{Im}(\mu_L)m_u + \frac{v\cos\beta}{\sqrt{2}}\text{Im}(\mu_{HL})\mathbf{Y}_{11}^{u0H}\right]\Big/ m_{\tilde u}^2, \tag{43}$$

$$\text{Im}(\delta_{11}^{d0})_{LR} = \left[\tan\beta\,\text{Im}(\mu_L)m_d + \frac{v\sin\beta}{\sqrt{2}}\text{Im}(\mu_{LH})\mathbf{Y}_{11}^{d0H}\right]\Big/ m_{\tilde d}^2, \tag{44}$$

where we have used (27) and (28), and $\mathbf{Y}^{u0H}$ and $\mathbf{Y}^{d0H}$ are defined in (23). In the last section Eqs. (43) and (44) will be used to relate the dark matter mass $m_{\rm DM}$, the neutron EDM, and the CP violation in B^0 mixing.

C. B^0 mixing

The tree-level contributions to the B^0 mixing coming from the heavy neutral Higgs boson exchange in this model are small if

$$\cos\beta M_H \gtrsim 1.2\ \text{TeV} \tag{45}$$

is satisfied [6,7], where M_H^2 is the $(\varphi_H^d - \varphi_H^d)$ element of the inverse of the mass squared matrix of the neutral Higgs bosons of the Z_2-even sector [φ_H^d is the scalar component of Φ_H^{d0} given in (10)]. In the following discussion we assume this is the case, so that the only relevant contribution comes from the SUSY breaking sector. Therefore, the total matrix element M_{12}^q in the neutral meson mixing can be written as $M_{12}^q = M_{12}^{\rm SM,q} + M_{12}^{\rm SUSY,q}$, where $M_{12}^{\rm SM,q}$ and $M_{12}^{\rm SUSY,q}$ are the SM contribution and the SUSY contribution, respectively.

We follow [10] to parametrize the new physics effects as

$$M_{12}^{\rm SM,q} + M_{12}^{\rm SUSY,q} = M_{12}^{\rm SM,q} \times \Delta_q, \tag{46}$$

and consider ΔM_q and the flavor specific CP asymmetry a_{sl}^q in terms of the complex number $\Delta_q = |\Delta_q|e^{i\phi_q^\Delta}$, where $q = d, s$, and

$$\Delta M_q = 2|M_{12}^{\rm SM,q}| \times |\Delta_q|, \quad a_{sl}^q = \frac{|\Gamma_{12}^q|}{|M_{12}^{\rm SM,q}|} \times \frac{\sin\phi_q}{|\Delta_q|}, \quad \phi_q = \phi_q^{\rm SM} + \phi_q^\Delta. \tag{47}$$

The SM values are given e.g. in [10], in which the results of [39–43] are used:

$$\begin{aligned} 2M_{12}^{{\rm SM},d} &= 0.56(1 \pm 0.45)\exp(i0.77)\ \text{ps}^{-1}, \\ 2M_{12}^{{\rm SM},s} &= 20.1(1 \pm 0.40)\exp(-i0.035)\ \text{ps}^{-1}, \\ \phi_d^{\rm SM} &= (-0.091^{+0.026}_{-0.038})\ \text{rad}, \\ \phi_s^{\rm SM} &= (4.2 \pm 1.4) \times 10^{-3}\ \text{rad}, \end{aligned} \tag{48}$$

where the errors are dominated by the uncertainty in the decay constants and bag parameters.[9]

As for the SUSY part, we take into account only the dominant contribution (gluino exchange) for $M_{12}^{\rm SUSY,q}$ given in [33] (see e.g. [44] for a more refined calculation):

$$\begin{aligned} M_{12}^{SUSY,s} = -\frac{\alpha_S^2}{324 m_{\tilde d}^2} M_s f_{B_s}^2 B_s \{ &[(\delta_{32}^d)_{LL}^2 + (\delta_{32}^d)_{RR}^2] \\ &\times [24 x f_6(x) + 66 \tilde f_6(x)] + (\delta_{32}^d)_{LL}(\delta_{32}^d)_{RR} \\ &\times [(384R_s + 72)x f_6(x) - (24R_s - 36)\tilde f_6(x)] \\ &- 132[(\delta_{32}^d)_{LR}^2 + (\delta_{23}^d)_{LR}^{*2}] R_s x f_6(x) \\ &- (\delta_{32}^d)_{LR}(\delta_{23}^d)_{LR}^*[144R_s + 84]\tilde f_6(x)\}, \end{aligned} \tag{49}$$

[9]Note that the values for $M_{12}^{{\rm SM},q}$ quoted above are those in the standard parametrization of the CKM matrix [22] and that the CKM matrix obtained from (16) is not in the standard parametrization. Therefore, we have to express the supersymmetric contribution $M_{12}^{{\rm SUSY},q}$ in the standard parametrization of the CKM matrix before actual calculations.

where

$$R_s = \left(\frac{M_s}{m_s + m_d}\right)^2,$$
$$f_6(x) = \frac{6(1+3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(x-1)^5},$$
$$\tilde{f}_6(x) = \frac{6x(1+x)\ln x - x^3 - 9x^2 + 9x + 1}{3(x-1)^5} \quad \text{with } x = m_{\tilde{g}}/m_{\tilde{d}}, \tag{50}$$

and similarly for K and B_d, and $m_{\tilde{g}}$ is the gluino mass. For the present calculations we assume that the bag parameters B_K, B_d, B_s are 1, and $\alpha_S = 0.12$. The other parameters are given in Table III.

We use the central values of (48) and Table III for our calculations, while requiring the (conservative) constraints

$$0.6 < \frac{\Delta M_{d,s}}{\Delta M_{d,s}^{\rm exp}} < 1.4, \qquad \frac{2|M_{12}^{\rm SUSY,K}|}{\Delta M_K^{\rm exp}} < 2,$$
$$\frac{{\rm Im} M_{12}^{\rm SUSY,K}\lambda_u^2}{\sqrt{2}\Delta M_K^{\rm exp}|\lambda_u|^2} < \epsilon_K = 2.2 \times 10^{-3}, \tag{51}$$

where $\lambda_u = (V_{\rm CKM})^*_{us}(V_{\rm CKM})_{ud}$.

The same sign dimuon asymmetry A^b_{sl} measured by the D0 Collaboration [9] is a linear combination of the semileptonic CP asymmetries in the B_d and B_s systems:

$$A^b_{sl} = (0.494 \pm 0.043) \times a^s_{sl} + (0.506 \pm 0.043) \times a^d_{sl}. \tag{52}$$

The SM value for A^b_{sl} is given by $A^b_{sl} = -(2.3^{+0.5}_{-0.6}) \times 10^{-4}$ [10], while the fit result yields [15]

$$A^b_{sl} = -(4.2^{+1.9}_{-1.8}) \times 10^{-3}. \tag{53}$$

VI. RESULT AND CONCLUSION

Most of the free parameters belong to the Higgs sector and to the SUSY breaking sector. The parameter space is so large that it will be beyond the scope of the present paper to analyze the complete parameter space. Instead, we first look for a benchmark point in the parameter space that satisfies all the requirements (34), (45), (51), and (53). Then we consider neighbor points and look for a border beyond which the constraints are no longer simultaneously satisfied. The border is extended by a certain amount, and the parameter space to be considered is defined such that it is surrounded by the extended border.

Note that a larger $\tan\beta$ means a smaller $\cos\beta$, which requires a finer fine-tuning in the Higgs sector in order to satisfy (45). For instance, $\tan\beta = 10$ would require $M_H \gtrsim 12$ TeV. In the following analysis we consider a benchmark value $\cos\beta = 0.3 (\tan\beta \simeq 3.18)$, which implies

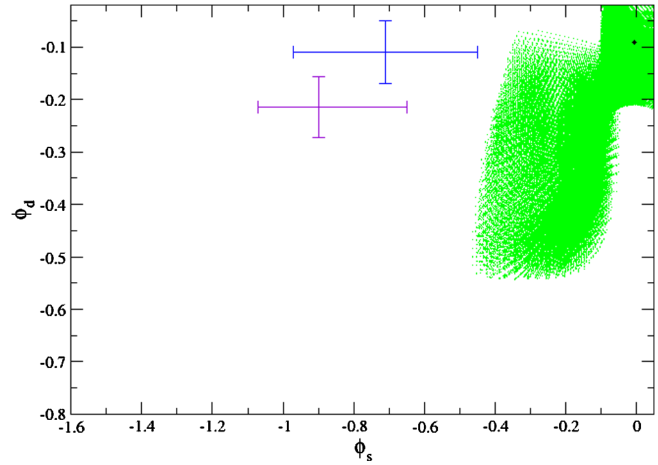

FIG. 1 (color online). The prediction in the $\phi_s - \phi_d$ plane. The fit result of the CKMfitter Group (purple, lower lines) [15] and that of the UTfit Group (blue, upper lines) [16] are also shown. The black dot is the SM value.

$M_H \gtrsim 4$ TeV. Further, $\Delta a^{q,d}_{L,R}$ in (31) are $O(1)$ free parameters, and so we assume that $|\Delta a^{q,d}_{L,R}| \lesssim 15$.

We start with the dark matter mass $m_{\rm DM}$ (the mass of the neutralino LSP). It is the smallest eigenvalue of (33) and depends on the gaugino masses M_1 and M_2, and the μ parameters. The μ parameters directly enter into the EDM [see (43) and (44)], while the tree-level mass insertions $(\delta^{d0}_{ij})_{RR,LL}$ given in (31) do not depend on the μ parameters. However, their one-loop corrections do depend on them [12]. So, the dark matter mass $m_{\rm DM}$ in the present model is constrained by the EDM and by the mixing of the neutral meson systems. We find that $m_{\rm DM}$ is indeed bounded above and below:

$$0.12\ [{\rm TeV}] \lesssim m_{\rm DM} \lesssim 0.33\ [{\rm TeV}], \tag{54}$$

where we have required (51) and (53) with $\cos\beta M_H \simeq 1.2$ TeV and used $m_{\tilde{g}} = m_{\tilde{u}} = m_{\tilde{d}} = m_{\tilde{q}} = 0.5$ TeV. The upper bound becomes larger if the size of the μ parameters increases. However, the size of the second term in the right-hand side of (28), in particular, for $(\delta^{d0}_{32})_{LR}$, increases, too. The upper bound given in (54) corresponds to $|(\delta^{d0}_{32})_{LR}| \sim O(10^{-2})$, which is about the upper limit to satisfy the constraint from $b \to s\gamma$ [29,33].[10] Similarly, if we increase $\cos\beta M_H$, the one-loop effect becomes larger because of a larger SUSY breaking in the extra Higgs sector, and consequently, (51) will be violated. To reduce the one-loop effect, we have to increase the size of the μ parameters to reduce the SUSY breaking. But this is not allowed because of the $b \to s\gamma$ constraint. Therefore, (54) should be regarded as the area of $m_{\rm DM}$ of the present model. The phenomenological feature of the dark matter of

[10] $|(\delta^{d0}_{23})_{LR}|$ is 2 orders of magnitude smaller than $|(\delta^{d0}_{32})_{LR}|$ in the present model [see (29)].

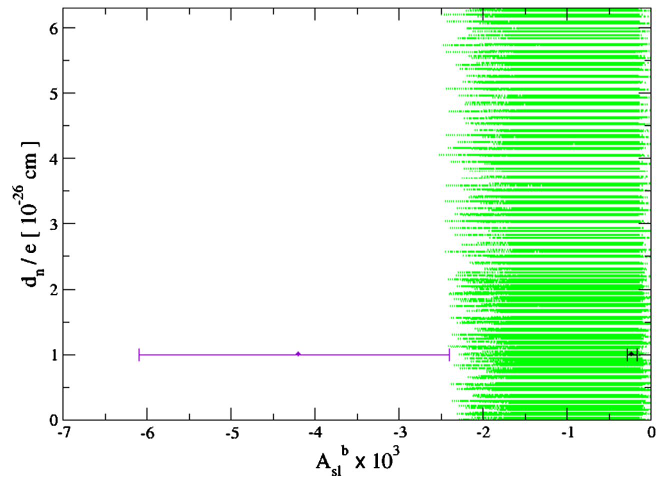

FIG. 2 (color online). The same sign dimuon asymmetry A^b_{sl} against d_n/e. The fit result for A^b_{sl} is $-(4.2^{+1.9}_{-1.8}) \times 10^{-3}$ (purple line) [15], and the D0 result [9] is $A^b_{sl} = -(9.57 \pm 2.51 \pm 1.46) \times 10^{-3}$. The SM value is shown in black.

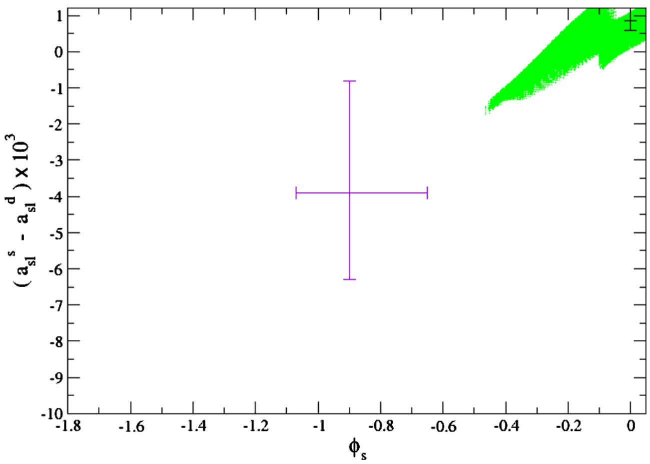

FIG. 3 (color online). The prediction of $a^s_{sl} - a^d_{sl}$, where the horizontal axis stands for ϕ_s. The fit result for $a^s_{sl} - a^d_{sl}$ is $-(3.9^{+2.4}_{-3.1}) \times 10^{-3}$ (purple lines), while the SM value is $(0.793^{+0.066}_{-0.214}) \times 10^{-3}$ (black).

the present model is basically the same as that of the MSSM. Therefore, it could be observed in various future experiments [45].

Next we consider the extra phases ϕ_s and ϕ_d defined in (47), which are shown in Fig. 1. Also shown are the fit results of the CKMfitter Group (purple) [15] and the UTfit Group (blue) [16]. As we see from the graphs, the theoretical values are comparable with the fit values and about 1 order of magnitude larger than the SM value (black dot). In calculating ϕ_s and ϕ_d we have neglected the contribution from the left-right insertions $(\delta^{d0}_{23,32})_{LR}(A)$ given in (29). This is because the contribution to the real part of $M^{\mathrm{SUSY},s}_{12}$ is very small; e.g. it is less than 1% of that from the $(\delta^d_{23})_{LL}$ and $(\delta^d_{23})_{RR}$ for $a^q_L = 1$, $a^d_R = -1.5$, $\tilde{A}^d_3 = \tilde{A}^d_2 = 1$, where use has been made of (31), (32), and (49).[11]

[11] $(\delta^{d0}_{23,32})_{LR}$ does not contribute to the imaginary part of M^{SUSY}_{12}.

The same sign dimuon asymmetry A^b_{sl} against d_n/e is shown in Fig. 2. A large imaginary part of the μ parameters, on one hand, produces a large CP violation in B^0 mixing. On the other hand, the large imaginary part implies a large EDM. Figure 2 shows that the SUSY contribution to d_n in this model can be made very small, while allowing a large A^b_{sl} which, in magnitude, is comparable with the fit result (53). As we see from Fig. 2 the error in A^b_{sl} is very crucial to test the prediction of the model. We hope that the error will be reduced by future experiments.

In Fig. 3 we plot the prediction of $a^s_{sl} - a^d_{sl}$ against ϕ_s. This combination of the asymmetries can be measured at the LHCb, and the experimental sensitivity with 1 fb^{-1}, which will be achieved in 2011 [46], is sufficient to test it.

ACKNOWLEDGMENTS

J. K. is partially supported by a Grant-in-Aid for Scientific Research (C) from Japan Society for Promotion of Science (No. 22540271).

[1] M. Dine, R. G. Leigh, and A. Kagan, Phys. Rev. D **48**, 4269 (1993); R. Barbieri, G. R. Dvali, and L. J. Hall, Phys. Lett. B **377**, 76 (1996); M. C. Chen and K. T. Mahanthappa, Phys. Rev. D **65**, 053010 (2002); S. F. King and G. G. Ross, Phys. Lett. B **574**, 239 (2003); G. G. Ross, L. Velasco-Sevilla, and O. Vives, Nucl. Phys. **B692**, 50 (2004).

[2] K. S. Babu and S. M. Barr, Phys. Lett. B **387**, 87 (1996); K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **83**, 2522 (1999).

[3] P. Pouliot and N. Seiberg, Phys. Lett. B **318**, 169 (1993); D. B. Kaplan and M. Schmaltz, Phys. Rev. D **49**, 3741 (1994); L. J. Hall and H. Murayama, Phys. Rev. Lett. **75**, 3985 (1995); C. D. Carone, L. J. Hall, and H. Murayama, Phys. Rev. D **53**, 6282 (1996); P. H. Frampton and T. W. Kephart, Int. J. Mod. Phys. A **10**, 4689 (1995); T. Kobayashi, J. Kubo, and H. Terao, Phys. Lett. B **568**, 83 (2003); T. Kobayashi, S. Raby, and R. J. Zhang, Nucl. Phys. **B704**, 3 (2005); M. C. Chen and K. T. Mahanthappa, Phys. Lett. B **652**, 34 (2007); I. de

Medeiros Varzielas, S. F. King, and G. G. Ross, Phys. Lett. B **648**, 201 (2007); H. Ishimori, T. Kobayashi, H. Okada, Y. Shimizu, and M. Tanimoto, J. High Energy Phys. 12 (2009) 054.

[4] K. S. Babu and J. Kubo, Phys. Rev. D **71**, 056006 (2005).

[5] Y. Kajiyama, E. Itou, and J. Kubo, Nucl. Phys. **B743**, 74 (2006).

[6] N. Kifune, J. Kubo, and A. Lenz, Phys. Rev. D **77**, 076010 (2008).

[7] K. S. Babu and Y. Meng, Phys. Rev. D **80**, 075003 (2009).

[8] H. Ishimori, T. Kobayashi, H. Ohki, H. Okada, Y. Shimizu, and M. Tanimoto, Prog. Theor. Phys. Suppl. **183**, 1 (2010).

[9] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. D **82**, 032001 (2010); Phys. Rev. Lett. **105**, 081801 (2010).

[10] A. Lenz and U. Nierste, J. High Energy Phys. 06 (2007) 072.

[11] O. Eberhardt, A. Lenz, and J. Rohrwild, Phys. Rev. D **82**, 095006 (2010); A. Dighe, A. Kundu, and S. Nandi, Phys. Rev. D **82**, 031502 (2010); B. A. Dobrescu, P. J. Fox, and A. Martin, Phys. Rev. Lett. **105**, 041801 (2010); C. H. Chen and G. Faisel, Phys. Lett. B **696**, 487 (2011); A. J. Buras, M. V. Carlucci, S. Gori, and G. Isidori, J. High Energy Phys. 10 (2010) 009; Z. Ligeti, M. Papucci, G. Perez, and J. Zupan, Phys. Rev. Lett. **105**, 131601 (2010); C. W. Bauer and N. D. Dunn, Phys. Lett. B **696**, 362 (2011); N. G. Deshpande, X. G. He, and G. Valencia, Phys. Rev. D **82**, 056013 (2010); B. Batell and M. Pospelov, Phys. Rev. D **82**, 054033 (2010); C. H. Chen, C. Q. Geng, and W. Wang, J. High Energy Phys. 11 (2010) 089; J. K. Parry, Phys. Lett. B **694**, 363 (2011); P. Ko and J. h. Park, Phys. Rev. D **82**, 117701 (2010); S. F. King, J. High Energy Phys. 09 (2010) 114; C. Delaunay, O. Gedalia, S. J. Lee, and G. Perez, Phys. Rev. D **83**, 115003 (2011); Y. Bai and A. E. Nelson, Phys. Rev. D **82**, 114027 (2010); A. J. Buras, K. Gemmler, and G. Isidori, Nucl. Phys. **B843**, 107 (2011); B. Dutta, Y. Mimura, and Y. Santoso, Phys. Rev. D **82**, 055017 (2010); M. Trott and M. B. Wise, J. High Energy Phys. 11 (2010) 157; M. Endo, S. Shirai, and T. T. Yanagida, Prog. Theor. Phys. **125**, 921 (2011); A. K. Alok, S. Baek, and D. London, arXiv:1010.1333; J. E. Kim, M. S. Seo, and S. Shin, Phys. Rev. D **83**, 036003 (2011); A. J. Buras, M. Nagai, and P. Paradisi, J. High Energy Phys. 05 (2011) 005; A. Datta, M. Duraisamy, and S. Khalil, Phys. Rev. D **83**, 094501 (2011); S. Nandi and A. Soni, Phys. Rev. D **83**, 114510 (2011).

[12] J. Kubo and A. Lenz, Phys. Rev. D **82**, 075001 (2010).

[13] L. Randall and S. F. Su, Nucl. Phys. **B540**, 37 (1999); W. S. Hou, M. Nagashima, and A. Soddu, Phys. Rev. D **76**, 016004 (2007); A. Lenz, Phys. Rev. D **76**, 065006 (2007); J. K. Parry, Phys. Rev. D **78**, 114023 (2008); M. Bobrowski, A. Lenz, J. Riedl, and J. Rohrwild, Phys. Rev. D **79**, 113006 (2009); K. Kawashima, J. Kubo, and A. Lenz, Phys. Lett. B **681**, 60 (2009); W. Altmannshofer, A. J. Buras, S. Gori, P. Paradisi, and D. M. Straub, Nucl. Phys. **B830**, 17 (2010); W. S. Hou, Y. Y. Mao, and C. H. Shen, Phys. Rev. D **82**, 036005 (2010).

[14] A. Hocker, H. Lacker, S. Laplace, and F. Le Diberder, Eur. Phys. J. C **21**, 225 (2001); J. Charles *et al.* (CKMfitter Group), Eur. Phys. J. C **41**, 1 (2005), http://ckmfitter.in2p3.fr/.

[15] A. Lenz *et al.*, Phys. Rev. D **83**, 036004 (2011).

[16] M. Bona *et al.*, arXiv:0906.0953, http://www.utfit.org/.

[17] E. Lunghi and A. Soni, Phys. Lett. B **697**, 323 (2011).

[18] P. H. Frampton and O. C. W. Kong, Phys. Rev. Lett. **75**, 781 (1995); **75**, 781 (1995); Phys. Rev. D **53**, R2293 (1996); P. H. Frampton and T. W. Kephart in [3].

[19] K. S. Babu, K. Kawashima, and J. Kubo, Phys. Rev. D **83**, 095008 (2011).

[20] T. Araki and J. Kubo, Int. J. Mod. Phys. A **24**, 5831 (2009).

[21] Z. z. Xing, H. Zhang, and S. Zhou, Phys. Rev. D **77**, 113016 (2008).

[22] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008), and 2009 partial update for the 2010 edition.

[23] J. Kubo, A. Mondragon, M. Mondragon, and E. Rodriguez-Jauregui, Prog. Theor. Phys. **109**, 795 (2003); **114**, 287(E) (2005); J. Kubo, Phys. Lett. B **578**, 156 (2004); **619**, 387(E) (2005).

[24] L. J. Hall, V. A. Kostelecky, and S. Raby, Nucl. Phys. **B267**, 415 (1986).

[25] F. Gabbiani and A. Masiero, Nucl. Phys. **B322**, 235 (1989).

[26] L. J. Hall, R. Rattazzi, and U. Sarid, Phys. Rev. D **50**, 7048 (1994).

[27] M. S. Carena, M. Olechowski, S. Pokorski, and C. E. M. Wagner, Nucl. Phys. **B426**, 269 (1994).

[28] S. Antusch and M. Spinrath, Phys. Rev. D **78**, 075020 (2008).

[29] A. Crivellin and U. Nierste, Phys. Rev. D **79**, 035018 (2009).

[30] J. Hisano, M. Nagai, and P. Paradisi, Phys. Rev. D **80**, 095014 (2009).

[31] A. Crivellin and U. Nierste, Phys. Rev. D **81**, 095007 (2010).

[32] A. Crivellin, L. Hofer, and J. Rosiek, arXiv:1103.4272.

[33] F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, Nucl. Phys. **B477**, 321 (1996).

[34] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. **267**, 195 (1996); P. Gondolo *et al.*, J. Cosmol. Astropart. Phys. 07 (2004) 008; P. Gondolo *et al.*, http://www.physto.se/edsjo/darksusy.

[35] M. Pospelov and A. Ritz, Ann. Phys. (N.Y.) **318**, 119 (2005).

[36] N. G. Deshpande and X. G. He, Phys. Rev. D **49**, 4812 (1994); N. G. Deshpande and X. G. He, Pramana **45**, S73 (1995).

[37] J. Polchinski and M. B. Wise, Phys. Lett. **125B**, 393 (1983).

[38] M. Endo and N. Yokozaki, J. High Energy Phys. 03 (2011) 130.

[39] M. Beneke, G. Buchalla, C. Greub, A. Lenz, and U. Nierste, Phys. Lett. B **459**, 631 (1999).

[40] M. Beneke, G. Buchalla, and I. Dunietz, Phys. Rev. D **54**, 4419 (1996).

[41] M. Beneke, G. Buchalla, C. Greub, A. Lenz, and U. Nierste, Nucl. Phys. **B639**, 389 (2002).

[42] M. Beneke, G. Buchalla, A. Lenz, and U. Nierste, Phys. Lett. B **576**, 173 (2003).

[43] M. Ciuchini, E. Franco, V. Lubicz, F. Mescia, and C. Tarantino, J. High Energy Phys. 08 (2003) 031.

[44] M. Gorbahn, S. Jager, U. Nierste, and S. Trine, arXiv:0901.2065.

[45] B. Altunkaynak, M. Holmes, and B. D. Nelson, J. High Energy Phys. 10 (2008) 013.

[46] F. Teubert, arXiv:1010.3696; O. Schneider (for the LHCb Collaboration), arXiv:1010.1851.