# Chromoelectric dipole moment of the top quark in models with vectorlike multiplets

Tarek Ibrahim<sup>1,\*</sup> and Pran Nath<sup>2,†</sup>

<sup>1</sup>Department of Physics, Faculty of Science, University of Alexandria, Alexandria, Egypt <sup>2</sup>Department of Physics, Northeastern University, Boston, Massachusetts 02115, USA

(Received 19 April 2011; published 5 July 2011)

The chromoelectric dipole moment of the top quark is calculated in a model with a vectorlike multiplet, which mixes with the third generation in an extension of the minimal supersymmetric standard model. Such mixings allow for new *CP* violating phases. Including these new *CP* phases, the chromoelectric dipole moment that generates an electric dipole of the top in this class of models is computed. The top chromoelectric dipole moment operator arises from loops involving the exchange of the *W*, the *Z*, as well as from the exchange involving the charginos, the neutralinos, the gluino, and the vectorlike multiplet and their superpartners. The analysis of the chromoelectric dipole moment operator of the top quark mass, cannot be ignored relative to the masses inside the loops. A numerical analysis is presented and it is shown that the contribution to the top electric dipole moment (EDM) could lie in the range  $(10^{-19}-10^{-18})$  ecm, consistent with the current limits on the EDM of the electron, the neutron and on atomic EDMs. A top EDM of size  $(10^{-19}-10^{-18})$  ecm could be accessible in collider experiments such as at the LHC and at the International Linear Collider.

DOI: 10.1103/PhysRevD.84.015003

PACS numbers: 13.40.Em, 11.30.Er, 14.65.Ha

#### I. INTRODUCTION

The electric dipole moment (EDM) of elementary particles provide an important window to possible new sources of *CP* violation (For recent reviews see [1]). This is so because in the standard model, the EDM of an elementary particle is rather small. Thus, for the top quark the EDM in the standard model is estimated to be less than  $10^{-30}$  ecm [2–4] and outside the realm of experiment in the foreseeable future (For a review of CP violation in top physics see [5]). However, much larger EDMs for elementary particles can arise in new physics models. One such model considered recently was where one has extra vectorlike generations, which can mix with the third generation [6-8]. Extra vectorlike generations can arise in many unified theories of particle physics [9,10] and if their masses lie in the TeV range they could mix with the third generation and produce observable effects. Such mixings are consistent with the current precision electroweak data [11], and thus the implications of such vector like multiplets have been analyzed in a number of works [12–20]. In [8], an analysis of the electric dipole operator for the top quark was given arising from the exchange of the extra vector like generations in the loops and it was found that a significantly larger EDM than in the standard model can arise for the top quark from such exchanges. In this work, we analyze the contribution to the chromoelectric dipole operator (CEDM) from the exchange of the vectorlike generations in the loops. Our analysis is done in an extension of the minimal supersymmetric standard model (MSSM),

including the extra vectorlike multiplets. The analysis shows that a top EDM as large as  $(10^{-19} - 10^{-18})$  ecm can arise from a constructive interference between the electric dipole moment and the chromoelectric dipole moment. A top EDM of this size lies within the realm of future experiment [21–24]. The role of EDMs in a variety of processes such as  $e^+e^- \rightarrow t\bar{t}$ ,  $\gamma\gamma \rightarrow t\bar{t}$  and other phenomena have been investigated by a number of authors [22,25–27,27–29], and thus the EDM of the top is of significant interest.

The outline of the rest of the paper is as follows: In Sec. II, we define the chromoelectric dipole moment of the quark and its connection with the electric dipole moment. In Sec. III, we give an analysis of the EDM of the top allowing for mixing between the vector like multiplet and the third generation quarks in the underlying model discussed in [8]. These mixings contain new sources of CP violation. Here, we compute the loops involving the exchanges of the W and the Z, of the charginos, of the neutralinos, of the gluino as well as exchanges involving the vectorlike multiplets and their superpartners. In Sec. IV, we discuss the parameter space of the model and list the new CP violating phases that enter in the analysis. A numerical analysis of the size of the EDM of the top is given in Sec. V. In this section, we also display the dependence of the top EMD on the CP phases arising from the mixings of the third generation quarks with the extra vector like generations. Conclusions are given in Sec. VI.

# II. CHROMOELECTRIC DIPOLE MOMENT OF THE TOP QUARK

The chromoelectric dipole moment  $\tilde{d}^C$  is defined in the effective dimension 5 operator

<sup>\*</sup>tarek-ibrahim@alex-sci.edu.eg

<sup>&</sup>lt;sup>†</sup>nath@neu.edu

TAREK IBRAHIM AND PRAN NATH

$$\mathcal{L}_{I} = -\frac{i}{2}\tilde{d}^{C}\bar{q}\sigma_{\mu\nu}\gamma_{5}T^{a}qG^{\mu\nu a}, \qquad (1)$$

where  $T^a$  are the SU(3) generators and  $G^{\mu\nu a}$  is the gluon field strength. The contribution of this operator to the EDM of quarks can be computed using dimensional analysis [30]. This technique can be expressed using the "reduced" coupling constant rule. Thus, the contribution of chromoelectric dipole moment operator to the EDM of the quarks is given as follows:

$$d^C = \frac{e}{4\pi} \tilde{d}^C.$$
 (2)

The alternative technique to estimate contributions of the chromoelectric operator is to use the QCD sum rules [31]. We note that the analysis of the top EDM is more complicated relative to EDM of the light quarks and of the light leptons (see e.g., [32,33]), because we cannot ignore the mass of the external fermion (i.e., of the top quark in this case) compared to the masses that run inside the loops. So, the form factors that enter the analysis of the top EDM are more complicated relative to the form factors that enter the loop integrals are functions of more than just one mass ratio.

## III. TOP CEDM FROM EXCHANGE OF VECTOR LIKE MULTIPLETS

Using the formalism of [8], one can compute the contributions to the chromoelectric dipole moment of the top quark. There are several contributions to it arising from the exchange of the charginos, of the neutralinos, of the gluinos, and of the W and Z boson. CP violation in these diagrams enters via the mass matrices involving the third generation and their mirrors, and similarly via the mass matrices involving their superpartners and via the interaction vertices. A full description of the CP phases and the dependence of CEDM on them is given in Sec. IV. We discuss now the various contributions to the CEDM of the top.

### A. Chargino exchange contribution

The chargino exchange contribution to the chromoelectric dipole moment of the top quark arises through the left loop diagram of Fig. 1. The relevant part of Lagrangian that generates this contribution is given by

$$-\mathcal{L}_{t-\tilde{b}-\chi^{+}} = \sum_{k=1}^{2} \sum_{i=1}^{2} \sum_{j=1}^{4} \bar{t}_{k} [\Gamma_{Lkji} P_{L} + \Gamma_{Rkji} P_{R}] \tilde{\chi}_{i}^{+} \tilde{b}_{j} + \text{H.c.}$$
(3)

where

$$\Gamma_{Lkji} = -g[V_{i2}^* \kappa_t D_{R1k}^{t*} \tilde{D}_{1j}^b - D_{R2k}^{t*} V_{i1}^* \tilde{D}_{4j}^b + D_{R2k}^{t*} \kappa_B V_{i2}^* \tilde{D}_{2j}^b],$$
  

$$\Gamma_{Rkji} = g[U_{i1} D_{L1k}^{t*} \tilde{D}_{1j}^b - D_{L1k}^{t*} \kappa_b U_{i2} \tilde{D}_{3j}^b - D_{L2k}^{t*} \kappa_T U_{i2} \tilde{D}_{4j}^b],$$
(4)



FIG. 1. Left: One loop contribution to the chromoelectric dipole moment of the top quark from the exchange of the chargino and from the exchange of sbottoms and mirror sbottoms. Right: Same as the left diagram, except that one has chromoelectric dipole moment arising from the exchange of the neutralinos and from the exchange of stops and mirror stops.

where  $\tilde{D}^b$  is the diagonalizing matrix of the 4 × 4 sbottom mixed with scalar mirrors mass<sup>2</sup> matrix as defined in the appendix of [8]. These elements contain *CP* violating phases can also contribute to the chromoelectric dipole moment of the top. The couplings  $\kappa_f$  are defined as

$$(\kappa_T, \kappa_b) = \frac{(m_T, m_b)}{\sqrt{2}M_W \cos\beta}, \qquad (\kappa_B, \kappa_t) = \frac{(m_B, m_t)}{\sqrt{2}M_W \sin\beta}.$$
 (5)

Here, U and V are the matrices that diagonalize the chargino mass matrix  $M_C$  so that

$$U^* M_C V^{-1} = \text{diag}(m_{\tilde{\chi}_1}^+, m_{\tilde{\chi}_2}^+).$$
(6)

Using the above interaction, we get from the left loop diagram of Fig. 1 the contribution

$$\tilde{d}^{C}(\chi^{+}) = \frac{g_{s}}{16\pi^{2}} \sum_{i=1}^{2} \sum_{j=1}^{4} \frac{m_{\chi_{i}^{+}}}{m_{\tilde{b}_{j}}^{2}} \operatorname{Im}(\Gamma_{L1ji}\Gamma_{R1ji}^{*}) I_{3}\left(\frac{m_{\chi_{i}^{+}}^{2}}{m_{\tilde{b}_{j}}^{2}}, \frac{m_{t_{1}}^{2}}{m_{\tilde{b}_{j}}^{2}}\right),$$
(7)

where  $I_3(r_1, r_2)$  is given by

$$I_3(r_1, r_2) = \int_0^1 dx \frac{x - x^2}{1 + (r_1 - r_2 - 1)x + r_2 x^2}.$$
 (8)

We note that the limit of  $I_3(r_1, r_2)$  for  $r_2 \sim 0$  is the well known form factors  $B(r_1)$  in the case of light quarks [33]. While our analysis is quite general, we will limit ourselves for simplicity to the case where there is mixing between the third generation and the mirror part of the vector multiplet. The inclusion of the nonmirror part is essentially trivial, as it corresponds to an extension of the CKM matrix from a  $3 \times 3$  to a  $4 \times 4$  matrix in the standard model sector, and similar straightforward extensions in the supersymmetric sector. In the rest of the analysis, we will focus just on the mixings with the mirrors, which is rather nontrivial.

### **B.** Neutralino exchange contribution

The neutralino exchange contribution to the chromoelectric dipole moment of the top quark through the right loop diagram of Fig. 1. The relevant part of Lagrangian that generates this contribution is given by

$$-\mathcal{L}_{t-\tilde{t}-\chi^{0}} = \sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{2} \bar{t}_{j} [C_{Ljki}P_{L} + C_{Rjki}P_{R}] \tilde{\chi}_{i}^{0} \tilde{t}_{k} + \text{H.c.},$$
(9)

where

$$C_{Ljki} = \sqrt{2} [\alpha_{ti} D_{R1j}^{t*} \tilde{D}_{1k}^{t} - \gamma_{ti} D_{R1j}^{t*} \tilde{D}_{3k}^{t} + \beta_{Ti} D_{R2j}^{t*} \tilde{D}_{4k}^{t} - \delta_{Ti} D_{R2j}^{t*} \tilde{D}_{2k}^{t}],$$
  
$$C_{Rjki} = \sqrt{2} [\beta_{ti} D_{L1j}^{t*} \tilde{D}_{1k}^{t} - \delta_{ti} D_{L1j}^{t*} \tilde{D}_{3k}^{t} + \alpha_{Ti} D_{L2j}^{t*} \tilde{D}_{4k}^{t} - \gamma_{Ti} D_{L2j}^{t*} \tilde{D}_{2k}^{t}].$$
(10)

The matrix  $\tilde{D}^t$  is the diagonalizing matrix of the 4 × 4 stop mixed with scalar mirrors mass<sup>2</sup> matrix as defined in the appendix of [8]. The couplings that enter the above equations are given by

$$\alpha_{tj} = \frac{gm_t X_{4j}}{2m_W \sin\beta}, \quad \beta_{tj} = \frac{2}{3} eX_{1j}^{\prime*} + \frac{g}{\cos\theta_W} X_{2j}^{\prime*} \left(\frac{1}{2} - \frac{2}{3} \sin^2\theta_W\right)$$
$$\gamma_{tj} = \frac{2}{3} eX_{1j}^{\prime} - \frac{2}{3} \frac{g\sin^2\theta_W}{\cos\theta_W} X_{2j}^{\prime}, \quad \delta_{tj} = -\frac{gm_t X_{4j}^*}{2m_W \sin\beta}. \tag{11}$$

Here,

$$\begin{aligned} \alpha_{Tj} &= \frac{gm_T X_{3j}^*}{2m_W \cos\beta}, \\ \beta_{Tj} &= -\frac{2}{3} e X_{1j}' + \frac{g}{\cos\theta_W} X_{2j}' \left( -\frac{1}{2} + \frac{2}{3} \sin^2\theta_W \right), \\ \gamma_{Tj} &= -\frac{2}{3} e X_{1j}'^* + \frac{2}{3} \frac{g \sin^2\theta_W}{\cos\theta_W} X_{2j}'^*, \\ \delta_{Tj} &= -\frac{gm_T X_{3j}}{2m_W \cos\beta}, \end{aligned}$$
(12)

where

$$X'_{1j} = (X_{1j}\cos\theta_W + X_{2j}\sin\theta_W),$$
  

$$X'_{2j} = (-X_{1j}\sin\theta_W + X_{2j}\cos\theta_W),$$
(13)

and where the matrix X diagonlizes the neutralino mass matrix so that

$$X^{T}M_{\tilde{\chi}^{0}}X = \operatorname{diag}(m_{\chi_{1}^{0}}, m_{\chi_{2}^{0}}, m_{\chi_{3}^{0}}, m_{\chi_{4}^{0}}).$$
(14)

Using the above interaction, we get from the right loop diagram Fig. 1 the neutralino contributions to the top chromoelectric dipole moment to be

$$\tilde{d}^{C}(\chi^{0}) = \frac{g_{s}}{16\pi^{2}} \sum_{i=1}^{4} \sum_{k=1}^{4} \frac{m_{\chi_{i}^{0}}}{m_{\tilde{t}_{k}}^{2}} \operatorname{Im}(C_{L1ki}C_{R1ki}^{*}) I_{3}\left(\frac{m_{\chi_{i}^{0}}^{2}}{m_{\tilde{t}_{k}}^{2}}, \frac{m_{t_{1}}^{2}}{m_{\tilde{t}_{k}}^{2}}\right).$$
(15)

#### C. Gluino exchange contribution

The gluino contribution to the chromoelectric dipole moment of the top comes from the two loop diagrams of Fig. 2. The relevant part of Lagrangian that generates this contribution is given by

$$-\mathcal{L}_{t\tilde{t}\tilde{g}} = \sqrt{2}g_s \sum_{a=1}^{8} \sum_{j,k=1}^{3} \sum_{n=1}^{2} \sum_{m=1}^{4} \times T_{jk}^a \tilde{t}_n^j [K_{L_{nm}} P_L + K_{R_{nm}} P_R] \tilde{g}_a \tilde{t}_m^k + \text{H.c.}$$
(16)

where

$$K_{L_{nm}} = e^{-i\xi_3/2} [D_{R_{2n}}^{t*} \tilde{D}_{4m}^t - D_{R_{1n}}^{t*} \tilde{D}_{3m}^t],$$
  

$$K_{R_{nm}} = e^{i\xi_3/2} [D_{L_{1n}}^{t*} \tilde{D}_{1m}^t - D_{L_{2n}}^{t*} \tilde{D}_{2m}^t],$$
(17)

where  $\xi_3$  is the phase of the gluino mass.

The above Lagrangian gives a contribution

$$\tilde{d}^{C}(\tilde{g}) = \frac{g_{s}\alpha_{s}}{12\pi} \sum_{j=1}^{4} \frac{m_{\tilde{g}}}{m_{\tilde{t}_{j}}^{2}} \operatorname{Im}(K_{L_{1j}}K_{R_{1j}}^{*}) I_{5}\left(\frac{m_{\tilde{g}}^{2}}{m_{\tilde{t}_{j}}^{2}}, \frac{m_{\tilde{t}_{1}}^{2}}{m_{\tilde{t}_{j}}^{2}}\right), \quad (18)$$

where  $I_5(r_1, r_2)$  is given by

$$I_5(r_1, r_2) = \int_0^1 dx \frac{x + 8x^2}{1 + (r_1 - r_2 - 1)x + r_2 x^2}.$$
 (19)

We note that the limit of  $I_5(r_1, r_2)$  for  $r_2 \sim 0$  is the well known form factors  $3C(r_1)$  in the case of light quarks [33].

### **D.** *W* and *Z* exchange contributions

The *W* boson exchange contribution to the chromoelectric dipole moment of the top quark arises through the left loop diagram of Fig. 3. The relevant part of Lagrangian that generates this contribution is given by

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W^{+}_{\mu} \sum_{i} \sum_{j} \bar{t}_{j} \gamma^{\mu} [D^{**}_{L1j} D^{b}_{L1i} P_{L} + D^{**}_{R2j} D^{b}_{R2i} P_{R}] b_{i} + \text{H.c.}$$
(20)

where *i*, *j* run over the set of quarks and mirror quarks, including those from the third generation and from the vector multiplet,  $t_1$  is the physical top quark, and  $D_{L,R}^{t,b}$  are the diagonalizing matrices defined in the appendix of [8]. These matrices contain phases, and these phases



FIG. 2. Left: One loop contribution to the chromoelectric dipole moment of the top quark from gluino exchange and from the exchange of stops and mirror stops. Here, the external gluon line connects to the stops and mirror stops in the loop. Right: Same as the left diagram except that the external gluon line connects to gluinos in the loop, i.e., one has a gluino-gluino-gluon vertex in this case.

generate the chromoelectric dipole moment of the top quark. Using the above interaction, we get from the left loop diagram of Fig. 3, the contribution

$$\tilde{d}^{C}(W^{+}) = \frac{g_{s}}{16\pi^{2}M_{W}^{2}} \sum_{i=1}^{2} m_{b_{i}} \operatorname{Im}(\Gamma_{i}^{tb}) I_{1}\left(\frac{m_{b_{i}}^{2}}{M_{W}^{2}}, \frac{m_{t_{1}}^{2}}{M_{W}^{2}}\right).$$
(21)

Here  $\Gamma_i^{tb}$  is given

$$\Gamma_i^{tb} = \frac{g^2}{2} D_{L11}^{t*} D_{L1i}^b D_{R21}^t D_{R2i}^{b*}, \qquad (22)$$

and  $I_1(r_1, r_2)$  is given by

$$I_1(r_1, r_2) = \int_0^1 dx \frac{(4+r_1-r_2)x - 4x^2}{1+(r_1-r_2-1)x + r_2x^2}.$$
 (23)

Finally, we consider the right loop of Fig. 3, which produces the chromoelectric dipole moment of the top quark through the interaction with the Z boson. The relevant part of Lagrangian that generates this contribution is given by

$$\mathcal{L}_{\rm NC} = -Z_{\mu} \sum_{i=1}^{2} \sum_{j=1}^{2} \bar{t}_{j} \gamma^{\mu} [S_{Lji} P_{L} + S_{Rji} P_{R}] t_{i}, \quad (24)$$

where

$$S_{Lji} = -\frac{g}{6\cos\theta_W} [-3D_{L1j}^{t*}D_{L1i}^t + 4\sin^2\theta_W (D_{L1j}^{t*}D_{L1i}^t + D_{L2j}^{t*}D_{L2i}^t)],$$
  

$$S_{Rji} = -\frac{g}{6\cos\theta_W} [-3D_{R2j}^{t*}D_{R2i}^t + 4\sin^2\theta_W (D_{R1i}^{t*}D_{R1i}^t + D_{R2j}^{t*}D_{R2i}^t)].$$
 (25)

Using the above interaction, we get from the right loop of Fig. 3, the contribution

$$\tilde{d}^{C}(Z) = \frac{g_{s}}{16\pi^{2}M_{Z}^{2}} \sum_{i=1}^{2} m_{t_{i}} \operatorname{Im}(S_{L1i}S_{R1i}^{*}) I_{1}\left(\frac{m_{t_{i}}^{2}}{M_{Z}^{2}}, \frac{m_{t_{1}}^{2}}{M_{Z}^{2}}\right).$$
(26)

The total chromoelectric dipole moment of the top in the model is then given by the sum of the contributions computed in this section so that



FIG. 3. Left: One loop contribution to the chromoelectric dipole moment of the top quark from  $W^+$  exchange and from the exchange of the bottom quark and from the mirror bottom. Right: Same as the left diagram except that one has chromoelectric dipole moment arising from *Z* exchange and from the exchange of the top and from the mirror top.

$$\tilde{d}^{C} = \tilde{d}^{C}(\chi^{+}) + \tilde{d}^{C}(\chi^{0}) + \tilde{d}^{C}(\tilde{g}) + \tilde{d}^{C}(W^{+}) + \tilde{d}^{C}(Z).$$
(27)

# IV. PARAMETER SPACE OF THE MODEL AND *CP* PHASES

The mass matrices for quarks and mirrors including their mixings are diagonalized using bi-unitary transformations  $D_L^b$  and  $D_R^b$  for the bottom quarks and mirrors and  $D_L^t$  and  $D_R^t$  for the diagonalization of the top quarks and mirrors. We parametrize  $D_L^t$  and  $D_R^t$  as follows:

$$D_{L}^{t} = \begin{pmatrix} \cos\theta_{L} & -\sin\theta_{L}e^{-i\chi_{L}} \\ \sin\theta_{L}e^{i\chi_{L}} & \cos\theta_{L} \end{pmatrix},$$
$$D_{R}^{t} = \begin{pmatrix} \cos\theta_{R} & -\sin\theta_{R}e^{-i\chi_{R}} \\ \sin\theta_{R}e^{i\chi_{R}} & \cos\theta_{R} \end{pmatrix}.$$
(28)

Thus, the mixing between t and T is parameterized by the angles  $\theta_L$ ,  $\theta_R$ ,  $\chi_L$  and  $\chi_R$  where the angles  $\theta_L$ ,  $\theta_R$  are given by

$$\tan 2\theta_L = \frac{2|m_t h_5^* - m_T h_3|}{m_t^2 + |h_3|^2 - m_T^2 - |h_5|^2},$$

$$\tan 2\theta_R = \frac{2|-m_t h_3 + m_T h_5^*|}{m_t^2 + |h_5|^2 - m_T^2 - |h_3|^2},$$
(29)

and  $\chi_L$  and  $\chi_R$  are the *CP* violating phases defined by

$$\chi_R = \arg(-m_t h_3 + m_T h_5^*), \chi_L = \arg(m_t h_5^* - m_T h_3).$$
(30)

Similarly,  $D_L^b$  and  $D_R^b$  are given by

$$D_{L}^{b} = \begin{pmatrix} \cos\theta_{L} & -\sin\phi_{L}e^{-i\xi_{L}} \\ \sin\phi_{L}e^{i\xi_{L}} & \cos\phi_{L} \end{pmatrix},$$
$$D_{R}^{b} = \begin{pmatrix} \cos\phi_{R} & -\sin\phi_{R}e^{-i\xi_{R}} \\ \sin\phi_{R}e^{i\xi_{R}} & \cos\phi_{R} \end{pmatrix},$$
(31)

where the mixing between *b* and *B* is parametrized by the angle  $\phi_L$ ,  $\phi_R$ ,  $\xi_L$  and  $\xi_R$ . Here, the angles  $\phi_L$  and  $\phi_R$  are given by

$$\tan 2\phi_L = \frac{2|m_b h_4^* + m_B h_3|}{m_b^2 + |h_3|^2 - m_B^2 - |h_4|^2},$$
  
$$\tan 2\phi_R = \frac{2|m_b h_3 + m_B h_4^*|}{m_b^2 + |h_4|^2 - m_B^2 - |h_3|^2}$$
(32)

and the phases  $\xi_{L,R}$  arise from the couplings  $h_4$  and  $h_3$  through the relations

$$\xi_R = \arg(m_b h_3 + m_B h_4^*), \qquad \xi_L = \arg(m_b h_4^* + m_B h_3).$$
(33)

For the case of top and bottom masses arising from hermitian matrices, i.e., when  $h_5 = -h_3^*$  and  $h_4 = h_3^*$ , we have  $\theta_L = \theta_R$ ,  $\phi_L = \phi_R$ ,  $\chi_L = \chi_R = \chi$  and  $\xi_L = \xi_R = \xi$ . Further, here we have the relation  $\xi = \chi + \pi$  and thus the W-exchange and the Z-exchange terms in the EDM for the top vanish. However, more generally, the top and the bottom mass matrices are not hermitian and they generate nonvanishing contributions to the EDMs. Thus, the input parameters for this sector of the parameter space are  $m_{t1}$ ,  $m_T$ ,  $h_3$ ,  $h_5$ ,  $m_{b1}$ ,  $m_B$ ,  $h_4$  with  $h_3$ ,  $h_4$  and  $h_5$  being complex masses with the corresponding *CP* violating phases  $\chi_3$ ,  $\chi_4$ and  $\chi_5$ . For the sbottom and stop mass<sup>2</sup> matrices, we need the extra input parameters of the susy breaking sector,  $\tilde{M}_q$ ,  $\tilde{M}_B$ ,  $\tilde{M}_b$ ,  $\tilde{M}_Q$ ,  $\tilde{M}_t$ ,  $\tilde{M}_T$ ,  $A_b$ ,  $A_T$ ,  $A_t$ ,  $A_B$ ,  $\mu$ ,  $\tan\beta$ . The chargino, neutralino, and gluino sectors need the extra parameters  $\tilde{m}_1$ ,  $\tilde{m}_2$  and  $m_{\tilde{g}}$ . We will assume that the only parameters that have phases in the above set are  $A_T$ ,  $A_B$ ,  $A_t$ , and  $A_b$  with the corresponding phases given by  $\alpha_T$ ,  $\alpha_B$ ,  $\alpha_t$ , and  $\alpha_b$ .

### V. NUMERICAL ESTIMATE OF THE CEDM OF THE TOP

To simplify the analysis further, we set some of the phases to zero, i.e., specifically, we set  $\alpha_t = \alpha_b = 0$ . With this in mind, the only contributions to the chromoelectric dipole moment CEDM of the top quark arises from mixing terms between the scalars and the mirror scalars, between the fermions—and the mirror fermions, and finally, among the mirror scalars themselves. Thus, in the absence of the mirror part of the lagrangian, the top CEDM vanishes and so we can isolate the role of the *CP* violating phases in this sector and see the size of its contribution. The  $4 \times 4$  mass<sup>2</sup> matrices of stops and sbottoms are diagonlized numerically. Thus, the *CP* violating phases that would play a role in this analysis are

$$\chi_3, \ \chi_4, \ \chi_5, \ \alpha_T, \ \alpha_B. \tag{34}$$

To reduce the number of input parameters, we assume  $\tilde{M}_a = m_0, a = q, B, b, Q, T, t \text{ and } |A_i| = |A_0|, i = T, B,$ t, b. In the left panel of Fig. 4, we give a numerical analysis of the top EDM and discuss its variation with the phase  $\alpha_B$ . We note that the only component that varies with this phase is the chargino component. This is expected since  $\alpha_{R}$ enters the scalar bottom mass<sup>2</sup> matrix and the chargino contribution to the EDM is controlled by  $\tilde{D}^b$  which depends on  $\alpha_B$  while the other contributions are independent of this phase. Further, the chargino component exhibits a minimum where the different terms of it can have destructive cancellation. In the right panel of Fig. 4, we study the variation of the different components of  $d_t$  on the phase  $\alpha_T$ . We observe that the components that vary with this phase are the neutralino and the gluino contributions while the W, Z and chargino contributions have no dependence on this phase. The reason for the above is that  $\alpha_T$  enters the scalar top mass<sup>2</sup> matrix and the EDM arising from W, Z and chargino exchanges are independent of  $\tilde{D}^t$ . However, the neutralino and the gluino contributions are affected by it. It is clear that we see here the cancellation mechanism [32–35] working since the components are close to each other with different signs, so we have the possibility of a destructive cancellation.

In the left panel of Fig. 5, we show the behavior of the different components of the chromoelectric dipole moment contributions to the top EDM as a function of the phase  $\chi_3$ . We note that  $\chi_3$  enters  $D^t$ ,  $D^b$ ,  $\tilde{D}^t$  and  $\tilde{D}^b$ , and as a consequence, all diagrams in Fig. 1–3 that contribute to the top EDM have a  $\chi_3$  dependence. Further, the various diagrams that contribute to the top EDM may add constructively or destructively as shown in the *Z*, *W*, neutralino and chargino contributions. In the case of destructive



FIG. 4 (color online). Left: An exhibition of the dependence of  $d_t$  on  $\alpha_B$  when  $\tan\beta = 5$ ,  $m_T = 250$ ,  $|h_3| = 70$ ,  $|h_4| = 80$ ,  $m_B = 120$ ,  $|h_5| = 90$ ,  $m_0 = 220$ ,  $|A_0| = 200$ ,  $\tilde{m}_1 = 50$ ,  $\tilde{m}_2 = 100$ ,  $\mu = 150$ ,  $\tilde{m}_g = 350$ ,  $\chi_4 = 0.3$ ,  $\chi_5 = -0.8$ ,  $\alpha_T = 0.4$ , and  $\chi_3 = 0.4$ . (The six curves correspond to the contributions from the *Z*, *W*, neutralino, chargino, gluino and total CEDM. They are shown in ascending order at  $\alpha_B = 0$ .) Here and in subsequent figures all masses are in GeV and all angles are in rad. Right: An exhibition of the dependence of  $d_t$  on  $\alpha_T$  when  $\tan\beta = 25$ ,  $m_T = 200$ ,  $|h_3| = 85$ ,  $|h_4| = 75$ ,  $m_B = 150$ ,  $|h_5| = 85$ ,  $m_0 = 200$ ,  $|A_0| = 200$ ,  $\tilde{m}_1 = 50$ ,  $\tilde{m}_2 = 100$ ,  $\mu = 150$ ,  $\tilde{m}_g = 400$ ,  $\chi_4 = 0.5$ ,  $\chi_5 = 0.7$ ,  $\chi_3 = 0.8$ , and  $\alpha_B = 0.2$ . (The six curves correspond to the contributions from the neutralino, *Z*, chargino, *W*, total CEDM and gluino. They are shown in ascending order at  $\alpha_T = 0$ .)



FIG. 5 (color online). Left: An exhibition of the dependence of  $d_t$  on  $\chi_3$  when  $\tan\beta = 10$ ,  $m_T = 150$ ,  $|h_3| = 75$ ,  $|h_4| = 90$ ,  $m_B = 180$ ,  $|h_5| = 80$ ,  $m_0 = 300$ ,  $|A_0| = 300$ ,  $\tilde{m}_1 = 50$ ,  $\tilde{m}_2 = 100$ ,  $\mu = 150$ ,  $\tilde{m}_g = 400$ ,  $\chi_4 = 0.7$ ,  $\chi_5 = -0.4$ ,  $\alpha_T = 0.2$ , and  $\alpha_B = 0.7$ . (The six curves correspond to the contributions from the *Z*, neutralino, *W*, chargino, gluino and total CEDM. They are shown in ascending order at  $\chi_3 = 0$ ). Right: An exhibition of the dependence of  $d_t$  on  $\chi_4$  when  $\tan\beta = 15$ ,  $m_T = 350$ ,  $|h_3| = 80$ ,  $|h_4| = 70$ ,  $m_B = 200$ ,  $|h_5| = 100$ ,  $m_0 = 400$ ,  $|A_0| = 400$ ,  $\tilde{m}_1 = 50$ ,  $\tilde{m}_2 = 100$ ,  $\mu = 150$ ,  $\tilde{m}_g = 300$ ,  $\chi_3 = 0.6$ ,  $\chi_5 = 0.8$ ,  $\alpha_T = 0.7$ , and  $\alpha_B = 0.2$ . (The six curves correspond to the contributions from the *Z*, neutralino, chargino, *W*, total CEDM and gluino. They are shown in ascending order at  $\chi_4 = 0$ .)

interference, we have large cancellations again reminiscent of the cancellation mechanism for the EDM of the electron and for the neutron [32–35]. Of course, the desirable larger contributions for the top EDM occur away from the cancellation regions. In the right panel of Fig. 5, we study the variation of the different components of  $d_t$  as the magnitude of the phase  $\chi_4$  varies. The sparticle masses and couplings in the bottom sector and thus the top EDM arising from the exchange of the W and the charginos are sensitive to  $\chi_4$ , and thus only these two contributions to the top EDM have dependence on this parameter. In Fig. 6, we study the variation of the different components of  $d_t$  as the phase  $\chi_5$  changes. This phase enters the top quark mass matrix and the scalar top mass<sup>2</sup> matrix and consequently the matrices  $D_{L,R}^t$  and  $\tilde{D}^t$ . Thus, the contributions to the EDM of the top arising from the W, Z, neutralino, chargino, and gluino exchanges all have a dependence on  $\chi_5$  as exhibited in Fig. 6.

A comparison between the contributions of the chromoelectric dipole moment operator of the top EDM and that of the electric dipole moment operator [8], shows that they could be the same order of magnitude with like or unlike signs. That would provide an extra element for constructive or destructive interference of EDM components. To exhibit this, we give in Table I the values of EDM for the top quark coming from the electric dipole moment operator and the chromoelectric dipole moment operator. The first entry of Table I shows a destructive interference between the electric and the chromoelectric dipole moments, while the last two entries show a constructive interference. With constructive interference, a value of the top EDM as large as  $\sim 6 \times 10^{-19}$  ecm in magnitude (see the middle entry) can be gotten. It is very possible that a full search of the parameter space of phases can lead to a top EDM of size  $O(10^{-18})$  ecm.

Table caption: A sample illustration of the electric and chromoelectric dipole operator contributions to the electric dipole moment of the top quark. The inputs are:  $m_T = 350$ ,  $|h_3| = 100$ ,  $|h_4| = 175$ ,  $m_B = 100$ ,  $|h_5| = 190$ ,  $m_0 = 200$ ,  $|A_0| = 200$ ,  $\tilde{m}_1 = 50$ ,  $\tilde{m}_2 = 100$ ,  $\mu = 150$ ,  $\tilde{m}_g = 450$  and  $\tan\beta = 5$  (top row), 30 (middle row), 40 (bottom row). All masses are in units of GeV and all angles are in radian.

Constraints on the top chromo EDM have been obtained using the combined CDF and D0 data and the CMS and



FIG. 6 (color online). An exhibition of the dependence of  $d_t$  on  $\chi_5$  when  $\tan\beta = 20$ ,  $m_T = 300$ ,  $|h_3| = 90$ ,  $|h_4| = 85$ ,  $m_B = 250$ ,  $|h_5| = 95$ ,  $m_0 = 100$ ,  $|A_0| = 200$ ,  $\tilde{m}_1 = 50$ ,  $\tilde{m}_2 = 100$ ,  $\mu = 150$ ,  $\tilde{m}_g = 300$ ,  $\chi_4 = -0.6$ ,  $\chi_3 = 0.4$ ,  $\alpha_T = 0.7$ , and  $\alpha_B = 0.4$ . (The six curves correspond to the contributions from the W, Z, neutralino, chargino, gluino and total CEDM. They are shown in ascending order at  $\chi_5 = 0.$ )

 TABLE I.
 Electric and chromoelectric dipole operator contributions.

X3	$\chi_4$	X5	$\alpha_T$	$\alpha_B$	$d_t^E$ e.cm	$d_t^C$ e.cm
0.3	-0.5	1.0	0.8	-0.4	$8.04 \times 10^{-19}$	$-9.8 \times 10^{-19}$
0.8	0.4	-1.5	-0.6	0.3	$-1.57 \times 10^{-19}$	$-4.6 \times 10^{-19}$
-0.3	1.5	0.1	0.5	-1.2	$-1.73 \times 10^{-19}$	$-9.4 \times 10^{-19}$

ATLAS data on the total  $t\bar{t}$  pair production cross section in [36,37]. Further, it is shown in [38,39] that with 10 fb<sup>-1</sup> of data at  $\sqrt{s} = 14$  TeV at the LHC a 5 $\sigma$  statistical sensitivity to a top quark chromo electric dipole moment of about  $5 \times 10^{-18}$  g<sub>s</sub>.cm can be reached.

# VI. CONCLUSION

Currently, the physics at the TeV scale is largely unknown, and it is hoped that the LHC will provide us with an insight in this energy regime. It is fully conceivable that this energy regime contains extra anomaly free vectorlike quark multiplets, which can mix with the third generation. In this work, we analyze the effect of this mixing on the chromoelectric dipole moment of the top quark. In this case, one finds that there are contributions that arise from the exchange of the extra vectorlike multiplets in the loops. We specifically focus on the exchange of the mirrors since their exchange can produce more dramatic contributions. Several sets of diagrams were computed for this analysis. These include the chargino exchange, the neutralino exchange, the gluino exchange, as well as exchange of the W and the Z bosons. In the analysis, new sources of CPviolation enter. They arise from the complex mixing parameters of the third generation with the mirrors and from the soft parameter involving interactions of the third generation with the mirrors. Numerical analysis shows that an EDM as large as  $10^{-18}$  ecm can be obtained for the top quark from the electric and chromoelectric dipole contributions. An EDM of this size could be accessible in future experiments such as at the ILC.

### ACKNOWLEDGMENTS

We thank German Valencia for bringing to our attention the works of [38,39]. One of us (T. I.) acknowledges support from the D. Sc. committee at Alexandria University. This research is supported in part by NSF Grants No. PHY-0757959 and No. PHY-0704067.

- T. Ibrahim and P. Nath, Rev. Mod. Phys. **80**, 577 (2008); arXiv:hep-ph/0210251; J.R. Ellis, J.S. Lee, and A. Pilaftsis, J. High Energy Phys. 10 (2008) 049; M. Pospelov and A. Ritz, Ann. Phys. (N.Y.) **318**, 119 (2005).
- [2] F. Hoogeveen, Nucl. Phys. B341, 322 (1990); M.E. Pospelov and I.B. Khriplovich, Yad. Fiz. 53, 1030 (1991) [Sov. J. Nucl. Phys. 53, 638 (1991)].
- [3] A. Soni and R. M. Xu, Phys. Rev. Lett. 69, 33 (1992).
- [4] The analysis of [2] was for the electron and the EDM of the top is obtained by scaling as noted in [3].
- [5] D. Atwood, S. Bar-Shalom, G. Eilam, and A. Soni, Phys. Rep. 347, 1 (2001).
- [6] T. Ibrahim and P. Nath, Phys. Rev. D 78, 075013 (2008); arXiv:0806.3880; Nucl. Phys. B, Proc. Suppl. 200–202, 161 (2010).
- [7] T. Ibrahim and P. Nath, Phys. Rev. D 81, 033007 (2010).
- [8] T. Ibrahim and P. Nath, Phys. Rev. D 82, 055001 (2010).
- [9] H. Georgi, Nucl. Phys. B156, 126 (1979); F. Wilczek and A. Zee, Phys. Rev. D 25, 553 (1982); J. Maalampi, J. T. Peltoniemi, and M. Roos, Phys. Lett. B 220, 441 (1989); J. Maalampi and M. Roos, Phys. Rep. 186, 53 (1990); K. S. Babu, I. Gogoladze, P. Nath, and R. M. Syed, Phys. Rev. D 74, 075004 (2006); 74, 075004 (2006); P. Nath and R. M. Syed, Phys. Rev. D 81, 037701 (2010).
- [10] G. Senjanovic, F. Wilczek, and A. Zee, Phys. Lett. B 141, 389 (1984).
- M. Jezabek and J. H. Kuhn, Phys. Lett. B 329, 317 (1994);
   C. A. Nelson, B. T. Kress, M. Lopes, and T. P. McCauley,

Phys. Rev. D **56**, 5928 (1997); V.M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **100**, 062004 (2008).

- [12] V. Barger, J. Jiang, P. Langacker, and T. Li, Int. J. Mod. Phys. A 22, 6203 (2007).
- [13] L. Lavoura and J. P. Silva, Phys. Rev. D 47, 1117 (1993).
- [14] N. Maekawa, Phys. Rev. D 52, 1684 (1995).
- [15] D.E. Morrissey and C.E.M. Wagner, Phys. Rev. D 69, 053001 (2004).
- [16] D. Choudhury, T. M. P. Tait, and C. E. M. Wagner, Phys. Rev. D 65, 053002 (2002).
- [17] C. Liu, Phys. Rev. D 80, 035004 (2009).
- [18] K. S. Babu, I. Gogoladze, M. U. Rehman, and Q. Shafi, Phys. Rev. D 78, 055017 (2008).
- [19] S. P. Martin, Phys. Rev. D 81, 035004 (2010); 82, 055019 (2010); 83, 035019 (2011).
- [20] P.W. Graham, A. Ismail, S. Rajendran, and P. Saraswat, Phys. Rev. D 81, 055016 (2010).
- [21] R. Frey et al., arXiv:hep-ph/9704243.
- [22] D. Atwood, A. Aeppli, and A. Soni, Phys. Rev. Lett. 69, 2754 (1992).
- [23] P. Poulose and S.D. Rindani, Phys. Rev. D 57, 5444 (1998); 61, 119902(E) (2000).
- [24] S. Y. Choi and K. Hagiwara, Phys. Lett. B 359, 369 (1995).
- [25] A. Soni and R. M. Xu, Phys. Rev. D 45, 2405 (1992).
- [26] A. Bartl, E. Christova, T. Gajdosik, and W. Majerotto, Nucl. Phys. B, Proc. Suppl. 66, 75 (1998).
- [27] W. Hollik, J. I. Illana, S. Rigolin, C. Schappacher, and D. Stockinger, Nucl. Phys. B551, 3 (1999); B557, 407(E) (1999).

- [28] H. Novales-Sanchez and J. J. Toscano, AIP Conf. Proc. 1116, 443 (2009).
- [29] C.S. Huang and T.J. Li, Z. Phys. C 68, 319 (1995).
- [30] A. Manohar and H. Georgi, Nucl. Phys. B234, 189 (1984).
- [31] I.B. Khriplovich and K.N. Zyablyuk, Phys. Lett. B 383, 429 (1996).
- [32] T. Ibrahim and P. Nath, Phys. Lett. B 418, 98 (1998).
- [33] T. Ibrahim and P. Nath, Phys. Rev. D 57, 478 (1998).
- [34] T. Ibrahim and P. Nath, Phys. Rev. D 58, 111301 (1998);
  58, 111301 (1998); T. Falk and K. A. Olive, Phys. Lett. B
  439, 71 (1998); M. Brhlik, G. J. Good, and G. L. Kane, Phys. Rev. D 59, 115004 (1999); A. Bartl, T. Gajdosik, W. Porod, P. Stockinger, and H. Stremnitzer, Phys. Rev. 60, 073003 (1999); S. Pokorski, J. Rosiek, and C. A. Savoy, Nucl. Phys. B570, 81 (2000); E. Accomando, R. Arnowitt, and B. Dutta, Phys. Rev. D 61, 115003 (2000); U. Chattopadhyay, T. Ibrahim, and D. P. Roy, Phys. Rev. D
  64, 013004 (2001); C. S. Huang and W. Liao, Phys. Rev. D

**61**, 116002 (2000); **62**, 016008 (2000); M. Brhlik, L. Everett, G. Kane, and J. Lykken, Phys. Rev. Lett. **83**, 2124 (1999); Phys. Rev. D **62**, 035005 (2000); T. Ibrahim and P. Nath, Phys. Rev. D **61**, 093004 (2000); T. Ibrahim, Phys. Rev. D **64**, 035009 (2001); T. Falk, K. A. Olive, M. Prospelov, and R. Roiban, Nucl. Phys. **B560**, 3 (1999); V. D. Barger, T. Falk, T. Han, J. Jiang, T. Li, and T. Plehn, Phys. Rev. D **64**, 056007 (2001); T. Ibrahim and P. Nath, Phys. Rev. D **67**, 016005 (2003).

- [35] Y. Li, S. Profumo, and M. Ramsey-Musolf, J. High Energy Phys. 08 (2010) 062.
- [36] Z. Hioki and K. Ohkuma, arXiv:1104.1221.
- [37] Z. Hioki and K. Ohkuma, Eur. Phys. J. C 71, 1 (2011).
- [38] S. K. Gupta, A. S. Mete, and G. Valencia, Phys. Rev. D 80, 034013 (2009), and Private Communication with German Valencia.
- [39] O. Antipin and G. Valencia, Phys. Rev. D **79**, 013013 (2009).