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# Threshold effects in the decay of heavy $b'$ and $t'$ quarks

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A sequential fourth generation is still viable, but the $t'$ and $b'$ quarks are constrained not to be too far apart in mass. The $t' \to bW$ and $b' \to tW$ decay channels are still being pursued at the Tevatron, which will soon be surpassed by the LHC. We use a convolution method with up to five-body final states to study $t'$ and $b'$ decays. We show how the two decay branches for $m_{b'}$ below the $tW$ threshold, $b' \to tW^*$ and $t^*W$, merge with $b' \to tW$ above the threshold. We then consider the heavy-to-heavy transitions $b' \to t'^{(*)}W^{(*)}$ (or $t' \to b'^{(*)}W^{(*)}$), as they are not suppressed by quark mixing. We find that, because of the threshold sensitivity of the branching fraction of $t' \to b'W^*$ (or $b' \to t'W^*$), it is possible to measure the strength of the CKM mixing element $V_{t'b}$ (or $V_{tb'}$), especially when it is rather small. We urge experimenters to pursue and separate the $t' \to b'W^*$ (or $b' \to t'W^*$) decay in their search programs.

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## I. INTRODUCTION

As first pointed out by Kobayashi and Maskawa [1] (KM), if nature possesses three generations of quarks, then there would exist an irremovable *CP* violating phase in the charge current. With the emergence of the $\tau$ lepton and the $b$ quark, this picture quickly became the basis for the flavor part of the standard model (SM). Remarkably, this KM theory can explain all phenomena observed so far, culminating in the confirmation of the *CP* phase by the B factory experiments in 2001 [2]. But the 3 generation SM cannot be a complete theory even in regards to *CP* violation, as it falls far short of what is needed for the matter dominance of our universe. However, extending to a fourth generation of quarks, which does not add any new dynamics to the SM, one could attain enough *CP* violation for matter dominance [3]. In any case, despite the usual prejudice, a fourth generation of quarks is still quite viable [4,5]. Throughout this report, we use $t'$ and $b'$ to represent the sequential up- and down-type fourth generation quarks, respectively.

The Tevatron has held the energy frontier for two decades, but was surpassed by the LHC at the end of 2009. Utilizing the collision of $p$ and $\bar{p}$ beams at $\sqrt{s} = 1.96$ TeV, the CDF and D0 experiments have performed direct searches [2] for the fourth generation quarks. The best limits depend on the search channel. For the search of a top-like heavy quark, i.e. $t' \to qW$, CDF analyzed 4.6 fb$^{-1}$ collected data, giving a mass limit [6] of $m_{t'} > 335$ GeV at 95% confidence level (CL). This has been updated recently to 5.6 fb$^{-1}$, with or without tagging for a $b$-quark jet. The limit obtained [7] for $t' \to bW$ is at 358 GeV, while for $t' \to qW$ the limit is at 340 GeV, not much different from the previous result. The D0 experiment has reported recently a similar study of $t' \to qW$ with 5.3 fb$^{-1}$ collected data, giving a mass limit of $m_{t'} > 285$ GeV at 95% CL [8].

CDF has also searched for pair production of $b'$ quarks, followed by $b' \to tW$ decay. In the first study based on 2.7 fb$^{-1}$ collected data, CDF exploited the low background nature of the same-sign dilepton signature (together with associated jets, one of which $b$-tagged, plus missing transverse energy), and gave [9] the bound of 338 GeV. A better limit was obtained recently in the lepton plus jets study based on 4.8 fb$^{-1}$ collected data. CDF searched for an excess of events with an electron or a muon, at least five jets (one tagged as a $b$ or $c$), and an imbalance of transverse momentum. The observed events were consistent with background expectations, giving the upper limit of $m_{b'} > 372$ GeV at 95% CL [10].

The LHC has seen remarkable performance since 2010. Already, a search for pair-produced heavy bottom-like quarks in proton-proton collisions at $\sqrt{s} = 7$ TeV has been reported. The CMS experiment searched for $b'\bar{b}' \to tW^-\bar{t}W^+$ with same-sign dileptons, using a data set of 34 pb$^{-1}$ collected in 2010. No events were found in the signal region, and the $b'$ mass range from 255 to 361 GeV was excluded at the 95% confidence level [11]. For $t'$ (or $b'$) $\to qW$ search, the ATLAS experiment reported recently a study of dilepton events with 37 pb$^{-1}$ collected in 2010, using a boosted $W$ approach in a "collinear mass" variable. The reported preliminary [12] limit is 270 GeV. These studies are clearly a harbinger of the passing of the torch from the Tevatron to the LHC, as far as heavy chiral quark search is concerned.

In this report, we first take closer scrutiny of the $b' \to tW$ decay process to illustrate the width effect involving two unstable daughters. The decay widths of $b' \to t^{(*)}W^{(*)}$ are obtained using the convolution method [13–17] at tree level. If the $b'$ mass is below the $tW$ threshold, then $b' \to tW$ decay is phase space forbidden, and $b'$ decays via $b' \to tW^*$ and $t^*W$, where either $t$ or $W$ is off-shell. The former case was missed in a previous analysis [18]. Note, however, that each of these two decay widths would turn into the $b' \to tW$ decay width when $m_{b'}$ is above the $tW$ threshold. We therefore investigate how double counting is avoided as the threshold is approached. In so doing, we elucidate how, for different $b'$ mass scenarios, the decay rate of $b' \to t^{(*)}W^{(*)}$ can be effectively five-body, four-body, three-body, or finally, the two-body $b' \to tW$ process above $tW$ threshold of 255 GeV. The method is applied to investigate $b' \to t'^{(*)}W^{(*)}$ or $t' \to b'^{(*)}W^{(*)}$ decays, depending on mass hierarchy. In fact, because one expects the $t'$-$b'$ mass splitting to be less than $M_W$, the dominant process would be $b' \to t'W^*$ or $t' \to b'W^*$ decay, where the $W$ is virtual. We focus on studying the effect of the CKM mixing element on $b'$ and $t'$ decays, especially the near and below threshold behavior. If the CKM mixing element $|V_{t'b}|$ (or $|V_{tb'}|$) is small enough, the decay channel $b' \to tW$ (or $t' \to bW$) would be suppressed. Thus, a measurement of the $b' \to t'W^*$ (or $t' \to b'W^*$) branching fraction would allow one to in principle measure the strength of $|V_{t'b}|$ (or $|V_{tb'}|$). Finally, in an Appendix we compare the calculated decay widths for $b' \to tW^*$ decay across different thresholds with results obtained from PYTHIA.

## II. WIDTH EFFECT OF UNSTABLE DAUGHTERS

The threshold effect results from the finite widths of daughter particles, which can be described by the Breit-Wigner (BW) distribution. We illustrate the Breit-Wigner distribution with the top quark itself in the upper plot of Fig. 1. We shall subsequently use an artificial distinction of whether a particle is real or virtual: if the available energy is lower than the central mass value by $3\Gamma$, we consider it as a virtual particle in the decay final state. On the other hand,

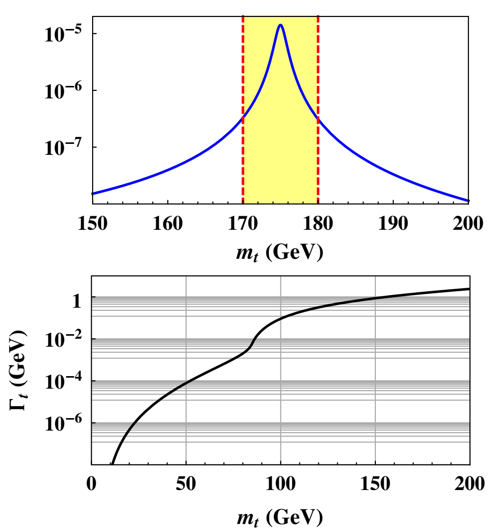


FIG. 1 (color online). Upper plot: the top quark resonance width in the form of a Breit-Wigner distribution; lower plot: the decay width of $t \to bW$ as a function of $m_t$. A clear finite width effect of the $W$-boson mass threshold is seen.

if the available energy is more than the central mass value by $3\Gamma$, it is considered as a real particle. This is indicated as the vertical dashed band in Fig. 1.

Let us use the simpler case of the decay width for the top quark [19] to illustrate threshold effects involving unstable daughter particles. In the lower part of Fig. 1, the decay width of the top quark, assuming 100% branching fraction into a bottom quark and a $W$-boson, is calculated with the convolution method [15], treating the $W$ boson as a BW distribution. If $m_t < m_W + m_b$, the top quark does not have enough rest energy to produce a real $W$ boson. As described by the Breit-Wigner distribution, this means that the top quark mass does not fully cover the distribution of the $W$ boson. Once $m_t > m_W + m_b$, then most of the $W$ boson distribution gets included. We therefore have a threshold effect:

$$m_t \gtrsim m_W + m_b \Rightarrow t \to bW,$$
$$m_t \lesssim m_W + m_b \Rightarrow t \to bW^*.$$

This threshold effect caused by the $W$ width can be seen for $m_t$ around $m_W + m_b \sim 85$ GeV. Without the finite width of the $W$ boson, i.e. assuming the $W$ is as stable as the $b$ quark, the top quark width would drop towards zero below the $bW$ threshold.

In the case of the fourth generation $b' \to tW$ decay, we have to consider not only the width of the $W$ boson, but also the finite width of the top quark. The latter was not considered in the previous study [18], where the oversight can be traced to Ref. [20]. Considering $t^{(*)}W^{(*)}$ as the only final state, one expects:

$$m_{b'} \gtrsim m_t + m_W \Rightarrow b' \to tW,$$
$$m_{b'} \lesssim m_t + m_W \Rightarrow b' \to tW^* \quad \text{and} \quad t^*W.$$

## III. $b' \to tW$ DECAY WIDTH

Let us analyze the $b' \to t^{(*)}W^{(*)}$ decay width for different $b'$ mass values. The corresponding Feynman diagrams are shown in Figs. 2(a)–2(e). By dividing the range for $m_{b'}$ into three regions via $m_b + 2m_W \lesssim m_t$ and $m_t + m_W$ thresholds, heuristically one can evaluate the decay width of $b'$ using the following approximations:

(i) $m_{b'} \lesssim m_b + 2m_W$: $b' \to t^*W^{(*)} \to bf_if_jW$ or $bWf_kf_l$ quasi four-body decay;

(ii) $m_t \lesssim m_{b'} \lesssim m_t + m_W$: $b' \to t^{(*)}W^{(*)} \to bWW$ or $tf_kf_l$ quasi three-body decay;

(iii) $m_{b'} \gtrsim m_t + m_W$: $b' \to tW$ two-body decay.

However, a direct calculation of the five-body decay width that covers the full kinematic range for $b' \to t^{(*)}W^{(*)} \to bW^{(*)}W^{(*)} \to bf_if_jf_kf_l$ at tree level, can be obtained via the convolution method [15–17], by treating $t$ and $W$ as unstable particles through a Breit-Wigner distribution. That is

$$\Gamma_{b'\to bf_if_jf_kf_l} = \int_0^{(m_{b'}-m_b)^2}\int_0^{(m_{b'}-m_b-\sqrt{q^2})^2} dq^2dp^2\rho(q^2, m_W, \Gamma_W^0)\rho(p^2, m_W, \Gamma_W^0)\times\Gamma^0(b'\to bW(q^2)W(p^2)). \quad (1)$$

The three-body width $\Gamma^0(b'\to bW(q^2)W(p^2))$ is for variable effective $W$ masses $q^2$ and $p^2$, and

$$\rho(q^2, m_W, \Gamma_W^0) = \frac{1}{\pi}\frac{\frac{q^2}{m_W}\Gamma_W^0}{(q^2-m_W^2)^2+(\frac{q^2}{m_W}\Gamma_W^0)^2} \quad (2)$$

is the BW distribution for the $W$ boson, where $\frac{q^2}{m_W}\Gamma_W^0$ is derived from $\sqrt{q^2}\sum_{i,j}\Gamma^0(W(p^2)\to f_if_j)$ in the limit of massless final state fermions [16]. That is, $\Gamma_W^0 = \sum_{k,l}|V_{kl}|^2\frac{N_cg_W^2m_W}{48\pi}$, with $N_c = 3$ for quarks, and $N_c = 1$ for leptons. We sum over CKM dominant $u\bar{d}$ and $c\bar{s}$ quark final states only.

The three-body width $\Gamma^0(b'\to bW(q^2)W(p^2))$ can be put as a convolution of two-body decays,

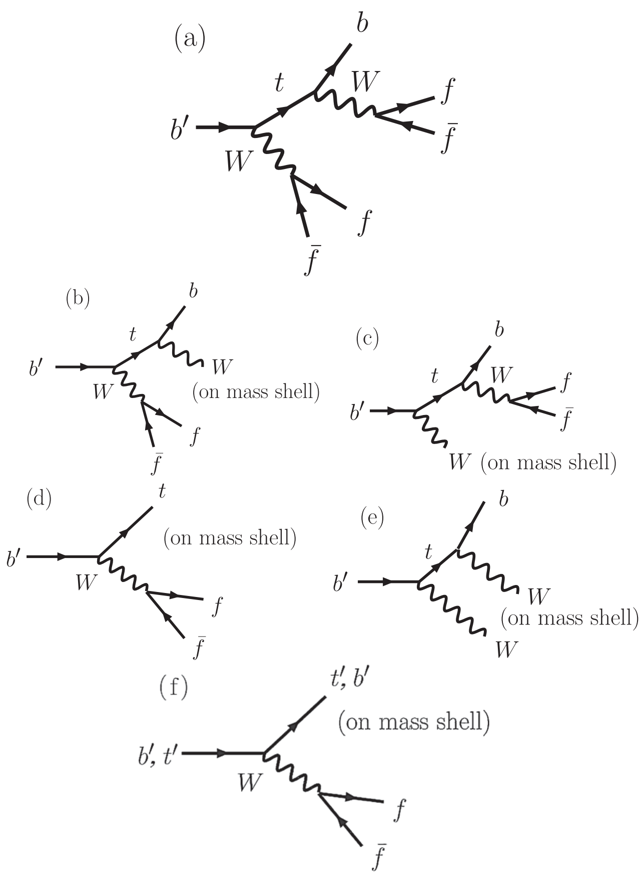


FIG. 2. Feynman diagrams for (a) five-body $b'\to t^{(*)}W^{(*)}$ decay, as well as $n$-body approximation diagrams under the convolution method, where (b) and (c) are for four-body, and (d) and (e) are for three-body approximations. The dominant three-body $b'\to t'W^*$ (or $t'\to b'W^*$) heavy-to-heavy transition is illustrated in (f).

$$\begin{aligned}\Gamma^0(b'\to bW(q^2)W(p^2)) &= \int_0^{(m_{b'}-\sqrt{p^2})^2}dk^2\Gamma(b'\to t(k^2)W(p^2))\\ &\quad\times\rho(k^2, m_t)\Gamma(t(k^2)\to b(s^2)W(q^2)),\end{aligned} \quad (3)$$

where $s^2 = m_b^2$, and the two-body decay width is

$$\begin{aligned}\Gamma(t(k^2)\to b(s^2)W(q^2)) &= \frac{G_Fk^{3/2}}{8\pi\sqrt{2}}|V_{tb}|^2\lambda^{1/2}\Big(1, \frac{s^2}{k^2}, \frac{q^2}{k^2}\Big)\\ &\quad\times\Big[\Big(1-\frac{s^2}{k^2}\Big)^2+\Big(1+\frac{s^2}{k^2}\Big)\frac{q^2}{k^2}-2\frac{q^4}{k^4}\Big],\end{aligned} \quad (4)$$

with $\lambda(x, y, z) \equiv x^2+y^2+z^2-2(xy+yz+xz)$, and $\Gamma^0(b'\to t(k^2)W(p^2))$ is analogous.

After rearranging the function, using the probability distribution, for a top quark in this case,

$$\rho_t(q^2, m_t) = \frac{1}{\pi}\frac{q\Gamma_t(q^2)}{(q^2-m_t^2)^2+(q\Gamma_t(q^2))^2}. \quad (5)$$

Inserting Eqs. (2)–(5) into Eq. (1), we get the decay width of $b'$, where the numerical result is given in Fig. 3 as the black solid curve. Note that Eq. (5) is the general form of the probability distribution for unstable particles [17], but $q\Gamma_W(q^2) = \frac{q^2}{m_W}\Gamma_W^0$ in the limit of vanishing final state fermion masses for $W\to f_if_j$. Therefore, the analog of Eq. (5) for the W boson reduces to Eq. (2) (for further discussion, see Ref. [16]).

To make contact with various threshold effects, note that just above a kinematic threshold, the narrow width assumption (BW distribution becomes a $\delta$ function) can be used for some daughter particle. Thus, in different $b'$ mass regions, we can take the finite width effects into account and deal with the $b'$ decay processes as a $n$-body decay, with $n < 5$. In this way, we recover the heuristic view as depicted in Figs. 2(a)–2(e). The calculation is simpler, as there are fewer phase space integrals.

In the following, we compare the full five-body decay with the fewer body decay scenarios (see lower part of Fig. 3). By dividing the $m_{b'}$ into the three regions with $m_b + 2m_W \lesssim m_t$ and $m_t + m_W$ thresholds as mentioned earlier, we can evaluate the $b'$ width using the following approximations:

(i) For $m_{b'} \gtrsim m_t + m_W \sim 255$ GeV, one has an effective two-body decay.

(ii) For 180 GeV $\lesssim m_{b'} \lesssim 245$ GeV, since it is $3\Gamma_t$ away from $m_t$ as well as $m_t + m_W$ thresholds, either $t$ or $W$ must be decaying off-shell. Hence, the $b'$ decay width can be estimated with a

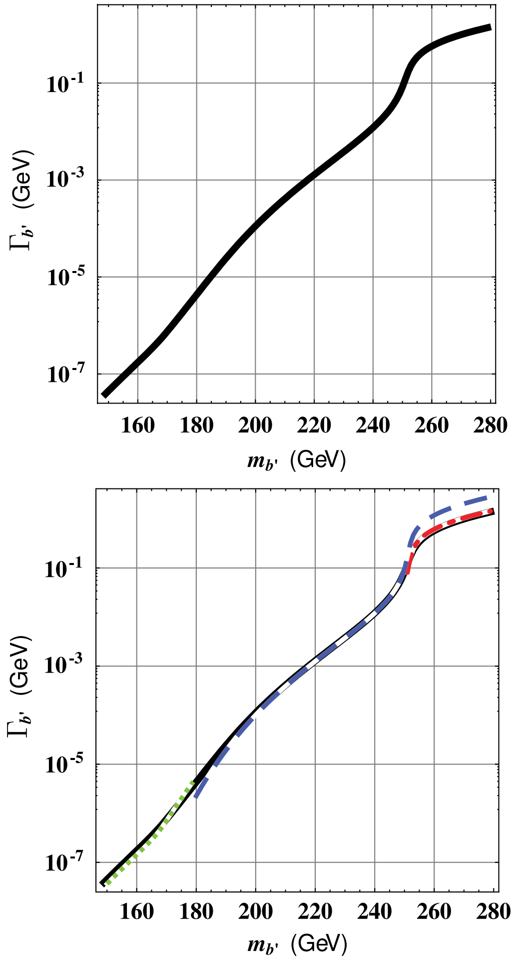


FIG. 3 (color online). The decay width of $b' \to tW$, modulo $|V_{tb'}|^2$. The black solid curve is the full five-body decay result. The (green) dotted, (blue) dashed and (red) dot-dashed curves show the four-, three- and two-body decay approximations, valid for different kinematic regions. Note that extending the three-body curve above the $tW$ threshold would result in double-counting.

quasi-three-body decay model, with contributions from mainly $b' \to t^* W \to bWW$ and $b' \to tW^* \to tf_jf_k$, added incoherently. We can see from Fig. 3 that this three-body model serves quite well within this region, as compared with the full five-body result. However, if one extends this approximation to $m_{b'} \gtrsim 250$ GeV, $t$ and $W$ will be both turning on-shell, such that $b' \to bWW$ and $b' \to tf_jf_k$ become equivalent. This would give an overestimate of $\Gamma_{b'}$ by a factor of two when comparing with the five-body (or $tW$ two-body) calculation, because of the incoherent sum assumption. The two decay "branches" are merging, and there should be some interesting interference effects, which would require a full five-body calculation to uncover.

(iii) For $m_{b'} \lesssim 160$ GeV, i.e. about $3\Gamma_W$ away from $m_b + 2M_W$, the $t$ has to decay off-shell, but only one $W$ boson can be on-shell. The effective four-body $b' \to bWf_1f_2$ decay approximation gives consistent results with the full five-body decay.

## IV. MEASURING QUARK MIXING VIA $b'$ AND $t'$ DECAYS

Different assumptions on the magnitudes of the CKM mixing elements will lead to different dominant $t'$ and $b'$ decay channels. The tree diagram processes $t' \to bW$ and $b' \to tW$ have been treated in experimental searches so far as the dominant decays. But even the loop suppressed FCNC decays through penguin diagrams [18], $t' \to tZ$, $cZ$ and $b' \to bZ$, $sZ$ (even with the $Z$ replaced by $g$ and $\gamma$) might be significant in certain kinematic or quark mixing parameter regions.

One would naively expect the decay branching ratios of $t' \to sW$ and $b' \to cW$ to be relatively small, because of the jump over two generations. But we should stress that $V_{t's}$ and $V_{cb'}$ are yet unmeasured CKM elements, and could be unexpectedly large. We illustrate this in Fig. 4: for $m_{b'}$ considerably above $tW$ threshold, $b' \to cW$ would dominate over $b' \to tW$ if $|V_{cb'}|/|V_{tb'}|$ is larger than 0.7. If $|V_{cb'}|/|V_{tb'}|$ is larger than one, $b' \to cW$ will always be larger than $b' \to tW$ because of phase space. It would therefore be important for the experiments to separate $t' \to bW$ and $t' \to qW$ (where $q = s, d$), as CDF has just started doing, as well as separate $b' \to cW$ (even $b' \to uW$) from the above two processes while pursuing $b' \to tW$. If a fourth generation is discovered, we would be just at the beginning of measuring relevant CKM elements, as in the early $B$ physics program.

It should be noted that, if one takes seriously the possible hint for a sizable $t'$ effect in $b \to s$ transitions ($B \to K\pi$

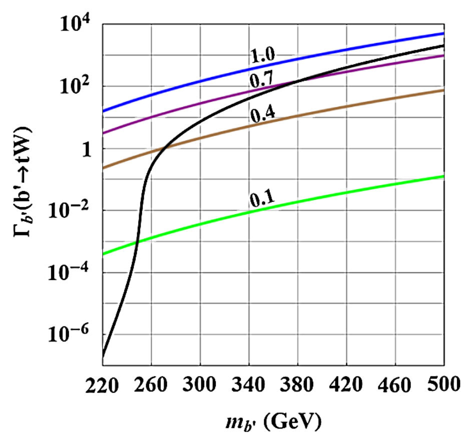


FIG. 4 (color online). The decay width of $b' \to tW$ (modulo $|V_{tb'}|^2$) and $b' \to cW$ for different $b'$ mass assumptions. The $b' \to tW$ curve rises from the lower left, while the four flatter $b' \to cW$ curves are for $|V_{cb'}|/|V_{tb'}| = 0.1$, 0.4, 0.7, 1, respectively.

direct CPV difference, and mixing-dependent CPV in $B_s \to J/\psi\phi$) [21,22], then $V_{cb'}$ (related to $V_{t's}$ that enters $b \to s$) could be comparable to $V_{tb'}$. If this is the case, $b' \to cW$ could compete with, even dominate over $b' \to tW$ far above the $tW$ threshold!

In the same vein, it is important to keep in mind the CKM-allowed $b' \to t'W$ decay, or $t' \to b'W$ decay, depending on whichever quark is heavier. Electroweak precision tests (EWPrT) constrain $|m_{t'} - m_{b'}| < m_W$ [2,4,5]. However, direct search should not be confined to the parameter space allowed by EWPrT. Since $|V_{t'b'}| \simeq 1$ is rather likely, which could be much larger than $|V_{tb'}|$ and $|V_{t'b}|$, we turn to compare the CKM allowed versus the CKM suppressed $t'$ and $b'$ decays. We find that the EWPrT constraint makes the CKM allowed intrafourth generation transitions rather interesting, precisely because of the strong threshold dependence.

First, let us consider $m_{t'} > m_{b'}$. The width for the top-like decay $t' \to bW$ is proportional to $|V_{t'b}|^2$. As $|V_{t'b'}| \simeq |V_{tb}| \simeq 1$, while the value of $|V_{tb'}|^2$ is expected to be of order 1% or less, $t' \to b'^{*}W$ (where $b'$ is virtual) is suppressed by both phase space and $|V_{tb'}|^2$. Thus, to very good approximation, we can treat $b'$ as on-shell in the final state (we have verified this by direct computation of $t' \to b'^{*}W$). The EWPrT bound that $m_{t'} - m_{b'} < M_W$ then implies that the associated $W$ is off-shell, as illustrated in Fig. 2(f). The three-body phase space suppression could be compensated by the CKM allowed coupling, as compared with the two-body phase space but CKM suppressed $t' \to bW$ decay.

Assuming $t' \to b'W^*$ and $t' \to bW$ to be the two dominant modes, we plot the branching fraction (BF) of $t' \to b'W^*$ in the upper two plots of Fig. 5 as a function of $m_{t'}$, for $m_{b'} = 350$ and 450 GeV, and for $|V_{t'b}| = 0.005$, 0.01, 0.02, 0.05, 0.1, and 0.2. For $m_{b'}$ just above $m_{t'}$, the BF for $t' \to b'W^*$ is severely suppressed. However, depending on how small $|V_{t'b}|$ is, and hence how much $t' \to bW$ rate gets suppressed, the BF for $t' \to b'W^*$ can be considerably enhanced. So, if one knows $b'$ mass already, then a measurement of BF($t' \to b'W^*$) would provide a measurement of $|V_{t'b}|$. It is particularly sensitive to small $|V_{t'b}|$ values (this sensitivity drops somewhat as the $t'$, $b'$ system becomes heavier), which provides a complementary program to the indirect measurement through loop-induced $b \to s$

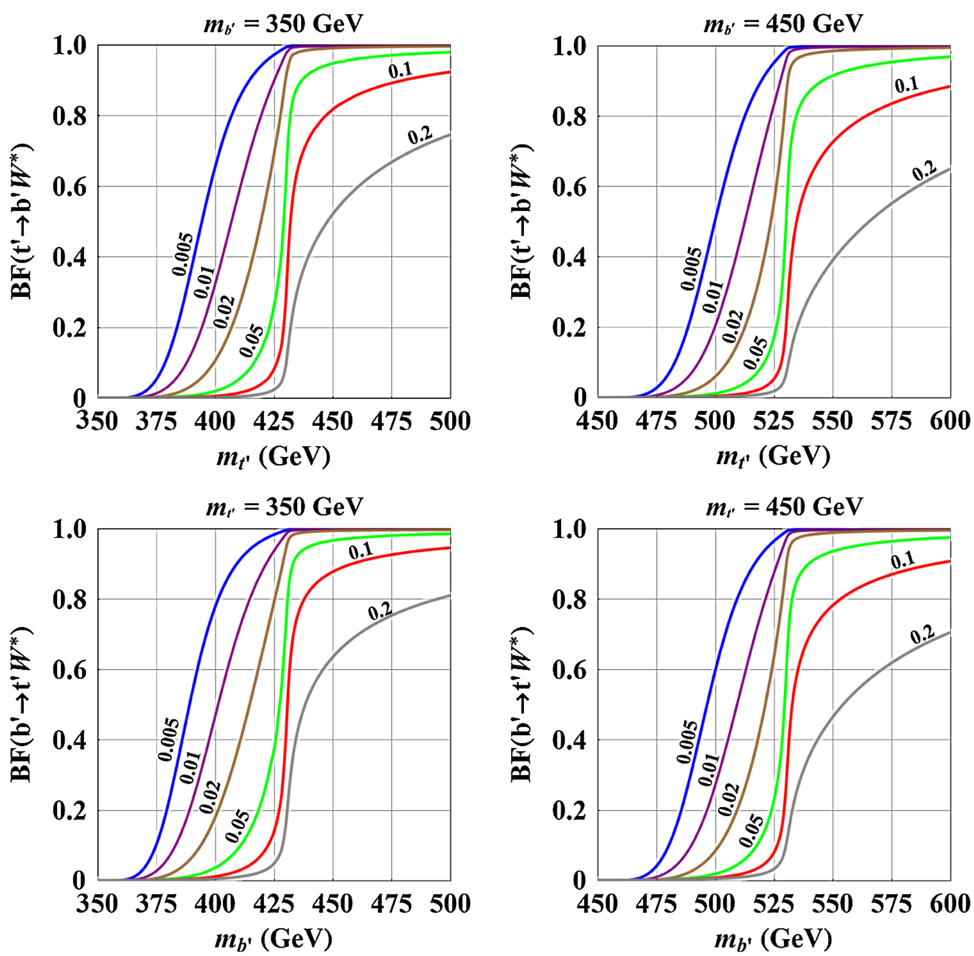


FIG. 5 (color online). Branching fractions for $t' \to b'W$ (upper) and $b' \to tW$ (lower) decays. The different curves represent different magnitudes of $|V_{t'b}|$ and $|V_{tb'}|$. These curves would be modified if $t' \to qW$ ($q = s$, $d$) and $b' \to cW$ are significant.

transitions. If it so happens that the EWPrT constraint of $m_{t'} - m_{b'} < M_W$ is violated, and hence the $W$ in $t' \to b'W$ decay is on-shell, then this heavy-to-heavy process would in fact dominate for small $|V_{t'b}|$ values. One should then measure BF($t' \to bW$) instead for the determination of $|V_{t'b}|$.

The case for $m_{b'} > m_{t'}$ is analogous. The decay width of $b' \to tW$ is proportional to $|V_{tb'}|^2$, which is almost the same as $|V_{t'b}|^2$ and not more than 1%, while $|V_{t'b'}| \simeq 1$. We give BF($b' \to t'W^*$) in the lower two plots of Fig. 5 as a function of $m_{b'}$, for $m_{t'} = 350, 450$ GeV, and for $|V_{tb'}| = 0.005, 0.01, 0.02, 0.05, 0.1, 0.2$. When the EWPrT constraint is violated (i.e. $m_{b'} > m_{t'} + m_W$), $b' \to t'W$ decay could dominate. But for $m_{b'} - m_{t'} < M_W$, a measurement of BF($b' \to t'W^*$) provides a measure of $|V_{tb'}|$. Because $b' \to tW$ decay itself is kinematically limited compared to $t' \to bW$ decay, BF($b' \to t'W^*$) is slightly more sensitive to $|V_{tb'}|$ than BF($t' \to b'W$) is to $|V_{t'b}|$.

As we have mentioned the potential importance of $b' \to cW$ and $t' \to sW$ transitions as compared to $b' \to tW$ and $t' \to bW$, we remind the fourth generation searchers at the colliders that the true target is quite broad: either $t' \to sW$, $bW$ and $b'W^*$, and $b' \to cW$, $tW$; or $t' \to sW$, $bW$, and $b' \to cW$, $tW$ and $t'W^*$ (there can still be $t' \to dW$ and $b' \to uW$). In the first case where $t'$ is heavier, $b'$ would likely be first discovered via $b' \to cW$ and especially $b' \to tW$. But one has to not only separate $t' \to sW$, $t' \to bW$ and $b' \to cW$ (which all feed the current $Q \to qW$ search signature [23]), but also disentangle the complicated $t' \to b'W^*$, which, if dominant, would involve a $t'\bar{t}' \to t\bar{t}W^+W^-W^{*+}W^{*-} \to b\bar{b}W^+W^-W^+W^-W^{*+}W^{*-}$ final state. In the second case where $b'$ is heavier, one should first discover a new “heavy top” quark, then try to separate $t' \to sW$, $t' \to bW$ and $b' \to cW$, while disentangling $b' \to tW$ (with $t \to bW$) from $b' \to t'W^*$ (with $t' \to sW$, $bW$). That is, for $b'$ production, besides $b' \to cW$ possibility which becomes part of the $t'$ program, in the same-sign dilepton approach arising from $q_1\bar{q}_2WWW^{(*)}W^{(*)}$, the signature is potentially rather complex, where there could be anywhere from zero to two $b$-tagged jets, while one or two $W$ bosons could be off-shell.

## V. DISCUSSION AND CONCLUSION

Our starting point was noting that, in the case of the fourth generation $b' \to tW$ decay, not only the width of the $W$ boson, but also the width of the top quark have to be considered. A direct calculation of the five-body decay width, obtained via the convolution method, can cover the full kinematic range for $b' \to t^{(*)}W^{(*)} \to bW^{(*)}W^{(*)} \to bf_if_jf_kf_l$ at tree level. One can also check the various effective four-, three- and two-body decay processes. In so doing, we clarified how the two branches of $b' \to t^*W$ and $b' \to tW^*$ merge to $b' \to tW$, where both $t$ and $W$ are on-shell. This is, in fact, already incorporated in PYTHIA. We compare our results with that of PYTHIA 6 in the Appendix. Though the general trend is consistent, some difference is noticed. We note that a full five-body calculation is needed to uncover the interesting interference effects which happen while the above-mentioned two “branches” are merging.

Our computation, though easily adapted to the $b' \to t'W^*$ (or $t' \to b'W^*$) case, certainly did not consider initial and final state interactions. For these, and to make experimental contact, one would need to link with the pair production, as well as jet fragmentation processes. One would then need to incorporate various QCD corrections, which is certainly outside the scope of this work. Note that the case of $b' \to t'W^*$ (or $t' \to b'W^*$) would be in a somewhat different kinematic regime than $b' \to tW$ for these correction, given that $b'$ and $t'$ are semi-degenerate, especially when their mass scale becomes higher.

The dominant decay channels of $t'$ and $b'$ quarks depend not only on their masses and mass difference, but on the CKM mixing elements, $V_{t'b}$, $V_{t's}$ (and $V_{t'd}$), or $V_{tb'}$, $V_{cb'}$ (and $V_{ub'}$) as well. We have illustrated with the two cases of suppressed $t' \leftrightarrow b'$ transitions (Fig. 4), or suppressed $b' \to cW$ and $t' \to sW$ decays (Fig. 5). In the former case, separating, say, $t' \to sW$ from $t' \to bW$ (measuring BF($t' \to sW$)) could provide a measurement of $|V_{t's}/V_{t'b}|$, while in the latter case, a measurement of BF($t' \to b'W^*$) would provide a measurement of $|V_{t'b}|$, with the sensitivity geared towards small $|V_{t'b}|$ values. The complete program, alluded to at the end of the previous section, involving fourth generation quark decays to fourth, third, second and even first generation quarks, is much more complex.

Let us just consider the case of “classical splitting” [23], i.e. $t'$ heavier than $b'$, and satisfying the EWPrT constraint of $m_{t'} < m_{b'} + M_W$. One should consider the decays $b' \to tW$, $cW$, $uW$, and $t' \to b'W^*$, $bW$, $sW$, $dW$. For simplicity, let us drop the first generation. Reference [23] discussed how the various channels feed the two current search channels of lepton plus multijets and missing transverse momentum, and same-sign dileptons plus jets and missing transverse momentum. Their main point is that the limits would be, in general, improved by combining the two studies. But they noted that if BF($t' \to b'W$) is sizable, it would weaken the $b'$ mass bound.

Our point is that beyond [23], there is a wealth of information on CKM quark mixing involving the fourth generation, if one could separate the various $t'$ and $b'$ quark decay modes. The $b' \to tW$ mode would be the easiest to uncover via the same-sign dilepton signature, even if $b' \to cW$ decay is sizable. But the presence of the latter would complicate the agenda, with further complication through the $t' \to b'W^*$ mode. New methods would have to be developed (e.g. the boosted-$W$ technique used by ATLAS [12]) to disentangle such complicated events of $t\bar{t}WW$, $t\bar{c}WW$, with the possible addition of up to two, perhaps off-shell, $W$ bosons. The analysis should certainly go beyond the discovery method of same-sign dileptons; for example, a fully reconstructed top plus a boosted-$W$ imbedded in a rather complicated event. At the same time, there would be a separate analysis track of $c\bar{c}WW$, $b\bar{b}WW$, $b\bar{s}WW$ and $s\bar{s}WW$, similar to the current $Q \to qW$ search program. The $b$ (and $c$) tagging separation, already invoked by CDF [7], must be applied, together with an effort to separate the mass and charge of the decaying parent. In the case of prominent $b' \to cW$, the $t' \to b'W^*$ mode could result in $c\bar{c}WWW^*W^*$ events that have no trace of top quarks in them.

A main consequence of the consideration of threshold behavior for $t' \to b'W^*$ is that, if this decay mode can be separated in the above analysis (and with $t'$ and $b'$ decays separated), we have an ''amplifier'' for the measurement of small $|V_{t'b}|$ values in the measurement of BF($t' \to b'W^*$), if $m_{t'} < m_{b'} + M_W$ is satisfied. Consideration of $t' \to sW$ implies that one really measures the ratio of $t' \to bW$ and $t' \to b'W^*$ decay rates, while the ratio with $t' \to sW$ would provide information on $|V_{t's}|$. The analysis can be extended if in fact $m_{t'} > m_{b'} + M_W$ is found, though one would then be more sensitive to modest, rather than very small, $|V_{t'b}|$ values.

In conclusion, with the fast rise in accumulated luminosity at the LHC, one expects great progress in the search for fourth generation $t'$ and $b'$ quarks, with good potential for discovery. If discovery is made, the next task would be to sort out all decay modes. For this matter, it is important to cover, and separate, the CKM suppressed decays $t' \to sW$ and $b' \to cW$. Equally important would be to search for either $t' \to b'W^{(*)}$, or $b' \to t'W^{(*)}$, where the electroweak precision test constraint of $|m_{t'} - m_{b'}| < M_W$ would imply that the associated $W$ boson is virtual. The decay final states of these very heavy chiral quarks would be rather complex, but the threshold sensitivity studied in this work suggests that a measurement of the $t' \to b'W^{(*)}$ (or $b' \to t'W^{(*)}$) decay branching fraction, as compared with $t' \to bW$ and $sW$ (or $b' \to tW$ and $cW$), would provide a sensitive measurement of $|V_{t'b}|$ or the combined strength of $V_{t'b}$ and $V_{t's}$ ($|V_{tb'}|$ or the combined strength of $V_{tb'}$ and $V_{cb'}$). This would complement the indirect studies of loop-induced $b \leftrightarrow s$ transitions for an enlarged quark mixing sector.

## APPENDIX A: COMPARISON WITH PYTHIA

In Sec. III, we compared our various calculations of the $b' \to t^{(*)}W^{(*)}$ decay width with the result for five-body final state. Even though $m_{b'} \lesssim 300$ GeV is excluded by experimental data, it is of interest to compare the calculated decay widths with those from the PYTHIA 6 generator [24], at least as a cross-check. The results are shown side-by-side in Fig. 6. For the PYTHIA results, we identify an on-shell or off-shell decay with the definition described in Sec. II of this report. That is, by comparing the mass of the decaying $t$ or $W$ with its central value, if the applied energy is larger than the central value of the particle mass of three times the natural width, the decay is identified as on-shell, as shown in Fig. 1. The same definition is used in the integrations of $n$-body model calculations. Although the trends of our calculation and the PYTHIA results are similar, we can see quantitative deviations. Clearly, the effects of initial and final state interactions (ISR/FSR) as well as other corrections that are built into PYTHIA are not considered in our calculation. The actual cause of the deviations would need further investigation to clarify.

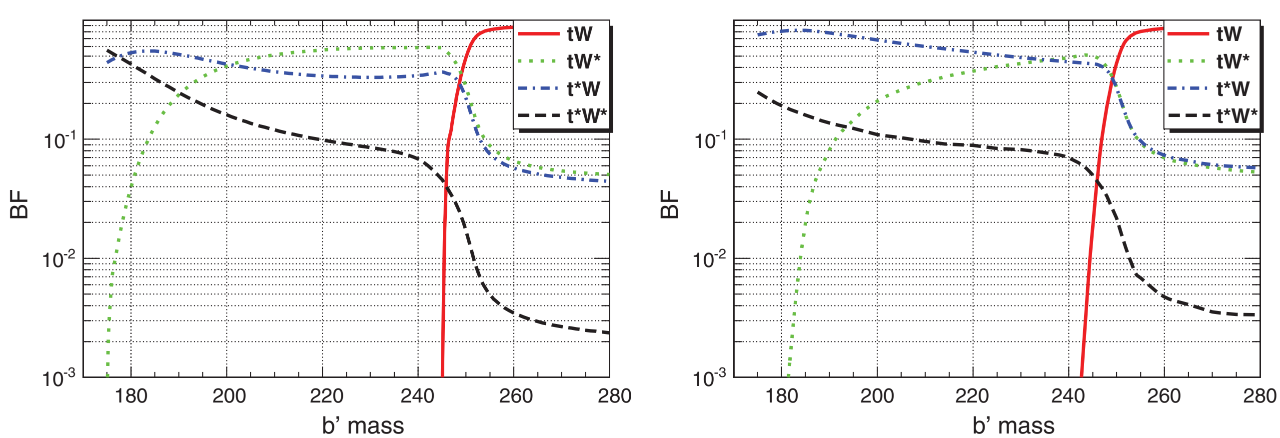


FIG. 6 (color online). The branching fractions for $b' \to tW$, $tW^*$, $t^*W$, and $t^*W^*$ processes, corresponding to two-body, three-body and five-body processes as separated by kinematics. The (red) solid, (green) dotted, (blue) dot-dashed and the (black) dashed lines are for $b' \to tW$, $tW^*$, $t^*W$ and $t^*W^*$, respectively. The left plot is from our five-body calculation, while the right plot is obtained from PYTHIA 6 [24]. The trends of the two plots are similar, but differ in the details.

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