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Rise of Kp total cross section and universality

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The increase of the measured hadronic total cross sections at the highest energies is empirically described by the squared log of center-of-mass energy $\sqrt{s}$ as $\sigma_{\rm tot} \simeq B\log^2 s$, consistent with the energy dependence of the Froissart unitarity bound. The coefficient B is argued to have a universal value, but this is not proved directly from QCD. In the previous tests of this universality, the $p(\bar{p})p$, $\pi^{\mp}p$, and $K^{\mp}p$ forward scatterings were analyzed independently and found to be consistent with $B_{pp} \simeq B_{\pi p} \simeq B_{Kp}$, although the determined value of B_{Kp} had large uncertainty. In the present work, we have further analyzed forward $K^{\mp}p$ scattering to obtain a more exact value of B_{Kp}. Making use of continuous-moment sum rules we have fully exploited the information of low-energy scattering data to predict the high-energy behavior of the amplitude through duality. The estimation of B_{Kp} is improved remarkably, and our result strongly supports the universality of B.

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I. INTRODUCTION AND SUMMARY

The increase of total cross sections $\sigma_{\rm tot}$ in a high-energy region is described [1] by the squared log of center-of-mass energy $\sqrt{s}$ as $\sigma_{\rm tot} \simeq B\log^2 s$ consistent with the energy dependence of the Froissart unitarity bound [2]. The COMPETE Collaboration has further assumed $\sigma_{\rm tot} \simeq B\log^2(s/s_0)$ to apply to all hadron total cross sections with a universal value of B [3]. The universality of B was theoretically anticipated in Refs. [4,5] and recently inferred from a color glass condensate [6,7] of QCD. However, there is still no rigorous proof based on QCD.

In a previous work [8], to test this universality empirically, the $p(\bar{p})p$, $\pi^{\mp}p$, and $K^{\mp}p$ forward scattering amplitudes were analyzed, and the values of B, denoted, respectively, as B_{pp}, $B_{\pi p}$, and B_{Kp}, were estimated independently. The resulting values were consistent with the universality, $B_{pp} \simeq B_{\pi p} \simeq B_{Kp}$, although there was still a rather large uncertainty in B_{Kp}, $B_{Kp} = 0.354 \pm 0.099$ mb.

In the present work, to reduce the uncertainty of B_{Kp} we have refined the analysis of forward $K^{\mp}p$ scattering. By employing continuous-moment sum rules (CMSRs) [9–12] for the crossing-even amplitude, we have fully exploited the information of low-energy scattering data to predict the high-energy behavior of $\sigma_{\rm tot}$ through duality. We use two CMSRs with parameters $\epsilon = -1$, -3, and following Ref. [12], the estimated values of the cross-section integrals are treated as data points. The unphysical region of the integral is problematic. The pole of $\Lambda(1405)$ in the unphysical region gives a relatively large contribution in K^-p amplitude. We estimate its contribution by the coupled-channel study by A. D. Martin [13]. The $\sigma_{\rm tot}$ and ρ ratios ($=\mathrm{Re}f/\mathrm{Im}f$) of $K^{\mp}p$ forward amplitudes in a high-energy region are fit simultaneously with two CMSRs' data points. The resulting value is $B_{Kp} = 0.328 \pm 0.045$ mb. The error is improved remarkably, less than half of the previous estimate. By comparing with previously determined values of B_{pp} and $B_{\pi p}$, the universality, $B_{Kp} \simeq B_{\pi p} \simeq B_{pp}$, is strongly supported.

II. FORMULAS

We inherit the notation and definitions from a previous work [8]: $\nu(k)$ is the energy (momentum) of the kaon beam in a laboratory system, $\nu = \sqrt{k^2 + m_K^2}$, where m_K is the mass of $K^{\mp}$. It is related to the center-of-mass energy $\sqrt{s}$ by

$$s = M^2 + m_K^2 + 2M\nu. \quad (1)$$

The $K^{\mp}p$ forward amplitudes are denoted as $f^{K^{\mp}p}(\nu)$. The total cross sections $\sigma_{\rm tot}^{K^{\mp}p}$ are given by $\mathrm{Im}f^{K^{\mp}p}(\nu) = (k/4\pi)\sigma_{\rm tot}^{K^{\mp}p}$ through an optical theorem. The crossing-even/odd amplitudes $f^{(\pm)}(\nu)$ are given by $f^{(\pm)}(\nu) = (f^{K^-p}(\nu) \pm f^{K^+p}(\nu))/2$. The $f^{(+)}(\nu)$ is related to the $A'(\nu, t)$ amplitude at $t = 0$ [12] as $f^{(+)}(\nu) = A'(\nu, t = 0)/4\pi$. Real and imaginary parts of $f^{(\pm)}(\nu)$ are assumed to take the forms [8]

$$\mathrm{Im} f_{\rm as}^{(+)} = \frac{\nu}{m_K^2}\left(c_2 \log^2\frac{\nu}{m_K} + c_1 \log\frac{\nu}{m_K} + c_0\right) + \frac{\beta_{P'}}{m_K}\left(\frac{\nu}{m_K}\right)^{\alpha_{P'}}, \quad (2)$$

$$\mathrm{Im}\, f_{\rm as}^{(-)} = \frac{\beta_V}{m_K}\left(\frac{\nu}{m_K}\right)^{\alpha_V}, \quad (3)$$

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$$\mathrm{Re}\, f_{\mathrm{as}}^{(+)} = \frac{\pi\nu}{2m_K^2}\left(2c_2 \log\frac{\nu}{m_K} + c_1\right) - \frac{\beta_{P'}}{m_K}\left(\frac{\nu}{m_K}\right)^{\alpha_{P'}} \times \cot\frac{\pi\alpha_{P'}}{2} + f^{(+)}(0), \tag{4}$$

$$\mathrm{Re}\, f_{\mathrm{as}}^{(-)} = \frac{\beta_V}{m_K}\left(\frac{\nu}{m_K}\right)^{\alpha_V} \tan\frac{\pi\alpha_V}{2}, \tag{5}$$

in the asymptotic high-energy region, where we assume $\mathrm{Im}f^{(+)}$ is given by $\log^2\nu$ (and $\log\nu$) terms in addition to the ordinary Pomeron (c_0) term and Reggeon ($\beta_{P'}$) term. Similarly $\mathrm{Im}f^{(-)}$ is given by exchange of ρ-, ω-meson trajectories in Regge theory with degenerate trajectory intercepts assumed. Their intercepts are taken to be an empirical value $\alpha_{P'} \simeq \alpha_V \simeq 0.5$. The real parts of $f_{\mathrm{as}}^{(\pm)}$ are obtained from their imaginary parts by using crossing symmetry, $f^{(\pm)}(-\nu_R - i\epsilon) = \pm f^{(\pm)}(\nu_R + i\epsilon)$, except for a subtraction constant $f^{(+)}(0)$. There is a total of six parameters: $c_{2,1,0}$, $\beta_{P',V}$, $f^{(+)}(0)$. The c_2 term in $\mathrm{Im}f_{\mathrm{as}}^{(+)}$ dominates σ_{tot} in the high-energy-region and it is related to B_{Kp} by

$$B_{Kp} = 4\pi c_2/m_K^2. \tag{6}$$

III. CONTINUOUS-MOMENT SUM RULES

We can exploit the low-energy scattering data to predict the amplitudes in the high-energy region by using CMSRs [11]. In Ref. [12,14] a compact form of CMSRs is given in πN scattering, where Regge-pole contributions are parametrized in a form satisfying crossing symmetry and convenient for CMSRs. The crossing-even $A'(\nu, t)$ amplitude was taken to be $A'(\nu,t) = \sum_{i=P,P',P''}[-\gamma_i(\nu_0^2 - \nu^2)^{\alpha_i/2}]$ in the asymptotic energy region $\nu \geq \nu_1$. Here ν_0 is the normal threshold $\nu_0 = \mu + t/(4M)$, where $\mu(M)$ is the charged pion (proton) mass, and $(\nu_0^2 - \nu^2)$ is analytically continued to $(\nu^2 - \nu_0^2)e^{-i\pi}$ in $\nu > \nu_0$ for $A'(\nu, t)$ to have the correct phase factor of Reggeon-exchange amplitudes. The finite-energy sum rule for $\nu A'(\nu, t)$ is given by

$$\int_{\nu_0}^{\nu_1} d\nu\nu\, \mathrm{Im}[(\nu_0^2 - \nu^2)^{-(\epsilon+1)/2} A'] = \sum_{i=P,P',P''} \gamma_i \frac{(\nu_1^2 - \nu_0^2)^{(\alpha_i - \epsilon+1)/2} \sin[\frac{\pi}{2}(\alpha_i - \epsilon - 1)]}{\alpha_i - \epsilon + 1}, \tag{7}$$

where ϵ is a continuous parameter and the nucleon pole term contribution should be added to the left-hand side (LHS).

If we know completely the scattering amplitudes in the low-energy region from experiments, the high-energy amplitudes are predicted via CMSRs through analyticity. In Ref. [12], by using the CERN results on πN scatterings up to 2 GeV, the low-energy integrals on the LHS of Eq. (7) were evaluated in the region $0 \leq -t < 1$ GeV^2 and $0(-1) \geq \epsilon \geq -5$ for $t = 0(t < 0)$ in steps of 0.5. These numbers were treated as data points, and simultaneously fit as data to the Reggeon parametrization of the asymptotic high-energy region.

This method can be applied to the analysis of forward $K^{\mp}p$ scattering. Our $f^{(+)}(\nu)$ amplitude corresponds to $A'/(4\pi)$ with $t = 0$, and the CMSR is given by

$$\int_0^{\nu_1} d\nu\nu\, \mathrm{Im}[(m_K^2 - \nu^2)^{-(\epsilon+1)/2} f^{(+)}(\nu)] = \int_0^{\nu_1} d\nu\nu\, \mathrm{Im}[(m_K^2 - \nu^2)^{-(\epsilon+1)/2} f_{\mathrm{as}}^{(+)}(\nu)]. \tag{8}$$

The LHS should be evaluated from low-energy experimental data, while the right-hand side (RHS) is analytically calculated by using the formulas (2) and (4).

If we take nonodd values of ϵ, $\mathrm{Re}f^{(+)}(\nu)$ data in the low-energy region are necessary as inputs. However, experimental data [15] of $\mathrm{Re}f(\nu)$ (or ρ ratios) for $K^{\mp}p$ are poorly known for $k \leq 5$ GeV. In this situation we are forced to select ϵ as odd integers; specifically we take $\epsilon = -1, -3$.

The CMSRs with $\epsilon = -1, -3$ are equivalent to the $n = 1, 3$ moment sum rule [10],

$$\frac{2}{\pi}\int_0^{\nu_1} d\nu\nu^n\, \mathrm{Im}f^{(+)}(\nu) = \frac{2}{\pi}\int_0^{\nu_1} d\nu\nu^n\, \mathrm{Im}f_{\mathrm{as}}^{(+)}(\nu), \tag{9}$$

where $\frac{2}{\pi}$ is from our convention. The LHS of Eq. (9) is evaluated in the next section.

IV. EVALUATION OF INTEGRALS FROM EXPERIMENTAL DATA

The LHS of Eq. (9) is obtained by averaging the integrals of $\mathrm{Im}f^{K^+p}$ and $\mathrm{Im}f^{K^-p}$, which are evaluated separately from experimental data.

The K^+p channel is exotic and it has no contribution below threshold, $\nu < m_K$. Thus, its integral region is m_K to ν_1. By changing the variable from ν to k, the relevant integral is given by

$$\frac{2}{\pi}\int_0^{\nu_1} d\nu\nu^n\, \mathrm{Im}f^{K^+p} = \frac{2}{\pi}\int_0^{\bar{\nu}_1} dkk\nu^{n-1}\frac{k}{4\pi}\sigma_{\mathrm{tot}}^{K^+p}, \tag{10}$$

where $\bar{\nu}_1 \equiv \sqrt{\nu_1^2 - m_K^2}$, which is taken to be some value in the asymptotic high-energy region. We take $\bar{\nu}_1 = 5$ GeV. Actually the integral of the RHS of Eq. (10) is estimated by dividing its region into two parts: In the low-energy part, from $k = 0$ to k_d, there are many data points [15]. They are connected by straight lines and the area of this polygonal line graph can be regarded as the relevant integral. In high-energy part, from k_d to $\bar{\nu}_1$, we use the phenomenological fit used in our previous work [8]. The dividing momentum k_d is taken to be 3 GeV. As a result we obtain

$$\frac{2}{\pi}\int_0^{\nu_1} d\nu \nu^n \mathrm{Im} f^{K^+p} = \frac{2}{\pi}\int_0^{k_d} dk\cdots + \frac{2}{\pi}\int_{k_d}^{\bar{\nu}_1} dk\cdots = \begin{cases} 20.348(41) + 72.435(184) = 692.795(189)\,\mathrm{GeV} \\ 115.15(27) + 1294.90(3.48) = 1410.05(3.49)\,\mathrm{GeV}^3 \end{cases} \tag{11}$$

for $n = 1, 3$, respectively.

On the other hand, the K^-p amplitude includes Born terms of Σ^0, Λ^0 poles, which correspond to the nucleon pole term in the πN amplitude. K^-p is an exothermic reaction with open channels, $\Lambda\pi$ and $\Sigma\pi$, below threshold which give contributions to $\mathrm{Im} f^{K^-p}(\nu)$ in unphysical regions, $\nu_{\Lambda\pi} \leq \nu \leq m_K$ and $\nu_{\Sigma\pi} \leq \nu \leq m_K$, respectively, where $\nu_{\Lambda\pi}/\nu_{\Sigma\pi}$ is the $\Lambda\pi/\Sigma\pi$ threshold energy, $\nu_{\Lambda\pi/\Sigma\pi} = [(M_{\Lambda^0/\Sigma^0} + \mu)^2 - M^2 - m_K^2]/(2M)$. A large contribution from $I = 0$ $\Lambda(1405)$ is expected to be in the unphysical region. In order to estimate these contributions we adopt a coupled-channel analysis by A. D. Martin [13]. In his analysis the cross sections of $K^-p \to K^-p$, $\bar{K}^0 n$, $\Sigma\pi$, $\Lambda\pi^0$; $K_2^0 p \to K_1^0 p$, $\Lambda\pi^+$; $\sigma_{\mathrm{tot}}(K_2^0 p)$; and $\mathrm{Re} f^{K^-p, K^-n}$ were reproduced successfully. Martin included a correction from Coulomb scattering following the scheme of Dalitz and Tuan [16] for channels like K^-p, which gives a fairly large effect in a very low energy region. Here we use the purely strong-interaction part of Martin's amplitude to estimate the unphysical region contribution. We consider that this is the most reliable way to estimate the unphysical region. The relevant channels are the three $I = 1$ channels $\bar{K}N$, $\Sigma\pi$, $\Lambda\pi$ and the two $I = 0$ channels $\bar{K}N$, $\Sigma\pi$. The S-wave $I = 1(0)$ scattering amplitudes $\boldsymbol{T}^{I=1}(\boldsymbol{T}^{I=0})$ are parametrized by the K matirix [M matrix] as $\boldsymbol{T}^{I=1} = \boldsymbol{K}(1 - i\boldsymbol{q}\boldsymbol{K})^{-1}$ [$\boldsymbol{T}^{I=0} = (\boldsymbol{M} - i\boldsymbol{q})^{-1}$], where $\boldsymbol{q}$ is a diagonal matrix of the channel c.m. momenta denoted as $\boldsymbol{q} = \mathrm{diag}\{p, p_\Sigma, p_\Lambda\}(\mathrm{diag}\{p, p_\Sigma\})$. The real symmetric 3-by-3 matrix $\boldsymbol{K}$ is taken to be constant, including six parameters. The $\boldsymbol{M}$ matrix is taken to be an effective range form, $\boldsymbol{M} = \boldsymbol{A} + \boldsymbol{R}p^2$, which makes it possible to describe the $\Lambda(1405)$ resonance. Here p is the $\bar{K}N$ c.m. momentum which is continued below threshold as $p = i|p|$. Real symmteric 2-by-2 matrices $\boldsymbol{A}$, $\boldsymbol{R}$ are taken to be constant, and $\boldsymbol{M}$ includes six parameters.

By using the elements of $\boldsymbol{K}$ and $\boldsymbol{M}$, the inverse of $I = 1$, 0 $\bar{K}N$ scattering amplitudes $\boldsymbol{T}_{KK}^{I=1,0}$ is given explicitly [17] by

$$\begin{aligned}
(\boldsymbol{T}_{KK}^{I=1,0})^{-1} &= A_{I=1,0}^{-1} + ip, \\
A_{I=1} &= K_{KK} + \frac{1}{B_1}(K_{K\Sigma}B_2 + K_{K\Lambda}B_3), \\
B_1 &= (1 - ip_\Sigma K_{\Sigma\Sigma})(1 - ip_\Lambda K_{\Lambda\Lambda}) + p_\Sigma p_\Lambda K_{\Sigma\Lambda}^2, \\
B_2 &= p_\Sigma p_\Lambda (K_{K\Sigma}K_{\Lambda\Lambda} - K_{K\Lambda}K_{\Sigma\Lambda}) + ip_\Sigma K_{K\Sigma}, \\
B_3 &= p_\Sigma p_\Lambda (K_{K\Lambda}K_{\Sigma\Sigma} - K_{K\Sigma}K_{\Sigma\Lambda}) + ip_\Lambda K_{K\Lambda}, \\
A_{I=0}^{-1} &= M_{KK} - \frac{M_{K\Sigma}^2}{M_{\Sigma\Sigma} - ip_\Sigma},
\end{aligned} \tag{12}$$

where the subscript KK means a (1,1) element corresponding to $\bar{K}N \to \bar{K}N$. Other subscripts are used similarly. $A_I = a_I + ib_I$ are the S-wave $\bar{K}N$ scattering lengths and $M_{KK} = A_{KK} + R_{KK}p^2$, etc. The best-fit values of the relevant 12 parameters are given [18] with errors in Ref. [13].

Our K^-p forward amplitude $f^{K^-p}(\nu)$ is related [19] to the $\boldsymbol{T}_{KK}^I$ by

$$f^{K^-p}(\nu) = \frac{\sqrt{s}}{2M}[\boldsymbol{T}_{KK}^{I=0} + \boldsymbol{T}_{KK}^{I=1}]. \tag{13}$$

There are no $\sigma_{\mathrm{tot}}^{K^-p}$ data reported below $k \equiv k_s = 0.245$ GeV by the Particle Data Group [15]. The $\mathrm{Im} f^{K^-p}$ obtained by Martin is also utilized in this energy region $m_K \leq \nu \leq \nu_s$ where $\nu_s = \sqrt{k_s^2 + m_K^2}$.

The relevant integral is evaluated as follows:

$$\begin{aligned}
\frac{2}{\pi}\int_0^{\nu_1} \nu^n \mathrm{Im} f^{K^-p}(\nu) d\nu &= \sum_{R=\Lambda^0,\Sigma^0} \frac{g_R^2}{M}\nu_{B,R}^n(-M_R + M + \nu_{B,R}) + \frac{2}{\pi}\int_{\nu_{\Lambda\pi}}^{\nu_s} d\nu\cdots + \frac{2}{\pi}\int_{k_s}^{k_d} dk\cdots + \frac{2}{\pi}\int_{k_d}^{\bar{\nu}_1} dk\cdots \\
&= \begin{cases} (-0.106 \pm 0.060) + (0.508 \pm 0.095) + (35.481 \pm 0.069) + (108.499 \pm 0.405) \\ (-0.0004 \pm 0.0010) + (0.110 \pm 0.017) + (191.28 \pm 0.54) + (1928.04 \pm 7.46) \end{cases} \qquad (14) \\
&= \begin{cases} 144.382 \pm 0.426\ \mathrm{GeV} & n = 1 \\ 2119.43 \pm 7.48\ \mathrm{GeV}^3 & n = 3, \end{cases} \qquad (15)
\end{aligned}$$

where $\nu_{B,R} = (M_R^2 - M^2 - m_K^2)/(2M)$. The first bracket of Eq. (14) is the Born term, which is estimated by using $g_{\Lambda^0}^2 = 13.7 \pm 1.9$ and $g_{\Sigma^0}^2 = 0$. The error comes from the upper limit of $g_{\Sigma^0}^2 < 3.7 \pm 1.3$ [13]. The second bracket, corresponding to the integral region $\nu_{\Lambda\pi} \leq \nu \leq \nu_s$, is also estimated by Eq. (13) of Martin's amplitude. The error comes from $R_{KK} = 0.41 \pm 0.10$ fm [13], which gives the largest uncertainty among all the parameters. The third and fourth brackets are estimated from the polygonal line graph and phenomenological fit [8], respectively. For $n = 1$ a small but sizable contribution comes from the first term and second term. The corresponding errors affect the final value of Eq. (15) but they are not the main sources of its error. For $n = 3$ the first and second terms are both negligible. Thus, the uncertainty from the unphysical region, especially from the

TABLE I. Best-fit parameters and χ^2: $\sigma_{\rm tot}^{K^\mp p}$ and ${\rm Re} f^{K^\mp p}$ in $k \geq 5$ GeV are fit simultaneously with (i) $n = 1$ CMSR datum, (ii) $n = 3$ CMSR datum, (iii) $n = 1$ and 3 CMSR data. The results are compared with case (iv) with no CMSR data and with the previous analysis [8], where $\sigma_{\rm tot}^{K^\mp p}$ in $k \geq 20$ GeV and ${\rm Re} f^{K^\mp p}$ in $k \geq 5$ GeV are fit using the finite-energy sum rule with the region of the integral $5 \leq k \leq 20$ GeV as a constraint.

Case	c_2	c_1	c_0	$\beta_{P'}$	β_V	$f^{(+)}(0)$	$\chi^2/(N_D - N_P - 1)$
(i) $n = 1$	0.01652(224)	−0.1241	1.152	0.2702	0.5741	1.180	$143.21/(165 - 6 - 1)$
(ii) $n = 3$	0.01724(256)	−0.1339	1.188	0.2110	0.5736	1.609	$143.54/(165 - 6 - 1)$
(iii) $n = 1$, 3 both	0.01634(223)	−0.1221	1.146	0.2736	0.5737	1.178	$144.05/(166 - 6 - 1)$
(iv) no CMSR	0.01522(385)	−0.1065	1.088	0.3726	0.5749	2.104	$143.04/(164 - 6 - 1)$
II [8]	0.01757(495)	−0.1388	1.207	0.1840	0.5684	1.660	$63.80/(111 - 5 - 1)$

$\Lambda(1405)$ pole, is considered to be insignificant in the use of the values of Eq. (15).

By averaging Eqs. (11) and (15) we obtain

$$\frac{2}{\pi}\int_0^{\nu_1} \nu^n \,{\rm Im} f^{(+)}(\nu) d\nu = \begin{cases} 118.589 \pm 0.233\,{\rm GeV} \\ 1764.74 \pm 4.13\,{\rm GeV}^3 \end{cases} \tag{16}$$

for $n = 1$, 3, respectively. These two values are treated as low-energy data points and they are fit simultaneously with the data in the asymptotic high-energy region.

V. RESULTS AND CONCLUDING REMARKS

The $\sigma_{\rm tot}^{K^- p}$, $\sigma_{\rm tot}^{K^+ p}$, $\rho^{K^- p}$, and $\rho^{K^+ p}$ (more precisely [20] ${\rm Re} f^{K^- p}$, and ${\rm Re} f^{K^+ p}$) with $k \geq 5$ GeV, which are given in Ref. [15], are fit by using the formula in Sec. II. Thenumber of parameters is six: $c_{2,1,0}$, $\beta_{P',V}$, $f^{(+)}(0)$. We fit to the CMSR data points of $n = 1$, 3 (16) simultaneously. We considered three cases: (i) the case including a $n = 1$ datum of (16), (ii) the case including a $n = 3$ datum of (16), and (iii) the case including both $n = 1$, 3 data of (16). The results are compared with (iv) the case with no use of CMSRs and also our previous analysis [Ishida and Igi (II) [8]].

The best-fit parameters and χ^2 values are given in Table I.

All fits are successful ($\chi^2/$ deg .freedom < 1). The best-fit χ^2 values of (i), (ii), and (iii) are almost the same as case (iv) with no use of CMSRs, suggesting that the CMSR works well in the fit. Results of the best fit in case (iii) are shown in Figs. 1 and 2.

The error of c_2 in case (iii) is much smaller than in case (iv), and is greatly improved from our previous analysis II [8]. Correspondingly the parameter B_{Kp} associated with the $\ln^2 s$ dependence is given by

$$\begin{array}{lcl} & \text{Present result (iii)} & & \text{Previous analysis II [8]} \\ c_2 = & 0.016\,34 \pm 0.002\,23 & \longleftarrow & 0.017\,57 \pm 0.004\,95 \\ B_{Kp} = & 0.328 \pm 0.045\ {\rm mb} & \longleftarrow & 0.354 \pm 0.099\ {\rm mb} \end{array} \tag{17}$$

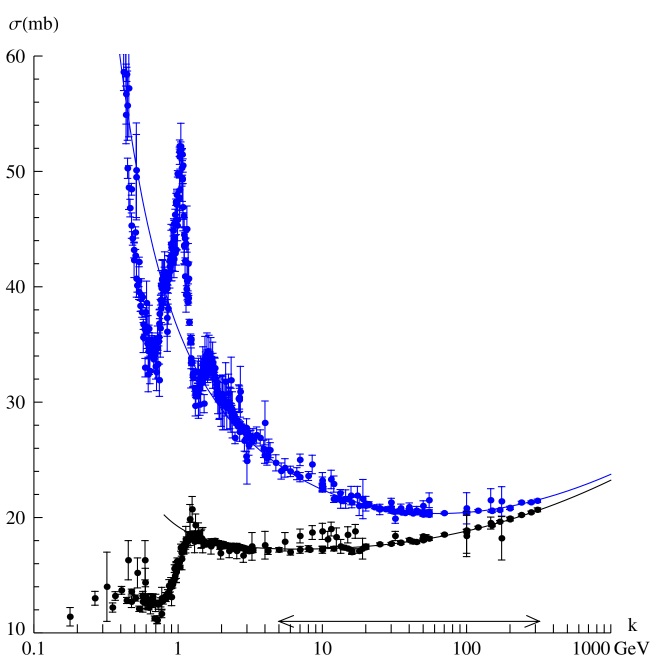

FIG. 1 (color online). Results of the fit to $\sigma_{\rm tot}^{K^\mp p}$ (mb). Upper (blue) data represent $K^- p$ and lower (black) data are $K^+ p$. The horizontal arrow represents the energy region of the fit to the asymptotic amplitude. The solid curves are the asymptotic Reggeon amplitudes, which are extrapolated down through the resonance region.

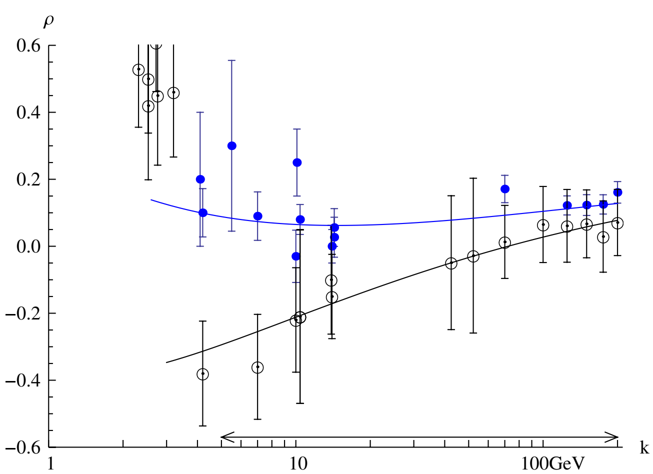

FIG. 2 (color online). Results of the fit to $\rho^{K^\mp p}$. The solid (blue) data points represent $K^- p$ and the open (black) data points are $K^+ p$. The horizontal arrow represents the energy region of the fit to the asymptotic amplitude.

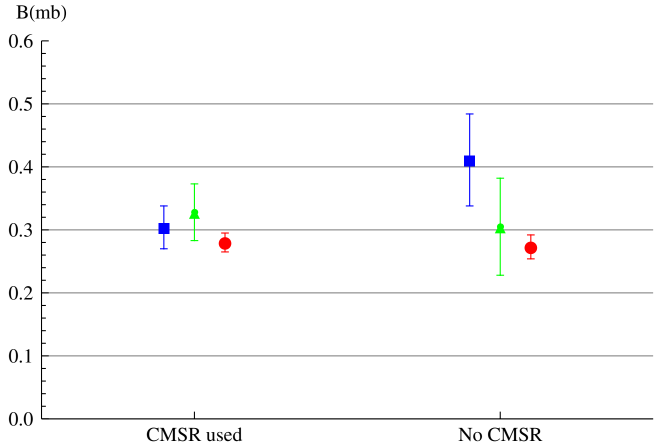

FIG. 3 (color online). Values of the asymptotic parameter B (mb) for the $p(\bar{p})p$, πp, and Kp total cross sections using CMSRs (left) in comparison with the cases with no CMSR constraint (right). Red circles and blue squares represent $p(\bar{p})p$ and πp, respectively, which were obtained in our previous analysis II [8]. Green triangles are Kp, obtained in the present analysis.

There are several comments that should be added: (i) In the present analysis we take $\bar{\nu}_1 = 5$ GeV, and the $\sigma_{\rm tot}$ and Ref data in $k \geq \bar{\nu}_1$ are fit. The results are almost independent of the choice of $\bar{\nu}_1$. If we take $\bar{\nu}_1 = 4$ GeV and the CMSRs with $\epsilon = -1, -3$ together with the data of $k \geq 4$ GeV are fit simultaneously, we obtain the result $B_{Kp} = 0.311 \pm 0.039$ mb, which is almost the same result as Eq. (17). (ii) In our previous analysis [8], $\bar{\nu}_1 = 20$ GeV was taken, and the data of σ in $k \geq 20$ GeV and Re$f(k)$ in $k \geq 5$ GeV were fit. The $n = 1$ integral with $\bar{\nu}_1 = 20$ GeV on the LHS of (11) [which is obtained as 6802.61 (10.90) GeV] is fit simultaneously to the data of the same energy regions as the previous analysis [8]. The resulting value is $B_{Kp} = 0.360 \pm 0.110$ mb, which is almost the same as II [8], $B_{Kp} = 0.354 \pm 0.099$ mb, so no improvement is obtained in this method. Hence we have adopted a different energy region in the present analysis.

The obtained value of B_{Kp} together with the previous estimates of $B_{\pi p}$ and B_{pp} in Ref. [8] are shown graphically in Fig. 3 compared with the case with no use of sum rules. Our results strongly suggest the universality of the coefficient B. It should be noted that the inclusion of low-energy data through the CMSRs is essential in reaching this conclusion, especially that $B_{Kp} \simeq B_{\pi p}$ as shown in the present work. Our conclusion is that the B does not depend on the flavor content of the particles scattered.

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[18] The best-fit values [13] are $K_{KK} = 1.01(7)$, $K_{K\Sigma} = -0.92(5)$, $K_{K\Lambda} = -0.66(5)$, $K_{\Sigma\Sigma} = 0.58(19)$, $K_{\Sigma\Lambda} = 0.62(24)$, $K_{\Lambda\Lambda} = 0.08(25)$, $A_{KK} = -0.10(5)$, $A_{K\Sigma} = -0.92(3)$, $A_{\Sigma\Sigma} = 1.60(16)$, $R_{KK} = 0.41(10)$,

$R_{K\Sigma} = -0.30(18)$, $R_{\Sigma\Sigma} = 0.07(35)$ where the elements of $\boldsymbol{K}$, $\boldsymbol{R}(\boldsymbol{A})$ are given in fm (fm^{-1}).

[19] The factor $\frac{\sqrt{s}}{M}$ comes from different normalizations of f and $\boldsymbol{T}$, $\mathrm{Im}f = \frac{k}{4\pi}\sigma$ and $\mathrm{Im}(\boldsymbol{T})_{KK} = \frac{p}{4\pi}\sigma$. The laboratory momentum k is related to the c.m. momentum p by $k = \frac{\sqrt{s}}{M}p$.

[20] In the analysis we obtained $\mathrm{Re}f^{K^{\mp}p}$ data points from original $\rho^{K^{\mp}p}$ data [15] by multiplying by $\mathrm{Im}f^{K^{\mp}p} = \frac{k}{4\pi}\sigma_{\mathrm{tot}}^{K^{\mp}p}(k)$, where we use as $\sigma_{\mathrm{tot}}^{K^{\mp}p}(k)$ the fit result of Ref. [15]. See Ref. [8] for more detail. In the analysis $\sigma_{\mathrm{tot}}^{K^{\mp}p}$ data and $\mathrm{Re}f^{K^{\mp}p}$ data are fit simultaneously.