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Twist-four corrections to parity-violating electron-deuteron scattering

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Parity-violating electron-deuteron scattering can potentially provide a clean access to electroweak couplings that are sensitive to physics beyond the standard model. However, hadronic effects can contaminate their extraction from high-precision measurements. Power-suppressed contributions are one of the main sources of uncertainties along with charge-symmetry violating effects in leading-twist parton densities. In this work we calculate the twist-four correlation functions contributing to the left-right polarization asymmetry making use of nucleon multiparton light-cone wave functions.

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I. INTRODUCTION

Even after decades of experimental studies, deep inelastic scattering (DIS) remains one of the most powerful tools for unraveling the partonic structure of nucleons and nuclei. DIS also allows for systematic searches for physics beyond the standard model. Parity violation in DIS (PV-DIS) at medium energies is particularly sensitive to effects of new physics. Historically, this process played an important role in verifying the standard model [1,2]. Today the search for new physics motivates a number of ongoing and planned experiments [3–9]. The physics reason for this great interest is that within the standard model the Weinberg angle θ_W should show a highly nontrivial characteristic scale dependence, which can be mapped out by combining experiments at different momentum scales. The SoLID experiment at JLab [10,11] (see also [12,13]) will be especially sensitive to the poorly measured weak neutral coupling constants C_{2q} in the low-energy electroweak Lagrangian

$$\mathcal{L}_{\rm PV} = \frac{G_F}{\sqrt{2}}[\bar{e}\gamma^\mu\gamma_5 e(C_{1u}\bar{u}\gamma_\mu u + C_{1d}\bar{d}\gamma_\mu d) + \bar{e}\gamma^\mu e(C_{2u}\bar{u}\gamma_\mu\gamma_5 u + C_{2d}\bar{d}\gamma_\mu\gamma_5 d)]. \quad (1)$$

To analyze the theoretical situation, effects of new physics are parameterized by $\delta C_{i\alpha}$ [14] according to $C_{1\alpha} = 2g_A^e g_V^\alpha + \delta C_{1\alpha}$ and $C_{2\alpha} = 2g_V^e g_A^\alpha + \delta C_{2\alpha}$, where the standard model coupling constants are $g_{V,A}^f = Q_{wf}^L \pm Q_{wf}^R$ in terms of the left and right ($\alpha = L, R$) weak charges

$$Q_{w,f}^\alpha = T_3(f_\alpha) - Q(f)\sin^2\theta_W. \quad (2)$$

The $(\delta C_{i\alpha})$ are inaccessible in other measurements, which gives PV-DIS its unique quality.

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The projected sensitivity of the SoLID experiment for an asymmetry discussed below is $\delta A/A = \pm 0.005$(stat) at an average Q^2 of 3.3 GeV^2 and an average x of $\langle x\rangle = 0.34$, which sets the scale for the size of acceptable theoretical uncertainties. At this level of precision several sources of systematic uncertainties can hamper a precise determination of the $C_{i\alpha}$, as discussed recently in Refs. [15,16]. Some of the most relevant are uncertainties in leading-twist parton distributions functions, in particular, charge-symmetry violation (CSV), contributions from higher-twist correlation functions, and kinematical target-mass corrections. Far from being a nuisance, higher-twist correlations encode very interesting and yet little known information on hadron structure. Therefore, all cases in which leading-twist contributions are absent or reduced, such that one has a good chance to determine higher-twist ones, are of great interest. If the relevant higher-twist contributions are measurable with a given experimental sensitivity, as we will claim they are not in this case, one is in a win-win situation: PV-DIS can be regarded either as a tool for finding new physics, in case the effects of the latter are prominent, or it can be seen as a venue to access unknown aspects of strong interaction physics.

Parity-violating weak interactions give rise to an asymmetry in the inclusive cross sections for scattering of left- and right-handed electrons off a deuteron

$$A_{RL} = \frac{d\sigma^R - d\sigma^L}{d\sigma^R + d\sigma^L}. \quad (3)$$

This is the main medium-energy observable in PV-DIS that will be scrutinized at Jefferson Lab [10–12]. Among all uncertainties of the theoretical prediction of this asymmetry we will focus on the power-suppressed contributions. Two recent studies of it reached somewhat different conclusions [15,16]. Our results turn out to be very similar to those from [15].

It was demonstrated by Bjorken and Wolfenstein [17,18] that twist-four corrections to the asymmetry are due to a single (nonlocal) four-quark operator. The first estimates of the matrix element of the spin-two part of this operator were obtained in the framework of the MIT bag model [19,20]. This technique was extended in Ref. [15] to include the effects of higher spin operators. It was found that their effect is negligible within the model used. Renormalon analysis offers yet another technique for modeling the momentum fraction dependence of certain higher-twist matrix elements [21,22]. These renormalon-based studies demonstrate [21] that higher-twist correlation functions (involving two quarks and a gluon) tend to grow at large x, i.e., like $(1-x)^{-1}$, in qualitative agreement with experimental measurements of electroweak structure functions [23]. However, the four-quark operators we consider are free from ultraviolet renormalons [24] and thus this approach is not applicable. The absence of renormalon contributions might explain the qualitatively different behavior of such correlators and gluonic ones. In this work we calculate twist-four corrections employing a model for the nucleon wave functions in the light-cone formalism that was proposed by [25–27].

The paper is organized as follows: Section II contains basic definitions and notations. In Sec. III we give a detailed discussion of power corrections to the asymmetry (3) . In Sec. IV the necessary ingredients of the light-cone formalism are given. Results of our calculation and our prediction for the twist-four corrections to the asymmetry are collected in Sec. V. Finally we give our conclusions. Two appendices contain technical details and formulas left out in the body of the paper.

II. PRELIMINARIES

Let us briefly discuss the physical observables we will be analyzing below. The cross section for polarized electron scattering off an unpolarized deuteron target, with kinematics shown in Fig. 1, is given by the sum of three terms

$$d\sigma^{L/R} = d\sigma^{L/R}_{ee} + d\sigma^{L/R}_{ww} + 2d\sigma^{L/R}_{ew}, \tag{4}$$

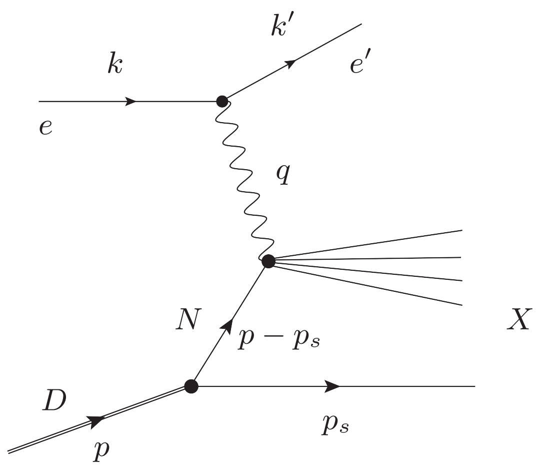

FIG. 1. Kinematics in deep inelastic deuteron-electron scattering.

which describe the contributions due to the electromagnetic and weak interactions and their interference. Each term is a function of the standard kinematical variables

$$Q^2 = -q^2, \quad \nu = (p \cdot q), \quad x = \frac{Q^2}{2\nu}, \quad y = \frac{(p \cdot q)}{(p \cdot k)}. \tag{5}$$

Each term in Eq. (4) is given by the convolution of a leptonic and hadronic tensor. This reads in the laboratory frame

$$\frac{d\sigma^{L/R}_{ab}}{d\Omega dk'_0} = \frac{k'_0}{k_0} A_{ab}(Q^2)(L^{L/R}_{ab})_{\mu\nu} W^{\mu\nu}_{ab}, \tag{6}$$

where the repeated Latin indices imply summation over electromagnetic and weak exchanges $a, b = (e, w)$. The coefficients

$$A_{ee}(Q^2) = \frac{2\alpha^2}{Q^4}, \quad A_{ew}(Q^2) = \frac{\sqrt{2} G_F \alpha}{\pi Q^2}, \quad A_{ww}(Q^2) = \frac{G_F^2}{\pi^2}$$

encode the products of gauge boson propagators and interaction strengths. The leptonic tensor admits the conventional form

$$(L^{L/R}_{ab})_{\mu\nu} = Q^{L/R}_a Q^{L/R}_b \ell^{L/R}_{\mu\nu}, \tag{7}$$

which is a product of the electromagnetic (weak) charge [28] $Q^{L/R}_{e(w)}$ for the left (right) handed electron

$$Q^{L/R}_e = -1, \qquad Q^L_w = -\tfrac{1}{2} + \sin^2\theta_W, \qquad Q^R_w = \sin^2\theta_W,$$

and

$$\ell^{L/R}_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}(k \cdot k') \pm i\varepsilon_{\mu\nu\rho\sigma} k^\rho k'^\sigma. \tag{8}$$

The hadronic tensor $W^{\mu\nu}_{ab}$ is the deuteron matrix element of the product of currents

$$W^{\mu\nu}_{ab}(p, q) = \frac{1}{8\pi M_D} \int d^4 z e^{iq\cdot z} \langle D(p)|\{j^\mu_a(z) j^\nu_b(0) + j^\mu_b(z) j^\nu_a(0)\}|D(p)\rangle,$$

where M_D is the deuteron mass and averaging over deuteron polarizations is implied. The electromagnetic and neutral quark current are defined as [cf. Eq. (2)]

$$j^\mu_e = \bar{q} Q \gamma^\mu q, \qquad j^\mu_w = \bar{q}_L \tau_3 \gamma^\mu q_L - \sin^2\theta_W j^\mu_e, \tag{10}$$

where $q = (u, d)$. It was demonstrated by Bjorken [17] that if one assumes valence quark dominance in the region $x > 0.4$ and neglects all sea quark and isospin breaking effects (which should be justified for large virtual mass Q^2), the asymmetry (3) becomes free of hadronic physics contaminations and is given by the Cahn-Gilman formula [2]

$$A_{RL} = -\frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \frac{9}{10} \left[\left(1 - \frac{20}{9}\sin^2\theta_W\right) + (1 - 4\sin^2\theta_W) \frac{1-(1-y)^2}{1+(1-y)^2}\right]. \tag{11}$$

New physics is best parameterized by allowing for nonstandard values for the coefficients $C_{i\alpha}$, which are reintroduced in Eq. (11) by replacing

$$1 - \tfrac{20}{9}\sin^2\theta_W \to -\tfrac{2}{3}(2C_{1u} - C_{1d}),$$
$$1 - 4\sin^2\theta_W \to -\tfrac{2}{3}(2C_{2u} - C_{2d}).$$

However, the assumptions leading to vanishing hadronic effects are only valid approximately and have to be abandoned in the analysis of high-precision experiments. The main hadronic effects are caused by CSV and power-suppressed correlators. The central point behind our work, and that of others, is that these effects have a strong x dependence which allows, if precisely known, to isolate and subtract them and thus to increase the sensitivity of experiments like SoLID to new physics. Thus one has to go beyond leading approximations and has to take into account higher-order electroweak effects, sea quark effects, target mass, and higher-twist corrections at least at a level matching the accuracy of experimental measurements.

CSV arises from isospin violation of u and d quark distributions in the proton and neutron, i.e., by $\delta u = u_p - d_n \neq 0$ and $\delta d = d_p - u_n \neq 0$. Modern global analysis of parton distribution functions incorporate CSV effects, which are found to become more significant as x decreases [29], $R_{\rm CSV} \sim (\delta u - \delta d)/(u + d) \sim (1 - x)^4\sqrt{x}$. CSV effects might explain a significant fraction of the discrepancy between the NuTeV results [30] and predictions based on the standard model and isospin symmetry.

The other source of corrections are power-suppressed contributions from multiparticle correlation functions. Obviously the nucleon wave function is a complex state containing many highly entangled Fock states, only partially characterized by parton distribution functions. The isolation and determination of specific multiple-field correlators is the logical next step to exploring hadrons and is therefore of great interest in its own right. In contrast to mere one-particle probability distributions, they contain information on relative phases. As they are typically power suppressed, high luminosity experiments at medium-large Q^2 are needed to extract them. These are requirements that are perfectly fit by Jefferson Lab, especially after the energy upgrade.

III. TWIST-FOUR CORRECTIONS

In the region of low transferred momentum $Q^2 \ll M_W^2$ one has $d\sigma_{ww} \ll d\sigma_{ew} \ll d\sigma_{ee}$ and the asymmetry takes the form

$$A_{RL} = \frac{d\sigma_{ew}^R - d\sigma_{ew}^L}{d\sigma_{ee}}, \qquad (12)$$

where we took into account that $d\sigma_{ee}^L = d\sigma_{ee}^R \equiv d\sigma_{ee}$.

Introducing the scalar, isovector, and axial isovector currents

$$S^\mu = \tfrac{1}{2}\bar{q}\gamma^\mu q, \quad V^\mu = \bar{q}\gamma^\mu\tau^3 q, \quad A^\mu = \bar{q}\gamma^\mu\gamma_5\tau^3 q, \qquad (13)$$

one can represent the electromagnetic (weak) hadronic tensors as follows:

$$W_{ee}^{\mu\nu}(p, q) = W_V^{\mu\nu}(p, q) + \tfrac{1}{9}W_S^{\mu\nu}(p, q),$$
$$W_{ew}^{\mu\nu}(p, q) = (\tfrac{1}{2} - \sin^2\theta)W_V^{\mu\nu}(p, q) - \tfrac{1}{9}\sin^2\theta W_S^{\mu\nu}(p, q) - \tfrac{1}{2}W_A^{\mu\nu}(p, q), \qquad (14)$$

where

$$W_V^{\mu\nu}(p, q) = \frac{1}{4\pi M_D}\int d^4z e^{iq\cdot z}\langle D(p)|V^\mu(z)V^\nu(0)|D(p)\rangle,$$
$$W_S^{\mu\nu}(p, q) = \frac{1}{4\pi M_D}\int d^4z e^{iq\cdot z}\langle D(p)|S^\mu(z)S^\nu(0)|D(p)\rangle,$$
$$W_A^{\mu\nu}(p, q) = \frac{1}{8\pi M_D}\int d^4z e^{iq\cdot z}\langle D(p)|A^\mu(z)V^\nu(0) + V^\mu(z)A^\nu(0)|D(p)\rangle. \qquad (15)$$

Here we took into account that the deuteron matrix elements of nonsinglet terms, i.e., involving the product of isovector and isosinglet currents VS and AS, vanish by isospin symmetry since the deuteron is an isoscalar state. Keeping only twist-two terms in the operator product expansion expansion of the hadronic tensors (15), one arrives at the Cahn-Gilman formula (11); the first and the second term in the square brackets in (11) arise from vector-vector (W_{ew}^v) and axial-vector (W_{ew}^a) correlators, respectively. The corrections to the Cahn-Gilman formula can be parameterized as follows:

$$A_{RL} = -\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\frac{3}{5}\left[\tilde{a}_1 + \tilde{a}_2\frac{1 - (1 - y)^2}{1 + (1 - y)^2}\right], \qquad (16)$$

where ($i = 1, 2$)

$$\tilde{a}_i = -(2C_{iu} - C_{id})[1 + R_i]. \qquad (17)$$

Here the functions R_i ($i = 1, 2$) alluded to before receive contributions from several sources of hadronic effects. The precision measurement of the mixing angle at low Q^2 gives $\sin^2\theta_W \simeq 0.2397$ [5]. Thus the axial current contribution ($\tilde{a}_2$) to the asymmetry is relatively small, and we will focus on the calculation of twist-four corrections to $\tilde{a}_1$. They can be easily identified. Indeed, neglecting effects of isospin breaking one gets (see Ref. [17])

$$\langle D|S^\mu(z)S^\nu(0) - V^\mu(z)V^\nu(0)|D\rangle = \tfrac{1}{2}\langle D|\bar{u}(z)\gamma^\mu u(z)\bar{d}(0)\gamma^\nu d(0) + (u \leftrightarrow d)|D\rangle. \qquad (18)$$

The expansion of the operator at the right-hand side of this equation starts from twist four. In terms of

$$W_{ud}^{\mu\nu}(p,q) = \frac{1}{8\pi M_D}\int d^4z e^{iq\cdot z}\langle D(p)|\bar{u}(z)\gamma^\mu u(z)\bar{d}(0)\gamma^\nu d(0) + (u\leftrightarrow d)|D(p)\rangle \qquad (19)$$

we define the structure functions $F_{i=1,2}^a$ as coefficients in the tensor decomposition

$$M_D W_a^{\mu\nu}(p,q) = \Big(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\Big)F_1^a + \frac{1}{\nu}\Big(p^\mu - \frac{(pq)}{q^2}q^\mu\Big) \times \Big(p^\nu - \frac{(pq)}{q^2}q^\nu\Big)F_2^a, \quad (20)$$

where the index runs over $a = V, S, ud$. The twist-four contribution to R_1 takes the form

$$R_1^{\text{tw-4}} = -\frac{1}{10(1-\frac{20}{9}\sin^2\theta_W)}\frac{\mathcal{F}^{ud}}{\mathcal{F}^S}, \quad (21)$$

where

$$\mathcal{F}^a = xyF_1^a - \Big[1 - \frac{1}{y} + \frac{xM_D}{2E}\Big]F_2^a. \quad (22)$$

Keeping in $\mathcal{F}^S$ and $\mathcal{F}^{ud}$ the dominant contributions only, i.e., twist-two and twist-four, respectively, and taking into account that they both satisfy the Callan-Gross relation $F_2 = 2xF_1$, one finds

$$\frac{\mathcal{F}^{ud}}{\mathcal{F}^S} \simeq \frac{F_1^{ud}}{F_1^S}. \quad (23)$$

The expression for F_1^S at lowest order of perturbation theory is given by the sum of parton densities in the deuteron

$$F_1^S(x) = \tfrac{1}{8}[u_D(x) + d_D(x) + \bar{u}_D(x) + \bar{d}_D(x)], \quad (24)$$

where as usual $\bar{q}_D(x) = -q_D(-x)$. The quark distribution functions are defined by the matrix elements of nonlocal light-cone operators,

$$\langle D|\bar{q}(z)\not{z}q(-z)|D\rangle = 2(p\cdot z)\int_{-1}^1 dx e^{2i(p\cdot z)x}q_D(x). \quad (25)$$

To evaluate F_1^{ud} we represent the hadronic tensor $W_{ud}^{\mu\nu}$ via the dispersion relation as a time-ordered product of electroweak currents

$$W_{ud}^{\mu\nu}(p,q) = \text{Im}\Big[\frac{i}{4\pi M_D}\int d^4z e^{iq\cdot z}\langle D(p)|T\{\bar{u}(z)\gamma^\mu u(z)\bar{d}(0) \times \gamma^\nu d(0) + (u\leftrightarrow d)\}|D(p)\rangle\Big] \quad (26)$$

and make use of the operator product expansion [31]

$$T\{\bar{u}(z)\gamma_\mu u(z)\bar{d}(-z)\gamma_\nu d(-z) + (u\leftrightarrow d)\}^{\text{tw-4}} = \frac{\alpha_s}{16\pi i}\Big\{-\log z^2\partial_\mu\partial_\nu\int_0^1 du\frac{\bar{u}}{u^2}\mathcal{Q}(uz) + \frac{1}{z^2}S_{\mu\alpha\nu\beta}z^\alpha\partial^\beta\int_0^1\frac{du}{u}\mathcal{Q}(uz)\Big\}, \quad (27)$$

where $S_{\mu\alpha\nu\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\nu\alpha}g_{\mu\beta} - g_{\mu\nu}g_{\alpha\beta}$.

The operator $\mathcal{Q}$ ($\mathcal{Q}_2$ in the notations of Ref. [31]) is given by the following expression:

$$\mathcal{Q}(z) = i\int_{-1}^1 dv\int_{-1}^v dt[\Pi_{12}^-\Pi_{34}^-\mathcal{Q}_V(1,v,t,-1) + \Pi_{12}^+\Pi_{34}^+\mathcal{Q}_A(1,v,t,-1)] + (z\leftrightarrow -z). \quad (28)$$

Here

$$\mathcal{Q}_A(a) = (\bar{u}(a_1z)t^a\not{z}\gamma_5u(a_2z))(\bar{d}(a_3z)t^a\not{z}\gamma_5d(a_4z)),$$
$$\mathcal{Q}_V(a) = (\bar{u}(a_1z)t^a\not{z}u(a_2z))(\bar{d}(a_3z)t^a\not{z}d(a_4z)), \quad (29)$$

and $\Pi_{ik}^\pm = (1\pm P_{ik})$, where P_{ik} is the permutation operator, e.g., $P_{12}\mathcal{Q}_V(a_1,a_2,a_3,a_4) = \mathcal{Q}_V(a_2,a_1,a_3,a_4)$. For later convenience we rewrite (28) as follows:

$$\mathcal{Q}(z) = i\int_{-1}^1 dv\int_{-1}^v dt[\hat{\mathcal{Q}}_+(1,v,t,-1) - \hat{\mathcal{Q}}_-(1,v,t,-1)], \quad (30)$$

where

$$\hat{\mathcal{Q}}_+(a) = (1+P_{12}P_{34})(1+P_{14}P_{23})\mathcal{Q}_+(a),$$
$$\hat{\mathcal{Q}}_-(a) = (P_{12}+P_{34})(1+P_{14}P_{23})\mathcal{Q}_-(a) \quad (31)$$

and

$$\mathcal{Q}_\pm(a) = \mathcal{Q}_V(a) \pm \mathcal{Q}_A(a). \quad (32)$$

Let us define the twist-four distribution $\tilde{\mathcal{Q}}_D(x)$ as a deuteron matrix element of the operator $\mathcal{Q}$

$$\langle D|\mathcal{Q}(z)|D\rangle = i\int_{-1}^1 dx e^{2i(p\cdot z)x}\tilde{\mathcal{Q}}_D(x). \quad (33)$$

It follows from (28) and (32) that $\tilde{\mathcal{Q}}_D(x)$ is an even function of x with vanishing first moment,

$$\int_{-1}^1 dx\tilde{\mathcal{Q}}_D(x) = 0.$$

Inserting (27) and (28) into (26) one finds after some algebra

$$F_1^{ud}(x) = -\frac{\alpha_s\pi}{4Q^2}x\tilde{\mathcal{Q}}_D(x). \quad (34)$$

Then, keeping in F_1^S the valence quark contribution only we obtain the following expression for the twist-four correction to the asymmetry:

$$R_1^{\text{tw-4}} = \frac{1}{Q^2}\frac{\alpha_s\pi}{5(1-\frac{20}{9}\sin^2\theta_W)}\frac{x\tilde{\mathcal{Q}}_D(x)}{u_D(x)+d_D(x)}. \quad (35)$$

The deuteron is a weakly coupled state of the proton and neutron with the binding energy $E_B \simeq 2.2$ MeV. In the incoherent impulse approximation its hadronic tensor in the deuteron's rest frame can be represented as [32]

$$W_{\mu\nu}^D(p,q) \simeq \int\frac{d^3\boldsymbol{p}_s}{(2\pi)^3E_{p_s}/M_N}|f(\boldsymbol{p}_s)|^2(W_{\mu\nu}^{(p)}(p-p_s,q) + W_{\mu\nu}^{(n)}(p-p_s,q)), \quad (36)$$

where $p_s = (E_{p_s}, \boldsymbol{p}_s)$ and the integration is performed over the spectator three-momentum $\boldsymbol{p}_s$, see Fig. 1. Here $f(\boldsymbol{p}_s)$ is the deuteron wave function in its rest frame, normalized as $[(2\pi)^{-3}\int d^3\boldsymbol{p}_s|f(\boldsymbol{p}_s)|^2 = 1]$ and $W_{\mu\nu}^{(p(n))}$ are the proton (neutron) hadronic tensors. The function $f(\boldsymbol{p}_s)$ is strongly peaked at $\boldsymbol{p}_s = 0$ [32]. Thus one can simplify the above expression by neglecting terms of order $\sim|\boldsymbol{p}_s|/M_N$ and higher under the integral. Then one finds

$$W^D_{\mu\nu}(p,q) \simeq W^{(p)}_{\mu\nu}(p/2,q) + W^{(n)}_{\mu\nu}(p/2,q), \tag{37}$$

and as a consequence $d\sigma_d \simeq d\sigma_p + d\sigma_n$. Then Eq. (37) yields the following relation between the structure functions of deuteron and nucleons,

$$F_2^d(x/2) \simeq F_2^p(x) + F_2^n(x).$$

It turns out that this approximation overestimates the deuteron structure function by $5 \div 10\%$ [32,33]. This is acceptable for our purposes. For the parton densities the corresponding relation reads (cf. Eqs. (47) and (48) in Ref. [15])

$$\tfrac{1}{2} q_D(x/2) \simeq q_p(x) + q_n(x). \tag{38}$$

Similarly, defining the proton (neutron) twist-four distributions by

$$\langle N|\mathcal{Q}(z)|N\rangle = i\int_{-1}^{1} dx e^{2i(p\cdot z)x}\tilde{\mathcal{Q}}_p(x) \tag{39}$$

one gets for the deuteron twist-four function $\tilde{\mathcal{Q}}_D(x)$

$$\tfrac{1}{4}\tilde{\mathcal{Q}}_D(x/2) = \tilde{\mathcal{Q}}_p(x) + \tilde{\mathcal{Q}}_n(x) = 2\tilde{\mathcal{Q}}_p(x). \tag{40}$$

Here we took into account that $\tilde{\mathcal{Q}}_p(x) = \tilde{\mathcal{Q}}_n(x)$ due to isospin symmetry.

We also define the nucleon twist-four distribution $\mathcal{Q}_\pm(\xi)$ [and similarly $\hat{\mathcal{Q}}_\pm(\xi)$] by

$$\langle N|\mathcal{Q}_\pm(a)|N\rangle = (p\cdot z)^2 \int \mathcal{D}\xi e^{-i(p\cdot z)\sum_k a_k\xi_k}\mathcal{Q}_\pm(\xi), \tag{41}$$

where ξ cumulatively denotes the array of four variables $\xi = (\xi_1, \xi_2, \xi_3, \xi_4)$ and the integration measure stands for $\mathcal{D}\xi = \prod_{k=1}^4 d\xi_k \delta(\sum_i \xi_i)$. Then it follows from Eq. (30) that

$$\tilde{\mathcal{Q}}_p(x) = \int \frac{\mathcal{D}\xi}{\xi_2\xi_3(\xi_2+\xi_3)}\{(\xi_2+\xi_3)\delta(x+\xi_1+\xi_2) - \xi_3\delta(x+\xi_1) - \xi_2\delta(\xi_4 - x)\}[\hat{\mathcal{Q}}_+(\xi) - \hat{\mathcal{Q}}_-(\xi)]. \tag{42}$$

IV. NUCLEON LIGHT-CONE WAVE FUNCTIONS

Our lack of information on the magnitude of higher-twist matrix elements is the main obstacle for a quantitative analysis of power-suppressed contributions to hadronic cross sections. Hadron structure models provide estimates for the size of nonperturbative matrix elements, but their predictions vary strongly. This is understandable in view of the fact that confinement is incorporated rather differently. The first estimates of twist-four corrections to the asymmetry (3) were obtained within the MIT bag model [19,20], which incorporates confinement quite *ad hoc* (see also Refs. [15,34] for recent developments). In this work we use another approach, the light-cone formalism [35], for the evaluation of twist-four corrections.

In the light-cone formalism the nucleon is represented by a superposition of multiparton Fock state wave functions. The latter are functions of the parton longitudinal momentum fractions x_i, transverse momenta $\boldsymbol{k}_{\perp i}$, and parton helicities. The light-cone wave functions (LCWFs) are eigenfunctions of the QCD Hamiltonian quantized in the light-cone gauge [36,37]. Models for LCWFs of various degree of sophistication have been considered in different contexts in the vast literature on the subject, see, e.g., Refs. [25,26,35,38–41]. In this work we will follow the formalism developed in Refs. [25–27,38] and will take into account only the lowest components of the nucleon LCWFs: the three-quark and three-quark-gluon component. The details of the light-cone formalism relevant for our further discussion are collected in Appendix A.

The three-quark component of the nucleon state is parameterized in terms of corresponding LCWF $\Psi^{(0)}_{123}$ as follows:

$$|p,+\rangle_{3q} = -\frac{\epsilon^{ijk}}{\sqrt{6}}\int [\mathcal{D}X]_3 \Psi^{(0)}_{123}(X)(u^\dagger_{i\uparrow}(1)u^\dagger_{j\downarrow}(2)d^\dagger_{k\uparrow}(3) - u^\dagger_{i\uparrow}(1)d^\dagger_{j\downarrow}(2)u^\dagger_{k\uparrow}(3))|0\rangle. \tag{43}$$

Here and below for notational simplicity arguments like ℓ in $u^\dagger_{i\uparrow}(\ell)$, stand for the collection of all relevant arguments, i.e., $u^\dagger_{\uparrow i}(\ell) = u^\dagger_{\uparrow i}(x_\ell, \boldsymbol{k}_{\perp\ell})$. The creation (annihilation) operators of a quark with helicity λ and momentum p satisfy the commutation relation (A10). As usual, the momentum fraction x_i is defined as the ratio of the longitudinal (i.e., "+") momentum of the ith parton and the one of the nucleon. The integration measure has the following form:

$$[\mathcal{D}X]_N = \frac{1}{\sqrt{x_1\ldots x_N}}[dx]_N[d^2\boldsymbol{k}_\perp]_N,$$
$$[dx]_N = \prod_{i=1}^N dx_i\delta\Big(1-\sum x_i\Big), \tag{44}$$
$$[d^2\boldsymbol{k}_\perp]_N = \frac{1}{(16\pi^3)^{N-1}}\prod_{i=1}^N d^2\boldsymbol{k}_{\perp i}\delta^{(2)}\Big(\sum \boldsymbol{k}_{\perp i}\Big).$$

Here we accept the Bolz-Kroll *Ansatz* [25] for the function $\Psi^{(0)}_{123}$

$$\Psi^{(0)}_{123} = \frac{f_N}{4\sqrt{6}}\phi(x_1,x_2,x_3)\Omega_3(a_3,x_i,\boldsymbol{k}_{\perp i}). \tag{45}$$

The transverse momentum dependence is encoded in the function Ω_N

$$\Omega_N(a_N,x_i,\boldsymbol{k}_{\perp i}) = \frac{(16\pi^2 a_N^2)^{N-1}}{x_1x_2\ldots x_N}\exp\Big[-a_N^2\sum_i \boldsymbol{k}^2_{\perp i}/x_i\Big], \tag{46}$$

which is normalized such that

$$\int [d^2\boldsymbol{k}_\perp]_N \Omega_N(a_N, x_i, \boldsymbol{k}_{\perp i}) = 1,$$
$$\int [d^2\boldsymbol{k}_\perp]_N \Omega_N^2(a_N, x_i, \boldsymbol{k}_{\perp i}) = \frac{\rho_N}{x_1 \ldots x_N}, \tag{47}$$

where $\rho_N = (8\pi^2 a_N^2)^{N-1}$. The function $\phi(x_i)$, entering (45), depends only on the longitudinal momentum fractions of constituent partons and coincides with the leading-twist, i.e., twist-three, nucleon distribution amplitude defined at the low-energy scale $\mu_0 = 1$ GeV. We use the following *Ansatz* for $\phi(x)$ [25]:

$$\phi(x_1, x_2, x_3) = 60 x_1 x_2 x_3 (1 + 3x_1), \tag{48}$$

which emerges from the truncation of the conformal partial wave expansion after the lowest few terms. The normalization constant f_N in Eq. (45) is determined by the matrix element of the corresponding local three-quark operator. The analysis within the framework of QCD sum rules [42] yields in the following estimate for f_N [43–47] at the scale $\mu_0 = 1$ GeV

$$f_N = (5.0 \pm 0.5) \times 10^{-3}\ \text{GeV}^2. \tag{49}$$

On the other hand, the parameter a_3 determines the smearing of the wave function in the transverse plane and, e.g., the average quark transverse momentum. Following Ref. [27] we take $a_3 = 0.73\ \text{GeV}^{-1}$ in our estimates. With this set of parameters, the contribution of the three-quark Fock state to the norm of the nucleon state is about 17%,

$$P_{3q} = \frac{435}{112} f_N^2 \rho_3 \simeq 0.17. \tag{50}$$

The four-parton quark-gluon contributions with zero angular momentum to the nucleon states have the following form [27]:

$$|p,+\rangle_{uudg_\downarrow} = \epsilon^{ijk} \int [\mathcal{D}X]_4 \Psi^{\downarrow}_{1234}(X) a_\downarrow^{a,\dagger}(4) \times [t^a u_\uparrow(1)]_i^\dagger u_{j\uparrow}^\dagger(2)) d_{k\uparrow}^\dagger(3)|0\rangle,$$
$$|p,+\rangle_{uudg^\uparrow} = \epsilon^{ijk} \int [\mathcal{D}X]_4 \{\Psi^{\uparrow(1)}_{1234}(X)[t^a u_\downarrow(1)]_i^\dagger \times (u_{j\uparrow}^\dagger(2) d_{k\downarrow}^\dagger(3) - d_{j\uparrow}^\dagger(2) u_{k\downarrow}^\dagger(3)) a_\uparrow^{a,\dagger}(4) + \Psi^{\uparrow(2)}_{1234}(X) u_{i\downarrow}^\dagger(1)([t^a u_\downarrow(2)]_j^\dagger d_{k\uparrow}^\dagger(3) - [t^a d_\downarrow(2)]_j^\dagger u_{k\uparrow}^\dagger(3)) a_\uparrow^{a,\dagger}(4)\}|0\rangle, \tag{51}$$

where the four-parton LCWFs are again taken in the Bolz-Kroll form

$$\Psi^{\downarrow}_{1234} = \frac{1}{\sqrt{2x_4}} \phi_g(x_1, x_2, x_3, x_4) \Omega_4(a_g, x_i, \boldsymbol{k}_{\perp i}),$$
$$\Psi^{\uparrow(1)}_{1234} = \frac{1}{\sqrt{2x_4}} \psi_g^{(1)}(x_1, x_2, x_3, x_4) \Omega_4(a_g, x_i, \boldsymbol{k}_{\perp i}), \tag{52}$$
$$\Psi^{\uparrow(2)}_{1234} = \frac{1}{\sqrt{2x_4}} \psi_g^{(2)}(x_1, x_2, x_3, x_4) \Omega_4(a_g, x_i, \boldsymbol{k}_{\perp i}).$$

The functions ϕ_g, $\psi_g^{(i)}$ that depend on the light-cone momentum fractions of the partons can be expressed in terms of the twist-four quark-gluon nucleon distribution amplitudes Ξ_4^g, Ψ_4^g, Φ_4^g introduced in Ref. [48]. Keeping only the lowest terms in the conformal expansion of the corresponding distribution amplitudes one arrives at the following expressions [27]:

$$g\phi_g(x_1, x_2, x_3, x_4) = -210 m_N \lambda_1^g x_1 x_2 x_3 x_4^2,$$
$$g\psi_g^{(1)}(x_1, x_2, x_3, x_4) = -105 m_N (\lambda_2^g + \lambda_3^g) x_1 x_2 x_3 x_4^2, \tag{53}$$
$$g\psi_g^{(2)}(x_1, x_2, x_3, x_4) = -105 m_N (\lambda_2^g - \lambda_3^g) x_1 x_2 x_3 x_4^2.$$

The sum rule technique was found to give the following estimates for the coupling constants λ_k^g at low-energy scale 1 GeV [27]

$$\lambda_1^g = (2.6 \pm 1.2) \times 10^{-3}\ \text{GeV}^2,$$
$$\lambda_2^g = (2.3 \pm 0.7) \times 10^{-3}\ \text{GeV}^2, \tag{54}$$
$$\lambda_3^g = (0.54 \pm 0.21) \times 10^{-3}\ \text{GeV}^2.$$

We choose $a_g = a_3/2^{1/6} = 0.65\ \text{GeV}^{-1}$ and $\alpha_s = 0.5$ at the scale 1 GeV, which results in the following probabilities for the quark-gluon components within the nucleon state [27]

$$P_{g^\downarrow} = \frac{35}{8g^2} m_N^2 \rho_4 (\lambda_1^g)^2 \simeq 0.15,$$
$$P_{g^\uparrow} = \frac{105}{16g^2} m_N^2 \rho_4 [(\lambda_2^g)^2 + (\lambda_3^g)^2] \simeq 0.185. \tag{55}$$

V. RESULTS AND DISCUSSION

Now that we have models for the nucleon LCWFs, it is straightforward to evaluate the matrix elements of the four-fermion operators $\mathcal{Q}_\pm$ and constrain the momentum fraction dependence of the corresponding higher-twist correlator $\tilde{\mathcal{Q}}^{(p)}$. The distributions $\mathcal{Q}_\pm(\xi)$ defined by Eq. (32) possess the following support properties:

$$\mathcal{Q}_\pm(\xi) = \theta(-\xi_1)\theta(-\xi_3)\theta(\xi_2)\theta(\xi_4)\theta(1 - \xi_2 - \xi_4) \times q_\pm(-\xi_1, \xi_2, -\xi_3, \xi_4). \tag{56}$$

Here the functions $q_\pm(\xi)$ are expressed in terms of integrals involving the nucleon wave functions; see Appendix B for explicit formulas, while below we quote expressions which correspond to the *Ansätze* (48) and (53). The structure of the Fock expansion corresponds to the decomposition of the twist-four distributions $q_\pm$ into the following three components:

$$q_\pm(\xi) = q_\pm^{3q}(\xi) + q_\pm^{g_\downarrow}(\xi) + q_\pm^{g_\uparrow}(\xi). \tag{57}$$

Each term in this sum corresponds to the contribution of the pertinent multiparton component of the nucleon wave function, i.e., three-quark and quark-gluon, respectively. Making use of the results derived in the previous section, one finds the following explicit momentum fraction dependence for the distributions $q_\pm^{3q}$,

$$q_-^{3q}(\xi) = c_{3q}\chi_1(\xi)[(4-3(\xi_2+\xi_4))^2 + (5-3\xi_3)(5-3\xi_4)],$$
$$q_+^{3q}(\xi) = c_{3q}\chi_1(\xi)(1+3\xi_1)(1+3\xi_2), \tag{58}$$

where

$$\chi_1(\xi) = \xi_1\xi_2\xi_3\xi_4 \frac{1-\xi_2-\xi_4}{\xi_2+\xi_4}, \tag{59}$$

and the overall normalization constant being

$$c_{3q} = P_{3q}\frac{560}{87\pi^2 a_3^2}. \tag{60}$$

For the four-parton quark-gluon functions $q_\pm^{g_\uparrow}$, $q_\pm^{g_\downarrow}$ one gets

$$q_\pm^{g_\uparrow}(\xi) = c_{g_\uparrow}^\pm \chi_2(\xi), \qquad q_\pm^{g_\downarrow}(\xi) = c_{g_\downarrow}^\pm \chi_2(\xi), \tag{61}$$

where

$$\chi_2(\xi) = \chi_1(\xi)(1-\xi_2-\xi_4)^3 \tag{62}$$

and

$$c_{g_\uparrow}^+ = P_{g_\uparrow}\frac{560}{\pi^2 a_g^2}\left[1 - \frac{5}{3}\frac{\lambda_3^2}{\lambda_2^2+\lambda_3^2}\right],$$
$$c_{g_\uparrow}^- = -P_{g_\uparrow}\frac{700}{\pi^2 a_g^2}\left[1 + \frac{\lambda_3}{\lambda_2^2+\lambda_3^2}\left[\frac{6}{5}\lambda_2 - \frac{4}{3}\lambda_3\right]\right], \tag{63}$$
$$c_{g_\downarrow}^+ = P_{g_\downarrow}\frac{280}{\pi^2 a_g^2},$$

while $c_{g_\downarrow}^- = 0$.

Furthermore, making use of Eq. (42) one obtains after some algebra the following representation for the function $\tilde{\mathcal{Q}}_p(x)$, $x > 0$:

$$\tilde{\mathcal{Q}}_p(x) = -2\int_0^{1-x} d\xi\Big\{\frac{1}{\xi}\log(x/\xi)\hat{q}_+(x,\xi,\xi,x) + \frac{1}{x+\xi}\Big\{\int_0^{x+\xi}\frac{d\eta}{\eta}\Big[\frac{x+\xi}{\eta-\xi}\Big(\hat{q}_+(x,\eta,\xi,x+\xi-\eta) - \frac{\eta}{\xi}\hat{q}_+(x,\xi,\xi,x)\Big) - \hat{q}_-(x,\eta,\xi,x+\xi-\eta)\Big] + \frac{1}{2}\int_0^{1-x-\xi}\frac{d\eta}{\eta}\Big[\Big[\frac{x+\xi}{\xi} + \frac{\eta}{\eta+x}\Big]\hat{q}_+(\eta+x,\eta,\xi,\xi+x) + \Big[1 + \frac{\eta}{\eta+x}\frac{x+\xi}{\xi}\Big] \times \hat{q}_-(\eta+x,\eta,\xi,\xi+x)\Big]\Big\}\Big\}, \tag{64}$$

where

$$\hat{q}_\pm(\xi) = \tfrac{1}{2}(1+P_{14}P_{23})(1+P_{13}P_{24})q_\pm(\xi). \tag{65}$$

Performing the final integration is straightforward and one can obtain a closed analytical form of $\tilde{\mathcal{Q}}_p(x)$ (however, the resulting expression is quite long and in order to save space it will not be displayed here). The twist-four distribution is displayed in the upper panel of Fig. 2. The dashed and dotted lines correspond to its three-quark and quark-gluon components, respectively. Both of them exhibit a global minimum at $x \simeq 0.4$. In the lower panel of Fig. 2, we blow up its high-x region to demonstrate the node structure of the three-quark contribution. As $x \to 1$ the four-parton quark-gluon component of $\tilde{\mathcal{Q}}_p(x)$ is suppressed by the decay factor $(1-x)^3$ with respect to the three-quark component. At the same time the twist-four distribution $\tilde{\mathcal{Q}}_p(x)$ is enhanced in comparison with the twist-two parton densities calculated within the same model, $\tilde{\mathcal{Q}}_p(x)/u_p(x) \sim \log(1-x)$ for $x \to 1$.

Our predictions for the twist-four correction $R_1^{\text{tw-4}}$ to the Cahn-Gilman formula is shown in Fig. 3. In order to make an comparison with the results of Ref. [15] easier, we display $R_1^{\text{tw-4}}$ for $Q^2 = 4, 6, 8, 10, 12\ \text{GeV}^2$. It turns out that our prediction for $R_1^{\text{tw-4}}$ is roughly twice as large as that of Ref. [15] with the minimum of the function being slightly shifted towards lower x' (i.e., from $x' \simeq 0.7$ to $x' \simeq 0.6$). Note that the x dependence of the twist-four contribution is much better determined than its normalization: The three-quark component of the nucleon wave functions is constrained by the existing experimental data (parton densities and nucleon form factor, [25]), but the *Ansatz* (53) for the quark-gluon wave functions has to be regarded as an exploratory estimate (see Ref. [27] for a discussion). Nevertheless, since for large x' the contribution due to the quark-gluon components of the wave functions are strongly suppressed, see Fig. 2, we believe that for $x' > 0.7$ our estimate for $R_1^{\text{tw-4}}(x')$ should be rather accurate. That is, the function $R_1(x')$ has to change sign around $x' \sim 0.8$. We also checked that our result, once we compute its Mellin moments, are in good agreement with earlier calculations of higher-twist corrections to the first moments of structure functions [19,20].

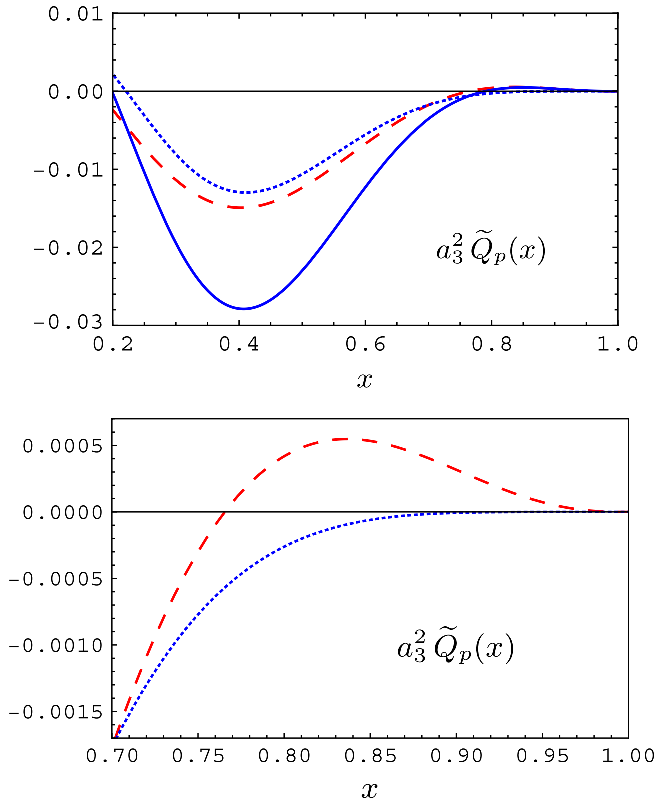

FIG. 2 (color online). The nucleon twist-four distribution $\tilde{Q}^{(p)}(x)$ multiplied by a_3^2 (solid line). The dashed and dotted lines show the contribution of three-quark and quark-gluon wave functions, respectively. The lower panel is a blowup of the high x region.

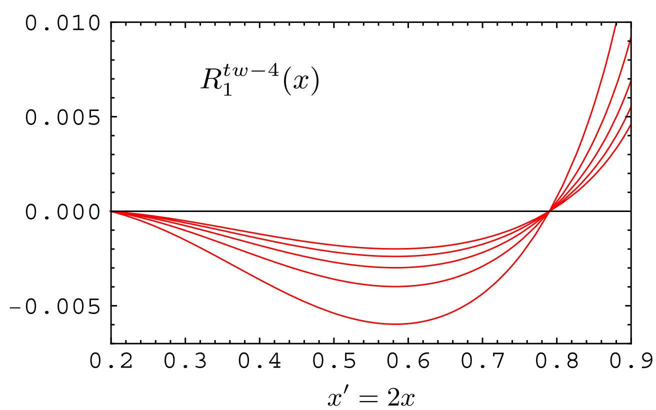

FIG. 3 (color online). The estimate $R_1^{\text{tw-4}}$ as a function of the Bjorken x for different values of Q^2. The curves from the bottom to top correspond to the values $Q^2 = 4,\ 6,\ 8,\ 10,\ 12\ \text{GeV}^2$, respectively. The experimental accuracy of SoLID is ± 0.005 for R_1^{tw-4} at an average Q^2 of 3.3 GeV^2 and $\langle x \rangle = 0.34$.

VI. CONCLUSION

Parity-violating deep inelastic scattering is a process of fundamental importance and, therefore, will be investigated by ever more precise experiments. It is sensitive to physics beyond the standard model as well as to specific aspects of strong interaction dynamics, encoded in higher-twist correlators. To disentangle both, the x dependence of the twist-four contribution must be known precisely, which seems to be in reach with present-day techniques. The task of determining these higher-twist contributions has a certain urgency in view of the upcoming JLab experiment SoLID [10]. In the current study we calculated the twist-four correction to the leading contribution $\tilde{a}_1$ to the parity-violating asymmetry by determining matrix elements of light-cone four-quark operators [17]. We found that within the framework of light-cone wave functions, the estimate for twist-four correlation functions has similar features as found in a recent calculation within the MIT bag model [15]. The size of the correction R_1 is about twice as large in our calculation and the form differs slightly, but these differences might well reflect the present-day theoretical uncertainties of such calculations. The size of the twist-four correction we obtain is borderline. It has to be taken into account to improve the sensitivity of SoLID for new physics, but it does not seem to be large enough for SoLID to test our prediction. However, as mapping out the running of $\sin^2\theta_W$ is a fundamentally important experiment, we are optimistic that still more precise experiments will be performed in the future, which should then be sensitive enough to observe the higher-twist contributions we analyzed.

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APPENDIX A: LIGHT-CONE EXPANSION

In this appendix, in order to make the paper self-consistent, we spell out our notations and conventions that we used to perform calculations of hadronic matrix elements in the body of the paper.

For an arbitrary four-vector a^μ, we define the light-cone coordinates as

$$a_+ = \frac{1}{\sqrt{2}}(a^0 + a^3), \qquad a_- = \frac{1}{\sqrt{2}}(a^0 - a^3), \qquad a = a^1 + ia^2, \qquad \bar{a} = a^1 - ia^2. \tag{A1}$$

We find it convenient to pass from four-dimensional vectors to two-dimensional matrix notations for all tensors. For a vector a_μ we introduce the matrix $a = a_\mu \sigma^\mu$, where $\sigma^\mu = (\mathbb{I}, \vec{\sigma})$,

$$a_{\alpha\dot{\alpha}} = a_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \begin{pmatrix} \sqrt{2}a_- & -\bar{a} \\ -a & \sqrt{2}a_+ \end{pmatrix}_{\alpha\dot{\alpha}}. \tag{A2}$$

In the Weil representation the Dirac γ matrices has the form

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix},$$

with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. In the two-component notation the Dirac spinors read

$$q = \begin{pmatrix} q_\downarrow \\ q_\uparrow \end{pmatrix}, \qquad \bar{q} = q^\dagger \gamma^0 = (\bar{q}_\downarrow, \bar{q}_\uparrow), \tag{A3}$$

where $q_{\uparrow(\downarrow)} = \frac{1}{2}(1 \pm \gamma_5)q$ are components with positive/negative helicity, respectively. The two independent light-like vectors

$$n^\mu = \frac{1}{\sqrt{2}}(1, 0, 0, -1), \qquad \tilde{n}^\mu = \frac{1}{\sqrt{2}}(1, 0, 0, 1), \tag{A4}$$

$n^2 = \tilde{n}^2 = 0$, $n \cdot \tilde{n} = 1$ can be parameterized in terms of two auxiliary Weil spinors:

$$n_{\alpha\dot{\alpha}} = \lambda_\alpha \bar{\lambda}_{\dot{\alpha}}, \qquad \tilde{n}_{\alpha\dot{\alpha}} = \mu_\alpha \bar{\mu}_{\dot{\alpha}}, \tag{A5}$$

which read explicitly

$$\begin{aligned} \lambda_\alpha &= 2^{1/4}\begin{pmatrix} -1 \\ 0 \end{pmatrix}, \qquad \mu_\alpha = 2^{1/4}\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\ \bar{\lambda}_{\dot{\alpha}} &= 2^{1/4}\begin{pmatrix} -1 \\ 0 \end{pmatrix}, \qquad \bar{\mu}_{\dot{\alpha}} = 2^{1/4}\begin{pmatrix} 0 \\ 1 \end{pmatrix}. \end{aligned} \tag{A6}$$

The following rules allow one to raise and lower spinor indices

$$\lambda^\alpha = \epsilon^{\alpha\beta}\lambda_\beta, \quad \lambda_\alpha = \lambda^\beta \epsilon_{\beta\alpha}, \quad \bar{\lambda}^{\dot{\alpha}} = \bar{\lambda}_{\dot{\beta}}\epsilon^{\dot{\beta}\dot{\alpha}}, \quad \bar{\lambda}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\lambda}^{\dot{\beta}},$$

with the antisymmetric Levi-Civita tensor having only the following nonzero components:

$$\epsilon_{12} = \epsilon^{12} = -\epsilon_{\dot{1}\dot{2}} = -\epsilon^{\dot{1}\dot{2}} = 1.$$

The auxiliary spinors λ and μ are normalized as

$$\begin{aligned} (\mu\lambda) &= \mu^\alpha \lambda_\alpha = -(\lambda\mu) = -\sqrt{2}, \\ (\bar{\mu}\,\bar{\lambda}) &= \bar{\mu}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}} = -(\bar{\lambda}\,\bar{\mu}) = +\sqrt{2} \end{aligned} \tag{A7}$$

and are used to project out the "plus" and "minus" components of the fields. For fermions, we define

$$\psi_+ = \lambda^\alpha \psi_\alpha, \quad \psi_- = \mu^\alpha \psi_\alpha, \quad \bar{\chi}_+ = \bar{\chi}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}}, \quad \bar{\chi}_- = \bar{\chi}_{\dot{\alpha}}\bar{\mu}^{\dot{\alpha}}. \tag{A8}$$

In the same fashion the light-cone decomposition of a vector (e.g., gluon) field takes the form

$$A_{\alpha\dot{\alpha}} = A_- \lambda_\alpha \bar{\lambda}_{\dot{\alpha}} + A_+ \mu_\alpha \bar{\mu}_{\dot{\alpha}} + \frac{\bar{A}}{\sqrt{2}}\lambda_\alpha \bar{\mu}_{\dot{\alpha}} + \frac{A}{\sqrt{2}}\mu_\alpha \bar{\lambda}_{\dot{\alpha}}.$$

The plus spinor fields ψ_+, $\bar{\chi}_+$ and transverse gluon fields A, $\bar{A}$ are assumed to be the dynamical degrees of freedom in the light-cone quantization framework. While the minus fields ψ_-, $\bar{\chi}_-$, A_- can be expressed in terms of these with the help of equations of motion. Finally, we use the gauge $A_+ = 0$.

The *good* components of the quark field have the following canonical expansion:

$$\begin{aligned} q_{\downarrow+}(x) &= \int \frac{dp_+}{\sqrt{2p_+}} \frac{d^2\boldsymbol{p}_\perp}{(2\pi)^3}\theta(p_+)[e^{-ip\cdot x}b_\downarrow(p) + e^{+ip\cdot x}d_\uparrow^\dagger(p)], \\ q_{\uparrow+}(x) &= \int \frac{dp_+}{\sqrt{2p_+}} \frac{d^2\boldsymbol{p}_\perp}{(2\pi)^3}\theta(p_+)[e^{-ip\cdot x}b_\uparrow(p) + e^{+ip\cdot x}d_\downarrow^\dagger(p)], \end{aligned} \tag{A9}$$

in terms of the annihilation operators of quark and antiquark of positive (negative) helicity $b_{\uparrow(\downarrow)}$, $d_{\uparrow(\downarrow)}$, respectively. They obey the standard anticommutation relations

$$\begin{aligned} \{b_\lambda(p), b^\dagger_{\lambda'}(p')\} &= \{d_\lambda(p), d^\dagger_{\lambda'}(p')\} \\ &= 2p_+(2\pi)^3\delta_{\lambda,\lambda'}\delta(p_+ - p'_+)\delta^{(2)}(\boldsymbol{p}_\perp - \boldsymbol{p}'_\perp). \end{aligned} \tag{A10}$$

Similarly the expansion for the dynamical transversely polarized gluon fields A and $\bar{A}$ reads

$$\begin{aligned} \bar{A}(x) &= \sqrt{2}\int \frac{dk_+}{2k_+}\frac{d^2\boldsymbol{k}_\perp}{(2\pi)^3}\theta(k_+)[e^{-ik\cdot x}a_\uparrow(k) + e^{+ik\cdot x}a_\downarrow^\dagger(k)], \\ A(x) &= \sqrt{2}\int \frac{dk_+}{2k_+}\frac{d^2\boldsymbol{k}_\perp}{(2\pi)^3}\theta(k_+)[e^{-ik\cdot x}a_\downarrow(k) + e^{+ik\cdot x}a_\uparrow^\dagger(k)]. \end{aligned} \tag{A11}$$

Here and below $A = \sum_a t^a A^a$ are matrices in the fundamental representation of $SU(3)$, and t^a are the usual generators, normalized as $\mathrm{tr}(t^a t^b) = \frac{1}{2}\delta^{ab}$. The creation and annihilation operators obey the commutation relation

$$\begin{aligned} [a^b_\lambda(p), (a^{b'}_{\lambda'}(p'))^\dagger] &= 2p_+(2\pi)^3\delta_{\lambda,\lambda'}\delta^{bb'}\delta(p_+ - p'_+) \\ &\quad \times \delta^{(2)}(\boldsymbol{p}_\perp - \boldsymbol{p}'_\perp). \end{aligned} \tag{A12}$$

As mentioned above, *bad* (i.e., minus) components can be expressed in terms of the dynamical fields using QCD equations of motion.

APPENDIX B: DISTRIBUTIONS $q_\pm$

As discussed in Sec. V, we represent the twist-four distributions $q_\pm(\xi)$ as shown in Eq. (57). We remind the reader that the arguments $\xi = (\xi_1, \xi_2, \xi_3, \xi_4)$ are subject to the constraints, $0 \le \xi_i \le 1$ and $\xi_1 + \xi_3 = \xi_2 + \xi_4$. A straightforward calculation of its components $q_\pm^{3q}(\xi)$, $q_\pm^{g_\downarrow}(\xi)$, $q_\pm^{g_\uparrow}(\xi)$, arising from three- and four-parton Fock states of the nucleon, yields the following expressions in terms of the LCWFs introduced in the main text,

$$q_+^{3q}(\xi) = -\frac{4}{9}\frac{(\pi a_3 f_N)^2}{(\xi_2+\xi_4)(1-\xi_2-\xi_4)}\phi(\xi_1, 1-\xi_1-\xi_3, \xi_3)\phi(\xi_2, 1-\xi_2-\xi_4, \xi_4),$$

$$q_-^{3q}(\xi) = -\frac{4}{9}\frac{(\pi a_3 f_N)^2}{(\xi_2+\xi_4)(1-\xi_2-\xi_4)}\{\phi(1-\xi_1-\xi_3, \xi_1, \xi_3)\phi(1-\xi_2-\xi_4, \xi_2, \xi_4) + (\phi(\xi_1, \xi_3, 1-\xi_1-\xi_3) + \phi(1-\xi_1-\xi_3, \xi_3, \xi_1))(\phi(\xi_2, \xi_4, 1-\xi_2-\xi_4) + \phi(1-\xi_2-\xi_4, \xi_4, \xi_2))\}, \tag{B1}$$

where ϕ is given by Eq. (48). Next, we got that $q_-^{g_\uparrow}(\xi) = 0$ and

$$q_+^{g_\uparrow}(\xi) = \frac{32(8\pi^2 a_g^2)^2}{3(\xi_2+\xi_4)}\int_0^1 \frac{dx_2}{x_2}\frac{dx_4}{x_4^2}\delta(1-\xi_2-\xi_4-x_2-x_4)\bigg\{\phi_g(\xi_1, x_2, \xi_3, x_4)\bigg[\phi_g(x_2, \xi_2, \xi_4, x_4) + \frac{1}{4}\phi_g(\xi_2, x_2, \xi_4, x_4)\bigg] + \phi_g(x_2, \xi_1, \xi_3, x_4)[\phi_g(\xi_2, x_2, \xi_4, x_4) - 2\phi_g(x_2, \xi_2, \xi_4, x_4)]\bigg\}. \tag{B2}$$

Finally,

$$q_+^{g_\downarrow}(\xi) = \frac{8(8\pi^2 a_g^2)^2}{3(\xi_2+\xi_4)}\int_0^1 \frac{dx_2}{x_2}\frac{dx_4}{x_4^2}\delta(1-\xi_2-\xi_4-x_2-x_4)\{\psi_g^{(1)}(\xi_1, x_2, \xi_3, x_4)[\psi_g^{(1)}(\xi_2, x_2, \xi_4, x_4) + 5\psi_g^{(2)}(\xi_2, \xi_4, x_2, x_4)] + \psi_g^{(2)}(\xi_1, \xi_3, x_2, x_4)[\psi_g^{(2)}(\xi_2, \xi_4, x_2, x_4) + 5\psi_g^{(1)}(\xi_2, x_2, \xi_4, x_4)]\},$$

$$q_-^{g_\downarrow}(\xi) = \frac{32(8\pi^2 a_g^2)^2}{3(\xi_2+\xi_4)}\int_0^1 \frac{dx_2}{x_2}\frac{dx_4}{x_4^2}\delta(1-\xi_2-\xi_4-x_2-x_4)\bigg\{\psi_g(\xi_1, x_2, \xi_3, x_4)\bigg[\psi_g(x_2, \xi_2, \xi_4, x_4) + \frac{1}{4}\psi_g(\xi_2, x_2, \xi_4, x_4)\bigg] + \psi_g(x_2, \xi_1, \xi_3, x_4)[\psi_g(\xi_2, x_2, \xi_4, x_4) - 2\psi_g(x_2, \xi_2, \xi_4, x_4)] - \bigg[\psi_g^{(1)}(x_2, \xi_2, \xi_4, x_4) - \frac{1}{4}\psi_g^{(2)}(x_2, \xi_4, \xi_2, x_4)\bigg] \times \psi_g^{(2)}(x_2, \xi_3, \xi_1, x_4) - \psi_g^{(1)}(x_2, \xi_1, \xi_3, x_4)[\psi_g^{(2)}(x_2, \xi_4, \xi_2, x_4) + 2\psi_g^{(1)}(x_2, \xi_2, \xi_4, x_4)]\bigg\}, \tag{B3}$$

where

$$\psi_g(x_1, x_2, x_3, x_4) = \psi_g^{(1)}(x_1, x_2, x_3, x_4) - \psi_g^{(2)}(x_3, x_1, x_2, x_4).$$

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