One-photon decay of the tetraquark state $X(3872) \to \gamma + J/\psi$ in a relativistic constituent quark model with infrared confinement

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We further explore the consequences of treating the $X(3872)$ meson as a tetraquark bound state by analyzing its one-photon decay $X \to \gamma + J/\psi$ in the framework of our approach developed in previous papers which incorporates quark confinement in an effective way. To introduce electromagnetism we gauge a nonlocal effective Lagrangian describing the interaction of the $X(3872)$ meson with its four constituent quarks by using the P-exponential path-independent formalism. We calculate the matrix element of the transition $X \to \gamma + J/\psi$ and prove its gauge invariance. We evaluate the $X \to \gamma + J/\psi$ decay width and the longitudinal/transverse composition of the J/ψ in this decay. For a reasonable value of the size parameter of the $X(3872)$ meson we find consistency with the available experimental data. We also calculate the helicity and multipole amplitudes of the process, and describe how they can be obtained from the covariant transition amplitude by covariant projection.

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I. INTRODUCTION

This paper is a direct continuation of our previous work [1] where we have analyzed the strong decays of the charmonium–like state $X(3872)$ in the framework of our relativistic constituent quark model which includes infrared confinement in an effective way [2]. In our approach the $X(3872)$ meson is interpreted as a tetraquark state with the quantum numbers $J^{PC} = 1^{++}$ as in [3]. In this paper we analyze the one-photon decay $X \to \gamma + J/\psi$ in the same tetraquark picture. The electromagnetic interaction is incorporated into our relativistic nonlocal effective Lagrangian in a gauge invariant way using the P-exponential path-independent formalism.

We begin by collecting the experimental data relevant for our purposes. A narrow charmonium–like state $X(3872)$ was observed in 2003 in the exclusive decay process $B^\pm \to K^\pm \pi^+\pi^- J/\psi$ [4]. The $X(3872)$ decays into $\pi^+\pi^- J/\psi$ and has a mass of $m_X = 3872.0 \pm 0.6(\text{stat}) \pm 0.5(\text{syst})$ very close to the $m_{D^0} + m_{D^{*0}} = 3871.81 \pm 0.25$ mass threshold [5]. Its width was found to be less than 2.3 MeV at 90% confidence level. The state was confirmed in B-decays by the *BABAR* experiment [6] and in $p\bar{p}$ production by the Tevatron experiments CDF [7] and D0 [8]. The most precise measurement up to now was done in [9] with $m_X = 3871.61 \pm 0.16 \pm 0.19$. The new average mass given in [7] is

$$m_X = 3871.51 \pm 0.22 \text{ MeV}. \tag{1}$$

The Belle Collaboration has reported [10] evidence for the decay modes $X(3872) \to \gamma + J/\psi$ and to $X \to \pi^+\pi^-\pi^0 J/\psi$:

$$\begin{aligned}
&\mathcal{B}(B \to XK) \cdot \mathcal{B}(X \to \gamma + J/\psi) \\
&\quad = (1.8 \pm 0.6(\text{stat}) \pm 0.1(\text{syst})) \times 10^{-6}, \\
&\frac{\Gamma(X \to \gamma + J/\psi)}{\Gamma(X \to \pi^+\pi^- J/\psi)} = 0.14 \pm 0.05, \\
&\frac{\mathcal{B}(X \to \pi^+\pi^-\pi^0 J/\psi)}{\mathcal{B}(X \to \pi^+\pi^- J/\psi)} = 1.0 \pm 0.4(\text{stat}) \pm 0.3(\text{syst}).
\end{aligned} \tag{2}$$

These observations imply strong isospin violation because the three-pion decay proceeds via an intermediate ω meson with isospin 0 whereas the two-pion decay proceeds via the intermediate ρ meson with isospin 1. It is evident that the two-pion decay via the intermediate ρ meson is very difficult to explain by using an interpretation of the $X(3872)$ as a simple $c\bar{c}$ charmonium state with isospin 0.

In an analysis of $B^+ \to J/\psi \gamma K^+$ decays, the *BABAR* Collaboration [11] found evidence for the radiative decay $X(3872) \to \gamma + J/\psi$ with a statistical significance of 3.4σ. They reported the following values for the product of branching fractions

$$\begin{aligned}
&\mathcal{B}(B^+ \to XK^+) \cdot \mathcal{B}(X \to \gamma + J/\psi) \\
&\quad = (3.3 \pm 1.0(\text{stat}) \pm 0.3(\text{syst})) \times 10^{-6}.
\end{aligned} \tag{3}$$

The Belle Collaboration reported [12] the first observation of a near-threshold enhancement in the $D^0\bar{D}^0\pi^0$

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system from $B \to D^0 \bar{D}^0 \pi^0 K$. The enhancement peaks at a mass of $M = 3875.2 \pm 0.7^{+0.3}_{-1.6} \pm 0.8$ MeV. The branching fraction for events in the peak is

$$\mathcal{B}(B \to D^0 \bar{D}^0 \pi^0 K) = (1.22 \pm 0.31^{+0.23}_{-0.30}) \times 10^{-4}. \quad (4)$$

All available experimental data up to 2007 were analyzed in [13]. The authors found that [13]

$$\mathcal{B}(B^+ \to XK^+) = 1.30^{+0.20}_{-0.34} \times 10^{-4}, \quad \frac{\Gamma(X \to \gamma + J/\psi)}{\Gamma(X \to \pi^+\pi^- J/\psi)} = 0.22 \pm 0.06. \quad (5)$$

The *BABAR* Collaboration found evidence for the decays $X \to \gamma + J/\psi$ and $X \to \gamma + \psi(2S)$ in their data sample of the $B \to c\bar{c}\gamma K$ decays. The measured products of branching fractions are [14]

$$\begin{aligned} &\mathcal{B}(B^\pm \to XK^\pm) \cdot \mathcal{B}(X \to \gamma + J/\psi) \\ &\quad = (2.8 \pm 0.8(\text{stat}) \pm 0.1(\text{syst})) \times 10^{-6}, \\ &\mathcal{B}(B^\pm \to XK^\pm) \cdot \mathcal{B}(X \to \gamma + \psi(2S)) \\ &\quad = (9.5 \pm 2.7(\text{stat}) \pm 0.6(\text{syst})) \times 10^{-6}. \end{aligned} \quad (6)$$

There have been many theoretical attempts to unravel the structure of the $X(3872)$ and its decays. Many of the theoretical predictions for the decay $X(3872) \to \gamma + J/\psi$ published up to now are very model dependent. We mention some of them in turn.

All possible $1D$ and $2P$ $c\bar{c}$ assignments for the $X(3872)$ were considered in [15]. The authors obtained $E1$ radiative widths for decays into charmonium $c\bar{c}$ states as well as for some strong decays taking the experimental mass as input. The conclusion was that many of the possible J^{PC} assignments can be eliminated due to the smallness of the observed total width. The suggestion was that radiative transitions could be used to test the remaining J^{PC} assignments.

Some tests of the hypothesis that the $X(3872)$ is a weakly bound $D^0 \bar{D}^{0*}$ molecule state were suggested in [16]. It was proposed that measuring the $3\pi J/\psi$, $\gamma + J/\psi$, $\gamma + \psi'$, $\bar{K}K^*$, and $\pi\rho$ decay modes of the X will serve as a definitive diagnostic tool to confirm or to rule out the molecule hypothesis.

Assuming that the $X(3872)$ state has the structure $(D^0 \bar{D}^{0*} - D^{0*} \bar{D}^0)/\sqrt{2}$ with quantum numbers $J^{PC} = 1^{++}$, the $X(3872) \to \gamma + J/\psi$ decay width was calculated using a phenomenological Lagrangian approach [17]. The calculated value of the radiative decay width varied from 125 KeV to 250 KeV depending on the model parameters.

QCD sum rules were used in [18] to calculate the width of the radiative decay of the meson $X(3872)$, which was assumed to be a mixture between charmonium and exotic molecular $[c\bar{q}][q\bar{c}]$ states with $J^{PC} = 1^{++}$. In a small range for the values of the mixing angle, one obtains

$$\frac{\Gamma(X \to \gamma + J/\psi)}{\Gamma(X \to J/\psi \pi^+ \pi^-)} = 0.19 \pm 0.13. \quad (7)$$

Our paper is organized as follows. In Sec. II we gauge a nonlocal effective Lagrangian describing the interaction of the $X(3872)$ meson with its constituent quarks by using the P-exponential path-independent formalism developed in [19,20]. In Sec. III we calculate the matrix element of the radiative transition $X \to \gamma + J/\psi$ and prove its gauge invariance analytically. In Sec. IV we present the results of our numerical analysis. First, we check numerically that the final amplitude is gauge invariant. Second, we introduce infrared confinement as was done in our previous papers Refs. [1,2] and evaluate the $X \to \gamma + J/\psi$ decay width. Finally, in Sec. V we summarize our results. In an Appendix we describe how the two helicity or the two multipole amplitudes of the process can be obtained from the gauge invariant transition amplitude by covariant projection.

II. THEORETICAL FRAMEWORK

The effective interaction Lagrangians describing the coupling of the charmonium-like meson such as the $X(3872)$ to four quarks, and the coupling of the charmonium J/ψ state to its two constituent quarks are written in the form (see Ref. [1])

$$\mathcal{L}_{\text{int}} = g_X X_{q\mu}(x) \cdot J^\mu_{X_q}(x) + g_{J/\psi} J/\psi_\mu(x) \cdot J^\mu_{J/\psi}(x) \quad (q = u, d). \quad (8)$$

The nonlocal interpolating quark currents read

$$\begin{aligned} J^\mu_{X_q}(x) &= \int dx_1 \ldots \int dx_4 \delta\Big(x - \sum_{i=1}^4 w_i x_i\Big) \Phi_X\Big(\sum_{i<j} (x_i - x_j)^2\Big) \\ &\quad \times \frac{1}{\sqrt{2}} \varepsilon_{abc} \varepsilon_{dec} \{[q_a(x_4) C\gamma^5 c_b(x_1)] \\ &\quad \times [\bar{q}_d(x_3) \gamma^\mu C \bar{c}_e(x_2)] + (\gamma^5 \leftrightarrow \gamma^\mu)\}, \\ w_1 &= w_2 = \frac{m_c}{2(m_q + m_c)} \equiv \frac{w_c}{2}, \\ w_3 &= w_4 = \frac{m_q}{2(m_q + m_c)} \equiv \frac{w_q}{2}, \\ J^\mu_{J/\psi}(y) &= \int dy_1 \int dy_2 \delta\Big(y - \frac{1}{2}(y_1 + y_2)\Big) \\ &\quad \times \Phi_{J/\psi}((y_1 - y_2)^2) \bar{c}_a(y_1) \gamma^\mu c_a(y_2). \end{aligned} \quad (9)$$

The matrix $C = \gamma^0 \gamma^2$ is related to the charge conjugation matrix: $C = C^\dagger = C^{-1} = -C^T$, $C\Gamma^T C^{-1} = \pm\Gamma$, ("+" for $\Gamma = S, P, A$ and "−" for $\Gamma = V, T$). We follow

[3] and take the tetraquark state to be a linear superposition of the X_u and X_d states according to

$$X_l \equiv X_{\rm low} = X_u \cos\theta + X_d \sin\theta,$$
$$X_h \equiv X_{\rm high} = -X_u \sin\theta + X_d \cos\theta. \tag{10}$$

The coupling constant g_X in Eq. (8) will be determined from the compositeness condition $Z_H = 0$ (see e.g. Refs. [21,22]). The compositeness condition requires that the renormalization constant Z_H of the elementary meson X is set to zero, i.e.

$$Z_H = 1 - \Pi'_H(p_H^2 = m_H^2) = 0, \tag{11}$$

where $\Pi_X(p^2)$ is the scalar part of the meson mass operator and the prime stands for the derivative w.r.t. p_H^2. For the spin one states $X(3872)$ and J/ψ the compositeness condition reads

$$\Pi_V^{\mu\nu}(p) = g^{\mu\nu}\Pi_V(p^2) + p^\mu p^\nu \Pi_V^{(1)}(p^2),$$
$$\Pi_V(p^2) = \frac{1}{3}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Pi_V^{\mu\nu}(p). \tag{12}$$

The X meson mass operator can be calculated from the self-energy three-loop sunrise-type diagram with four quark-antiquark propagators. The calculation is described in more detail in Ref. [1].

As in the case of baryons composed of three quarks it is convenient to transform to Jacobi coordinates in the integrals of Eq. (9). In the case of four quarks one has

$$\begin{aligned}
x_1 &= x + \frac{2w_2 + w_3 + w_4}{2\sqrt{2}}\rho_1 - \frac{w_3 - w_4}{2\sqrt{2}}\rho_2 + \frac{w_3 + w_4}{2}\rho_3 \equiv x + \sum_{j=1}^{3} c_{1j}\rho_j,\\
x_2 &= x - \frac{2w_1 + w_3 + w_4}{2\sqrt{2}}\rho_1 - \frac{w_3 - w_4}{2\sqrt{2}}\rho_2 + \frac{w_3 + w_4}{2}\rho_3 \equiv x + \sum_{j=1}^{3} c_{2j}\rho_j,\\
x_3 &= x - \frac{w_1 - w_2}{2\sqrt{2}}\rho_1 + \frac{w_1 + w_2 + 2w_4}{2\sqrt{2}}\rho_2 - \frac{w_1 + w_2}{2}\rho_3 \equiv x + \sum_{j=1}^{3} c_{3j}\rho_j,\\
x_4 &= x - \frac{w_1 - w_2}{2\sqrt{2}}\rho_1 - \frac{w_1 + w_2 + 2w_3}{2\sqrt{2}}\rho_2 - \frac{w_1 + w_2}{2}\rho_3 \equiv x + \sum_{j=1}^{3} c_{4j}\rho_j,
\end{aligned} \tag{13}$$

where $x = \sum_{i=1}^4 x_i w_i$ and $\sum_{1\le i<j\le 4}(x_i - x_j)^2 = \sum_{i=1}^3 \rho_i^2$. The inverse transformation reads

$$\rho_1 = \sqrt{2}(x_1 - x_2),$$
$$\rho_2 = \sqrt{2}(x_3 - x_4),$$
$$\rho_3 = x_1 + x_2 - x_3 - x_4.$$

In the case of two quarks as e.g. in the J/ψ case one has

$$y_1 = y + \frac{1}{2}\rho, \qquad y_2 = y - \frac{1}{2}\rho. \tag{14}$$

One then has

$$J^\mu_{X_q}(x) = \int d\vec{\rho}\,\Phi_X(\vec{\rho}^{\,2}) J^\mu_{4q}(x_1, \ldots, x_4),$$
$$J^\mu_{4q}(x_1, \ldots, x_4) = \frac{1}{\sqrt{2}}\varepsilon_{abc}\varepsilon_{dec}\{[q_a(x_4)C\gamma^5 c_b(x_1)] \times [\bar{q}_d(x_3)\gamma^\mu C\bar{c}_e(x_2)] + (\gamma^5 \leftrightarrow \gamma^\mu)\},$$
$$J^\mu_{J/\psi}(y) = \int d\rho\,\Phi_{J/\psi}(\rho^2) J^\mu_{2q}(y_1, y_2),$$
$$J^\mu_{2q}(y_1, y_2) = \bar{c}_a(y_1)\gamma^\mu c_a(y_2), \tag{15}$$

where $d\vec{\rho} = d\rho_1 d\rho_2 d\rho_3$ and $\vec{\rho}^{\,2} = \rho_1^2 + \rho_2^2 + \rho_3^2$. The Jacobian is absorbed into the coupling g_X.

The gauge invariant interaction of a bound quark state with the electromagnetic field has been described in some detail in Ref. [19]. For comprehensive purposes we recall some of the key points of the gauging process. Since the $X(3872)$ and J/ψ mesons are neutral mesons we will discuss the charged quarks only. The free Lagrangian of quarks is gauged in the standard manner by using minimal substitution:

$$\partial^\mu q \to (\partial^\mu - ie_q A^\mu)q, \quad \partial^\mu \bar{q} \to (\partial^\mu + ie_q A^\mu)\bar{q}, \tag{16}$$

where e_q is the quark's charge ($e_u = \frac{2}{3}e$, $e_d = -\frac{1}{3}e$, etc.). Minimal substitution gives us the first piece of the electromagnetic interaction Lagrangian

$$\mathcal{L}^{\rm em(1)}_{\rm int}(x) = \sum_q e_q A_\mu(x) J^\mu_q(x), \quad J^\mu_q(x) = \bar{q}(x)\gamma^\mu q(x). \tag{17}$$

In order to guarantee gauge invariance of the nonlocal strong interaction Lagrangian, one multiplies each quark field $q(x_i)$ in the relevant quark current $J^\mu(x)$ given by Eq. (15) by a gauge field exponential according to

$$
\begin{aligned}
q(x_i) &\to e^{-ie_q I(x_i,x,P)} q(x_i),\\
\bar{q}(x_i) &\to e^{ie_q I(x_i,x,P)} \bar{q}(x_i),\\
I(x_i, x, P) &= \int_x^{x_i} dz_\mu A^\mu(z).
\end{aligned} \tag{18}
$$

where P is the path taken from x to x_i. It is readily seen that the full Lagrangian Eq. (8) is invariant under the local gauge transformations

$$
\begin{aligned}
q(x_i) &\to e^{ie_q f(x_i)} q(x_i),\\
\bar{q}(x_i) &\to e^{-ie_q f(x_i)} \bar{q}(x_i),\\
A^\mu(z) &\to A^\mu(z) + \partial^\mu f(z), \quad \text{so that}\\
I(x_i, x, P) &\to I(x_i, x, P) + f(x_i) - f(x).
\end{aligned} \tag{19}
$$

The second term of the electromagnetic interaction Lagrangian $\mathcal{L}^{em}_{\rm int;2}$ arises when one expands the gauge exponential in powers of A_μ up to the order of perturbation theory that one is considering. Superficially the results appear to depend on the path P which connects the endpoints in the path integral in Eq. (18). However, one needs to know only derivatives of the path integrals when doing the perturbative expansion. One can make use of the formalism developed in [20] which is based on the path-independent definition of the derivative of $I(x, y, P)$:

$$
\lim_{dx^\mu\to 0} dx^\mu \frac{\partial}{\partial x^\mu} I(x, y, P) = \lim_{dx^\mu\to 0} [I(x + dx, y, P') - I(x, y, P)], \tag{20}
$$

where the path P' is obtained from P by shifting the endpoint x by dx. Use of the definition (20) leads to the key rule

$$
\frac{\partial}{\partial x^\mu} I(x, y, P) = A_\mu(x) \tag{21}
$$

which states that the derivative of the path integral $I(x, y, P)$ does not depend on the path P originally used in the definition. The nonminimal substitution (18) is therefore completely equivalent to the minimal prescription as is evident from the identities (20) or (21). The method of deriving Feynman rules for the nonlocal coupling of hadrons to photons and quarks was worked out before in Refs. [19,20] and will be discussed in the next section where we apply the formalism to the physical processes considered in this paper.

Expanding the Lagrangian up to the first order in A^μ one obtains

$$
\begin{aligned}
\mathcal{L}^{\rm em(2)}_{\rm int}(x) &= g_X X_{q\mu}(x)\cdot J^\mu_{X_q-\rm em}(x) + g_{J/\psi} J/\psi_\mu(x)\cdot J^\mu_{J/\psi-\rm em}(x) \qquad (q = u, d),\\
J^\mu_{X_q-\rm em} &= \int d\vec{\rho}\,\Phi_X(\vec{\rho}^{\,2}) J^\mu_{4q}(x_1, \ldots, x_4)\{ie_q[I^{x_3}_x - I^{x_4}_x] + ie_c[I^{x_2}_x - I^{x_1}_x]\},\\
J^\mu_{J/\psi-\rm em} &= \int d\rho\,\Phi_{J/\psi}(\rho^2) J^\mu_{2q}(x_1, x_2) ie_c[I^{x_1}_x - I^{x_2}_x], \qquad I^{x_i}_x \equiv I(x_i, x, P).
\end{aligned} \tag{22}
$$

In order to use the key rule Eq. (21) we take the Fourier-transforms for the vertex functions Φ and quark fields q

$$
\begin{aligned}
&\Phi_X(\vec{\rho}^{\,2}) = \int \frac{d^4\vec{\omega}}{(2\pi)^4} \tilde{\Phi}_X(-\vec{\omega}^2) e^{-i\vec{\rho}\,\vec{\omega}} = \tilde{\Phi}_X(\vec{\partial}^2_\rho)\delta^{(4)}(\vec{\rho}), \qquad \Phi_{J/\psi}(\rho^2) = \int \frac{d^4\omega}{(2\pi)^4} \tilde{\Phi}_{J/\psi}(-\omega^2) e^{-i\rho\omega} = \tilde{\Phi}_{J/\psi}(\partial^2_\rho)\delta^{(4)}(\rho),\\
&q(x_i) = \int \frac{d^4p_i}{(2\pi)^4} e^{-ip_ix_i}\tilde{q}(p_i), \qquad \bar{q}(x_i) = \int \frac{d^4p_i}{(2\pi)^4} e^{ip_ix_i}\tilde{\bar{q}}(p_i).
\end{aligned} \tag{23}
$$

One then writes down

$$
\begin{aligned}
J^\mu_{X_q-\rm em} &= \prod_{i=1}^4 \int \frac{d^4p_i}{(2\pi)^4} \tilde{J}^\mu_{4q}(p_1, \ldots, p_4) \int d\vec{\rho}\,\delta^{(4)}(\vec{\rho})\tilde{\Phi}_X(\vec{\partial}^2_\rho) e^{-i(p_1x_1 - p_2x_2 - p_3x_3 + p_4x_4)}\{ie_q[I^{x_3}_x - I^{x_4}_x] + ie_c[I^{x_2}_x - I^{x_1}_x]\}\\
&= \prod_{i=1}^4 \int \frac{d^4p_i}{(2\pi)^4} \tilde{J}^\mu_{4q}(p_1, \ldots, p_4) e^{-i(p_1 - p_2 - p_3 + p_4)x} \int d\vec{\rho}\,\delta^{(4)}(\vec{\rho}) e^{-i\vec{\rho}\,\vec{\omega}}\tilde{\Phi}_X(\vec{D}^2_\rho)\{ie_q[I^{x_3}_x - I^{x_4}_x] + ie_c[I^{x_2}_x - I^{x_1}_x]\},\\
J^\mu_{J/\psi-\rm em} &= \prod_{i=1}^2 \int \frac{d^4p_i}{(2\pi)^4} \tilde{J}^\mu_{2q}(p_1, p_2) \int d\rho\,\delta^{(4)}(\rho)\tilde{\Phi}_{J/\psi}(\partial^2_\rho) e^{i(p_1x_1 - p_2x_2)} ie_c[I^{x_1}_x - I^{x_2}_x]\\
&= \prod_{i=1}^2 \int \frac{d^4p_i}{(2\pi)^4} \tilde{J}^\mu_{2q}(p_1, p_2) e^{i(p_1 - p_2)x} \int d\rho\,\delta^{(4)}(\rho) e^{ip\rho}\tilde{\Phi}_{J/\psi}(D^2_\rho) ie_c[I^{x_1}_x - I^{x_2}_x],\\
D^\mu_{\rho_i} &= \partial^\mu_{\rho_i} - i\omega^\mu_i, \qquad D^\mu_\rho = \partial^\mu_\rho + ip^\mu,
\end{aligned} \tag{24}
$$

where

$$\omega_1 = c_{11}p_1 - c_{21}p_2 - c_{31}p_3 + c_{41}p_4, \qquad \omega_2 = c_{12}p_1 - c_{22}p_2 - c_{32}p_3 + c_{42}p_4,$$
$$\omega_3 = c_{13}p_1 - c_{23}p_2 - c_{33}p_3 + c_{43}p_4, \qquad p = \frac{1}{2}(p_1 + p_2). \tag{25}$$

Finally, we employ a convenient identity which was proven in [19]. The identity reads

$$F(D^2_{\rho_j})I^{x_i}_X = \int_0^1 d\tau F'(\tau D^2_{\rho_j} - (1-\tau)\omega_j^2)c_{ij}(\partial^\nu_{\rho_j}A_\nu(x_i) - 2i\omega^\nu_j A_\nu(x_i)) + F(-\omega_j^2)I^{x_i}_X. \tag{26}$$

The identity holds for any function $F(z)$ that is analytical at $z = 0$.

One obtains

$$J^\mu_{X_q-\mathrm{em}}(x) = \prod_{i=1}^4 \int d^4x_i \int d^4y J^\mu_{4q}(x_1, \ldots, x_4)A_\rho(y)E^\rho_X(x; x_1, \ldots, x_4, y),$$
$$E^\rho_X(x; x_1, \ldots, x_4, y) = \prod_{i=1}^4 \int \frac{d^4p_i}{(2\pi)^4} \int \frac{d^4r}{(2\pi)^4} e^{-ip_1(x-x_1)+ip_2(x-x_2)+ip_3(x-x_3)-ip_4(x-x_4)-ir(x-y)} \tilde{E}^\rho_X(p_1, \ldots, p_4, r),$$
$$\tilde{E}^\rho_X(p_1, \ldots, p_4, r) = \int_0^1 d\tau \sum_{j=1}^3 \{e_c[-\tilde{\Phi}'_X(-z_{1j})l^\rho_{1j} + \tilde{\Phi}'_X(-z_{2j})l^\rho_{2j}] + e_q[-\tilde{\Phi}'_X(-z_{4j})l^\rho_{4j}$$
$$+ \tilde{\Phi}'_X(-z_{3j})l^\rho_{3j}]\} l_{ij} = c_{ij}(c_{ij}r + 2\omega_j), \qquad (i = 1, \ldots, 4; j = 1, \ldots, 3), \tag{27}$$
$$z_{i1} = \tau(c_{i1}r + \omega_1)^2 + (1-\tau)\omega_1^2 + \omega_2^2 + \omega_3^2,$$
$$z_{i2} = (c_{i1}r + \omega_1)^2 + \tau(c_{i2}r + \omega_2)^2 + (1-\tau)\omega_2^2 + \omega_3^2,$$
$$z_{i3} = (c_{i1}r + \omega_1)^2 + (c_{i2}r + \omega_2)^2 + \tau(c_{i3}r + \omega_3)^2 + (1-\tau)\omega_3^2.$$

$$J^\nu_{J/\psi-\mathrm{em}}(y) = \int d^4y_1 \int d^4y_2 \int d^4z J^\nu_{2q}(y_1, y_2)A_\rho(z)E^\rho_{J/\psi}(y; y_1, y_2, z),$$
$$E^\rho_{J/\psi}(y; y_1, y_2, z) = \int \frac{d^4p_1}{(2\pi)^4} \int \frac{d^4p_2}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} e^{-ip_1(y_1-y)+ip_2(y_2-y)+iq(z-y)} \tilde{E}^\rho_{J/\psi}(p_1, p_2, q),$$
$$\tilde{E}^\rho_{J/\psi}(p_1, p_2, q) = e_c \int_0^1 d\tau\{-\tilde{\Phi}'_{J/\psi}(-z_-)l^\rho_- - \tilde{\Phi}'_{J/\psi}(-z_+)l^\rho_+\}, \tag{28}$$
$$z_\mp = \tau(p \mp \frac{1}{2}q) - (1-\tau)p^2,$$
$$l_\mp = p \mp \frac{1}{4}q,$$
$$p = \frac{1}{2}(p_1 + p_2).$$

For calculational convenience we will choose a simple Gaussian form for the vertex function $\bar{\Phi}_X(-\Omega^2)$. The minus sign in the argument of the Gaussian function is chosen to emphasize that we are working in Minkowski space. One has

$$\bar{\Phi}_X(-\Omega^2) = \exp(\Omega^2/\Lambda_X^2) \tag{29}$$

where the parameter Λ_X characterizes the size of the X meson. Since Ω^2 turns into $-\Omega^2$ in Euclidean space the form (29) has the appropriate falloff behavior in the Euclidean region. We emphasize that any choice for Φ_X is appropriate as long as it falls off sufficiently fast in the ultraviolet region of Euclidean space to render the corresponding Feynman diagrams ultraviolet finite. As mentioned before we shall choose a Gaussian form for Φ_X in our numerical calculation for calculational convenience.

III. MATRIX ELEMENT FOR THE DECAY $X \to \gamma + J/\psi$

The matrix element of the decay $X(3872) \to \gamma + J/\psi$ can be calculated from the Feynman diagrams shown in Fig. 1. The invariant matrix element for the decay is given by

$$M(X_q(p) \to J/\psi(q_1)\gamma(q_2)) = i(2\pi)^4\delta^{(4)}(p - q_1 - q_2)\varepsilon^\mu_X \varepsilon^\rho_\gamma \varepsilon^\nu_{J/\psi} T_{\mu\rho\nu}(q_1, q_2), \tag{30}$$

where

$$T_{\mu\rho\nu}(q_1,q_2)=\sum_{i=a,b,c,d}T^{(i)}_{\mu\rho\nu}(q_1,q_2),$$

$$T^{(a)}_{\mu\rho\nu}=6\sqrt{2}g_Xg_{J/\psi}e_q\int\frac{d^4k_1}{(2\pi)^4i}\int\frac{d^4k_2}{(2\pi)^4i}\tilde{\Phi}_X(-K_a^2)\tilde{\Phi}_{J/\psi}\Big(-\Big(k_1+\frac{1}{2}q_1\Big)^2\Big)$$
$$\times\frac{1}{2}\mathrm{tr}[\gamma_5S_c(k_1)\gamma_\nu S_c(k_1+q_1)\gamma_\mu S_q(k_2)\gamma_\rho S_q(k_2+q_2)-(\gamma_5\leftrightarrow\gamma_\mu)],$$
$$K_a^2=\frac{1}{2}\Big(k_1+\frac{1}{2}q_1\Big)^2+\frac{1}{2}\Big(k_2+\frac{1}{2}q_2\Big)^2+\frac{1}{4}(w_qq_1-w_cq_2)^2,$$
$$T^{(b)}_{\mu\rho\nu}=6\sqrt{2}g_Xg_{J/\psi}\int\frac{d^4k_1}{(2\pi)^4i}\int\frac{d^4k_2}{(2\pi)^4i}\tilde{\Phi}_{J/\psi}\Big(-\Big(k_2+\frac{1}{2}q_1\Big)^2\Big)\tilde{E}_{X\rho}(p_1,\ldots,p_4,r)$$
$$\times\frac{1}{2}\mathrm{tr}[\gamma_5S_q(k_1)\gamma_\mu S_c(k_2)\gamma_\nu S_c(k_2+q_1)-(\gamma_5\leftrightarrow\gamma_\mu)],$$
$$p_1=k_2,\quad p_2=k_2+q_1,\quad p_3=p_4=-k_1,\quad r=-q_2,$$
$$T^{(c)}_{\mu\rho\nu}=6\sqrt{2}g_Xg_{J/\psi}e_c\int\frac{d^4k_1}{(2\pi)^4i}\int\frac{d^4k_2}{(2\pi)^4i}\tilde{\Phi}_X(-K_c^2)\tilde{\Phi}_{J/\psi}\Big(-\Big(k_2+q_2+\frac{1}{2}q_1\Big)^2\Big)$$
$$\times\frac{1}{2}\mathrm{tr}[\gamma_5S_q(k_1)\gamma_\mu S_c(k_2)\gamma_\rho S_c(k_2+q_2)\gamma_\nu S_c(k_2+p)-(\gamma_5\leftrightarrow\gamma_\mu)],$$
$$K_c^2=\frac{1}{2}k_1^2+\frac{1}{2}\Big(k_2+\frac{1}{2}p\Big)^2+\frac{1}{4}w_q^2p^2,$$
$$T^{(d)}_{\mu\rho\nu}=6\sqrt{2}g_Xg_{J/\psi}e_c\int\frac{d^4k_1}{(2\pi)^4i}\int\frac{d^4k_2}{(2\pi)^4i}\tilde{\Phi}_X(-K_c^2)\tilde{E}_{J/\psi\rho}(p_1,p_2,q)\frac{1}{2}\mathrm{tr}[\gamma_\mu S_q(k_1)\gamma_5S_c(k_2)\gamma_\nu S_c(k_2+p)-(\gamma_5\leftrightarrow\gamma_\mu)],$$
$$p_1=-k_2-p,\quad p_2=-k_2,\quad q=-q_2.$$

We have analytically checked on the gauge invariance of the unintegrated transition matrix element by contraction with the photon momentum q_2 which yields $q_2^\rho T_{\mu\rho\nu}(q_1,q_2)=0$ using the identities

$$S(k_2)\not{q}_2S(k_2+q_2)=S(k_2+q_2)-S(k_2),$$
$$\int_0^1d\tau\tilde{\Phi}'(-\tau a-(1-\tau)b)(a-b)=\tilde{\Phi}(-b)-\tilde{\Phi}(-a).$$

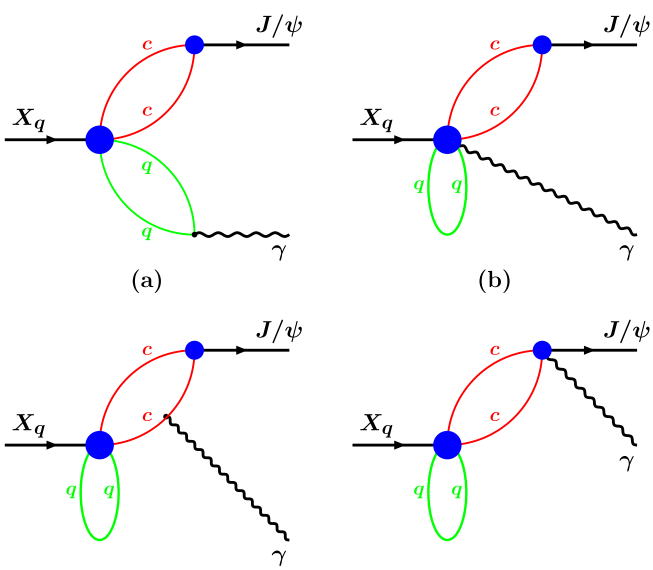

FIG. 1 (color online). Feynman diagrams describing the decay $X\to\gamma+J/\psi$.

IV. NUMERICAL RESULTS

The evaluation of the loop integrals in Eq. (30) proceeds as described in our previous paper [1]. If one takes the on-mass shell conditions into account

$$\varepsilon_X^\mu p_\mu=0,\qquad\varepsilon^\nu_{J/\psi}q_{1\nu}=0,\qquad\varepsilon^\rho_\gamma q_{2\rho}=0\tag{31}$$

one can write down five seemingly independent Lorentz structures

$$T_{\mu\rho\nu}(q_1,q_2)=\varepsilon_{q_2\mu\nu\rho}(q_1\cdot q_2)W_1+\varepsilon_{q_1q_2\nu\rho}q_{1\mu}W_2+\varepsilon_{q_1q_2\mu\rho}q_{2\nu}W_3+\varepsilon_{q_1q_2\mu\nu}q_{1\rho}W_4+\varepsilon_{q_1\mu\nu\rho}(q_1\cdot q_2)W_5.\tag{32}$$

Using the gauge invariance condition

$$q_2^\rho T_{\mu\rho\nu}=(q_1\cdot q_2)\varepsilon_{q_1q_2\mu\nu}(W_4+W_5)=0\tag{33}$$

one has $W_4=-W_5$ which reduces the set of independent covariants to four:

$$T_{\mu\rho\nu}(q_1, q_2) = (q_1 \cdot q_2)\varepsilon_{q_2\mu\nu\rho}W_1 + \varepsilon_{q_1q_2\nu\rho}q_{1\mu}W_2 + \varepsilon_{q_1q_2\mu\rho}q_{2\nu}W_3 + (\varepsilon_{q_1q_2\mu\nu}q_{1\rho} - (q_1 \cdot q_2)\varepsilon_{q_1\mu\nu\rho})W_4. \quad (34)$$

The gauge invariance condition $W_4 = -W_5$ provides for a numerical check on the gauge invariance of our calculation as described further on.

However, there are two nontrivial relations among the four covariants which can be derived by noting [23] that the tensor

$$T_{\mu[\nu_1\nu_2\nu_3\nu_4\nu_5]} = g_{\mu\nu_1}\varepsilon_{\nu_2\nu_3\nu_4\nu_5} + \text{cycl.}(\nu_1\nu_2\nu_3\nu_4\nu_5) \quad (35)$$

vanishes in four dimensions since it is totally antisymmetric in the five indices $(\nu_1, \nu_2, \nu_3, \nu_4, \nu_5)$. Upon contraction with $q_1^\mu q_1^{\nu_1} q_2^{\nu_2}$ and $q_2^\mu q_1^{\nu_1} q_2^{\nu_2}$ one finds (between polarization vectors)

$$q_1^2\varepsilon_{q_2\mu\nu\rho} + \varepsilon_{q_1q_2\nu\rho}q_{1\mu} + (\varepsilon_{q_1q_2\mu\nu}q_{1\rho} - (q_1 \cdot q_2)\varepsilon_{q_1\mu\nu\rho}) = 0, \quad (36)$$

$$(q_1 \cdot q_2)\varepsilon_{q_2\mu\nu\rho} - \varepsilon_{q_1q_2\nu\rho}q_{1\mu} - \varepsilon_{q_1q_2\mu\rho}q_{2\nu} = 0. \quad (37)$$

The two conditions reduce the set of independent covariants to two. This is the appropriate number of independent covariants since the photon transition is described by two independent amplitudes as e.g. by the $E1$ and $M2$ transition amplitudes.

Using the two constraint Eqs. (36) and (37) the expansion (34) can be written in the form

$$T_{\mu\rho\nu} = \left(W_1 + W_3 - \frac{m_{J/\psi}^2}{(q_1 \cdot q_2)}W_4\right)\varepsilon_{q_1q_2\mu\rho}q_{2\nu} + \left(W_1 + W_2 - \left(1 + \frac{m_{J/\psi}^2}{(q_1 \cdot q_2)}\right)W_4\right)\varepsilon_{q_1q_2\nu\rho}q_{1\mu}. \quad (38)$$

By comparing with the corresponding expressions in the Appendix one notes that the first and second terms in (38) describe transitions into the longitudinal and transverse components of the J/ψ.

The quantities W_i are represented by the four-fold integrals

$$W_i = \int_0^\infty dt \int_0^1 d^3\beta F_i(t, \beta_1, \beta_2, \beta_3), \quad (39)$$

where we have suppressed the additional dependence of the integrand F_i on the set of variables p^2, q_1^2, q_2^2; m_q, m_c, s_X, $s_{J/\psi}$ with $s_X = 1/\Lambda_X^2$ and $s_{J/\psi} = 1/\Lambda_{J/\psi}^2$. The integrals in Eq. (39) have branch points at $p^2 = 4(m_q + m_c)^2$ [diagram in Fig. 1(a)] and at $p^2 = 4m_c^2$ [diagrams in Figs. 1(b)–1(d)]. At these points the integrals become nonanalytical in the conventional sense when $t \to \infty$. In order to check on the gauge invariance of the amplitude $T_{\mu\rho\nu}(q_1, q_2)$, we have taken the X-meson momentum squared to be below the closest unitarity threshold, i.e. $p^2 < 4m_c^2$. We have checked explicitly that, for $m_X = 3.1$ GeV and $m_{J/\psi} = 2.9$ GeV, the gauge condition $W_4 = -W_5$ is numerically satisfied to very high accuracy. Note that the gauge invariance condition is independent of the overall couplings g_X and $g_{J/\psi}$ and thus the numerical check can be done irrelevant of their values.

In the next step we introduce an infrared cutoff $1/\lambda^2$ on the upper limit of the t-integration in Eq. (39). In this manner one removes all possible nonanalytic structures and thereby one obtains entire functions for the amplitudes, i.e. one has effectively instituted quark confinement, see Refs. [1,2]. The value of $\lambda = 181$ MeV was found by fitting the calculated basic quantities to the experimental data. However, for such a value of λ the contributions coming from the bubble diagrams in Figs. 1(b)–1(d) blow up at $p^2 = m_X^2$ compared with the contribution from the diagram Fig. 1(a). The bubble diagrams are needed only to guarantee the gauge invariance of the matrix element. For physical applications one should take into account only the gauge invariant part of the diagram Fig. 1(a).

It is convenient to present the decay width via helicity or multipole amplitudes. The projection of the Lorentz amplitudes to the helicity amplitudes is given in Appendix. One has

$$\Gamma(X \to \gamma J/\psi) = \frac{1}{12\pi}\frac{|\vec{q}_2|}{m_X^2}(|H_L|^2 + |H_T|^2) = \frac{1}{12\pi}\frac{|\vec{q}_2|}{m_X^2}(|A_{E1}|^2 + |A_{M2}|^2), \quad (40)$$

where the helicity amplitudes H_L and H_T are expressed in terms of the Lorentz amplitudes as

$$H_L = i\frac{m_X^2}{m_{J/\psi}}|\vec{q}_2|^2\left[W_1 + W_3 - \frac{m_{J/\psi}^2}{m_X|\vec{q}_2|}W_4\right],$$
$$H_T = -im_X|\vec{q}_2|^2\left[W_1 + W_2 - \left(1 + \frac{m_{J/\psi}^2}{m_X|\vec{q}_2|}\right)W_4\right], \quad (41)$$
$$|\vec{q}_2| = \frac{m_X^2 - m_{J/\psi}^2}{2m_X}.$$

The $E1$ and $M2$ multipole amplitudes are obtained via $A_{E1/M2} = (H_L \mp H_T)/\sqrt{2}$. If we choose $\Lambda_X = 3.0$ GeV for the size parameter of the $X(3872)$ we obtain $A_{M2}/A_{E1} = 0.11$, i.e. the electric multipole amplitude A_{E1} dominates the transition, as expected. Nevertheless our predicted angular decay distribution $W(\vartheta) \sim 1 - 0.52\cos^2\vartheta$ differs noticeably from its form $W(\vartheta) \sim 1 - 0.333\cos^2\vartheta$ for $E1$ dominance. It would be interesting to experimentally check on this prediction of our tetraquark model.

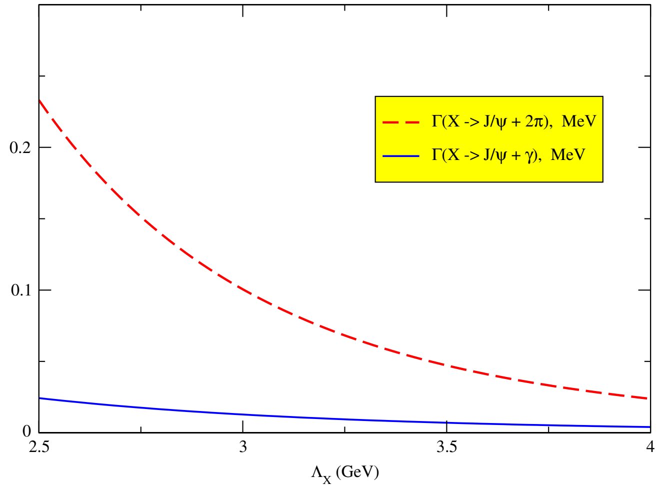

FIG. 2 (color online). The dependence of the decay widths $\Gamma(X_l \to \gamma + J/\psi)$ and $\Gamma(X_l \to J/\psi 2\pi)$ on the size parameter Λ_X.

In Fig. 2 we show a plot of the size parameter dependence of the decay width $\Gamma(X_l \to J/\psi + \gamma)$ together with the decay width $\Gamma(X_l \to J/\psi + 2\pi)$ taken from [1]. We correct an error of Ref. [1] in the normalization condition of the X meson, which led to a $\lesssim 30\%$ underestimate of the strong decay widths. Both decay widths become smaller as the size parameter increases. Note that the radiative decay width for $X_h = -X_u \sin\theta + X_d \cos\theta$ is almost an order of magnitude smaller than that for $X_l = X_u \cos\theta + X_d \sin\theta$. If one takes $\Lambda_X \in (3, 4)$ GeV with the central value $\Lambda_X = 3.5$ GeV then our prediction for the ratio of widths reads

$$\frac{\Gamma(X_l \to \gamma + J/\psi)}{\Gamma(X_l \to J/\psi + 2\pi)}\bigg|_{\rm theor} = 0.15 \pm 0.03 \qquad (42)$$

which fits very well the experimental data from the Belle Collaboration [10].

$$\frac{\Gamma(X \to \gamma + J/\psi)}{\Gamma(X \to J/\psi 2\pi)} = \begin{cases} 0.14 \pm 0.05 & \text{Belle}\,[10] \\ 0.22 \pm 0.06 & Babar[13] \end{cases}. \qquad (43)$$

V. SUMMARY AND CONCLUSION

We have used our relativistic constituent quark model which includes infrared confinement in an effective way to calculate the radiative decay $X \to \gamma + J/\psi$. We take the $X(3872)$ meson to be a tetraquark state with the quantum numbers $J^{\rm PC} = 1^{++}$. In order to introduce electromagnetic interactions we have gauged a nonlocal effective Lagrangian which describes the interaction of the $X(3872)$ meson with its four constituent quarks by using the P-exponential path-independent formalism. We have calculated the matrix element of the transition $X \to \gamma + J/\psi$ and have shown its gauge invariance. We have evaluated the $X \to \gamma + J/\psi$ decay width and the polarization of the J/ψ in the decay. The calculated decay width is consistent with the available experimental data for reasonable values of the size parameter of the $X(3872)$ meson.

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APPENDIX A: HELICITY AND MULTIPOLE AMPLITUDES

The material presented in this Appendix is adapted from similar material written down in [24] in a slightly different context. There are two independent helicity amplitudes $H_{\lambda_X;\lambda_\gamma\lambda_{J/\psi}}$ which we denote by $H_i (i = L, T)$ according to the helicity of the final meson state J/ψ, where $\lambda_{J/\psi} = 0$ and $\lambda_{J/\psi} = \pm 1$ stand for the longitudinal and transverse helicities of the J/ψ. From parity one has $H_{+;-0} = -H_{-;+0} = H_L$ and $H_{0;++} = -H_{0;--} = H_T$.

We seek a covariant representation for the longitudinal and transverse projectors $\mathrm{IP}^{\mu\rho\nu}_{L,T}$ which, when applied to the transition amplitude $T_{\mu\rho\nu}$, project onto the helicity amplitudes $H_{L,T}$ according to

$$H_i = \mathrm{IP}^{\mu\rho\nu}_i T_{\mu\rho\nu}, \qquad (i = L, T). \qquad (A1)$$

The projectors are defined by

$$\begin{aligned} \mathrm{IP}^{\mu\rho\nu}_L &= \frac{1}{2}(\varepsilon^\mu_X(+)\bar{\varepsilon}^{\dagger\rho}_\gamma(-) - \varepsilon^\mu_X(-)\bar{\varepsilon}^{\dagger\rho}_\gamma(+))\varepsilon^{\dagger\nu}_{J/\psi}(0), \\ \mathrm{IP}^{\mu\rho\nu}_T &= \frac{1}{2}\varepsilon^\mu_X(0)(\bar{\varepsilon}^{\dagger\rho}_\gamma(+)\varepsilon^{\dagger\nu}_{J/\psi}(+) - \bar{\varepsilon}^{\dagger\rho}_\gamma(-)\varepsilon^{\dagger\nu}_{J/\psi}(-)), \end{aligned} \qquad (A2)$$

where we use the Jacob-Wick convention for the helicity polarization four-vectors as written down in [25]. The z-direction is defined by the momentum of the J/ψ. The bars in the polarization four-vectors $\bar{\varepsilon}^\rho_\gamma(\lambda_\gamma)$ of the photon are a reminder that the photon helicities are defined relative to the negative z-direction. In the present context it is important to take into account both parity configurations related by a helicity reflection in the definition of Eq. (A2). In explicit form one has in the X-rest frame

TABLE I. Helicity and multipole projections of the basic tensors $K^{(i)}_{\mu\rho\nu}$. The tensors $K^{(i)}_{\mu\rho\nu}$ ($i = 2, 4, 5, 6$) are gauge invariant. They satisfy $q_2^\rho K^{(i)}_{\mu\rho\nu} = 0$.

i	$K^{(i)}_{\mu\rho\nu}$	$H_L^{(i)} = \mathrm{IP}_L^{\mu\rho\nu} K^{(i)}_{\mu\rho\nu}$	$H_T^{(i)} = \mathrm{IP}_T^{\mu\rho\nu} K^{(i)}_{\mu\rho\nu}$	$A_{E1}^{(i)} = \mathrm{IP}_{E1}^{\mu\rho\nu} K^{(i)}_{\mu\rho\nu}$	$A_{M2}^{(i)} = \mathrm{IP}_{M2}^{\mu\rho\nu} K^{(i)}_{\mu\rho\nu}$
1	$\varepsilon_{\mu\rho\nu q_1}$	$im_{J/\psi}$	$-i\frac{m_X^2+m_{J/\psi}^2}{2m_X}$	$\frac{i}{\sqrt{2}}\frac{(m_X+m_{J/\psi})^2}{2m_X}$	$-\frac{i}{\sqrt{2}}\frac{2m_X}{(m_X+m_{J/\psi})^2}\lvert\vec{q}_2\rvert^2$
2	$q_{1\mu}\varepsilon_{\rho\nu q_1 q_2}$	0	$im_X\lvert\vec{q}_2\rvert^2$	$-\frac{i}{\sqrt{2}}m_X\lvert\vec{q}_2\rvert^2$	$\frac{i}{\sqrt{2}}m_X\lvert\vec{q}_2\rvert^2$
3	$q_{1\rho}\varepsilon_{\mu\nu q_1 q_2}$	0	0	0	0
4	$q_{2\nu}\varepsilon_{\mu\rho q_1 q_2}$	$i\frac{m_X^2}{m_{J/\psi}}\lvert\vec{q}_2\rvert^2$	0	$\frac{i}{\sqrt{2}}\frac{m_X^2}{m_{J/\psi}}\lvert\vec{q}_2\rvert^2$	$\frac{i}{\sqrt{2}}\frac{m_X^2}{m_{J/\psi}}\lvert\vec{q}_2\rvert^2$
5	$\varepsilon_{\mu\rho\nu q_2}$	$i\frac{m_X}{m_{J/\psi}}\lvert\vec{q}_2\rvert$	$-i\lvert\vec{q}_2\rvert$	$\frac{i}{\sqrt{2}}\frac{m_X+m_{J/\psi}}{m_{J/\psi}}\lvert\vec{q}_2\rvert$	$\frac{i}{\sqrt{2}}\frac{2m_X}{m_{J/\psi}(m_X+m_{J/\psi})}\lvert\vec{q}_2\rvert^2$
6	$K^{(3)}_{\mu\rho\nu} - (q_1 q_2)K^{(1)}_{\mu\rho\nu}$	$-im_X m_{J/\psi}\lvert\vec{q}_2\rvert$	$i\frac{m_X^2+m_{J/\psi}^2}{2}\lvert\vec{q}_2\rvert$	$-\frac{i}{\sqrt{2}}\frac{(m_X+m_{J/\psi})^2}{2}\lvert\vec{q}_2\rvert$	$\frac{i}{\sqrt{2}}\frac{2m_X^2}{(m_X+m_{J/\psi})^2}\lvert\vec{q}_2\rvert^3$

$$
\begin{aligned}
\varepsilon_{X\mu}(\pm) &= \frac{1}{\sqrt{2}}(0; \pm 1, i, 0),\\
p^\alpha &= (m_X; 0, 0, 0),\\
\varepsilon_{X\mu}(0) &= (0; 0, 0, -1),\\
\varepsilon^\dagger_{J/\psi\nu}(\pm) &= \frac{1}{\sqrt{2}}(0; \pm 1, -i, 0),\\
q_1^\alpha &= \left(\frac{m_X^2 + m_{J/\psi}^2}{2m_X}; 0, 0, \lvert\vec{q}_2\rvert\right),\\
\varepsilon^\dagger_{J/\psi\nu}(0) &= \frac{1}{m_{J/\psi}}\left(\lvert\vec{q}_2\rvert; 0, 0, -\frac{m_X^2 + m_{J/\psi}^2}{2m_X}\right),\\
\bar{\varepsilon}^\dagger_{\gamma\rho}(\pm) &= \frac{1}{\sqrt{2}}(0; \mp 1, -i, 0),\\
q_2^\alpha &= \lvert\vec{q}_2\rvert(1; 0, 0, -1).
\end{aligned}
\tag{A3}
$$

A convenient covariant representation of the projectors can be obtained in the form

$$
\mathrm{IP}_i^{\mu\rho\nu} = h_i^{\mu'\rho'\nu'} S_X^{(1)\mu}{}_{\mu'}(p)(-g^\rho_{\rho'}) S^{(1)\nu}_{J/\psi\,\nu'}(q_1), \tag{A4}
$$

where

$$
\begin{aligned}
h_L^{\mu\rho\nu} &= \frac{i}{2}\frac{m_{J/\psi}}{(q_1\cdot q_2)^2}\varepsilon^{\mu\rho q_1 q_2} q_2^\nu,\\
h_T^{\mu\rho\nu} &= -\frac{i}{2}\frac{m_X}{(q_1\cdot q_2)^2} q_2^\mu \varepsilon^{\rho\nu q_1 q_2},
\end{aligned}
\tag{A5}
$$

and where the massive propagator functions are given by ($V = X, J/\psi$)

$$
S_V^{(1)\alpha}{}_{\alpha'}(p_V) = -g^\alpha{}_{\alpha'} + \frac{p_V^\alpha p_{V\alpha'}}{m_V^2}. \tag{A6}
$$

The massive propagator functions are needed in the projectors Eq. (A4) to project out the appropriate three-dimensional subspaces in the respective rest systems of the spin 1 particles. For the photon one exploits the gauge freedom to write the propagator function as $(-g^\rho_{\rho'})$. Note that the compact form (A4) is only obtained if one uses the summed form (A2). The projection operators are orthonormal in the sense that $\mathrm{IP}_i^{\mu\rho\nu}\mathrm{IP}^\dagger_{j\mu\rho\nu} = -\frac{1}{2}\delta_{ij}$.

The angular decay distribution in the decay $X(3872) \to \gamma + J/\psi(\to \ell^+\ell^-)$ is given by

$$
\begin{aligned}
\frac{d\Gamma}{d\cos\vartheta} &= \mathrm{BR}(J/\psi \to \ell^+\ell^-)\frac{1}{4\pi}\frac{1}{2S_X + 1}\frac{\lvert\vec{q}_2\rvert}{m_X^2}\\
&\quad \times \left(\frac{3}{4}\sin^2\vartheta\lvert H_L\rvert^2 + \frac{3}{8}(1 + \cos^2\vartheta)\lvert H_T\rvert^2\right),
\end{aligned}
\tag{A7}
$$

where ϑ is the polar angle of either of the leptons $\ell^\pm$ relative to the original flight direction of the J/ψ, all in the rest system of the J/ψ.

One can alternatively describe the transition in terms of the two multipole amplitudes A_{E1} and A_{M2}. The multipole amplitudes are related to the helicity amplitudes via [26]

$$
A_{E1} = \frac{1}{\sqrt{2}}(H_L - H_T), \qquad A_{M2} = \frac{1}{\sqrt{2}}(H_L + H_T). \tag{A8}
$$

The corresponding projectors onto the multipole amplitudes are given by

$$
\begin{aligned}
\mathrm{IP}_{E1}^{\mu\rho\nu} &= \frac{1}{\sqrt{2}}(\mathrm{IP}_L^{\mu\rho\nu} - \mathrm{IP}_T^{\mu\rho\nu}),\\
\mathrm{IP}_{M2}^{\mu\rho\nu} &= \frac{1}{\sqrt{2}}(\mathrm{IP}_L^{\mu\rho\nu} + \mathrm{IP}_T^{\mu\rho\nu}).
\end{aligned}
\tag{A9}
$$

In Table I we have summarized the helicity and multipole amplitudes resulting from the relevant projections of the basic covariants Eq. (32). The entries can be seen to satisfy the constraint equations Eqs. (36) and (37). The multipole amplitudes $A_{E1,M2}$ calculated from the gauge invariant structures $K^{(i)}_{\mu\rho\nu}$ ($i = 2, 4, 5, 6$) show the appropriate lowest-order power behavior $A_{E1} \sim \lvert\vec{q}_2\rvert$ and $A_{M2} \sim \lvert\vec{q}_2\rvert^2$.

The leading $\lvert\vec{q}_2\rvert$ contribution to the angular decay distribution proportional to $\lvert A_{E1}\rvert^2$ is thus given by $W(\cos\vartheta) \propto (3 - \cos^2\vartheta)$. The next-to-leading contribution proportional to $2\mathcal{R}(A_{E1}A^*_{M2})$ is down by one power of $\lvert\vec{q}_2\rvert$. The nonleading angular distribution is given by $W(\cos\vartheta) \propto (1 - 3\cos^2\vartheta)$ (in the same units).

[1] S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov, and J. G. Körner, Phys. Rev. D **81**, 114007 (2010); S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov, J. G. Körner, and G. G. Saidullaeva, "A relativistic quark model with infrared confinement and the tetraquark state," AIP Conf. Proc. **1343**, 385 (2011) [arXiv:1011.4417];

[2] T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij, Phys. Rev. D **81**, 034010 (2010).

[3] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Phys. Rev. D **71**, 014028 (2005).

[4] S. K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262001 (2003).

[5] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).

[6] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **93**, 041801 (2004).

[7] D. E. Acosta *et al.* (CDF Collaboration), Phys. Rev. Lett. **93**, 072001 (2004).

[8] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **93**, 162002 (2004).

[9] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **103**, 152001 (2009); Kai Yi, arXiv:0910.3163.

[10] K. Abe *et al.* (Belle Collaboration), arXiv:hep-ex/0505037.

[11] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **74**, 071101 (2006).

[12] G. Gokhroo *et al.* (Belle Collaboration), Phys. Rev. Lett. **97**, 162002 (2006).

[13] E. Klempt and A. Zaitsev, Phys. Rep. **454**, 1 (2007).

[14] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **102**, 132001 (2009).

[15] T. Barnes and S. Godfrey, Phys. Rev. D **69**, 054008 (2004).

[16] E. S. Swanson, Phys. Lett. B **598**, 197 (2004).

[17] Y. -b. Dong, A. Faessler, and T. Gutsche *et al.*, Phys. Rev. D **77**, 094013 (2008).

[18] M. Nielsen and C. M. Zanetti, Phys. Rev. D **82**, 116002 (2010).

[19] M. A. Ivanov, M. P. Locher, and V. E. Lyubovitskij, Few-Body Syst. **21**, 131 (1996).

[20] S. Mandelstam, Ann. Phys. (N.Y.) **19**, 1 (1962); J. Terning, Phys. Rev. D **44**, 887 (1991).

[21] G. V. Efimov and M. A. Ivanov, *The Quark Confinement Model of Hadrons* (IOP Publishing, Bristol & Philadelphia, 1993).

[22] A. Salam, Nuovo Cim. **25**, 224 (1962); S. Weinberg, Phys. Rev. **130**, 776 (1963); for review, see K. Hayashi *et al.*, Fortschr. Phys. **15**, 625 (1967).

[23] J. G. Körner and M. C. Mauser, Lect. Notes Phys. **647**, 212 (2004).

[24] J. G. Körner, J. H. Kühn, M. Krammer, and H. Schneider, Nucl. Phys. **B229**, 115 (1983).

[25] P. R. Auvil and J. J. Brehm, Phys. Rev. **145**, 1152 (1966).

[26] W. N. Cottingham and B. R. Pollard, Ann. Phys. (N.Y.) **105**, 111 (1977).