Hierarchical neutrino masses and mixing in nonminimal $SU(5)$

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We consider the problem of neutrino masses and mixing within the framework of a nonminimal supersymmetric $SU(5)$ model extended by adding a set of $\underline{\mathbf{1}}$, $\underline{\mathbf{24}}$ chiral superfields accommodating three right-handed neutrinos. A type I + III seesaw mechanism can then be realized, giving rise to a hierarchical mass spectrum for the light neutrinos of the form $m_3 > m_2 \gg m_1$, consistent with present data. The extra colored states are pushed to the unification scale by proton stability constraints, while the intermediate seesaw energy scale and the unification scale are maintained in phenomenologically acceptable ranges. We also examine the issue of the large neutrino mixing hierarchy $\theta_{23} > \theta_{12} \gg \theta_{13}$ in the above framework of hierarchical neutrino masses.

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I. INTRODUCTION

The discovery of neutrino oscillations is the first encounter with physics beyond the standard model (SM). Data coming from various sources [1] are conclusive that neutrinos are massive (at least two of them) and that they mix, exhibiting oscillation phenomena [2]. This implies a mismatch between flavor and mass eigenstates in an obvious analogy with the Cabibbo-Kobayashi-Maskawa matrix in the quark sector of the SM. In order to explain the new evidence on the overall scale and structure of the neutrino mass matrix, several proposals have been put forward, among which the most interesting appears to be the so-called seesaw mechanism [3]. It provides an elegant answer to the smallness of the observed neutrino masses, although it leaves open the issue of the underlying structure of the neutrino mass matrix. The seesaw mechanism is a general term and can be realized in a number of forms and variations, with the basic idea relying on the fact that a large energy scale $M \gg m_W$ is introduced through the coupling of the left-handed neutrinos to a sector of heavy fields. By integrating these heavy degrees of freedom out, an effective operator is produced, giving small masses to the neutrinos of order $\sim m_W^2/M$. A heavy scale in the neighborhood of $M \sim 10^{14}$ GeV leads to an overall neutrino mass scale of $\sim 10^{-1}$ eV, in general agreement with observations. The usual classification of seesaw types in the literature is based on the gauge properties of the heavy particles used to mediate the seesaw mechanism.[1] Types I, II, and III correspond to fermion singlets, scalar charged isotriplets, and fermion neutral isotriplets, respectively.

Since the seesaw mechanism requires a sector of particles with masses well above the scales at which the validity of the SM has been established, it is natural to consider its realization in the framework of general approaches for the extension of the SM, such as supersymmetry and grand unified theories (GUTs). The simplest choice of considering minimal supersymmetric $SU(5)$ and introducing the required heavy fields as singlets retains all the arbitrariness of realizing the seesaw mechanism without introducing any new constraint on scales and structure. In addition, a phenomenologically viable scenario within this context has to overcome the problems of the original model, such as proton decay and unrealistic fermion masses. A way around the former would be to tune the Yukawa couplings and biunitary transformations with the soft sector through certain relations, something unnecessary if the unification scale could be shifted. For realistic fermion masses the presence of nonrenormalizable operators would be required.

In order to obtain potentially interesting constraints on the scale and structure of neutrino masses, the sector of heavy fields has to partake in the GUT. This can be realized in other GUTs [4], such as $SO(10)$ and flipped $SU(5)$ [5], or by extending the gauge nonsinglet field content of $SU(5)$. The realization of the so-called type I seesaw mechanism in the SM introduces right-handed neutrinos as gauge singlet fields. In contrast, in the type III seesaw mechanism, right-handed neutrinos are nontrivially introduced as the neutral components of isotriplet fields [6]. This can be promoted to extended versions of $SU(5)$ that feature additional chiral superfields in the $\underline{\mathbf{24}}$ representation, each containing two suitable right-handed neutrino candidates.[2] A mixed "type $\mathbf{I} + \mathbf{III}$" seesaw mechanism can then be realized with an extra $\underline{\mathbf{24}}$ [8], while the most appealing three generation scenario with three right-handed neutrinos requires additional $\underline{\mathbf{24}}$'s or $\underline{\mathbf{1}}$'s. In the present article we consider a version of supersymmetric $SU(5)$ extended through the introduction of extra chiral superfields $\mathcal{S}(\mathbf{1})$, $\mathcal{T}(\mathbf{24})$, $\mathcal{T}'(\mathbf{24})$, which provide us with three right-handed neutrino candidates. Our basic assumption is

[1] An alternative classification based on the explicit mathematical expression for the light neutrino masses is also common.

[2] Fermions in a single $\underline{\mathbf{24}}$ representation have been introduced in the framework of nonsupersymmetric $SU(5)$ in [7], where the seesaw mechanism was realized with two right-handed neutrinos at a predicted low energy scale.

that these right-handed neutrino fields obtain a Majorana mass at a high but still intermediate scale a few orders of magnitude below the unification scale. This assumption is supported by a renormalization group analysis, incorporating proton lifetime constraints [9], and allows for an intermediate scale in the vicinity of 10^{13}–10^{14} GeV. Not all of the scales involved in the right-handed neutrino Majorana mass matrix are constrained by the renormalization group. Depending on assumptions, several possibilities emerge leading to a different dependence of the resulting light neutrino masses on these scales. Furthermore, the fact that two of the right-handed neutrinos are members of the same $SU(5)$ representation leads to a particular rank 2 structure of the resulting light neutrino mass matrix that is accompanied by a massless eigenvalue. Although this fact is modified by nonrenormalizable terms, there is a definite prediction for one superlight neutrino, not in conflict with observations. Next, we examine the possibility of a hierarchical light neutrino mass spectrum $m_\nu^{(3)} > m_\nu^{(2)} \gg m_\nu^{(1)}$. This can be achieved in a variety of ways, depending on assumptions either for the mass scales involved or for the hierarchy of the Yukawa-type couplings. We also consider whether the observed large neutrino mixing can be accommodated in the framework of the model [10]. We conclude that hierarchical mixing patterns with $\theta_{13} \ll \theta_{12} \sim \theta_{23}$ can be obtained with generic choices of Yukawa couplings exhibiting a certain structure.

II. THE MODEL

The renormalizable part of the minimal $SU(5)$ superpotential, in terms of the chiral superfields $\mathcal{Q}_i^c(10)$, $\mathcal{Q}_i(\bar{5})$, $\mathcal{H}(\bar{5})$, $\mathcal{H}^c(5)$, $\Sigma(24)$, is

$$\mathcal{W}_0 = \mathcal{Y}_{ij}^u \mathcal{Q}_i^c \mathcal{Q}_j^c \mathcal{H}^c + \mathcal{Y}_{ij}^d \mathcal{Q}_i \mathcal{Q}_j^c \mathcal{H} + \frac{M}{2}\mathrm{Tr}(\Sigma^2) + \frac{\lambda}{3!}\mathrm{Tr}(\Sigma^3) + \lambda' \mathcal{H}^c \Sigma \mathcal{H} + M' \mathcal{H}^c \mathcal{H}, \tag{1}$$

where we have suppressed $SU(5)$ indices and display only the family indices i, j. Let us now introduce extra matter supermultiplets $\mathcal{S}(1)$, $\mathcal{T}(24)$, $\mathcal{T}'(24)$ with the standard matter parity assignment.[3] An extra Z_2 discrete symmetry, under which only $\mathcal{T}'(24)$ changes sign, differentiates between them so that $\mathcal{T}'$ does not couple to standard matter fields. The renormalizable contributions of the new fields to the superpotential are

$$\mathcal{W}_1 = \mathcal{Y}_i^S \mathcal{Q}_i \mathcal{H}^c \mathcal{S} + \mathcal{Y}_i^T \mathcal{Q}_i \mathcal{H}^c \mathcal{T} + \frac{\mu}{2}\mathcal{S}^2 + \frac{\mu'}{2}\mathrm{Tr}(\mathcal{T}^2) + \frac{\mu''}{2}\mathrm{Tr}(\mathcal{T}'^2) + f\,\mathrm{Tr}(\mathcal{T}^2\Sigma) + f'\,\mathrm{Tr}(\Sigma\mathcal{T})\mathcal{S} + f''\,\mathrm{Tr}(\mathcal{T}'^2\Sigma). \tag{2}$$

[3] We have $\mathcal{Q}$, $\mathcal{Q}^c$, S, $\mathcal{T} \to -1$, while Σ, $\mathcal{H}$, $\mathcal{H}^c \to 1$.

The decomposition of the new matter multiplet $\mathcal{T}(24)$ is

$$\mathcal{T}(24) = B(1,1,0) + T(1,3,0) + O(8,1,0) + \mathcal{X}(3,2,-5/6) + \mathcal{X}^c(\bar{3},2,5/6),$$

where the $SU(3)\times SU(2)\times U(1)$ identification of each component is self-explanatory. Analogous is the decomposition of the primed field $\mathcal{T}'(24)$. Denoting by T^0 the neutral component of the isotriplet $T(1,3,0)$, we can identify the three right-handed neutrino candidates as $N_i^c = (\mathcal{S}, B, T^0)$.

Symmetry breaking of $SU(5)$ down to $SU(3)\times SU(2)\times U(1)$ is realized in the standard fashion through a nonzero vacuum expectation value of Σ in the direction $\langle\Sigma\rangle = \frac{V}{\sqrt{30}}\mathrm{diag}(2,2,2,-3,-3)$. Note that the absence of cubic terms for the new fields, due to their parity assignment, does not allow them to acquire a nonzero vacuum expectation value and, thus, symmetry breaking proceeds exactly as in the minimal case. All components of Σ are either Higgsed away or obtain masses of the order of the GUT scale. The splitting between the masses of the Higgs isodoublets H_d, H_u and the Higgs colored triplets D, D^c contained in $\mathcal{H} = (H_d, D^c)$ and $\mathcal{H}^c = (H_u, D)$ is produced by the usual fine-tuning $M' = \frac{3\lambda' V}{\sqrt{30}}$, resulting in massless doublets and superheavy triplets. Then, the effective superpotential relevant for masses below the unification scale M_G reads

$$\begin{aligned}\mathcal{W}_{\mathrm{eff}} = {} & Y_{ij}^u u_i^c Q_j H_u + Y_{ij}^d d_i^c Q_j H_d + Y_{ij}^e e_i^c L_j H_d \\ & + Y_i^{\mathcal{S}} L_i \mathcal{S} H_u + Y_i^B L_i B H_u + Y_i^T L_i T H_u + Y_i^{\mathcal{X}} d_i^c \mathcal{X} H_u \\ & + \frac{M_{\mathcal{S}}}{2}\mathcal{S}^2 + \frac{M_B}{2}B^2 + M_{\mathcal{S}B}\mathcal{S}B + \frac{M_T}{2}\mathrm{Tr}(T^2) \\ & + M_{\mathcal{X}}\mathcal{X}\mathcal{X}^c + \frac{M_O}{2}\mathrm{Tr}(O^2) + \frac{M_{T'}}{2}\mathrm{Tr}(T'^2) \\ & + \frac{M_{O'}}{2}\mathrm{Tr}(O'^2) + M_{\mathcal{X}'}\mathcal{X}'\mathcal{X}'^c + \frac{M_{B'}}{2}B'^2.\end{aligned} \tag{3}$$

Matching the effective and the $SU(5)$-symmetric theory at M_G leads to the following relations for the Yukawa couplings:

$$Y^u = 2\mathcal{Y}^u, \qquad (Y^e)^\perp = Y^d = \mathcal{Y}^d, \qquad Y^{\mathcal{S}} = \mathcal{Y}^{\mathcal{S}}, \qquad Y^{\mathcal{X}} = Y^T = \frac{\sqrt{30}}{3}Y^B = \mathcal{Y}^{\mathcal{T}}, \tag{4}$$

while for the mass parameters we get

$$M_{\mathcal{S}} = \mu, \qquad M_B = \mu' - \frac{2fV}{\sqrt{30}}, \qquad M_T = \mu' - \frac{6fV}{\sqrt{30}}, \tag{5}$$

$$M_O = \mu' + \frac{4fV}{\sqrt{30}}, \qquad M_X = \mu' - \frac{fV}{\sqrt{30}}, \qquad (6)$$
$$M_{SB} = -f'V$$

and

$$M_{T'} = \mu'' - \frac{6f''V}{\sqrt{30}}, \qquad M_{B'} = \mu'' - \frac{2f''V}{\sqrt{30}},$$
$$M_{O'} = \mu'' + \frac{4f''V}{\sqrt{30}}, \qquad M_{X'} = \mu'' - \frac{f''V}{\sqrt{30}}. \qquad (7)$$

The seesaw scale is the scale of the right-handed neutrino mass matrix expressed in terms of the parameters M_S, M_B, M_T, and M_{SB}, related through the four parameters μ, μ', fV, and f'/f. The allowed range for these parameters will be strongly constrained by the requirements of unification at a sufficiently high scale. This will follow shortly from a renormalization group analysis.

In addition to the renormalizable contributions above, nonrenormalizable contributions to the superpotential

$$\mathcal{W}_{\rm NR} = \frac{\lambda_{IJKL}}{M_P}\Phi_I\Phi_J\Phi_K\Phi_L + O(1/M_P^2) + \ldots$$

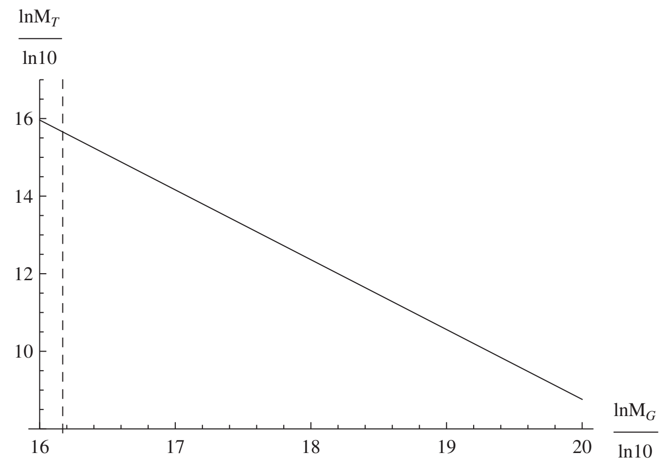

FIG. 1. Isotriplet mass M_T vs the unification scale M_G. The octet mass satisfying $M_{O'} \leq M_G$ sets a lower bound for unification at $M_G \approx 1.5 \times 10^{16}$ GeV.

can, in principle, affect masses, especially whenever we have mass degeneracies. We have denoted the scale of nonrenormalizable interactions generically by M_P, expecting their scale to be the Planck scale. The lowest order terms in $\mathcal{W}_{\rm NR}$ are

$$\mathcal{Q}\mathcal{T}\Sigma\mathcal{H}^c + \mathcal{Q}\Sigma\mathcal{H}^c\mathcal{S} + \mathcal{T}\mathcal{Q}^c\mathcal{H}\mathcal{H} + \mathcal{Q}^c\mathcal{Q}^c\Sigma\mathcal{H}^c + \Sigma\mathcal{Q}^c\mathcal{H}\mathcal{Q} + \mathcal{H}^c\mathcal{Q}\mathcal{Q}\mathcal{H}^c + \mathcal{T}\mathcal{Q}^c\mathcal{Q}\mathcal{Q} + \mathcal{Q}^c\mathcal{Q}\mathcal{Q}\mathcal{S}$$
$$+ \mathcal{Q}^c\mathcal{Q}^c\mathcal{Q}^c\mathcal{Q} + \mathcal{T}^2\Sigma^2 + \Sigma^2\mathcal{T}\mathcal{S} + \mathcal{H}\mathcal{T}^2\mathcal{H}^c + \Sigma^2\mathcal{S}^2 + \mathcal{H}\mathcal{T}\mathcal{H}^c\mathcal{S} + \mathcal{H}\mathcal{H}^c\mathcal{S}^2 + \mathcal{T}^4 + \mathcal{T}^3\mathcal{S} + \mathcal{T}^2\mathcal{S}^2$$
$$+ \mathcal{S}^4 + \Sigma^4 + \mathcal{H}\Sigma^2\mathcal{H}^c + \mathcal{H}\mathcal{H}^c\mathcal{H}\mathcal{H}^c + \mathcal{T}'^2\Sigma^2 + \mathcal{H}\mathcal{T}'^2\mathcal{H}^c + \mathcal{T}'^4 + \mathcal{T}'^2\mathcal{T}^2 + \mathcal{T}\mathcal{T}'^2\mathcal{S} + \mathcal{T}'^2\mathcal{S}^2, \qquad (8)$$

suppressing the factor $1/M_P$ and the dimensionless couplings in front of each term, all assumed to be of the same order. Among these terms, those relevant for neutrino masses are the terms $\mathcal{H}^c\mathcal{Q}\mathcal{Q}\mathcal{H}^c$, leading to (tiny) Majorana masses for left-handed neutrinos; the terms $\mathcal{Q}\mathcal{T}\Sigma\mathcal{H}^c$, $\mathcal{Q}\Sigma\mathcal{H}^c\mathcal{S}$, contributing to Dirac masses; and the terms $\mathcal{T}^2\Sigma^2$, $\Sigma^2\mathcal{T}\mathcal{S}$, $\Sigma^2\mathcal{S}^2$, contributing to Majorana masses for the right-handed neutrinos.

III. ENERGY SCALES

The sector of additional superfields $\mathcal{T}$, $\mathcal{T}'$, $\mathcal{S}$ carries with it a set of extra parameters, namely, the mass parameters μ, μ', μ'' and the couplings f, f', f''. A basic assumption of the model is that the Majorana mass of right-handed neutrinos is at a high but still intermediate scale, a few orders of magnitude below M_G. Thus, we shall assume that the isotriplet component of $\mathcal{T}$ remains lighter than M_G. In addition, proton lifetime constraints translated to a high enough M_G require the presence of an additional light color octet. These requirements correspond to new fine-tunings of parameters, presumably, not worse than the standard GUT fine-tunings. As a working set of choices, we take ($M_G^2 = \frac{5g^2}{12}$ V^2)

$$\mu' = (3-\epsilon)M_G/2, \qquad \mu'' = (2+3\epsilon')M_G/5,$$
$$f = \frac{5g}{4\sqrt{2}}(1-\epsilon), \qquad f'' = -\frac{g}{2\sqrt{2}}(1-\epsilon'),$$

where $\epsilon \sim \epsilon' \ll 1$. These choices result in

$$M_T = \epsilon M_G, \qquad M_{O'} = \epsilon' M_G, \qquad (9)$$

while the rest of the masses are M_O, M_X, $M_{X'}$, $M_{T'} \sim O(M_G)$.

Thus, we assume that, apart from the minimal supersymmetric standard model fields and the color octet and isotriplet superfields that have intermediate masses $M_{O'}$ and M_T, all extra superfields decouple at M_G. In addition, we assume that supersymmetry is broken at an approximately common energy scale of $m_S = O(1\ \text{TeV})$ at which all superpartners decouple. From the one-loop renormalization group equations for the three $SU(3) \times SU(2) \times U(1)$ gauge couplings,[4] with the intermediate octet and isotriplet mass scales inserted, we obtain the following expressions for these couplings at M_Z,

[4]The triplet-octet splitting has been previously studied for $SU(5)$ models at one and two loops in [11].

TABLE I. Values (GeV) for the unification scale M_G, the colored octet mass $M_{O'}$, and the weak isotriplet mass M_T. The corresponding unified coupling α_G remains within the perturbative limit.

M_G	$M_{O'}$	M_T	α_G
3×10^{16}	3.1×10^{15}	1.3×10^{15}	0.040 23
5×10^{16}	1.0×10^{15}	5.2×10^{14}	0.041 12
8×10^{16}	3.6×10^{14}	2.3×10^{14}	0.041 97
1×10^{17}	2.2×10^{14}	1.5×10^{14}	0.042 39
3×10^{17}	2.0×10^{13}	2.1×10^{13}	0.044 57
5×10^{17}	6.4×10^{12}	8.3×10^{12}	0.045 66
8×10^{17}	2.3×10^{12}	3.6×10^{12}	0.046 71
1×10^{18}	1.4×10^{12}	2.4×10^{12}	0.047 23
3×10^{18}	1.2×10^{11}	3.3×10^{11}	0.049 96

$$\frac{2\pi}{\alpha_3(M_Z)} = \frac{2\pi}{\alpha_G} - 3\ln\left(\frac{M_G}{M_Z}\right) - 4\ln\left(\frac{m_S}{M_Z}\right) + 3\ln\left(\frac{M_G}{M_{O'}}\right),$$
$$\frac{2\pi}{\alpha_2(M_Z)} = \frac{2\pi}{\alpha_G} + \ln\left(\frac{M_G}{M_Z}\right) - \frac{25}{6}\ln\left(\frac{m_S}{M_Z}\right) + 2\ln\left(\frac{M_G}{M_T}\right),$$
$$\frac{2\pi}{\alpha_1(M_Z)} = \frac{2\pi}{\alpha_G} + \frac{33}{5}\ln\left(\frac{M_G}{M_Z}\right) - \frac{5}{2}\ln\left(\frac{m_S}{M_Z}\right), \tag{10}$$

where α_G is the common value of the three couplings at the unification scale M_G. Inserting the existing recent data [12] for $\alpha_3(M_Z)$, $\alpha_2(M_Z)$, $\alpha_1(M_Z)$, we obtain M_G and α_G, as well as the octet mass $M_{O'}$ for various choices of the isotriplet mass treated as input. An octet mass below M_G sets a lower bound of 1.5×10^{16} GeV for the unification scale. In Fig. 1 we show the values of M_G obtained in terms of M_T. These values are tabulated in Table I together with the corresponding values of $M_{O'}$ and α_G. Note that the values of $M_{O'}$ follow M_T within a close range, indicating an approximately common intermediate scale. The values for M_T in the proximity of 10^{14} GeV, corresponding to a safe $M_G \sim 10^{17}$ GeV, have the correct order of magnitude required for the seesaw scale, since $(10^2)^2/10^{14} \sim 0.1$ eV.

IV. NEUTRINO MASSES

The terms relevant for neutrino masses can be easily singled out from the renormalizable part of the superpotential (3).[5] These terms are

$$Y_i^{\mathcal{S}} L_i \mathcal{S} H_u + Y_i^B L_i B H_u + Y_i^T L_i T H_u + \frac{M_{\mathcal{S}}}{2}\mathcal{S}^2 + \frac{M_B}{2}B^2 + M_{\mathcal{S}B}\mathcal{S}B + \frac{M_T}{2}T^2 \tag{11}$$

or

$$v_u\left(Y_i^{\mathcal{S}}\mathcal{S} + Y_i^B B - \frac{Y_i^T}{\sqrt{2}}\tau_0\right)\nu_i + \frac{M_{\mathcal{S}}}{2}\mathcal{S}^2 + \frac{M_B}{2}B^2 + M_{\mathcal{S}B}\mathcal{S}B + \frac{M_T}{2}\tau_0^2.$$

The corresponding terms for charged fermion masses are $M_T\tau_+\tau_- - v_u Y_i^T e_i \tau_+$. The full neutrino mass matrix, in an $(\nu_i, \mathcal{S}, B, \tau_0)$ basis, is

$$\mathcal{M}_N = \begin{pmatrix} 0 & \mathcal{M}_D \\ \mathcal{M}_D^{\perp} & \mathcal{M}_R \end{pmatrix}, \tag{12}$$

where

$$\mathcal{M}_D = v_u \begin{pmatrix} Y_1^{\mathcal{S}} & Y_1^B & -\frac{1}{\sqrt{2}}Y_1^T \\ Y_2^{\mathcal{S}} & Y_2^B & -\frac{1}{\sqrt{2}}Y_2^T \\ Y_3^{\mathcal{S}} & Y_3^B & -\frac{1}{\sqrt{2}}Y_3^T \end{pmatrix},$$
$$\mathcal{M}_R = \begin{pmatrix} M_{\mathcal{S}} & M_{\mathcal{S}B} & 0 \\ M_{\mathcal{S}B} & M_B & 0 \\ 0 & 0 & M_T \end{pmatrix}.$$

Note that $Y_i^B = \frac{3}{\sqrt{30}}Y_i^T$. The constraints on μ' and f imply that $M_B \approx M_G$, while $M_{\mathcal{S}}$ and $M_{\mathcal{S}B}$ remain undetermined.

The light neutrino mass matrix will be

$$\mathcal{M}_\nu \approx -\mathcal{M}_D \mathcal{M}_R^{-1} \mathcal{M}_D^{\perp}. \tag{13}$$

The inverse right-handed neutrino Majorana mass is

$$\mathcal{M}_R^{-1} = \frac{1}{\Delta}\begin{pmatrix} -M_B & M_{\mathcal{S}B} & 0 \\ M_{\mathcal{S}B} & -M_{\mathcal{S}} & 0 \\ 0 & 0 & \frac{\Delta}{M_T} \end{pmatrix}, \tag{14}$$

with $\Delta = M_{\mathcal{S}B}^2 - M_{\mathcal{S}} M_B$.

The determinant of $\mathcal{M}_D$ vanishes due to the $SU(5)$ relation $Y_i^T = \frac{\sqrt{30}}{3}Y_i^B$. This propagates to $\mathcal{M}_\nu$, resulting in one massless left-handed neutrino. Such a feature is shared by a wider class of models in which two right-handed neutrinos or more belong to the same GUT representation.

The resulting light neutrino mass matrix can be put in the form

$$(\mathcal{M}_\nu)_{ij} = \frac{v_u^2}{\Delta}(AY_iY_j + B(Y_iY_j' + Y_i'Y_j) + CY_i'Y_j'), \tag{15}$$

where

$$A = M_B, \qquad B = -\frac{3}{\sqrt{30}}M_{\mathcal{S}B}, \qquad C = \frac{3}{10}M_{\mathcal{S}} - \frac{\Delta}{2M_T}. \tag{16}$$

We have simplified the notation by denoting $Y_i^{\mathcal{S}} = Y_i$ and $Y_i^T = Y_i'$.

[5] For simplicity, in our treatment of masses and mixings we neglect CP violation.

By going to the orthogonal basis in flavor space,

$$\hat{X}^{(1)} = \frac{\vec{Y}' \times \vec{Y}}{|\vec{Y}' \times \vec{Y}|}, \qquad \hat{X}^{(2)} = \frac{\vec{Y}}{\sqrt{Y^2}}, \qquad \hat{X}^{(3)} = \hat{X}^{(1)} \times \hat{X}^{(2)}, \tag{17}$$

where $\hat{X}^{(1)}$ is the massless eigenvector, we can set the neutrino matrix in the form

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_{22} & M_{23} \\ 0 & M_{23} & M_{33} \end{pmatrix}, \tag{18}$$

with

$$\begin{aligned} M_{22} &= \frac{v_u^2}{\Delta}\left(M_B Y^2 - \frac{6}{\sqrt{30}} M_{\mathcal{S}B}(Y \cdot Y') + \frac{3}{10} M_{\mathcal{S}} \frac{(Y \cdot Y')^2}{Y^2}\right) \\ &\quad - \frac{v_u^2}{2M_T}\frac{(Y \cdot Y')^2}{Y^2}, \\ M_{23} &= \sqrt{Y^2 Y'^2 - (Y \cdot Y')^2}\left\{-\frac{v_u^2}{\Delta}\left(-\frac{3}{\sqrt{30}} M_{\mathcal{S}B} \right.\right. \\ &\quad \left.\left. + \frac{3}{10} M_{\mathcal{S}} \frac{(Y \cdot Y')}{Y^2}\right) + \frac{v_u^2}{2M_T}\frac{(Y \cdot Y')}{Y^2}\right\}, \\ M_{33} &= \frac{1}{Y^2}(Y'^2 Y^2 - (Y \cdot Y')^2)\left\{-\frac{v_u^2}{2M_T} + \frac{3 v_u^2}{10}\frac{M_{\mathcal{S}}}{\Delta}\right\}. \end{aligned} \tag{19}$$

Before we extract the light neutrino eigenvalues from this matrix, we must consider the scales involved in these expressions. For the mass scale M_B we have already made the choice $M_B = M_G$. The other two scales $M_{\mathcal{S}}$, $M_{\mathcal{S}B}$, associated with the singlet $\mathcal{S}$, are not constrained.

First approach.—We shall assume that these two scales are also of the order of M_G. Thus, the dominant entry in the neutrino matrix elements M_{ab} will be the term $-\frac{v_u^2}{M_T}$ contained in C of (16), while the rest of the contributions will all be of the order of $\frac{v_u^2}{M_G}$, which is 3 orders of magnitude smaller. We may write[6]

$$\begin{aligned} M_{22} &= \frac{v_u^2}{\sqrt{|\Delta|}}\hat{M}_{22} - \frac{v_u^2}{2M_T}\frac{(Y \cdot Y')^2}{Y^2}, \\ M_{23} &= \frac{v_u^2}{\sqrt{|\Delta|}}\hat{M}_{23} + \frac{v_u^2}{2M_T}\frac{(Y \cdot Y')}{Y^2}\sqrt{Y^2 Y'^2 - (Y \cdot Y')^2}, \\ M_{33} &= \frac{v_u^2}{\sqrt{|\Delta|}}\hat{M}_{33} - \frac{v_u^2}{2M_T}\frac{1}{Y^2}(Y'^2 Y^2 - (Y \cdot Y')^2). \end{aligned} \tag{20}$$

The resulting light neutrino mass eigenvalues are

$$\begin{aligned} m_\nu^{(3)} &\approx -\frac{v_u^2}{2M_T} Y'^2, \\ m_\nu^{(2)} &\approx \frac{v_u^2}{\sqrt{|\Delta|}}\left\{\hat{M}_{22}\left(1 - \frac{(Y \cdot Y')^2}{Y^2 Y'^2}\right) + \hat{M}_{33}\frac{(Y \cdot Y')^2}{Y^2 Y'^2} \right. \\ &\quad \left. + 2\hat{M}_{23}\frac{(Y \cdot Y')}{Y^2 Y'^2}\sqrt{Y^2 Y'^2 - (Y \cdot Y')^2}\right\}. \end{aligned} \tag{21}$$

As it stands, for $|Y| \sim |Y'|$, the mass hierarchy is $m_\nu^{(2)}/m_\nu^{(3)} \sim \frac{v^2}{M_G} O(Y^2) / \frac{v^2}{M_T} O(Y^2) \sim M_T/M_G \sim \epsilon$, which is too strong a hierarchy to satisfy the data, without any other adjustment of parameters. On the other hand, if the overall scale of the determinant $\sqrt{|\Delta|} = \sqrt{|M_{\mathcal{S}B}^2 - M_{\mathcal{S}} M_B|}$ is set to be $\sqrt{|\Delta|} \sim \lambda M_G$, with $\lambda \sim \mathcal{O}(10^{-1})$, the relation $v^2/M_T \gg v^2 \hat{M}_{ab}/\sqrt{|\Delta|}$ still holds and, thus, we obtain

$$m_\nu^{(2)} \sim \frac{v_u^2}{\lambda^2 M_G} O(Y^2), \qquad m_\nu^{(3)} \sim \frac{v_u^2}{M_T} O(Y^2). \tag{22}$$

This can give the correct overall scale of the neutrino masses and a suitable hierarchy

$$\frac{m_\nu^{(2)}}{m_\nu^{(3)}} \sim \frac{\epsilon}{\lambda^2}. \tag{23}$$

Second approach.—An alternative is to assume that the scales associated with the singlet $\mathcal{S}$ are of the same intermediate order as M_T, namely,

$$M_{\mathcal{S}} \sim M_{\mathcal{S}B} \sim M_T \tag{24}$$

and, thus, $\Delta \approx -M_{\mathcal{S}} M_B$. Despite naturalness objections, this assumption is technically feasible. In this case, we have to leading order

$$m_\nu^{(2,3)} \approx -\frac{v_u^2}{4M_T}\{(Y')^2 + \lambda' Y^2 \pm \sqrt{\mathcal{R}}\}, \tag{25}$$

where

$$\mathcal{R} \equiv (\lambda' Y^2 - Y'^2)^2 + 4\lambda'(Y \cdot Y')^2 \tag{26}$$

and $\lambda' \equiv 2M_T/M_{\mathcal{S}}$, a number of $O(1)$ by assumption. Note that a hierarchy can also arise in this approach in the case $Y^2 \gg Y'^2$, namely,

$$\frac{m_\nu^{(2)}}{m_\nu^{(3)}} \approx \frac{(Y^2 Y'^2 - (Y \cdot Y')^2)}{\lambda' Y^4} = \frac{Y'^2 \sin^2\alpha}{\lambda' Y^2}. \tag{27}$$

[6] We have set

$$\begin{aligned} \hat{M}_{22} &= \frac{1}{\sqrt{|\Delta|}}\left(M_B Y^2 - \frac{6M_{\mathcal{S}B}}{\sqrt{30}}(Y \cdot Y') + \frac{3M_{\mathcal{S}}}{10}\frac{(Y \cdot Y')^2}{Y^2}\right), \\ \hat{M}_{23} &= \frac{1}{\sqrt{|\Delta|}}\sqrt{Y^2 Y'^2 - (Y \cdot Y')^2}\left(\frac{3M_{\mathcal{S}B}}{\sqrt{30}} - \frac{3M_{\mathcal{S}}}{10}\frac{(Y \cdot Y')}{Y^2}\right), \\ \hat{M}_{33} &= \frac{3M_{\mathcal{S}}}{10\sqrt{|\Delta|}}\left(Y'^2 - \frac{(Y \cdot Y')^2}{Y^2}\right). \end{aligned}$$

We have denoted by α the angle $\cos^{-1}(\hat{Y}\cdot\hat{Y}')$. Similar results can also be obtained for $Y'^2 \gg Y^2$ but with

$$\frac{m_\nu^{(2)}}{m_\nu^{(3)}} \approx \frac{\lambda' Y^2 \sin^2\alpha}{Y'^2}. \tag{28}$$

In this approach there is also another possibility for the existence of a mass hierarchy, namely, the possibility of almost parallel couplings in generation space ($\alpha \approx 0$),

$$Y\cdot Y' = \sqrt{Y^2}\sqrt{Y'^2}\cos\alpha \approx \sqrt{Y^2}\sqrt{Y'^2}\left(1-\frac{\alpha^2}{2}\right).$$

In this case, keeping $Y \sim Y'$, we obtain

$$\frac{m_\nu^{(2)}}{m_\nu^{(3)}} \approx \frac{\lambda' Y^2 Y'^2 \alpha^2}{(Y'^2+\lambda' Y^2)^2}. \tag{29}$$

Finally, in this approach, there is a third possibility for a hierarchy if we assume that there is a small hierarchy in the scales M_S:M_T corresponding to $\lambda' \sim 0.1$. In this case we get the same expression for the mass ratio as in (28) but with the desired hierarchy now originating from λ' instead of Y^2/Y'^2.

The above conclusions rely only on the renormalizable part of the superpotential. There are, however, some contributions to neutrino masses from various lowest order nonrenormalizable terms in (8). These include left-handed neutrino Majorana masses from the term

$$\mathcal{H}^c\mathcal{Q}\mathcal{Q}\mathcal{H}^c \sim \lambda_{ij}\frac{v_u^2}{M_P}\nu_i\nu_j. \tag{30}$$

These masses are tiny (10^{-5} eV or less, depending on the couplings involved) but they remove the massless state arising from the previous analysis, giving a lower bound for light neutrino masses.

Right-handed neutrino Majorana masses come from the terms

$$\mathcal{T}^2\Sigma^2 + \Sigma^2\mathcal{T}\mathcal{S} + \mathcal{S}^2\Sigma^2 \sim \lambda'_{ij}\frac{V^2}{M_P}N_j^cN_j^c. \tag{31}$$

These terms could very well be of the same order of magnitude as the intermediate scale M_T or even larger but become subdominant for relatively small couplings, meaning $\lambda' < 10^{-2}$. In addition to these terms, negligible right-handed Majorana mass contributions $O(v^2/M_P)$ arise from the terms $\mathcal{H}(\mathcal{T}^2,\mathcal{T}\mathcal{S},\mathcal{S}^2)\mathcal{H}^c$.

Dirac neutrino masses come from the terms

$$\mathcal{Q}\mathcal{T}\Sigma\mathcal{H}^c + \mathcal{Q}\Sigma\mathcal{H}^c\mathcal{S} \sim \lambda'_{ij}\frac{v_uV}{M_P}\nu_iN_j^c. \tag{32}$$

These contributions, suppressed by the factor V/M_P in comparison with renormalizable contributions, can remove massless states that arise due to the symmetries encountered in the renormalizable part of the Dirac neutrino mass matrix $\mathcal{M}_D$. To be specific, the operator $\mathcal{Q}\mathcal{T}\Sigma\mathcal{H}^c$ representing the invariants $\mathcal{Q}_i\mathcal{H}^cTr(\mathcal{T}\Sigma)$, $\mathcal{Q}_i\mathcal{T}\Sigma\mathcal{H}^c$, $\mathcal{Q}_i\Sigma\mathcal{T}\mathcal{H}^c$ contributes to the superpotential as

$$\lambda''_{1i}\frac{v_uV}{M_P}\nu_iB + (\lambda''_{2i}+\lambda''_{3i})\frac{v_uV}{M_P}\left(\frac{3}{10}\nu_iB - \sqrt{\frac{3}{20}}\nu_i\tau_0\right). \tag{33}$$

The presence of these terms modifies the structure of $\mathcal{M}_D$ and removes the massless state. The resulting, from the seesaw mechanism, light neutrino mass will be suppressed at least by a factor of $(\lambda''V/M_P)^2 < 10^{-2}$ compared to the lightest massive neutrino.

V. NEUTRINO MIXING

The charged lepton and neutrino mass terms $M_{(\ell)}\ell\ell^c + \frac{1}{2}M_{(\nu)}\nu\nu$ can be diagonalized in terms of three unitary matrices $\mathbf{U}_{(\ell)}$, $\mathbf{V}_{(\ell^c)}$, and $\mathbf{U}_{(\nu)}$. These matrices rotate the above gauge eigenstates into mass eigenstates. If we express the neutrino charge current $J_\mu \propto \ell^\dagger\sigma_\mu\nu$ in terms of mass eigenstates, a combination of two of these matrices will appear, $\ell^{\dagger\prime}\sigma_\mu\mathcal{U}_{\rm PMNS}\nu'$, known as the Pontecorvo-Maki-Nakagawa-Sakata [13] matrix,

$$\mathcal{U}_{\rm PMNS} \equiv \mathbf{U}_{(\ell)}^\dagger\mathbf{U}_{(\nu)}. \tag{34}$$

In what follows we shall concentrate on $\mathbf{U}_{(\nu)}$ and put aside the charged lepton mixing matrix, for which, in any case very little is known.

The overall neutrino mixing matrix

$$\mathbf{U}_{(\nu)} = \mathbf{U}_1\mathbf{U}_2, \qquad ((\mathbf{U}_1)_{ij} = \hat{X}_j^{(i)}) \tag{35}$$

is composed of the unitary matrix $\mathbf{U}_1$ that rotates the neutrino mass matrix (15) into (18) and a unitary matrix

$$\mathbf{U}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{pmatrix} \tag{36}$$

that diagonalizes (18). The rotation angle β is related to the matrix entries through

$$\beta = \frac{1}{2}\cot^{-1}\left(\frac{M_{22}-M_{33}}{2M_{23}}\right). \tag{37}$$

Note that the mass eigenvalues are just

$$m_\nu^{(2,3)} = \frac{1}{2}\left(M_{22}+M_{33} \pm \sqrt{(M_{22}-M_{33})^2+4M_{23}^2}\right).$$

The overall diagonalizing matrix is

$$\mathbf{U}_\nu = \mathbf{U}_1\mathbf{U}_2 = \begin{pmatrix} \hat{X}_1^1 & \cos\beta\hat{X}_2^1+\sin\beta\hat{X}_3^1 & -\sin\beta\hat{X}_2^1+\cos\beta\hat{X}_3^1 \\ \hat{X}_1^2 & \cos\beta\hat{X}_2^2+\sin\beta\hat{X}_3^2 & -\sin\beta\hat{X}_2^2+\cos\beta\hat{X}_3^2 \\ \hat{X}_1^3 & \cos\beta\hat{X}_2^3+\sin\beta\hat{X}_3^3 & -\sin\beta\hat{X}_2^3+\cos\beta\hat{X}_3^3 \end{pmatrix}. \tag{38}$$

In order to obtain the corresponding relations between the $\hat{X}^{(a)}$ and the original Yukawa couplings Y_i and Y'_i, we note that, as a result of the definitions (17), we may write

$$\vec{Y}\,' = Y'(\cos\alpha\hat{X}_2 - \sin\alpha\hat{X}_3),$$

where $\alpha \equiv \cos^{-1}(\hat{Y}\cdot\hat{Y}')$. Substituting, we obtain

$$\mathbf{U}_{\nu} = (\sin\alpha)^{-1}\begin{pmatrix} \hat{Y}_3\hat{Y}'_2 - \hat{Y}_2\hat{Y}'_3 & \sin(\alpha+\beta)\hat{Y}_1 - \sin\beta\hat{Y}'_1 & \cos(\alpha+\beta)\hat{Y}_1 - \cos\beta\hat{Y}'_1 \\ \hat{Y}'_3\hat{Y}_1 - \hat{Y}'_1\hat{Y}_3 & \sin(\alpha+\beta)\hat{Y}_2 - \sin\beta\hat{Y}'_2 & \cos(\alpha+\beta)\hat{Y}_2 - \cos\beta\hat{Y}'_2 \\ \hat{Y}_2\hat{Y}'_1 - \hat{Y}_1\hat{Y}'_2 & \sin(\alpha+\beta)\hat{Y}_3 - \sin\beta\hat{Y}'_3 & \cos(\alpha+\beta)\hat{Y}_3 - \cos\beta\hat{Y}'_3 \end{pmatrix}. \tag{39}$$

Equating this matrix with the standard parametrization, we obtain the relations between the standard mixing angles θ_{23}, θ_{12}, θ_{13} and the above parameters. It is clear that, as long as we have not imposed any additional constraints on the Yukawa coupling directions in family space, we have no predictive restrictions on the mixing angles. In the particular case that we are close to bimaximal mixing,

$$\theta_{23} \approx \frac{\pi}{4} + \epsilon_{23}, \qquad \theta_{12} \approx \frac{\pi}{4} + \epsilon_{12}, \qquad \theta_{13} \approx \epsilon_{13},$$

from the standard parametrization, we obtain

$$\mathbf{U}_{(\nu)} \approx \begin{pmatrix} \frac{1}{\sqrt{2}} - \frac{\epsilon_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} + \frac{\epsilon_{12}}{\sqrt{2}} & \epsilon_{13} \\ -\frac{1}{2} - \frac{\epsilon_{12}}{2} + \frac{\epsilon_{23}}{2} - \frac{\epsilon_{13}}{2} & \frac{1}{2} - \frac{\epsilon_{12}}{2} - \frac{\epsilon_{23}}{2} - \frac{\epsilon_{13}}{2} & \frac{1}{\sqrt{2}} + \frac{\epsilon_{23}}{\sqrt{2}} \\ \frac{1}{2} + \frac{\epsilon_{12}}{2} + \frac{\epsilon_{23}}{2} - \frac{\epsilon_{13}}{2} & -\frac{1}{2} + \frac{\epsilon_{12}}{2} - \frac{\epsilon_{13}}{2} - \frac{\epsilon_{13}}{2} & \frac{1}{\sqrt{2}} - \frac{\epsilon_{23}}{\sqrt{2}} \end{pmatrix}. \tag{40}$$

Equating this expression to (39), we obtain

$$\hat{Y} = \begin{bmatrix} \frac{\cos\beta}{\sqrt{2}} \\ \frac{\cos\beta}{2} - \frac{\sin\beta}{\sqrt{2}} \\ -\frac{\cos\beta}{2} - \frac{\sin\beta}{\sqrt{2}} \end{bmatrix} + \begin{bmatrix} \epsilon_{12}\frac{\cos\beta}{\sqrt{2}} - \epsilon_{13}\sin\beta \\ -(\epsilon_{12} + \epsilon_{23} + \epsilon_{13})\frac{\cos\beta}{2} - \epsilon_{23}\frac{\sin\beta}{\sqrt{2}} \\ -(-\epsilon_{12} + \epsilon_{23} + \epsilon_{13})\frac{\cos\beta}{2} + \epsilon_{23}\frac{\sin\beta}{\sqrt{2}} \end{bmatrix} \tag{41}$$

and

$$\hat{Y}\,' = \begin{bmatrix} \frac{\cos(\alpha+\beta)}{\sqrt{2}} \\ \frac{\cos(\alpha+\beta)}{2} - \frac{\sin(\alpha+\beta)}{\sqrt{2}} \\ -\frac{\cos(\alpha+\beta)}{2} - \frac{\sin(\alpha+\beta)}{\sqrt{2}} \end{bmatrix} + \begin{bmatrix} \epsilon_{12}\frac{\cos(\alpha+\beta)}{\sqrt{2}} - \epsilon_{13}\sin(\alpha+\beta) \\ -(\epsilon_{12} + \epsilon_{23} + \epsilon_{13})\frac{\cos(\alpha+\beta)}{2} - \epsilon_{23}\frac{\sin(\alpha+\beta)}{\sqrt{2}} \\ -(-\epsilon_{12} + \epsilon_{23} + \epsilon_{13})\frac{\cos(\alpha+\beta)}{2} + \epsilon_{23}\frac{\sin(\alpha+\beta)}{\sqrt{2}} \end{bmatrix}. \tag{42}$$

Closing this chapter we note that the range of values for variables α, β, $|Y|$, $|Y'|$, which determine the Yukawa couplings, depends on the mass-hierarchy approach followed. Among the different options, the small angle scenario of the second approach exhibits the most restrictive structure with $\beta \sim \alpha$, while by assumption $|Y| \sim |Y'|$.

VI. CONCLUSIONS

In the present article we studied a realization of the seesaw mechanism in the framework of an extended renormalizable version of the supersymmetric $SU(5)$ model. The right-handed neutrino fields were introduced as members of chiral $\mathbf{24} + \mathbf{1}$ superfields. In particular, two $\mathbf{24}$ superfields were introduced, out of which, due to different discrete symmetry charges, only one couples to matter and its neutral singlet and isotriplet components are identified as two of the right-handed neutrinos. Our basic assumption is that right-handed neutrinos survive below the grand unification scale, having an intermediate mass in the neighborhood of 10^{13}–10^{14} GeV, a scale suitable to generate, through the seesaw mechanism, a light neutrino mass of the observed mass value of $O(0.1\text{ GeV})$. The assumption of an isotriplet of an intermediate mass scale is supported by renormalization group analysis incorporating proton stability constraints. In addition, the model requires a color octet of neighboring mass, which, however, does not couple to ordinary matter. The right-handed neutrino mass matrix, then, depends on the constrained isotriplet scale M_T as well as the free, from the renormalization group, scales M_B, M_S, and M_{SB} associated with the SM singlets of $\underline{\mathbf{1}}$, $\underline{\mathbf{24}}$. If these scales are of $O(M_G)$, an extra fine-tuning is required in order to obtain a light neutrino mass hierarchy in agreement with data (first approach). The alternative assumption according to which the scales M_S, M_{SB} are of $O(M_T)$ is also possible (second approach). In this approach a phenomenologically acceptable neutrino mass hierarchy is possible as a result of the Yukawa hierarchy $Y' \ll Y$ or $Y' \gg Y$, where Y and Y' are the overall scales of the neutrino couplings $\langle H_u\rangle\nu(Y\mathbf{1} + Y'\mathbf{24})$. A second possibility of a hierarchy within this approach arises also when the angle between the Yukawa coupling vectors in family space Y_i and Y'_i is small. Nevertheless, the limiting case of aligned Yukawas is excluded, since it corresponds to two massless neutrinos. Alternatively, the required neutrino mass hierarchy can also arise as a result of a slight hierarchy of the scales M_S:M_T. However, in all these approaches, one very light neutrino is always present as a result of the structure of the neutrino mass matrix. Finally, we also find that a hierarchical mixing angle structure $\theta_{23} \sim \theta_{12} \gg \theta_{13}$ can be easily accommodated within the free parameter structure of the model.

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