Low energy signatures of the TeV scale seesaw mechanism

A. Ibarra,[1] E. Molinaro,[2] and S. T. Petcov[3,4,5]

[1]*Physik-Department T30d, Technische Universität München, James-Franck-Straße, 85748 Garching, Germany*
[2]*Centro de Física Teórica de Partículas (CFTP), Instituto Superior Técnico, Avenida Rovisco Pais 1, 1049-001, Lisboa, Portugal*
[3]*SISSA and INFN-Sezione di Trieste, Via Bonomea 265, 34136 Trieste, Italy*
[4]*IPMU, University of Tokyo, Tokyo, Japan*
[5]*Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 1784 Sofia, Bulgaria*
(Received 4 May 2011; published 7 July 2011)

We study a type I seesaw scenario where the right-handed (RH) neutrinos, responsible for the light neutrino mass generation, lie at the electroweak scale. Under certain conditions, the strength of the charged current (CC) and neutral current (NC) weak interactions of the standard model particles with the heavy RH neutrinos can be large enough to allow the production of the latter at the LHC, opening also the possibility of observing other low energy signatures of the new physics in the electroweak precision observables as well as in searches for rare leptonic decays or neutrinoless double beta decay. In this scenario the flavor structure of the indicated CC and NC couplings of the heavy RH neutrinos is essentially determined by the low energy neutrino parameters, leading to fairly strong correlations among the new phenomena. In particular, we show that the present bound on the $\mu \to e + \gamma$ decay rate makes very difficult the observation of the heavy RH neutrinos at the LHC or the observation of deviations from the standard model predictions in the electroweak precision data. We also show that all present experimental constraints on this scenario still allow (i) for an enhancement of the rate of neutrinoless double beta decay, which thus can be in the range of sensitivity of the GERDA experiment even when the light Majorana neutrinos possess a normal hierarchical mass spectrum, and (ii) for the predicted $\mu \to e + \gamma$ decay rate to be within the sensitivity range of the MEG experiment.

DOI: 10.1103/PhysRevD.84.013005 PACS numbers: 14.60.Pq

I. INTRODUCTION

One of the best motivated extensions of the standard model consists of introducing extra fermions that are singlets under the standard model gauge group, which we will call for definiteness right-handed (RH) neutrinos. With this particle content, the Lagrangian contains a Yukawa coupling of the RH neutrinos with the left-handed leptonic doublets and the Higgs doublet, which leads to a Dirac neutrino mass when the electroweak symmetry is spontaneously broken. Besides, it contains a Majorana mass term for the RH neutrinos, which is *a priori* unrelated to the electroweak symmetry breaking scale. The most popular scenario in this model consists of assuming that the RH neutrino Majorana mass scale is much larger than the electroweak symmetry breaking scale, thus naturally leading to tiny Majorana neutrino masses through the renowned seesaw mechanism [1]. Furthermore, any other low energy effect of the RH neutrinos decouples at least with their mass squared, resulting in a tiny rate for the rare leptonic decays [2], a tiny leptonic electric dipole moment [3], and a tiny deviation of the electroweak observables from the standard model predictions [4–6], in excellent agreement with presently existing experimental data.

It should be borne in mind, however, that the mass of the RH neutrinos is a free parameter which can take any value between zero and the Planck scale. An interesting possibility arises when the RH neutrinos have masses in the range $\mathcal{O}(100\text{–}1000)$ GeV. If this is the case, the new particles could be produced and detected at the Large Hadron Collider (LHC), if their couplings to the standard model particles are sizable [7,8]. This situation, at first sight bizarre in view of the tininess of the neutrino masses, can be realized in some well-motivated scenarios, namely, when lepton number is approximately conserved (see, e.g., [9–11]). More importantly, the contributions from the new particles to the low energy phenomena are no longer suppressed, opening the possibility of finding additional low energy signatures of the new physics with experiments at the intensity frontier. Furthermore, the existence of RH neutrinos with masses in the range of (100–1000) GeV with lepton number violating interactions can dramatically enhance the rate of neutrinoless double beta decay [12,13], inducing rates that could possibly be observable at GERDA [14], even when the light neutrinos present a normal hierarchical mass spectrum.

In this paper we analyze the constraints from various experiments on the scenario where the RH neutrinos are accessible to collider searches, as well as the interrelation between the different constraints. More specifically, we will derive the constraints that follow from the present bounds on the rate of the process $\mu \to e + \gamma$, with a special emphasis on the relation to the neutrino mixing and oscillation parameters. We will also discuss the prospects to observe neutrinoless double beta decay in the next round of experiments, in view of all present experimental constraints. In Sec. II we discuss the formalism and define the parameter space of the theory. In Secs. III and IV we discuss the constraints on the parameter space which arise from radiative charged lepton decays and the implications for collider searches and electroweak precision observables. In the subsequent section we perform a detailed analysis of the possible enhancement of the neutrinoless double beta decay rate due to the exchange of the heavy (RH) Majorana neutrinos. In Sec. VI we combine the constraints on the parameter space that we obtained in Secs. III, IV, and V. Finally, we report our conclusions.

II. PRELIMINARY REMARKS

We denote the light and heavy Majorana neutrinos with definite masses as χ_i and N_k, respectively.[1] The charged and neutral current weak interactions involving the light Majorana neutrinos have the form

$$\mathcal{L}_{\rm CC}^{\nu} = -\frac{g}{\sqrt{2}}\bar{\ell}\gamma_\alpha \nu_{\ell L} W^\alpha + {\rm H.c.} = -\frac{g}{\sqrt{2}}\bar{\ell}\gamma_\alpha((1+\eta)U)_{\ell i}\chi_{iL} W^\alpha + {\rm H.c.}, \quad (2.1)$$

$$\mathcal{L}_{\rm NC}^{\nu} = -\frac{g}{2c_w}\bar{\nu}_{\ell L}\gamma_\alpha \nu_{\ell L} Z^\alpha = -\frac{g}{2c_w}\bar{\chi}_{iL}\gamma_\alpha (U^\dagger(1+\eta+\eta^\dagger)U)_{ij}\chi_{jL} Z^\alpha, \quad (2.2)$$

where $(1+\eta)U = U_{\rm PMNS}$ is the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) neutrino mixing matrix [16–18], U is a 3×3 unitary matrix which diagonalizes the Majorana mass matrix of the left-handed (LH) flavor neutrinos $\nu_{\ell L}$ (generated by the seesaw mechanism), and the matrix η characterizes the deviations from unitarity of the PMNS matrix. The elements of $U_{\rm PMNS}$ are determined in experiments studying the oscillations of the flavor neutrinos and antineutrinos, ν_ℓ and $\bar{\nu}_\ell$, $\ell = e, \mu, \tau$, at relatively low energies. In these experiments the initial states of the flavor neutrinos, produced in some weak process, are coherent superpositions of the states of the light massive Majorana neutrino χ_i only. The states of the heavy Majorana neutrino N_j are not present in the superpositions representing the initial flavor neutrino states and this leads to deviations from unitarity of the PMNS matrix.

The charged current (CC) and the neutral current (NC) weak interaction couplings of the heavy Majorana neutrinos N_j to the $W^\pm$ and Z^0 bosons read

$$\mathcal{L}_{\rm CC}^{N} = -\frac{g}{2\sqrt{2}}\bar{\ell}\gamma_\alpha ({\rm RV})_{\ell k}(1-\gamma_5)N_k W^\alpha + {\rm H.c.}, \quad (2.3)$$

$$\mathcal{L}_{\rm NC}^{N} = -\frac{g}{4c_w}\bar{\nu}_{\ell L}\gamma_\alpha ({\rm RV})_{\ell k}(1-\gamma_5)N_k Z^\alpha + {\rm H.c.} \quad (2.4)$$

Here V is the unitary matrix which diagonalizes the Majorana mass matrix of the heavy RH neutrinos and the matrix R is determined by (see [15]) $R^* \cong M_D M_N^{-1}$, M_D and M_N being the neutrino Dirac and the RH neutrino Majorana mass matrices, respectively, $|M_D| \ll |M_N|$. The matrix η which parametrizes the deviations from unitarity of the neutrino mixing matrix, can be expressed in terms of the matrix R,

$$\eta \equiv -\frac{1}{2}RR^\dagger = -\frac{1}{2}({\rm RV})({\rm RV})^\dagger = \eta^\dagger. \quad (2.5)$$

It is possible to constrain the elements of the Hermitian matrix η by using the existing neutrino oscillation data and data on electroweak (EW) processes [5,6] (e.g., on $W^\pm$ decays, invisible Z decays, universality tests of EW interactions). For $M_k \gtrsim 100$ GeV these constraints read [5,6]

$$|\eta| < \begin{pmatrix} 2.0\times 10^{-3} & 0.6\times 10^{-4} & 1.6\times 10^{-3} \\ 0.6\times 10^{-4} & 0.8\times 10^{-3} & 1.0\times 10^{-3} \\ 1.6\times 10^{-3} & 1.0\times 10^{-3} & 2.6\times 10^{-3}\end{pmatrix}. \quad (2.6)$$

The elements of the matrix RV and the masses M_k of the heavy Majorana neutrinos N_k should satisfy the approximate constraint on the elements of the Majorana mass matrix of the LH flavor neutrinos [19], $|(m_\nu)_{\ell'\ell}| \lesssim 1$ eV, $\ell, \ell' = e, \mu, \tau$. In the case of the type I seesaw mechanism under discussion this implies

$$\sum_k |({\rm RV})^*_{\ell' k} M_k ({\rm RV})^\dagger_{k\ell}| \lesssim 1\ {\rm eV}, \qquad \ell', \ell = e, \mu, \tau. \quad (2.7)$$

This relation can be satisfied in several situations. The most trivial way to satisfy it is by imposing that $|({\rm RV})_{\ell k}| \ll 1$ for all ℓ and k, which renders the observation of all RH neutrinos impossible at the LHC or in any low energy phenomena. However, this relation can also be fulfilled if one element of the matrix RV is sizable. This requires the existence of at least another large matrix element, in order to cancel the large contribution of the former one in Eq. (2.7). Whereas the possibility of cancellations between three different terms cannot be precluded, in the simplest case only two terms will cancel. Then, the large contribution to Eq. (2.7) from one of the RH neutrinos, say N_1 with mass M_1, is canceled by a negative contribution from another RH neutrino, say N_2 with mass M_2, provided

$$({\rm RV})_{\ell 2} = \pm i({\rm RV})_{\ell 1}\sqrt{\frac{M_1}{M_2}}, \quad (2.8)$$

which is naturally fulfilled if the RH neutrinos N_1 and N_2 form a pseudo-Dirac pair [20,21], e.g., if there exists an approximately conserved lepton charge (see, e.g., [15]). In this scenario, in order not to spoil the cancellation between these two terms, the contribution from the third neutrino to Eq. (2.7) should be negligible. Therefore in what follows we will work for simplicity in the 3×2 seesaw scenario, in which the indicated CC and NC weak interaction couplings of N_3 are set to zero and N_3 is decoupled. In this case the standard model is effectively extended by the addition of two RH neutrino fields only. In this class of models

[1] We use the same notations as in [15].

(see, e.g., [22–24]) one of the three light (Majorana) neutrinos is massless and hence the neutrino mass spectrum is hierarchical. Two possible types of hierarchical spectrum are allowed by the current neutrino data (see, e.g., [25]): (i) normal hierarchical (NH), $m_1 = 0$, $m_2 = \sqrt{\Delta m^2_\odot}$, and $m_3 = \sqrt{\Delta m^2_A}$, where $\Delta m^2_\odot \equiv m_2^2 - m_1^2 > 0$ and $\Delta m^2_A \equiv m_3^2 - m_1^2$; and (ii) inverted hierarchical (IH), $m_3 = 0$, $m_2 = \sqrt{|\Delta m^2_A|}$, and $m_1 = \sqrt{|\Delta m^2_A| - \Delta m^2_\odot} \cong \sqrt{|\Delta m^2_A|}$, where $\Delta m^2_\odot \equiv m_2^2 - m_1^2 > 0$ and $\Delta m^2_A = m_3^2 - m_2^2 < 0$. In both cases we have: $\Delta m^2_\odot/|\Delta m^2_A| \cong 0.03 \ll 1$.

The two heavy neutrino fields satisfy the Majorana condition: $C\bar{N}_k^T = N_k$, $k = 1, 2$. If the Majorana mass matrix M_N of the RH neutrinos is not CP invariant, one can always make the eigenvalues of M_N real and positive, $M_{1,2} > 0$. For M_N respecting the CP symmetry, the two real eigenvalues of M_N can have the same or opposite signs (see, e.g., [26]). Taking, e.g., $M_2 > 0$, without loss of generality, M_1 can be positive or negative: $M_1 > 0$ and $M_1 < 0$. One can show, however, that we get the same results for the observables of interest to this study in the two cases. Therefore in what follows we shall work with $M_{1,2} > 0$.

We assume that the heavy RH neutrinos $N_{1,2}$ have masses in the range $M_{1,2} = \mathcal{O}(100\text{–}1000)$ GeV, which makes possible, in principle, their production, e.g., at LHC. In order to be produced with observable rates at LHC, the CC and NC couplings of N_1 and N_2 in Eqs. (2.3) and (2.4) have to be sufficiently large. Under this condition the existing experimental upper bounds on the neutrinoless double beta ($(\beta\beta)_{0\nu}$) decay rate put stringent constraints on the mass spectrum of the RH neutrinos. It can be shown [15], in particular, that in the type I seesaw scenario of interest the two heavy (RH) Majorana neutrinos $N_{1,2}$ must be almost degenerate in mass: $M_2 \cong M_1$. If we assume that $M_2 > M_1 > 0$ and $M_2 \equiv (1 + z)M_1$, $z > 0$, in order to satisfy the experimental limit on the $(\beta\beta)_{0\nu}$-decay rate, one should have $z \lesssim 10^{-3}$ (10^{-2}) for $M_1 \approx 10^2 (10^3)$ GeV [15].

The charged current and the neutral current weak interaction couplings of the heavy Majorana neutrinos N_j, $(\text{RV})_{\ell k}$, are furthermore constrained by the requirement of reproducing the correct low energy neutrino oscillation parameters after the decoupling of the heavy degrees of freedom. Remarkably, under the condition that $|(\text{RV})_{\ell k}|$ are sufficiently large to produce observable effects of the RH neutrinos at low energies, these couplings take a very simple form [15].

Indeed, the Dirac mass matrix M_D in the case under study can be written as [27]

$$M_D = iU^*_{\text{PMNS}}\sqrt{\hat{m}}O\sqrt{\hat{M}}V^\dagger, \tag{2.9}$$

where $\hat{m} \equiv \text{diag}(m_1, m_2, m_3)$ and O is a complex orthogonal matrix. In the scheme with two heavy RH Majorana neutrinos the matrix O has the form [23]

$$O \equiv \begin{pmatrix} 0 & 0 \\ \cos\hat{\theta} & \pm\sin\hat{\theta} \\ -\sin\hat{\theta} & \pm\cos\hat{\theta} \end{pmatrix}, \quad \text{for NH mass spectrum}, \tag{2.10}$$

$$O \equiv \begin{pmatrix} \cos\hat{\theta} & \pm\sin\hat{\theta} \\ -\sin\hat{\theta} & \pm\cos\hat{\theta} \\ 0 & 0 \end{pmatrix}, \quad \text{for IH mass spectrum}, \tag{2.11}$$

where $\hat{\theta} = \omega - i\xi$. The RH neutrino mixing matrix entering into the CC and NC weak interaction Lagrangians (2.3) and (2.4) can be expressed as [15]

$$\text{RV} = -iU_{\text{PMNS}}\sqrt{\hat{m}}O^*\sqrt{\hat{M}^{-1}}. \tag{2.12}$$

The O-matrix in the case of, e.g., NH spectrum can be decomposed as follows:

$$O = \frac{e^{i\hat{\theta}}}{2}\begin{pmatrix} 0 & 0 \\ 1 & \mp i \\ i & \pm 1 \end{pmatrix} + \frac{e^{-i\hat{\theta}}}{2}\begin{pmatrix} 0 & 0 \\ 1 & \pm i \\ -i & \pm 1 \end{pmatrix} = O_+ + O_-. \tag{2.13}$$

One can get a similar expression for the IH spectrum. The Dirac neutrino mass matrix can be decomposed accordingly as $M_D = M_{D+} + M_{D-}$, in a self-explanatory notation. Taking for definiteness $\xi > 0$, it follows that M_{D+} (M_{D-}) grows (decreases) exponentially with ξ.[2] Therefore, for sufficiently large ξ it is possible to compensate the huge suppression in Eq. (2.9) from the tiny observed neutrino masses and the relatively light RH neutrino masses.[3]

We are interested in heavy Majorana neutrino couplings to charged leptons and gauge bosons, which are large enough to produce observable low energy signatures. In the limit of "large" ξ, the matrix O in Eq. (2.13) can be very well approximated by

$$O \approx \frac{e^{i\omega}e^{\xi}}{2}\begin{pmatrix} 0 & 0 \\ 1 & \mp i \\ i & \pm 1 \end{pmatrix}. \tag{2.14}$$

[2]Obviously, if $\xi < 0$, M_{D-} (M_{D+}) will grow (decrease) exponentially with ξ. It is possible to show that for sufficiently large values of $|\xi|$ of interest, the results for the different observables considered in our study do not depend on the choice of the sign of ξ.

[3]Note, however, that M_{D-} cannot be neglected in the calculation of the Majorana mass matrix of the LH flavor neutrinos even though it is exponentially suppressed compared to M_{D+}: the naive approximation $M_D \simeq M_{D+}$ leads to $m_\nu = 0$, due to $O_+O_+^T = 0$ (see [15] for details). Therefore, reproducing the correct light neutrino masses and mixing requires a large amount of fine-tuning, unless the RH neutrinos form a pseudo-Dirac pair. Demanding $(M_D)_{ij} \sim \mathcal{O}(1\text{ GeV})$ and random RH neutrino masses of $\mathcal{O}(100\text{ GeV})$, for instance, requires a tuning of one part in 10^9 in order to produce a neutrino mass $m_i \sim \mathcal{O}(10^{-2}\text{ eV})$.

For NH spectrum and large ξ, the matrix RV takes the form

$$\mathrm{RV} \approx -\frac{e^{-i\omega}e^{\xi}}{2}\sqrt{\frac{m_3}{|M_1|}}\begin{pmatrix} (U_{e3} + i\sqrt{m_2/m_3}U_{e2}) & \pm i(U_{e3} + i\sqrt{m_2/m_3}U_{e2})/\sqrt{1+z} \\ (U_{\mu 3} + i\sqrt{m_2/m_3}U_{\mu 2}) & \pm i(U_{\mu 3} + i\sqrt{m_2/m_3}U_{\mu 2})/\sqrt{1+z} \\ (U_{\tau 3} + i\sqrt{m_2/m_3}U_{\tau 2}) & \pm i(U_{\tau 3} + i\sqrt{m_2/m_3}U_{\tau 2})/\sqrt{1+z} \end{pmatrix}. \tag{2.15}$$

In a different context similar expressions for the corresponding neutrino Yukawa couplings were derived in [28] in a scheme in which a successful leptogenesis can take place at $T \sim 10^7$ GeV, and in [11] where a TeV scale seesaw model with approximately conserved lepton charge was proposed. In the case of IH light neutrino mass spectrum, the matrix RV is obtained by replacing $m_{2,3} \to m_{1,2}$ and $U_{\alpha 2,\alpha 3} \to U_{\alpha 1,\alpha 2}$ ($\alpha = e, \mu, \tau$) in Eq. (2.15). For both types of neutrino mass spectrum, the elements of the two columns of RV in the limit of large ξ are related by the condition

$$(\mathrm{RV})_{\alpha 2} = \pm i(\mathrm{RV})_{\alpha 1}/\sqrt{1+z} \quad \text{for } \alpha = e, \mu, \tau, \tag{2.16}$$

thus recovering Eq. (2.8).

If M_N is CP conserving and $M_1 < 0$, $M_2 > 0$, the expressions for $(\mathrm{RV})_{\alpha 1}$ in Eq. (2.15) have an additional factor $-i$ and instead of Eq. (2.16) we have: $(\mathrm{RV})_{\alpha 2} = \mp(\mathrm{RV})_{\alpha 1}/\sqrt{1+z}$. This relation and the relation given in Eq. (2.16) lead to the same expressions for the observables discussed further in the present study.

The overall size of the couplings Eq. (2.15) depends crucially on the value of the parameter ξ, which has no direct physical interpretation. It proves convenient to express ξ in terms of the largest eigenvalue y of the matrix of neutrino Yukawa couplings using the relation

$$y^2 v^2 \equiv \max\{\mathrm{eig}(M_D M_D^\dagger)\} = \max\{\mathrm{eig}(\sqrt{\hat{m}} O \hat{M} O^\dagger \sqrt{m})\} = \frac{1}{4}e^{2\xi}M_1(m_2 + m_3)(2+z), \tag{2.17}$$

with $v = 174$ GeV. In terms of y and for $z \ll 1$, the heavy Majorana neutrino couplings become

$$|(\mathrm{RV})_{\alpha 1}|^2 = \frac{1}{2}\frac{y^2 v^2}{M_1^2}\frac{m_3}{m_2 + m_3}|U_{\alpha 3} + i\sqrt{m_2/m_3}U_{\alpha 2}|^2, \quad \text{NH}, \tag{2.18}$$

$$|(\mathrm{RV})_{\alpha 1}|^2 = \frac{1}{2}\frac{y^2 v^2}{M_1^2}\frac{m_2}{m_1 + m_2}|U_{\alpha 2} + i\sqrt{m_1/m_2}U_{\alpha 1}|^2 \cong \frac{1}{4}\frac{y^2 v^2}{M_1^2}|U_{\alpha 2} + iU_{\alpha 1}|^2, \quad \text{IH}, \tag{2.19}$$

where we have used the fact that for the IH spectrum one has $m_1 \cong m_2$.

From Eqs. (2.18) or (2.19), using the unitarity of the matrix U, one can express the neutrino Yukawa eigenvalue y in (2.17) in terms of heavy neutrino to charged leptons coupling constants,

$$y^2 v^2 = 2M_1^2(|(\mathrm{RV})_{e1}|^2 + |(\mathrm{RV})_{\mu 1}|^2 + |(\mathrm{RV})_{\tau 1}|^2). \tag{2.20}$$

An upper limit on the neutrino Yukawa coupling y can be derived by assuming the validity of perturbative unitarity. Indeed, the requirement of perturbative unitarity can be easily fulfilled if the seesaw parameter space satisfies the condition

$$\frac{\Gamma_{N_i}}{M_i} < \frac{1}{2}, \tag{2.21}$$

where Γ_{N_i} is the total decay rate of the heavy Majorana neutrino N_i, which in the limit $M_{N_i} \gg v$ is given by the following expression:

$$\Gamma_{N_i} = \frac{g^2}{16\pi M_W^2}M_i^3\sum_\ell |(\mathrm{RV})_{\ell i}|^2. \tag{2.22}$$

Therefore, taking into account Eqs. (2.20) and (2.22) and the condition (2.21), we get the following upper limit on the parameter y from perturbative unitarity:

$$y < 4. \tag{2.23}$$

III. THE $\mu \to e + \gamma$ DECAY

The $\mu \to e + \gamma$ decay branching ratio in the scenario under discussion is given by [29,30]

$$B(\mu \to e + \gamma) = \frac{\Gamma(\mu \to e + \gamma)}{\Gamma(\mu \to e + \nu_\mu + \bar{\nu}_e)} = \frac{3\alpha_{\mathrm{em}}}{32\pi}|T|^2, \tag{3.1}$$

where α_{em} is the fine structure constant and

$$T = \sum_{j=1}^{3}[(1+\eta)U]^*_{\mu j}[(1+\eta)U]_{ej}G\left(\frac{m_j^2}{M_W^2}\right) + \sum_{k=1}^{2}(\mathrm{RV})^*_{\mu k}(\mathrm{RV})_{ek}G\left(\frac{M_k^2}{M_W^2}\right). \tag{3.2}$$

The loop integration function $G(x)$ has the form

$$G(x) = \frac{10 - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3\log(x)}{3(x-1)^4}. \tag{3.3}$$

It is easy to verify that $G(x)$ is a monotonic function which takes values in the interval $[4/3, 10/3]$, with $G(x) \cong \frac{10}{3} - x$

for $x \ll 1$. From the definition of the matrix η, Eq. (2.5), one can write the amplitude T as follows:

$$T \cong [(\mathrm{RV})^*_{\mu 1}(\mathrm{RV})_{e1} + (\mathrm{RV})^*_{\mu 2}(\mathrm{RV})_{e2}][G(X) - G(0)], \quad (3.4)$$

where $X \equiv (M_1/M_W)^2$ and we have assumed that the difference between M_1 and M_2 is negligibly small and used $M_1 \cong M_2$. Using Eqs. (3.4) and (2.16) we get

$$|T| \cong \frac{2+z}{1+z}|(\mathrm{RV})^*_{\mu 1}(\mathrm{RV})_{e1}||G(X) - G(0)|. \quad (3.5)$$

Finally, using the expressions of $|(\mathrm{RV})_{\mu 1}|^2$ and $|(\mathrm{RV})_{e1}|^2$ in terms of neutrino parameters, Eqs. (2.18) and (2.19), we obtain the $\mu \to e + \gamma$ decay branching ratio for the NH and IH spectra

$$\text{NH: } B(\mu \to e + \gamma) \cong \frac{3\alpha_{\mathrm{em}}}{32\pi}\left(\frac{y^2 v^2}{M_1^2}\frac{m_3}{m_2+m_3}\right)^2 \left|U_{\mu 3} + i\sqrt{\frac{m_2}{m_3}}U_{\mu 2}\right|^2 \times \left|U_{e3} + i\sqrt{\frac{m_2}{m_3}}U_{e2}\right|^2 [G(X) - G(0)]^2, \quad (3.6)$$

$$\text{IH: } B(\mu \to e + \gamma) \cong \frac{3\alpha_{\mathrm{em}}}{32\pi}\left(\frac{y^2 v^2}{M_1^2}\frac{1}{2}\right)^2 |U_{\mu 2} + iU_{\mu 1}|^2 |U_{e2} + iU_{e1}|^2 \times [G(X) - G(0)]^2. \quad (3.7)$$

Employing the standard parametrization of the neutrino mixing matrix [25] it is not difficult to obtain expressions for the factors $|U_{\ell 3} + i\sqrt{m_2/m_3}U_{\ell 2}|^2$ and $|U_{\ell 2} + iU_{\ell 1}|^2$, $\ell = e, \mu$, in terms of the neutrino mixing parameters and the solar and atmospheric neutrino mass squared differences,

$$\left|U_{e3} + i\sqrt{\frac{m_2}{m_3}}U_{e2}\right|^2 = s_{13}^2 + \left(\frac{\Delta m^2_\odot}{\Delta m^2_A}\right)^{1/2} c_{13}^2 s_{12}^2 - 2\left(\frac{\Delta m^2_\odot}{\Delta m^2_A}\right)^{1/4} c_{13}s_{13}s_{12}\sin\left(\delta + \frac{\alpha_{21} - \alpha_{31}}{2}\right), \quad (3.8)$$

$$\left|U_{\mu 3} + i\sqrt{\frac{m_2}{m_3}}U_{\mu 2}\right|^2 = c_{13}^2 s_{23}^2 + \left(\frac{\Delta m^2_\odot}{\Delta m^2_{\mathrm{A}}}\right)^{1/2}(c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - 2c_{12}c_{23}s_{12}s_{23}s_{13}\cos\delta) + 2\left(\frac{\Delta m^2_\odot}{\Delta m^2_{\mathrm{A}}}\right)^{1/4} c_{13}s_{23}\left[c_{12}c_{23}\sin\left(\frac{\alpha_{31} - \alpha_{21}}{2}\right) - s_{12}s_{23}s_{13}\sin\left(\frac{\alpha_{31} - \alpha_{21}}{2} - \delta\right)\right], \quad (3.9)$$

$$|U_{e2} + iU_{e1}|^2 = c_{13}^2\left[1 + 2c_{12}s_{12}\sin\left(\frac{\alpha_{21}}{2}\right)\right], \quad (3.10)$$

$$|U_{\mu 2} + iU_{\mu 1}|^2 = c_{23}^2 + s_{13}^2 s_{23}^2 - 2c_{12}s_{12}(c_{23}^2 - s_{13}^2 s_{23}^2)\sin\frac{\alpha_{21}}{2} + 2c_{23}s_{13}s_{23}\left[s_{12}^2\sin\left(\frac{\alpha_{21}}{2} + \delta\right) - c_{12}^2\sin\left(\frac{\alpha_{21}}{2} - \delta\right)\right], \quad (3.11)$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, θ_{12}, θ_{23}, and θ_{13} are, respectively, the solar neutrino, atmospheric neutrino, and the CHOOZ angles, δ is the Dirac *CP* violating phase, and α_{21} and α_{31} are the two Majorana *CP* violating phases [31].

We show in Fig. 1 the branching ratio of $\mu \to e + \gamma$ as a function of the RH neutrino mass M_1, for three different values of the neutrino Yukawa coupling eigenvalue: $y = 0.001(0.01)[0.1]$, blue $\circ$ (green $+$) [red $\times$]. The figure was obtained by performing a scan of the values of the neutrino mixing angles θ_{12}, θ_{23}, and θ_{13} and the solar and atmospheric neutrino mass squared differences $\Delta m^2_\odot$ and Δm^2_{A} within the corresponding 3σ bounds (see Table I).[4] The Majorana phases α_{21} and $(\alpha_{31} - \alpha_{21})$ are varied in the interval[5] $[0, 4\pi]$ [33] and the Dirac phase δ is varied in the interval $[0, 2\pi]$. Shown are also the current experimental upper limit on the $\mu \to e + \gamma$ decay branching ratio [34], $B(\mu \to e + \gamma) < 1.2 \times 10^{-11}$, as well as the prospective limit of the MEG experiment [35], $B(\mu \to e + \gamma) < 10^{-13}$.

It is apparent from Fig. 1 that the data on the process $\mu \to e + \gamma$ set very stringent constraints on the TeV scale seesaw mechanism. A neutrino Yukawa coupling $y = 0.1$ generates a rate for the $\mu \to e + \gamma$ decay, which is ruled out by the MEGA experiment, unless the RH neutrino mass is $M_1 \gtrsim 300$ GeV. Furthermore, if the MEG experiment reaches the sensitivity of 10^{-13} without finding a signal, for the same Yukawa coupling $y = 0.1$ the RH neutrino mass

[4] In all scatter plots included in our paper the neutrino observables are scanned assuming a Gaussian distribution with the corresponding mean value and standard deviation reported in Table I. All the other (unmeasurable) seesaw parameters are selected with a flat distribution.

[5] We note that the phases α_{21} and α_{31} enter into the expression for the neutrino mixing matrix in the form $\exp(i\alpha_{21}/2)$ and $\exp(i\alpha_{31}/2)$, respectively.

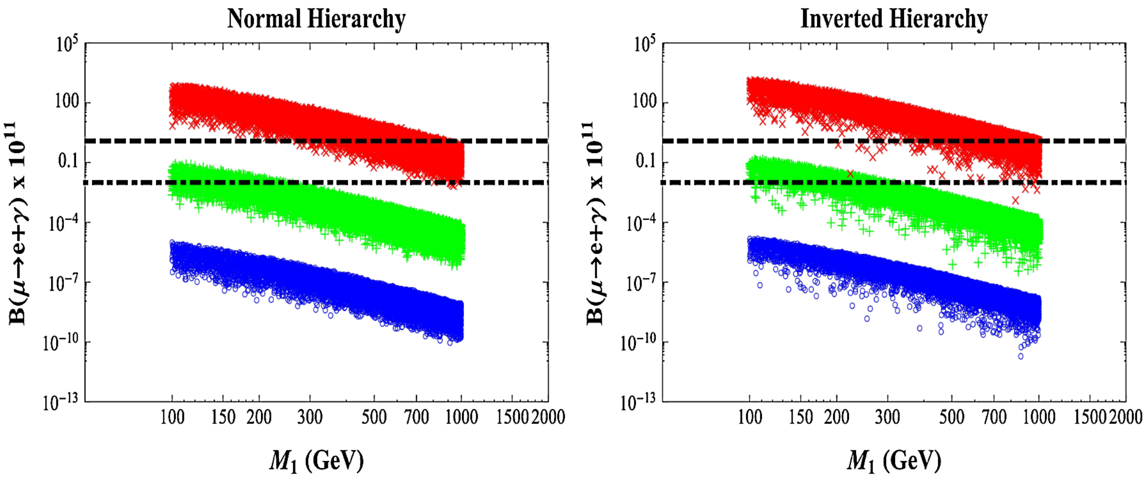

FIG. 1 (color online). The dependence of $B(\mu \to e + \gamma)$ on M_1 in the case of NH (left panel) and IH (right panel) light neutrino mass spectrum, for (i) $y = 0.001$ (blue ∘), (ii) $y = 0.01$ (green +), and (iii) $y = 0.1$ (red ×). The horizontal dashed line corresponds to the MEGA bound [34], $B(\mu \to e + \gamma) \leq 1.2 \times 10^{-11}$. The horizontal dot-dashed line corresponds to $B(\mu \to e + \gamma) = 10^{-13}$, which is the prospective sensitivity of the MEG experiment [35].

should be larger than 1 TeV. More generally, for $M_1 = 100$ GeV ($M_1 = 1$ TeV) and $z \ll 1$ we get the following upper limit on the product $|(\mathrm{RV})^*_{\mu 1}(\mathrm{RV})_{e1}|$ of the heavy Majorana neutrino couplings to the muon (electron) and the $W^\pm$ boson and to the Z^0 boson from the current upper limit [34] on $B(\mu \to e + \gamma)$:

$$|(\mathrm{RV})^*_{\mu 1}(\mathrm{RV})_{e1}| < 1.8 \times 10^{-4}(0.6 \times 10^{-4}), \tag{3.12}$$

where we have used Eqs. (3.1) and (3.5). This can be recast as an upper bound on the neutrino Yukawa coupling y. Taking, e.g. the best fit values of the solar and atmospheric oscillation parameters, we get

$$y \lesssim 0.036(0.21) \quad \text{for NH} \quad \text{with} \quad M_1 = 100\ \text{GeV}(1000\ \text{GeV}) \quad \text{and} \quad \sin\theta_{13} = 0, \tag{3.13}$$

$$y \lesssim 0.031(0.18) \quad \text{for IH} \quad \text{with} \quad M_1 = 100\ \text{GeV}(1000\ \text{GeV}) \quad \text{and} \quad \sin\theta_{13} = 0, \tag{3.14}$$

$$y \lesssim 0.094(0.54) \quad \text{for NH} \quad \text{with} \quad M_1 = 100\ \text{GeV}(1000\ \text{GeV}) \quad \text{and} \quad \sin\theta_{13} = 0.2, \tag{3.15}$$

$$y \lesssim 0.16(0.90) \quad \text{for IH} \quad \text{with} \quad M_1 = 100\ \text{GeV}(1000\ \text{GeV}) \quad \text{and} \quad \sin\theta_{13} = 0.2. \tag{3.16}$$

The upper limit on the neutrino Yukawa coupling derived for $\theta_{13} = 0$ is reached for $\alpha_{21} - \alpha_{31} \simeq \pi$ ($\alpha_{21} \simeq 3\pi$) in the case of NH (IH) light neutrino mass spectrum. For $\sin\theta_{13} = 0.2$, the upper limit on y is obtained for $\alpha_{21} - \alpha_{31} \simeq \pi$ ($\alpha_{21} \simeq \pi$) and $\delta \simeq 0$ ($\delta \simeq 0$), if the neutrino masses have normal (inverted) hierarchy.

A possible way to circumvent these stringent upper bounds consists in assuming a very fine cancellation between the different terms in one of the Eqs. (3.8), (3.9), (3.10), and (3.11) for the factors $|U_{\ell 3} + i\sqrt{m_2/m_3}U_{\ell 2}|^2$ and $|U_{\ell 2} + iU_{\ell 1}|^2$, $\ell = e, \mu$, in the expressions (3.6) and (3.7) for $B(\mu \to e + \gamma)$. Such a cancellation is possible in the case of NH spectrum (see also [28]). Indeed, we have

TABLE I. Best fit values with 1σ errors and 3σ intervals for the three flavor neutrino oscillation parameters (see [32] and references therein).

Parameter	Best fit $\pm 1\sigma$		3σ interval	
	NH	IH	NH	IH
$\Delta m^2_{21}(10^{-5}\ \mathrm{eV}^2)$	$7.59^{+0.20}_{-0.18}$		[7.09, 8.19]	
$\lvert\Delta m^2_{31}\rvert(10^{-3}\ \mathrm{eV}^2)$	$2.45^{+0.09}_{-0.09}$	$2.34^{+0.10}_{-0.09}$	[2.18, 2.73]	[2.08, 2.64]
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$		[0.27, 0.36]	
$\sin^2\theta_{23}$	0.51 ± 0.06	0.52 ± 0.06	[0.39, 0.64]	
$\sin^2\theta_{13}$	$0.010^{+0.009}_{-0.006}$	$0.013^{+0.009}_{-0.007}$	≤ 0.035	≤ 0.039

$|U_{e3} + i\sqrt{m_2/m_3}U_{e2}| = 0$ if

$$\sin\left(\delta + \frac{\alpha_{21} - \alpha_{31}}{2}\right) = 1, \quad \text{and} \quad \tan\theta_{13} = \left(\frac{\Delta m_\odot^2}{\Delta m_A^2}\right)^{1/4} \sin\theta_{12}. \tag{3.17}$$

In this case $B(\mu \to e + \gamma) = 0$ and thus the upper bound on the Yukawa coupling y is no longer valid. Assuming the neutrino parameters have values within the corresponding present 3σ ranges reported in Table I, we find that the values of θ_{13} implied by the condition in Eq. (3.17) satisfy $\tan\theta_{13} \gtrsim 0.21$ or $\sin^2\theta_{13} \gtrsim 0.043$. These values are outside the 3σ interval of the experimentally allowed values of $\sin^2\theta_{13}$ (see Table I).

We consider next the case in which the neutrino mass spectrum is with inverted hierarchy. The first thing to notice is that the factor $|U_{e2} + iU_{e1}|^2$ can be rather small for $\sin(\alpha_{21}/2) = -1$ since $2c_{12}s_{12} \cong 0.93$, where we have used the best fit value of $\sin^2\theta_{12} = 0.312$. In this case we have $|U_{e2} + iU_{e1}|^2 \cong 0.069(0.066)$ for $\sin^2\theta_{13} = 0(0.04)$. Second, we show that, as in the case of normal hierarchical spectrum, it is possible to have a strong suppression of the factor $|U_{\mu 2} + iU_{\mu 1}|^2$ if the CHOOZ mixing angle is close to the corresponding 3σ experimental upper bound. To be more quantitative, we take $\sin^2\theta_{12} = 1/3$ and $\sin^2\theta_{23} = 1/2$. Then, expression (3.11) takes the form

$$|U_{\mu 2} + iU_{\mu 1}|^2 = \frac{1}{6}\left[2(\sqrt{2}(-1 + s_{13}^2) - s_{13}\cos\delta)\sin\frac{\alpha_{21}}{2} + 3\left(1 + s_{13}^2 + 2s_{13}\cos\frac{\alpha_{21}}{2}\sin\delta\right)\right]. \tag{3.18}$$

It is not difficult to show that, for fixed values of the phases α_{21} and δ, $|U_{\mu 2} + iU_{\mu 1}|^2$ has a minimum for

$$\sin\theta_{13} = \frac{\cos\delta\sin\frac{\alpha_{21}}{2} - 3\cos\frac{\alpha_{21}}{2}\sin\delta}{3 + 2\sqrt{2}\sin\frac{\alpha_{21}}{2}}. \tag{3.19}$$

At the minimum, using Eqs. (3.18) and (3.19), we get

$$\min(|U_{\mu 2} + iU_{\mu 1}|^2) = \frac{(3\cos\delta\cos\frac{\alpha_{21}}{2} + \sin\delta\sin\frac{\alpha_{21}}{2})^2}{6(3 + 2\sqrt{2}\sin\frac{\alpha_{21}}{2})}. \tag{3.20}$$

We will find next for which values of the CP violating phases δ and α_{21} this lower bound is equal to zero and if the resulting θ_{13}, obtained from Eq. (3.19), is compatible with the existing limits from the neutrino oscillation data. We have $\min(|U_{\mu 2} + iU_{\mu 1}|^2) = 0$ if the Dirac and Majorana phases δ and α_{21} satisfy the following conditions: $\tan\delta\tan\frac{\alpha_{21}}{2} = -3$ and $\mathrm{sgn}(\cos\delta\cos\frac{\alpha_{21}}{2}) = -\mathrm{sgn}(\sin\delta\sin\frac{\alpha_{21}}{2})$. Taking $\cos\delta > 0$ ($\cos\delta < 0$) and using $\tan\delta = -3/\tan(\alpha_{21}/2)$ in Eq. (3.19) we get

$$\sin\theta_{13} = \mathrm{sgn}(\cos\delta)\frac{\sqrt{9 + \tan^2\frac{\alpha_{21}}{2}}}{3 + 2\sqrt{2}\sin\frac{\alpha_{21}}{2}}\cos\frac{\alpha_{21}}{2}. \tag{3.21}$$

The solution (3.21) is compatible with the 3σ upper limit of the CHOOZ mixing angle (see Table I). In general, one can always find a viable pair of CP violating phases α_{21} and δ satisfying the relations given above in order to set the right-hand side of Eq. (3.20) equal to zero, if the mixing angle θ_{13} is sufficiently large, namely, if $\sin\theta_{13} > 3 - 2\sqrt{2} \cong 0.17$. More precisely, one finds, for example, that $|U_{\mu 2} + iU_{\mu 1}|^2 \simeq 3.52 \times 10^{-8}(2.43 \times 10^{-6})$ for $s_{13} \simeq 0.2(0.17)$, $\alpha_{21} \simeq 2.732(\pi)$ and $\delta \simeq 5.725(10^{-3})$.

In order to interpret the results presented in Fig. 1, it proves convenient to use the analytic expressions of $B(\mu \to e + \gamma)$ in terms of the low energy neutrino parameters, the neutrino Yukawa coupling, and the RH neutrino mass, Eqs. (3.6), (3.7), (3.8), (3.9), (3.10), and (3.11). Taking for concreteness $\sin^2\theta_{23} \cong 1/2$, $\sin^2\theta_{12} \cong 1/3$ and using $\sin\theta_{13}$, $\Delta m_{\rm sol}^2/\Delta m_{\rm atm}^2 \ll 1$, Eqs. (3.6) and (3.7) approximately read

$$\text{NH: } B(\mu \to e + \gamma) \cong \frac{3\alpha_{\rm em}}{32\pi}\left(\frac{y^2 v^2}{M_1^2}\right)^2 [G(X) - G(0)]^2 \frac{1}{6}\left(\frac{\Delta m_\odot^2}{\Delta m_A^2}\right)^{1/4} \times \left|\left[\left(\frac{\Delta m_\odot^2}{\Delta m_A^2}\right)^{1/4} - 2\sqrt{3}\sin\theta_{13}\sin\left(\delta + \frac{\alpha_{21} - \alpha_{31}}{2}\right)\right]\left[1 - 2\sqrt{\frac{2}{3}}\left(\frac{\Delta m_\odot^2}{\Delta m_A^2}\right)^{1/4}\sin\left(\frac{\alpha_{21} - \alpha_{31}}{2}\right)\right]\right|, \tag{3.22}$$

$$\text{IH: } B(\mu \to e + \gamma) \cong \frac{3\alpha_{\rm em}}{32\pi}\left(\frac{y^2 v^2}{M_1^2}\frac{1}{2}\right)^2 [G(X) - G(0)]^2 \times \frac{1}{18}\left|5 + 4\cos\alpha_{21} - 2\sin\theta_{13}\left(3 + 2\sqrt{2}\sin\frac{\alpha_{21}}{2}\right)\left(\cos\delta\sin\frac{\alpha_{21}}{2} - 3\sin\delta\cos\frac{\alpha_{21}}{2}\right)\right|. \tag{3.23}$$

From these expressions it follows that there is a fairly strong dependence of the prediction of $B(\mu \to e + \gamma)$ on the Majorana phases. As a result, and since there is no proposal to constrain experimentally these phases, there is an uncertainty band of a factor of 5 for normal hierarchy and a factor of 9 for inverted hierarchy, which cannot be reduced even if the neutrino masses and mixing angles were known with arbitrarily high precision.

It is not difficult to get also expressions for the double ratios $R(21/31) = B(\mu \to e + \gamma)/B'(\tau \to e + \gamma)$ and $R(21/32) = B(\mu \to e + \gamma)/B'(\tau \to \mu + \gamma)$, where $B'(\tau \to e(\mu) + \gamma) \equiv B(\tau \to e(\mu) + \gamma)/B(\tau \to e(\mu) + \nu_\tau + \bar{\nu}_{e(\mu)})$, $B(\tau \to e(\mu) + \gamma)$ and $B(\tau \to e(\mu) + \nu_\tau + \bar{\nu}_{e(\mu)})$ being the branching ratios of the corresponding decays. The double ratios of interest depend only on the neutrino masses and the elements of the PMNS matrix

$$R(21/31) \cong \frac{|U_{\mu 3} + i\sqrt{\frac{m_2}{m_3}}U_{\mu 2}|^2}{|U_{\tau 3} + i\sqrt{\frac{m_2}{m_3}}U_{\tau 2}|^2}, \qquad \text{NH}, \tag{3.24}$$

$$R(21/31) \cong \frac{|U_{\mu 2} + iU_{\mu 1}|^2}{|U_{\tau 2} + iU_{\tau 1}|^2}, \qquad \text{IH}; \tag{3.25}$$

$$R(21/32) \cong \frac{|U_{e 3} + i\sqrt{\frac{m_2}{m_3}}U_{e 2}|^2}{|U_{\tau 3} + i\sqrt{\frac{m_2}{m_3}}U_{\tau 2}|^2} \qquad \text{NH}, \tag{3.26}$$

$$R(21/32) \cong \frac{|U_{e 2} + iU_{e 1}|^2}{|U_{\tau 2} + iU_{\tau 1}|^2}, \qquad \text{IH}. \tag{3.27}$$

For the NH (IH) light neutrino mass spectrum, the range of variability at 3σ of each of the two ratios defined above is $0.01 \lesssim R(21/31) \lesssim 20$ $[0.001 \lesssim R(21/31) \lesssim 300]$ and $5 \times 10^{-4} \lesssim R(21/32) \lesssim 3$ $[0.04 \lesssim R(21/32) \lesssim 5000]$.[6] Thus, in the case of NH spectrum, the predicted $\tau \to \mu + \gamma$ decay branching ratio $B'(\tau \to \mu + \gamma)$ in the scheme considered can be by several orders of magnitude larger than the $\mu \to e + \gamma$ decay branching ratio, $B(\mu \to e + \gamma)$.

IV. IMPLICATIONS FOR COLLIDER SEARCHES AND ELECTROWEAK PRECISION OBSERVABLES

Upper bounds on the couplings of RH neutrinos with standard model particles can be set by analyzing lepton number conserving processes like $\pi \to \ell \bar{\nu}_\ell$, $Z \to \nu\bar{\nu}$ and other tree-level processes involving light neutrinos in the final state [5]. From Eqs. (2.5) and (2.6), we get

$$|(\text{RV})_{e1}|^2 \lesssim 2 \times 10^{-3}, \tag{4.1}$$

$$|(\text{RV})_{\mu 1}|^2 \lesssim 0.8 \times 10^{-3}, \tag{4.2}$$

$$|(\text{RV})_{\tau 1}|^2 \lesssim 2.6 \times 10^{-3}. \tag{4.3}$$

Following the same rationale as in the previous section, one can translate the upper bounds on the RH neutrino couplings from electroweak precision observables into upper bounds on the neutrino Yukawa coupling y. For this purpose we observe that an upper limit on y, which is independent of the specific values of the neutrino oscillation parameters, can be easily derived from Eq. (2.20), taking into account the experimental constraints given above,

$$y \lesssim 0.06\left(\frac{M_1}{100\ \text{GeV}}\right). \tag{4.4}$$

On the other hand, using Eq. (2.19) and taking the best fit values of neutrino oscillation parameters, we obtain

$$y \lesssim 0.047(0.47) \quad \text{for NH} \quad \text{with} \quad M_1 = 100\ \text{GeV}(1000\ \text{GeV}) \quad \text{and} \quad \sin\theta_{13} = 0, \tag{4.5}$$

$$y \lesssim 0.046(0.46) \quad \text{for IH} \quad \text{with} \quad M_1 = 100\ \text{GeV}(1000\ \text{GeV}) \quad \text{and} \quad \sin\theta_{13} = 0, \tag{4.6}$$

$$y \lesssim 0.049(0.49) \quad \text{for NH} \quad \text{with} \quad M_1 = 100\ \text{GeV}(1000\ \text{GeV}) \quad \text{and} \quad \sin\theta_{13} = 0.2, \tag{4.7}$$

$$y \lesssim 0.053(0.53) \quad \text{for IH} \quad \text{with} \quad M_1 = 100\ \text{GeV}(1000\ \text{GeV}) \quad \text{and} \quad \sin\theta_{13} = 0.2. \tag{4.8}$$

The results reported in Eqs. (4.5) and (4.6) are obtained for $\alpha_{21} - \alpha_{31} \simeq \pi$ and $\alpha_{21} \simeq 0$, respectively. In the case of $\sin\theta_{13} = 0.2$ and NH (IH) light neutrino mass spectrum, the upper bound on y corresponds to $\alpha_{21} - \alpha_{31} \simeq \pi$ ($\alpha_{21} \simeq 0$) and $\delta \simeq \pi$ ($\delta \simeq 3\pi/2$).

The bounds for $\sin\theta_{13} = 0$ are *weaker* than those derived in the previous section from the nonobservation of the $\mu \to e + \gamma$ decay. Thus, the existing stringent upper bound on the $\mu \to e + \gamma$ decay rate makes very difficult the observation of deviations from the standard model in the electroweak precision data, predicted by the TeV scale seesaw scenario. In contrast, in the case of $\sin\theta_{13} = 0.2$, the constraint from the MEGA upper bound [34] can be avoided and we get a better upper limit on y from the electroweak precision data. More precisely, we find that for $M_1 = 100(1000)$ GeV, the limits given in Eqs. (4.1), (4.2), and (4.3) provide a better constraint on the neutrino Yukawa coupling y than the upper bound on $B(\mu \to e + \gamma)$ if $\sin\theta_{13} > 0.10(0.19)$ in the case of the NH neutrino mass spectrum, and for $\sin\theta_{13} > 0.13(0.17)$ if the spectrum is of the IH type. In particular, the parameter y can have a value as large as 4π that is still compatible with the current upper limit on $B(\mu \to e + \gamma)$ [34]. This is

[6] In order to get such estimates we assume that the neutrino observables have a Gaussian distribution with the corresponding mean value and standard deviation reported in Table I.

possible in the case of NH (IH) light neutrino mass spectrum for sufficiently large values of $\sin\theta_{13} \gtrsim 0.22(0.18)$. We note, however, that at $\sin\theta_{13} = 0.10(0.19)$ ($\sin\theta_{13} = 0.13(0.17)$) for the NH (IH) spectrum and $M_1 \approx 100(1000)$ GeV, we have $y \lesssim 0.05$ (0.5) both from the bound on $B(\mu \to e + \gamma)$ and the electroweak data limits. This is consistent with the absolute upper limit reported in (4.4).

These results are illustrated in Fig. 2, where we show, for $M_1 = 100$ GeV (upper panels) and $M_1 = 1000$ GeV (lower panels), the allowed ranges of the RH neutrino couplings $|(RV)_{\mu 1}|$ and $|(RV)_{e1}|$, in the case of NH (left panels) and IH (right panels) spectra. The region of the parameter space which is allowed by the electroweak precision data, Eqs. (4.1) and (4.2), is indicated with solid lines; the region allowed by the current bound on the $\mu \to e + \gamma$ decay rate is indicated with a dashed line. The projected MEG sensitivity reach is shown with a dot-dashed line. Finally, in the same figure we show a scatter plot of the points which are consistent with the 3σ allowed ranges of the neutrino oscillation parameters. This is done for four different values of the neutrino Yukawa coupling y: (i) $y = 0.001$ (blue $\circ$), (ii) $y = 0.01$ (green $+$), (iii) $y = 0.1$ (red $\times$), and (iv) $y = 1$ (orange $\diamond$). The cyan points in each plot correspond to an arbitrary value of the Yukawa coupling $y \leq 1$.

After imposing all the constraints on the parameter space discussed above, we find a fairly narrow band of values of $|(RV)_{\mu 1}|$ and $|(RV)_{e1}|$, which is allowed by the data, more precisely, by the requirement of reproducing the correct values of the neutrino oscillation parameters and by the constraint following from the upper bound on the $\mu \to e + \gamma$ decay rate. Interestingly, and as we have already mentioned, the limits from the electroweak precision data lie essentially in the excluded (shaded) region. Therefore, in view of the constraints from the data on the neutrino oscillation parameters and the $\mu \to e + \gamma$ decay, the discovery of significant deviations from the standard

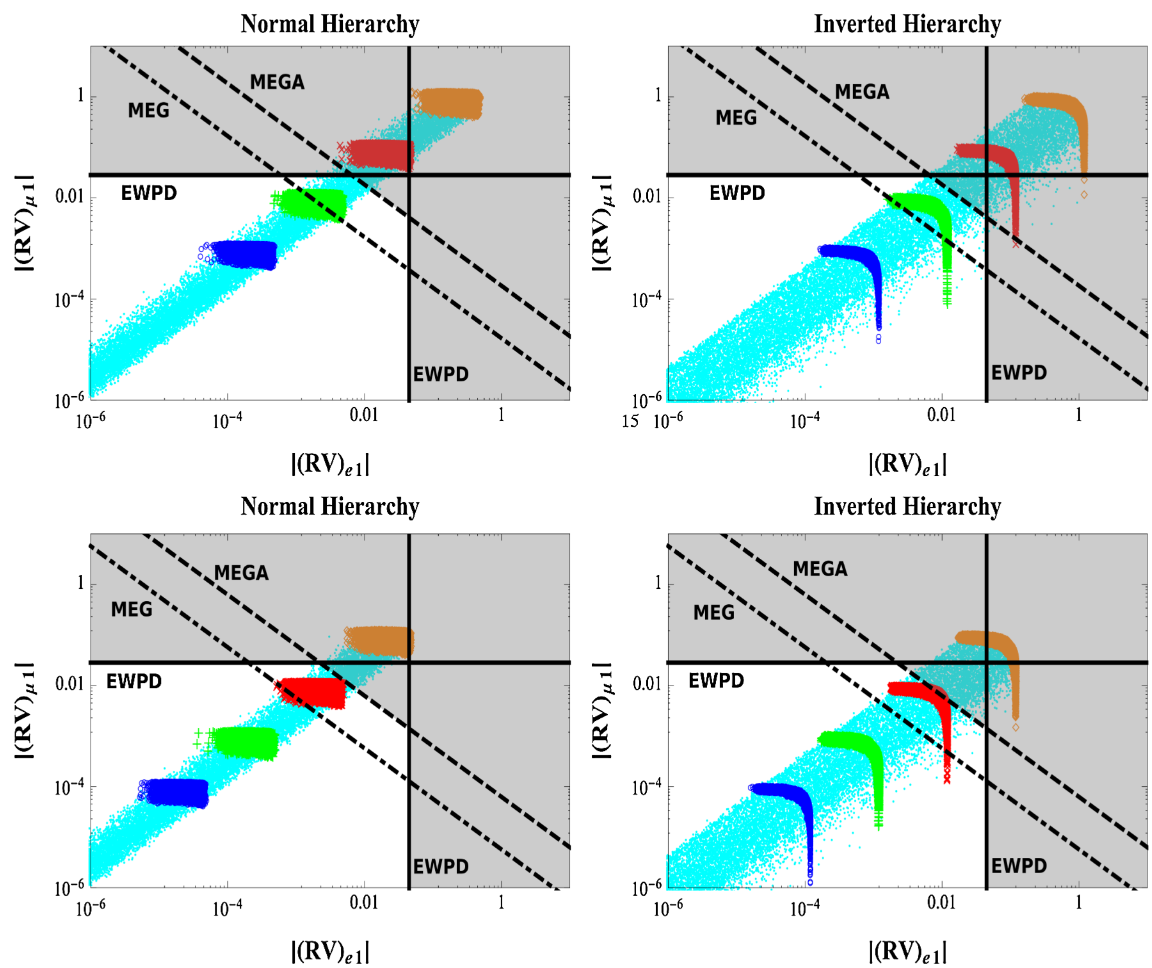

FIG. 2 (color online). Correlation between $|(RV)_{e1}|$ and $|(RV)_{\mu 1}|$ in the case of NH (left panels) and IH (right panels) light neutrino mass spectrum, for $M_1 = 100(1000)$ GeV, upper (lower) panels, (i) $y = 0.001$ (blue $\circ$), (ii) $y = 0.01$ (green $+$), (iii) $y = 0.1$ (red $\times$), and (iv) $y = 1$ (orange $\diamond$). The cyan points correspond to random values of $y \leq 1$. The dashed line corresponds to the MEGA bound [34], $B(\mu \to e + \gamma) \leq 1.2 \times 10^{-11}$. The dot-dashed line corresponds to $B(\mu \to e + \gamma) = 10^{-13}$, which is the prospective sensitivity of the MEG experiment [35].

model predictions in the electroweak precision observables appears highly improbable, unless a significant improvement in the precision of the data is achieved. The above conclusion will be strengthened if the MEG experiment reaches the sensitivity $B(\mu \to e + \gamma) \sim 10^{-13}$ without observing a signal.

It has been argued that present constraints from electroweak precision data allow the observation of pseudo-Dirac RH neutrinos at the LHC. More concretely, the following process with three charged lepton final state,

$$q\bar{q}' \to \mu^+ N_{\rm PD} \to \mu^+\mu^- W^+ \to \mu^+\mu^-\mu^+\nu_\mu, \quad (4.9)$$

could be observed at the LHC with a luminosity of 13 fb^{-1} with a 5σ significance if the RH neutrinos have a mass of $\sim$100 GeV [8]. A discovery reach in this channel implies a coupling $|({\rm RV})_{\mu 1}| \approx 0.04$ [8]. Taking $|({\rm RV})_{\mu 1}| \gtrsim 0.04$, we get from (2.18) and (2.19)

$$y \gtrsim 0.04 \quad \text{for NH} \quad \text{with} \quad M_1 = 100\ \text{GeV}, \quad (4.10)$$

$$y \gtrsim 0.05 \quad \text{for IH} \quad \text{with} \quad M_1 = 100\ \text{GeV}, \quad (4.11)$$

which should be compared with the upper limits on y we get from the upper bound on the $\mu \to e + \gamma$ decay rate, Eqs. (3.13) and (3.14), or from electroweak precision data, Eqs. (4.7) and (4.8). We find again a tension between the constraints on y obtained from the data on the $\mu \to e + \gamma$ decay or the electroweak processes and the values of y required for the production of RH neutrinos with observable rates at colliders. This is clearly seen in Fig. 2.

As a conclusion, the presently existing data on the neutrino mixing parameters and the present experimental upper bound $B(\mu \to e + \gamma) < 1.2 \times 10^{-11}$, basically rule out the possibility of producing pseudo-Dirac neutrinos at LHC with observable rates. A similar conclusion applies to the possibility of observing deviations to the standard model predictions in the electroweak precision observables. An improvement of the bound on the $\mu \to e + \gamma$ decay rate by a factor of $\sim$10 will make the observation of pseudo-Dirac neutrinos at the LHC completely impossible and the effects on the electroweak precision observables negligibly small, unless the RH neutrinos have additional (flavor conserving) couplings to the standard model particles, as in theories with extra $U(1)$ local gauge symmetry under which the pseudo-Dirac neutrinos are charged, or the TeV scale type III seesaw scenario.

V. PREDICTIONS FOR THE $(\beta\beta)_{0\nu}$-DECAY

In the scenario we are considering, the $(\beta\beta)_{0\nu}$-decay effective Majorana mass $|\langle m\rangle|$, which controls the $(\beta\beta)_{0\nu}$-decay rate, receives a contribution from the exchange of the heavy Majorana neutrino fields N_k [12], which may be not negligible for large neutrino Yukawa couplings. One has, in general,

$$|\langle m\rangle| \cong \left|\sum_{i=1}^{3} U_{ei}^2 m_i - \sum_{k=1}^{2} F(A, M_k)({\rm RV})_{ek}^2 M_k\right|, \quad (5.1)$$

where[7] the function $F(A, M_k)$ depends on the N_k masses and the type of decaying nucleus (A, Z). For $M_k =$ (100–1000) GeV, one can use the rather accurate approximate expression for $F(A, M_k)$ [13]: $F(A, M_k) \cong (M_a/M_k)^2 f(A)$, where $M_a \approx 0.9$ GeV and $f(A)$ depends on the decaying isotope considered. For, e.g., ^{76}Ge, ^{82}Se, ^{130}Te, and ^{136}Xe, the function $f(A)$ takes the values $f(A) \cong$ 0.079, 0.073, 0.085, and 0.068, respectively. In the case of ^{48}Ca, $f(A)$ has a smaller value [13]: $f(^{48}{\rm Ca}) \cong 0.033$. Using Eq. (2.16) we can write the heavy Majorana neutrino exchange contribution to $|\langle m\rangle|$ in a simplified form,

$$\langle m\rangle^{\rm N} \cong -\frac{2z + z^2}{(1+z)^2}({\rm RV})_{e1}^2 \frac{M_a^2}{M_1} f(A). \quad (5.2)$$

This contribution, as we will show below, can be even as large as $|\langle m\rangle^{\rm N}| \sim 0.2(0.3)$ eV in the case of NH (IH) light neutrino mass spectrum.[8]

In Fig. 3 we show the ratio between the total effective Majorana mass $|\langle m\rangle|$ given by Eq. (5.1) and the "standard" contribution due to the light Majorana neutrino exchange (see, e.g., [36,37]): $|\langle m\rangle^{\rm std}| \equiv |\sum_{i=1}^3 U_{ei}^2 m_i|$. In this plot we considered a nuclear matrix element factor $f(A)$ corresponding to ^{76}Ge, although the conclusions are analogous for other nuclei. The scan of the parameter space was done in the same way as in Fig. 1, selecting just the points that are in agreement with the present experimental bound on $B(\mu \to e + \gamma)$. In Fig. 4 we show the range of the possible values of $|\langle m\rangle|$ as function of $|({\rm RV})_{e1}|$ for $M_1 = 100$ GeV and $z < 10^{-2}$, in the case of $(\beta\beta)_{0\nu}$-decay of ^{76}Ge.

The results of the analysis illustrated graphically in Figs. 3 and 4 demonstrate that RH neutrinos can significantly enhance the rate of $(\beta\beta)_{0\nu}$-decay, without this being in conflict with the present upper bound on the $\mu \to e + \gamma$ decay rate. For example, $|\langle m\rangle|$ can be as large as 0.2(0.3) eV, if $z \cong 10^{-3}(10^{-2})$ and $M_1 \approx$ 100(1000) GeV. In contrast, in the limit $z \to 0$, the RH neutrinos behave as a Dirac pair and hence do not contribute to the $(\beta\beta)_{0\nu}$-decay amplitude to leading order.

Let us recall that in the scheme with two heavy Majorana neutrinos we are considering, the lightest neutrino mass is zero. This implies that for the NH and IH light neutrino

[7] Let us note that the interference between the contributions due to the light and heavy Majorana neutrino exchanges is not suppressed because both contributions are generated by the weak interaction involving currents of the same $(V - A)$ structure. The interference term of interest would be strongly suppressed if the heavy Majorana neutrino exchange is generated by $(V + A)$ currents [12].

[8] Note that in the approximation we use for $({\rm RV})_{ek}$ one has $\sum_{k=1}^2 ({\rm RV})_{ek}^2 M_k = 0$. The heavy Majorana neutrino exchange contribution $\langle m\rangle^{\rm N}$ to $|\langle m\rangle|$ is not zero due to the nontrivial dependence on M_k of the function $F(A, M_k)$; see Eq. (5.1).

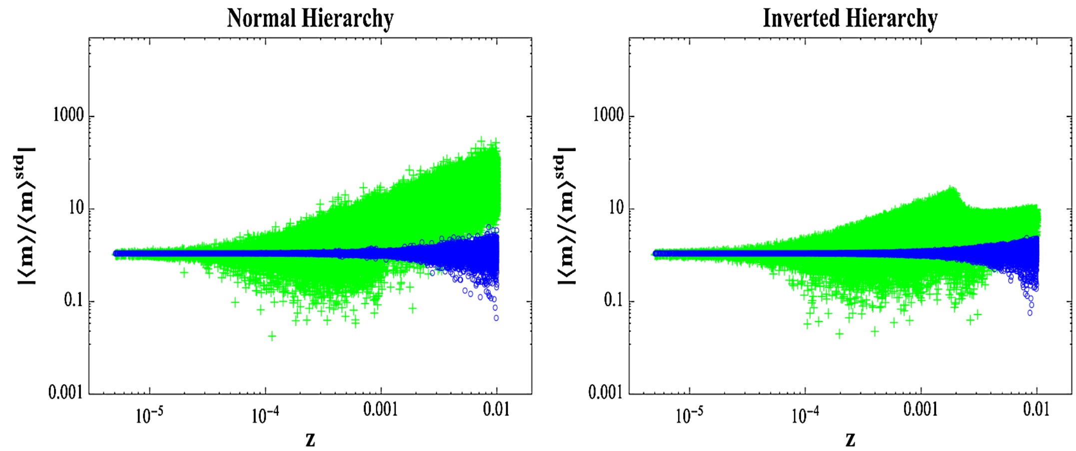

FIG. 3 (color online). The ratio between the effective Majorana mass $|\langle m\rangle|$ and the standard contribution $|\langle m\rangle^{\rm std}|$ for ^{76}Ge in the cases of NH (left panel) and IH (right panel) light neutrino mass spectrum, for $M_1 = 100$ GeV and (i) $y = 0.001$ (blue $\circ$) and (ii) $y = 0.01$ (green $+$).

mass spectra we have for the standard contribution to $|\langle m\rangle|$, respectively (see, e.g., [37]): $|\langle m\rangle^{\rm std}| \lesssim 0.005$ eV and 0.01 eV $\lesssim |\langle m\rangle^{\rm std}| \lesssim 0.05$ eV. Thus, if it is established that the light neutrino mass spectrum is hierarchical, a value of $|\langle m\rangle| > 0.05$ eV would signal the presence of a contribution to $|\langle m\rangle|$ beyond the standard one. In the scheme considered, $|\langle m\rangle|$ can be by a factor up to ~ 100 (~ 10) larger than the maximal value of $|\langle m\rangle^{\rm std}|$ predicted in the case of NH (IH) light neutrino mass spectrum.

It should be noted also that the predicted value of $|\langle m\rangle|$ in the cases of the NH (IH) spectrum exhibits a strong dependence on the PMNS parameters and especially on the Majorana phase [31] $\alpha_{21} - \alpha_{31}$ (α_{21}). If we have $|\langle m\rangle^{\rm std}| \cong |\langle m\rangle^{\rm N}|$, i.e., if the standard contribution to $|\langle m\rangle|$ is of the same order as the contribution due to the exchange of the heavy Majorana neutrinos $N_{1,2}$ [see Eq. (5.2)], $|\langle m\rangle|$ will depend also on the phase ω [see Eq. (2.15)].

Finally, if for a given decaying nucleus (A_1, Z_1) and certain values of the parameters of the problem there is a strong mutual compensation between the two contributions $\langle m\rangle^{\rm std}$ and $\langle m\rangle^{\rm N}$ in $|\langle m\rangle|$ and we have $|\langle m\rangle| \ll |\langle m\rangle^{\rm std}|$, $|\langle m\rangle^{\rm N}|$, similar cancellation will not happen, in general, for another decaying nucleus (A_2, Z_2) due to the dependence of $\langle m\rangle^{\rm N}$ on (A, Z) [12]. In the scheme considered, in which only hierarchical light neutrino mass spectrum is possible, and in view of the planned sensitivity of the next generation of $(\beta\beta)_{0\nu}$-decay experiments, this observation has practical importance if the light neutrino mass spectrum is with inverted hierarchy. In a more general context in which the third heavy Majorana neutrino is relevant for the seesaw mechanism and the light neutrino mass spectrum can be with partial hierarchy or even of the quasidegenerate type (see, e.g., [25,36]), the observation made above regards both types of light neutrino mass

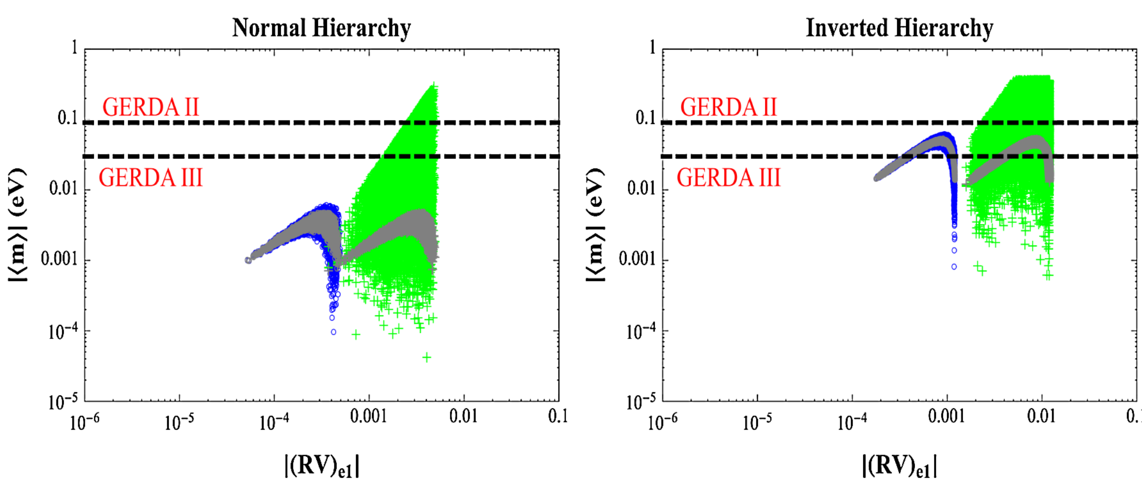

FIG. 4 (color online). The effective Majorana mass $|\langle m\rangle|$ vs $|(RV)_{e1}|$ for ^{76}Ge in the cases of NH (left panel) and IH (right panel) light neutrino mass spectrum, for $M_1 = 100$ GeV and (i) $y = 0.001$ (blue $\circ$) and (ii) $y = 0.01$ (green $+$). The gray markers correspond to $|\langle m\rangle^{\rm std}|$.

spectrum—with normal ordering and with inverted ordering. Indeed, $\langle m\rangle^{\rm N}$ in the general case can be written in the form

$$\langle m\rangle^{\rm N,A} \cong -f(A)\tilde{F}^N, \qquad \tilde{F}^N \equiv \sum_{k=1}^{3}({\rm RV})^2_{ek}\frac{M_a^2}{M_k}, \quad (5.3)$$

where $\tilde{F}^N$ does not depend on A. Suppose that for a given isotope (A_1, Z_1) the two terms in $|\langle m\rangle|$, $\langle m\rangle^{\rm std}$ and $\langle m\rangle^{\rm N,A_1}$, mutually compensate each other so that $|\langle m\rangle^{A_1}| \ll |\langle m\rangle^{\rm std}|$, $|\langle m\rangle^{\rm N,A_1}|$. That would imply that

$$\langle m\rangle^{\rm N,A_1} \equiv f(A_1)\tilde{F}^N \cong \sum_{i=1}^{3} U_{ei}^2 m_i. \quad (5.4)$$

In this case the effective Majorana mass corresponding to an isotope (A_2, Z_2) will be given by

$$|\langle m\rangle^{A_2}| \cong \left|1 - \frac{f(A_2)}{f(A_1)}\right| \left|\sum_{i=1}^{3} U_{ei}^2 m_i\right|. \quad (5.5)$$

If for example the cancellation between the two terms in $\langle m\rangle^{A_1}$ occurs for ^{48}Ca (for which $f(^{48}{\rm Ca}) \cong 0.033$), it will not take place for, e.g., ^{76}Ge, ^{82}Se, ^{130}Te, and ^{136}Xe because for these isotopes $f(A_2) \cong 0.079$, 0.073, 0.085, and 0.068, respectively. Actually, for ^{76}Ge, ^{82}Se, and ^{130}Te the factor $|1 - f(A_2)/f(A_1)| \cong 1.39$, 1.21, and 1.58, so we will have a somewhat larger $|\langle m\rangle^{A_2}|$ than the standard one $|\langle m\rangle^{\rm std}|$, while for ^{136}Xe the indicated factor is 1.06 and thus $|\langle m\rangle^{A_2}| \cong |\langle m\rangle^{\rm std}|$. If, however, the cancellation between the two terms in $\langle m\rangle^{A_1}$ takes place for, e.g., one of the nuclei ^{76}Ge, ^{82}Se, ^{130}Te, and ^{136}Xe, for which the function $f(A)$ has rather similar values, $|\langle m\rangle^{A_2}|$ for the other nuclei will be suppressed with respect to $|\langle m\rangle^{\rm std}|$ to various degrees. For instance, if the cancellation is operative for ^{136}Xe, $|\langle m\rangle^{A_2}|$ for ^{76}Ge, ^{82}Se, ^{130}Te, and ^{48}Ca will be suppressed with respect to the standard contribution $|\langle m\rangle^{\rm std}|$ by the factors 0.16, 0.07, 0.25, and 0.51, respectively.

In the case with two heavy Majorana neutrinos we are considering and for IH light neutrino mass spectrum, the condition for an exact cancellation between $\langle m\rangle^{\rm std}$ and $\langle m\rangle^{\rm N,A}$ can be easily derived in terms of the basic parameters of the scheme. Using Eqs. (2.18), (2.19), (5.1), and (5.2) we can write $|\langle m\rangle|$ as

$$|\langle m\rangle| \simeq |m_1 U_{e1}^2(1-K) + m_2 U_{e2}^2(1+K) + 2i\sqrt{m_1 m_2}K(U_{e2}U_{e1})|, \quad (5.6)$$

where K is given by

$$K \simeq \frac{z}{2}\frac{y^2 v^2}{M_1^2}\frac{M_a^2}{M_1\sqrt{\Delta {\rm m}_{\rm A}^2}}f(A)e^{-2i\omega}. \quad (5.7)$$

If we require $|\langle m\rangle| \simeq 0$, the factor K, which depends on the seesaw parameters y, z, ω, and M_1, is expressed only in terms of the neutrino oscillation parameters,

$$K \simeq \frac{\cos 2\theta_{12} + i\sin 2\theta_{12}\cos\frac{\alpha_{21}}{2}}{1 + \sin 2\theta_{12}\sin\frac{\alpha_{21}}{2}}. \quad (5.8)$$

If the Majorana phase α_{21} takes a CP conserving value we get

$$K \simeq \frac{\cos 2\theta_{12}}{1 + \eta_k \sin(2\theta_{12})} \quad \text{for } \alpha_{21} = (2k+1)\pi, \quad k = 0, \pm 1, \ldots \quad (5.9)$$

$$K \simeq e^{2i\eta_k\theta_{12}} \quad \text{for } \alpha_{21} = 2k\pi, \qquad k = 0, \pm 1, \ldots, \quad (5.10)$$

where $\eta_k = (-1)^k$. In the case of $\langle m\rangle^{A_1} = 0$ we obtain for $|\langle m\rangle^{A_2}|$

$$|\langle m\rangle^{A_2}| \cong \left|1 - \frac{f(A_2)}{f(A_1)}\right| c_{13}^2\sqrt{|\Delta m_{\rm A}^2|} \times\left(1 - \sin^2 2\theta_{12}\sin^2\frac{\alpha_{21}}{2}\right)^{1/2}. \quad (5.11)$$

The condition (5.8) [or (5.9) and (5.10)] for $\langle m\rangle^A = 0$ strongly constrains the phase ω and the size of K. In order to be satisfied, the condition of cancellation between $\langle m\rangle^{\rm std}$ and $\langle m\rangle^{\rm N,A}$ requires a correlation between the values of the seesaw parameters y, z, the phase ω and M_1, and the neutrino mass $\sqrt{|\Delta m_{\rm A}^2|} = m_2$, the solar neutrino mixing angle θ_{12}, and the Majorana phase α_{21}. *A priori*, such a correlation seems highly unlikely, making the cancellation in the effective Majorana mass in the scheme considered appear improbable. Thus, if the light neutrino mass spectrum is with inverted hierarchy, within the seesaw scheme considered with two heavy Majorana neutrinos, it appears quite unlikely that if, for example, the GERDA III experiment with ^{76}Ge observes a positive $(\beta\beta)_{0\nu}$-decay signal, another $(\beta\beta)_{0\nu}$-decay experiment which uses a nucleus different from ^{76}Ge will not see a signal due to a strong suppression of the effective Majorana mass caused by the cancellation under discussion for that nucleus, and vice-versa.

The dependence of the interplay between $\langle m\rangle^{\rm std}$ and $\langle m\rangle^{\rm N}$, i.e. between the contributions to $|\langle m\rangle|$ due to the exchange of light and heavy Majorana neutrinos, on z, ω and the effective Majorana phase α_{21} (IH spectrum) or $\alpha_{21} - \alpha_{31}$ (NH spectrum) is illustrated in Fig. 5 for $M_1 = 100$ GeV and $y = 0.01$. The solar and atmospheric neutrino oscillation parameters, i.e. $(\theta_{12}, \Delta m_\odot^2)$ and $(\theta_{23}, \Delta m_{\rm A}^2)$, respectively, are fixed to their corresponding best fit values, and we have set $\theta_{13} = 0.2$ and $\delta = 0$. Notice that the plots showing the correlation of $|\langle m\rangle|$ and the Majorana phase are symmetric with respect to α_{21} or $(\alpha_{21} - \alpha_{31})$ equal to $\pi/2$ and $3\pi/2$ if the phase ω takes the values $\omega = 0$, $\pi/2$, π. As Fig. 5 shows, we can have $|\langle m\rangle| \gtrsim 0.01$ eV in the case of NH spectrum for a relatively large range of values for the Majorana phase $\alpha_{21} - \alpha_{31}$ and certain values of the phase ω: for, e.g., $\omega = \pi/4$, we get $|\langle m\rangle| \gtrsim 0.01$ eV for $5 \lesssim \alpha_{21} - \alpha_{31} \lesssim 4\pi$ if $z = 10^{-3}$. Actually, for the indicated values of z and

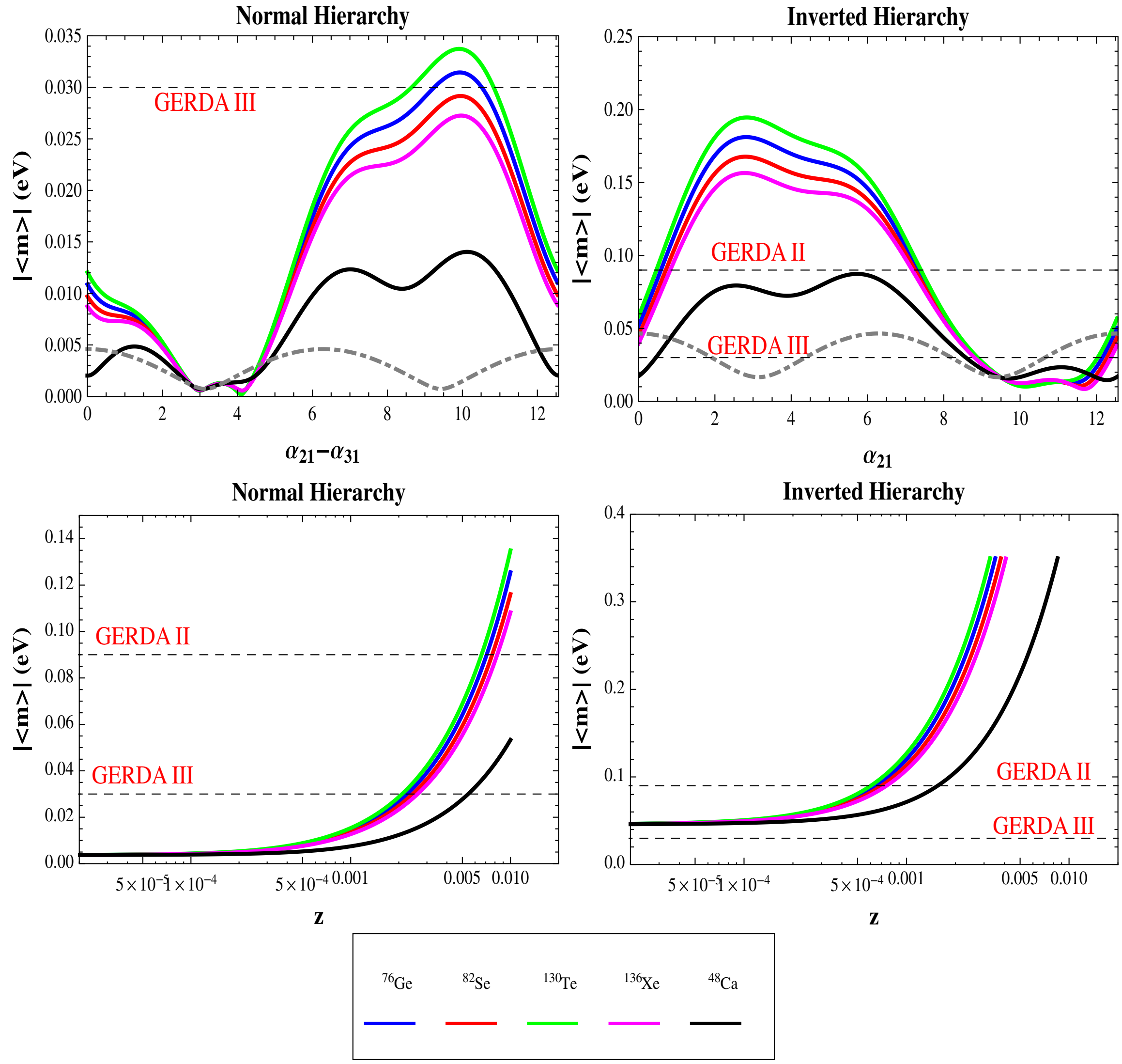

FIG. 5 (color online). Upper panels: the dependence of the effective Majorana mass $|\langle m\rangle|$ on the Majorana phase $\alpha_{21}-\alpha_{31}$ (left side), α_{21} (right side) for $M_1 = 100$ GeV, $\omega = \pi/4$, $y = 0.01$, and $z = 10^{-3}$. The gray dash-dotted line shows $|\langle m\rangle^{\rm std}|$. Lower panels: $|\langle m\rangle|$ versus the degeneracy parameter z for $M_1 = 100$ GeV, $\omega = 0$, $y = 0.01$, and $\alpha_{21} = 0$. The left (right) panels correspond to a NH (IH) light neutrino mass spectrum.

$\alpha_{21}-\alpha_{31}$ we have $|\langle m\rangle| \gtrsim 0.01$ eV for any value of ω from the interval $[0, 2\pi]$. The interplay between the Majorana phase and ω can induce also a minimum of $|\langle m\rangle|$ for a certain value of the degeneracy parameter z. For example, in the case of the IH spectrum, for $\alpha_{21} = 0$ and $\omega = \pi/3$, the predicted effective Majorana mass can be smaller than 0.03 eV: we have for the different nuclei considered $0.007\ \text{eV} \lesssim |\langle m\rangle| \lesssim 0.03\ \text{eV}$ if $2\times 10^{-4} \lesssim z \lesssim 10^{-3}$.

VI. THE $(\beta\beta)_{0\nu}$-DECAY EFFECTIVE MAJORANA MASS AND $B(\mu \to e + \gamma)$

We now combine the information on the seesaw parameter space that we obtained from the lepton flavor ($\mu \to e + \gamma$) and lepton number ($(\beta\beta)_{0\nu}$) violating processes studied in the previous sections. In the simple extension of the standard model considered so far, i.e. with the addition of two heavy RH neutrinos N_1 and N_2 at the TeV scale, which behave as a pseudo-Dirac particle and at the same time are responsible for the generation of neutrino masses via the seesaw mechanism, a sizable (dominant) contribution of N_1 and N_2 to the $(\beta\beta)_{0\nu}$-decay rate would imply a large effect in the muon radiative decay rate. Indeed, if $|\langle m\rangle| \cong |\langle m\rangle^{\rm N}|$, where $\langle m\rangle^{\rm N}$ is given in Eq. (5.2), using Eqs. (3.1) and (3.5) it is easy to show that, given the splitting $10^{-4} \lesssim z \ll 1$ between M_1 and M_2, $|\langle m\rangle| \cong |\langle m\rangle^{\rm N}|$ can be directly related to the $\mu \to e + \gamma$-decay branching ratio. More explicitly, we have

$$B(\mu \to e + \gamma) \cong \frac{3\alpha_{\rm em}}{64\pi}|G(0) - G(X)|^2|r|^2\frac{M_1^2}{M_a^4}\frac{|\langle m\rangle^{\rm N}|^2}{z^2(f(A))^2}, \tag{6.1}$$

where $r \equiv (U_{\mu 2} - i\sqrt{m_3/m_2}U_{\mu 3})/(U_{e2} - i\sqrt{m_3/m_2}U_{e3})$ for the NH mass spectrum. As was pointed out earlier, the corresponding expression for the case of IH spectrum is obtained by replacing $m_{2,3} \to m_{1,2}$ and $U_{\alpha 2,\alpha 3} \to U_{\alpha 1,\alpha 2}$ ($\alpha = e, \mu$). For the NH (IH) neutrino mass spectrum one has $0.5 \lesssim |r| \lesssim 30$ ($0.01 \lesssim |r| \lesssim 5$). Therefore, for $M_1 \approx 100$ GeV and NH spectrum, the predicted rate of the $\mu \to e + \gamma$ decay can even be larger by up to 1 order of magnitude than the upper bound on $B(\mu \to e + \gamma)$ if a positive signal in the current experiments searching for $(\beta\beta)_{0\nu}$-decay is detected, implying $|\langle m\rangle| \sim 0.1$ eV.

A lower bound on $B(\mu \to e + \gamma)$ in (6.1) can be derived for both types of light neutrino mass spectrum. In the case of NH spectrum, such lower bound is within the sensitivity of the MEG experiment, provided a $(\beta\beta)_{0\nu}$-decay corresponding to $|\langle m\rangle| \gtrsim 2 \times 10^{-2}$ eV is observed.

The analytic relation between $B(\mu \to e + \gamma)$ and $|\langle m\rangle|$ in Eq. (6.1) is confirmed by the results of numerical computation, reported in Fig. 6. The plot shows the correlation between the $\mu \to e + \gamma$ branching ratio and the effective Majorana mass in the case of large couplings between the RH (pseudo-Dirac pair) neutrinos and charged leptons in the Lagrangian (2.3). The effective Majorana mass $|\langle m\rangle|$ was computed for $z = 10^{-3}$ using the general expression (5.1). The $(\beta\beta)_{0\nu}$-decay nucleus was assumed to be ^{76}Ge. The neutrino oscillation parameters are taken, again, within the corresponding 3σ experimental intervals reported in Table I. The Majorana phase α_{21} ($\alpha_{31} - \alpha_{21}$) and the phase ω in the IH (NH) case were varied in the intervals $[0, 4\pi]$ and $[0, 2\pi]$, respectively. The neutrino Yukawa coupling takes values $y \lesssim 0.1$. The correlation between $B(\mu \to e + \gamma)$ and $|\langle m\rangle| \cong |\langle m\rangle^{\rm N}|$ reported in Eq. (6.1) is satisfied for values $y \gtrsim 0.01$. This is in agreement with Figs. 3 and 4, where it is shown that a signal compatible with the GERDA sensitivity reach is possible, provided $y \gtrsim 10^{-3}$, for both types of neutrino mass spectrum. Moreover, in the case of IH light neutrino mass spectrum, such correlation depends strongly on the value of the Majorana phase α_{21}. Indeed, for $M_1 \cong 100(1000)$ GeV and $y \cong 0.01(0.1)$ we expect that the MEG experiment [35] is able to measure the $\mu \to e + \gamma$ decay rate (see Fig. 2). If lepton flavor violation is discovered by MEG, according to Eqs. (5.2) and (3.10), a positive signal detected by GERDA II, i.e. $|\langle m\rangle| \cong |\langle m\rangle^N| \gtrsim 0.1$ eV, implies $10^{-3}(10^{-2}) \lesssim z(1 + 0.94\sin(\alpha_{21}/2)) \lesssim 4 \times 10^{-3}(4 \times 10^{-2})$. In the case of $M_1 = 100$ GeV and $z = 10^{-3}$, used to obtain Fig. 6, we would expect, in general, positive signals to be observed in both MEG and GERDA II experiments if $\alpha_{21} \cong 0, \pi$; in the case of $\alpha_{21} \cong 3\pi$, the $(\beta\beta)_{0\nu}$ and $\mu \to e + \gamma$ decays are predicted to proceed with rates below the sensitivity of these two experiments.

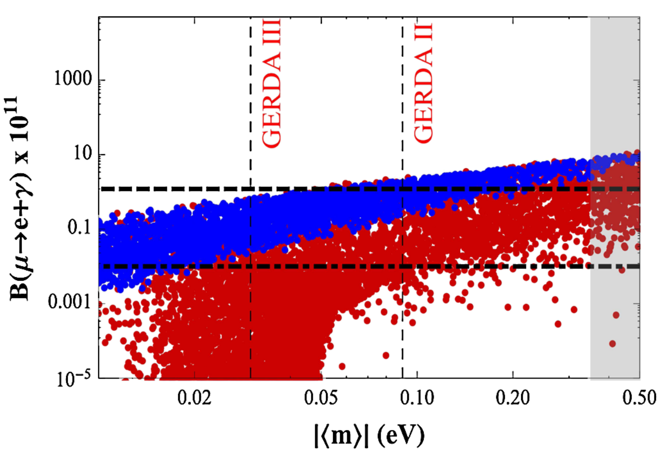

FIG. 6 (color online). $B(\mu \to e + \gamma)$ vs $|\langle m\rangle|$ for $M_1 = 100$ GeV, $z = 10^{-3}$ and (i) NH neutrino mass spectrum (upper blue dots) and (ii) IH neutrino mass spectrum (lower red dots).

We note, however, that it is not possible to get independent constraints on the degeneracy parameter z and the Majorana phase from the data on $(\beta\beta)_{0\nu}$ and $\mu \to e + \gamma$ decays. Finally, we notice also that the strong correlation exhibited in Fig. 6 is a consequence of the constraints imposed by the neutrino oscillation data on the type I seesaw parameter space in the case investigated by us.

VII. CONCLUSIONS

We have analyzed the low energy implications of a type I seesaw scenario with right-handed neutrino masses at the electroweak scale and sizable charged and neutral current weak interactions. This class of scenarios has the attractive feature that the RH neutrinos could be directly produced at the Large Hadron Collider, thus allowing one to test in collider experiments the mechanism of neutrino mass generation. Furthermore, and in contrast to the high-scale seesaw mechanism, the rates for the rare leptonic decays are unsuppressed in this scenario, which opens up the possibility of detecting signatures of new physics with experiments at the intensity frontier.

Present low energy data set very stringent constraints on this scenario. Namely, reproducing the small neutrino masses requires, barring cancellations, that two of the heavy RH neutrinos must form a pseudo-Dirac pair even in the case when there is no conserved lepton charge in the limit of zero splitting at tree level between the masses of the pair. Besides, reproducing the experimentally determined values of the low energy neutrino oscillation parameters (mixing angles and neutrino mass squared differences) fixes the weak charged current and neutral current couplings of the heavy Majorana neutrinos to the charged leptons and $W^{\pm}$ and light neutrinos and Z^0, $(RV)_{\ell k}$ [see Eqs. (2.3) and (2.4)], up to an overall scale which can be related to the largest eigenvalue y of the matrix of neutrino Yukawa couplings. This allows one to derive explicit expressions for the rates of the lepton flavor violating (LFV) charged lepton radiative decays, $\mu \rightarrow e + \gamma$, $\tau \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, in terms of the low energy (in principle measurable) neutrino mixing parameters (including the Dirac and Majorana *CP* violating phases), the neutrino Yukawa coupling y, and the RH neutrino mass scale. Using the present constraint on the rate of the process $\mu \rightarrow e + \gamma$ we have obtained an upper bound on the coupling y under the assumption that the RH neutrino mass scale is in the range (100–1000) GeV. Our analysis shows that the restrictions on this scenario from the data on the neutrino mixing parameters and the upper bound on the $\mu \rightarrow e + \gamma$ decay rate imply that the CC and NC couplings $|(\mathrm{RV})_{\ell k}|$ of the heavy RH Majorana neutrinos are too small to allow their production at the LHC with an observable rate. Other lepton flavor violating processes, such as $\mu - e$ conversion in nuclei, give complementary constraints on the parameter space of this model, which will be discussed elsewhere [38].

We have also analyzed the enhancement of the rate of neutrinoless double beta $(\beta\beta)_{0\nu}$-decay induced by the RH neutrinos. We have shown that even after imposing the restrictions on the parameter space implied by the data on the neutrino oscillation parameters and on the LFV charged lepton radiative decays, the contribution due to the exchange of the RH neutrinos in the $(\beta\beta)_{0\nu}$-decay amplitude can substantially enhance the $(\beta\beta)_{0\nu}$-decay rate. As a consequence, the latter can be in the range of sensitivity of the GERDA experiment even when the light neutrinos possess a normal hierarchical mass spectrum. Finally, the rate of the $\mu \rightarrow e + \gamma$ decay, generated by the exchange of the heavy RH neutrinos can naturally be within the sensitivity of the MEG experiment. Thus, the observation of $(\beta\beta)_{0\nu}$-decay in the next generation of experiments which are under preparation at present, and of the $\mu \rightarrow e + \gamma$ decay in the MEG experiment, could be the first indirect evidence for the TeV scale type I seesaw mechanism of neutrino mass generation.

ACKNOWLEDGMENTS

This work was supported in part by the INFN program on "Astroparticle Physics," by the Italian MIUR program on "Neutrinos, Dark Matter and Dark Energy in the Era of LHC," by the World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan (S. T. P.), and by the DFG Cluster of Excellence, Origin and Structure of the Universe (A. I.). The work of E. M. is supported by Fundação para a Ciência e a Tecnologia (FCT, Portugal) through the projects PTDC/FIS/098188/2008, CERN/FP/116328/2010, and CFTP-FCT Unit 777, which are partially funded through POCTI (FEDER).

[1] P. Minkowski, Phys. Lett. **67B**, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, *Proceedings of the Supergravity Stony Brook Workshop, New York, 1979*, edited by P. Van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan, 1979*, edited by A. Sawada and A. Sugamoto (KEK, Tsukuba, Japan, 1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[2] S. M. Bilenkii, S. T. Petcov, and B. Pontecorvo, Phys. Lett. B **67**, 309 (1977); T. P. Cheng and L. Li, Phys. Rev. Lett. **45**, 1908 (1980); W. J. Marciano and A. I. Sanda, Phys. Lett. **67B**, 303 (1977); B. W. Lee, S. Pakvasa, R. E. Shrock, and H. Sugawara, Phys. Rev. Lett. **38**, 937(E) (1977); **38**, 937 (1977).

[3] J. P. Archambault, A. Czarnecki, and M. Pospelov, Phys. Rev. D **70**, 073006 (2004).

[4] P. Langacker and D. London, Phys. Rev. D **38**, 886 (1988); E. Nardi, E. Roulet, and D. Tommasini, Phys. Lett. B **327**, 319 (1994); D. Tommasini, G. Barenboim, J. Bernabeu, and C. Jarlskog, Nucl. Phys. **B444**, 451 (1995); S. Bergmann and A. Kagan, Nucl. Phys. **B538**, 368 (1999); F. del Aguila, J. de Blas, and M. Perez-Victoria, Phys. Rev. D **78**, 013010 (2008).

[5] S. Antusch, J. P. Baumann, and E. Fernandez-Martinez, Nucl. Phys. **B810**, 369 (2009).

[6] S. Antusch *et al.*, J. High Energy Phys. 10 (2006) 084.

[7] T. Han and B. Zhang, Phys. Rev. Lett. **97**, 171804 (2006); F. del Aguila, J. A. Aguilar-Saavedra, and R. Pittau, J. High Energy Phys. 10 (2007) 047; A. Atre *et al.*, J. High Energy Phys. 05 (2009) 030; F. del Aguila and J. A. Aguilar-Saavedra, Nucl. Phys. **B813**, 22 (2009).

[8] F. del Aguila and J. A. Aguilar-Saavedra, Phys. Lett. B **672**, 158 (2009).

[9] M. C. Gonzalez-Garcia and J. W. F. Valle, Phys. Lett. B **216**, 360 (1989).

[10] J. Kersten and A. Y. Smirnov, Phys. Rev. D **76**, 073005 (2007).

[11] M. B. Gavela, T. Hambye, D. Hernandez, and P. Hernandez, J. High Energy Phys. 09 (2009) 038.

[12] A. Halprin, S. T. Petcov, and S. P. Rosen, Phys. Lett. **125B**, 335 (1983).

[13] J. Vergados, Nucl. Phys. **B218**, 109 (1983); W. C. Haxton and J. Stephenson, Prog. Part. Nucl. Phys. **12**, 409 (1984); M. Blennow *et al.*, J. High Energy Phys. 07 (2010) 096.

[14] I. Abt *et al.*, arXiv:hep-ex/0404039; A. A. Smolnikov (GERDA Collaboration), *Proceedings of the XIV International Baksan School: Particles and Cosmology,*

Kabardino-Balkaria, Russia, April 2007 (INR RAS, Moscow, 2008).

[15] A. Ibarra, E. Molinaro, and S. T. Petcov, J. High Energy Phys. 09 (2010) 108.

[16] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **33**, 549 (1957) [JETP **34**, 247 (1958)].

[17] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[18] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **53**, 1717 (1967) [Sov. Phys. JETP **26**, 984 (1968)].

[19] A. Merle and W. Rodejohann, Phys. Rev. D **73**, 073012 (2006).

[20] L. Wolfenstein, Nucl. Phys. **B186**, 147 (1981).

[21] S. T. Petcov, Phys. Lett. **110B**, 245 (1982).

[22] A. Kleppe, in *Proceedings of the 3rd Tallinn Symposium on Neutrino Physics, October 1995, Lohusalu, Estonia* (Estonian Academy of Sciences, Tallinn, Estonia, 1995), p. 118; E. Ma, D. P. Roy, and U. Sarkar, Phys. Lett. B **444**, 391 (1998); P.H. Frampton, S.L. Glashow, and T. Yanagida, Phys. Lett. B **548**, 119 (2002); M. Raidal and A. Strumia, Phys. Lett. B **553**, 72 (2003); V. Barger, D. A. Dicus, H. J. He, and T. j. Li, Phys. Lett. B **583**, 173 (2004); T. Endoh, S. Kaneko, S. K. Kang, T. Morozumi, and M. Tanimoto, Phys. Rev. Lett. **89**, 231601 (2002).

[23] A. Ibarra and G. G. Ross, Phys. Lett. B **591**, 285 (2004); **575**, 279 (2003).

[24] S. T. Petcov, W. Rodejohann, T. Shindou, and Y. Takanishi, Nucl. Phys. **B739**, 208 (2006).

[25] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).

[26] S. M. Bilenky and S. T. Petcov, Rev. Mod. Phys. **59**, 671 (1987).

[27] J. A. Casas and A. Ibarra, Nucl. Phys. **B618**, 171 (2001).

[28] M. Raidal, A. Strumia, and K. Turzynski, Phys. Lett. B **609**, 351 (2005); **632**, 752(E) (2006).

[29] S. T. Petcov, Yad. Fiz. **25**, 641 (1977) [Sov. J. Nucl. Phys. **25**, 340 (1977)]; S.M. Bilenky, S.T. Petcov, and B. Pontecorvo, Phys. Lett. **67B**, 309 (1977).

[30] T. P. Cheng and L. F. Li, Phys. Rev. Lett. **45**, 1908 (1980).

[31] S. M. Bilenky, J. Hosek, and S. T. Petcov, Phys. Lett. **94B**, 495 (1980).

[32] T. Schwetz, M. Tortola, and J. W. F. Valle, arXiv:1103.0734.

[33] E. Molinaro and S. T. Petcov, Eur. Phys. J. C **61**, 93 (2009).

[34] M.L. Brooks *et al.* (MEGA Collaboration), Phys. Rev. Lett. **83**, 1521 (1999).

[35] A. Maki, AIP Conf. Proc. **981**, 363 (2008).

[36] S.M. Bilenky, S. Pascoli, and S.T. Petcov, Phys. Rev. D **64**, 053010 (2001).

[37] S. Pascoli and S. T. Petcov, Phys. Lett. B **544**, 239 (2002); **580**, 280 (2004); Phys. Rev. D **77**, 113003 (2008); see also, e.g., S. T. Petcov, Phys. Scr. **T121**, 94 (2005).

[38] A. Ibarra, E. Molinaro, and S. T. Petcov (unpublished).