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Interacting $N = 1$ vector-spinor multiplet in 3D

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We present an $N = 1$ supersymmetric multiplet with a vector-spinor field in three dimensions. We call this the vector-spinor multiplet with the field content $(\psi_\mu, A_\mu, \lambda)$, where ψ_μ is a vector spinor, A_μ is a vector, while λ is a gaugino. Based on on-shell component field formulation, we can accommodate $N = 1$ supersymmetric Dirac-Born-Infeld (SDBI) interactions consistently with supersymmetry. This is possible even in the presence of the vector spinor. The ψ_μ-field equation contains a nontrivial interaction term with A_μ. Moreover, it turns out that in the presence of mass terms, one physical degree of freedom in the original λ is transferred to that of ψ_μ, making the latter propagating. In other words, our model presents nontrivial rewriting of SDBI interaction in terms of (ψ_μ, A_μ) instead of (A_μ, λ).

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I. INTRODUCTION

In four dimensions (4D), there have been supersymmetric formulations for multiplets with vector spinors ψ_μ with the minimal spin content $(3/2, 1)$ [1,2]. Here the important point is that the vector spinor has *no* spin 2 superpartner, but has only a spin 1 counterpart. In other words, the system has only global supersymmetry, but *no* local supersymmetry or supergravity. One of the ultimate aims is to establish the foundation for more general supersymmetric higher-spin interactions [3].

However, there have been so far no consistent interactions introduced for the $(3/2, 1)$ multiplet, at least in terms of component fields. This situation is understandable through the conventional wisdom that once spin 3/2 is introduced, there should be spin 2 graviton, resulting necessarily in supergravity for consistency. Another technical reasons is that the auxiliary field structure in superfields [1,2,4] is so involved that the corresponding formulations in component fields is impractically complicated.

This problem appears to be easily solved in superspace. In superspace, the $(3/2, 1)$ multiplet is represented by a spinor superfield Ψ_α and a real scalar superfield V [1,2,4]. We can conjecture, for example, a total action to be $I \equiv I_1 + I_2$, where the free action I_1 [1] and the supersymmetric Dirac-Born-Infeld (SDBI) action I_2 [5,6] can be written down as

$$I_1 \equiv \int d^8z[(D^\alpha \bar{\Psi}^{\dot\beta})(\bar{D}_{\dot\beta}\Psi_\alpha) + \frac{1}{4}(\bar{D}^{\dot\beta}\Psi^\alpha)(\bar{D}_{\dot\beta}\Psi_\alpha) + \frac{1}{4}(D_\alpha \bar{\Psi}_{\dot\beta})(D^\alpha\bar{\Psi}^{\dot\beta}) + \Psi^\alpha W_\alpha + \bar{\Psi}^{\dot\beta}\bar{W}_{\dot\alpha}] + \int d^6z\left(\frac{1}{4}W^\alpha W_\alpha + \frac{1}{4}\bar{W}^{\dot\alpha}\bar{W}_{\dot\alpha}\right), \tag{1.1}$$

$$I_2 \equiv \int d^8z\left[1 - \frac{1}{2}(K + \bar{K}) + \sqrt{1 - (K+\bar{K}) + \frac{1}{4}(K - \bar{K})^2}\right]^{-1} W^2\bar{W}^2, \tag{1.2}$$

where $K \equiv 2D^2(W^2)$, and we use the notation in [7]. The two actions, I_1 and I_2, are invariant under the following Λ, K, and Ω-gauge transformations [1]:

$$\begin{gathered}\delta_{\Lambda,K}\Psi_\alpha = \Lambda_\alpha + i\partial_{\alpha\dot\beta}D^2\bar{D}^{\dot\beta}K, \qquad \delta_\Omega V = +i(\Omega - \bar{\Omega}), \\ K = \bar{K}, \qquad V = \bar{V}, \qquad \bar{D}_{\dot\alpha}\Lambda_\beta = 0, \\ D_\alpha\bar{\Lambda}_{\dot\beta} = 0, \qquad D_\alpha\bar{\Omega}_{\dot\beta} = 0, \qquad \bar{D}_{\dot\alpha}\Omega_\beta = 0.\end{gathered} \tag{1.3}$$

However, the drawback here is that unless we write down the explicit component total Lagrangian after eliminating auxiliary fields, we cannot easily see the total consistency of the whole system. There was a superspace Lagrangian also proposed by Ogievetsky and Sokatchev in [2], but its corresponding component total Lagrangian has not been presented, to our knowledge. Unless we present the explicit component total Lagrangian by eliminating auxiliary fields, we cannot easily see the total consistency of the whole system. From this viewpoint, interaction Lagrangians in terms of component fields is not just for curiosity, but it is for physical significance.

In this brief report, instead of addressing the problem directly in 4D, we study an analogous multiplet in three dimensions (3D), taking advantage of simplification of supersymmetry in 3D. We consider the multiplet $(\psi_\mu, A_\mu, \lambda)$,[1] where ψ_μ is a vector spinor, A_μ is a vector, and λ is a Majorana spinor.

There are two important ingredients about our vector spinor ψ_μ in 3D. First, a *massless* vector spinor ψ_μ has 0

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[1] From now on, we use the indices $\mu, \nu, \cdots = 0, 1, 2$ for the 3D space-time indices.

on-shell degrees of freedom (DOF), based on the conventional counting: $(D-3)\times 2^{[D/2]-1} = (3-3)\times 2^0 = 0$.[2] However, as will be seen, when SDBI interactions are introduced, the original field equation $\mathcal{R}_{\mu\nu} \doteq 0$ for field strength $\mathcal{R}_{\mu\nu}$ is no longer zero, but modified by SDBI interactions. Second, due to mass terms present, the counting should be for a *massive* vector spinor: $(3-2)\times 2^0 = 1$. We will see in Sec. IV how this one DOF of ψ_μ is accounted for, transferred from our gaugino field. We will also see that ψ_μ for a massive case has one propagating DOF.

II. LAGRANGIAN AND TRANSFORMATION RULE

Our multiplet is $(\psi_\mu, A_\mu, \lambda)$, where λ is a Majorana spinor as the superpartner of A_μ, while ψ_μ is a vector spinor in the Majorana representation in 3D. We start with free fields with the action $I_0 \equiv \int d^3x \mathcal{L}_0$, where [3]

$$\mathcal{L}_0 = +\tfrac{1}{2}\epsilon^{\mu\nu\rho}(\bar{\psi}_\mu \partial_\nu \psi_\rho) - \tfrac{1}{4}(F_{\mu\nu})^2 - \tfrac{1}{2}(\bar{\lambda}\gamma^\mu\partial_\mu\lambda) - \tfrac{1}{2}m(\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu) - m(\bar{\psi}_\mu\gamma^\mu\lambda) + \tfrac{1}{4}m\epsilon^{\rho\sigma\tau}F_{\rho\sigma}A_\tau. \tag{2.1}$$

The first term is the kinetic term for the ψ_μ in 3D. The second line is for mass terms and a Chern-Simons term.

Our action I_0 is invariant under global $N=1$ supersymmetry [4]

$$\delta_Q A_\mu = -(\bar{\epsilon}\psi_\mu) + (\bar{\epsilon}\gamma_\mu\lambda), \tag{2.2a}$$
$$\delta_Q \psi_\mu = +\tfrac{1}{2}\epsilon_\mu{}^{\rho\sigma}F_{\rho\sigma} = +\epsilon\tilde{F}_\mu, \tag{2.2b}$$
$$\delta_Q \lambda = -\tfrac{1}{2}(\gamma^{\mu\nu}\epsilon)F_{\mu\nu} = -(\gamma^\mu\epsilon)\tilde{F}_\mu. \tag{2.2c}$$

We use the Hodge-dual quantities, such as

$$\tilde{F}_\mu \equiv +\tfrac{1}{2}\epsilon_\mu{}^{\rho\sigma}F_{\rho\sigma}, \qquad \tilde{\mathcal{R}}_\mu \equiv +\tfrac{1}{2}\epsilon_\mu{}^{\rho\sigma}\mathcal{R}_{\rho\sigma}, \tag{2.3}$$

where $\mathcal{R}_{\mu\nu} \equiv \partial_\mu\psi_\nu - \partial_\nu\psi_\mu$.

We now consider possible interactions for this system. A typical interaction for the vector field is SDBI interaction [5]. We use the real constant parameter α for SDBI terms, so that the total action is now $I \equiv I_0 + I_\alpha \equiv \int d^3x \mathcal{L} \equiv \int d^3x(\mathcal{L}_0 + \mathcal{L}_\alpha)$, where $\mathcal{L}_\alpha$ gives our $\mathcal{O}(\alpha)$ interactions:

$$\mathcal{L}_\alpha \equiv +\tfrac{1}{4}\alpha(F_{\mu\nu}^2)^2 + \alpha\epsilon^{\mu\nu\rho}(\bar{\psi}_\mu\mathcal{R}_{\nu\rho})F_{\sigma\tau}^2 + 4\alpha(\bar{\psi}_\mu\tilde{\mathcal{R}}^\mu)^2 + \alpha(\bar{\lambda}\not{\partial}\lambda)(F_{\rho\sigma})^2 + \alpha\tilde{F}_\mu[\bar{\lambda}\gamma^\mu\not{\partial}(\gamma^\nu\lambda\tilde{F}_\nu)] + \alpha(\bar{\lambda}\not{\partial}\lambda)^2 + \tfrac{1}{4}\alpha(\bar{\lambda}\lambda)\partial_\mu^2(\bar{\lambda}\lambda). \tag{2.4}$$

The first term is proportional to the usual DBI term $(F^4)_\mu{}^\mu - (1/4)(F_{\mu\nu}^2)^2$ [8], because of the particular feature in 3D.

The total action I is invariant up to $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(m\alpha)$-terms, under the modified supersymmetry transformation at $\mathcal{O}(\alpha)$:

$$\delta_Q A_\mu = -(\bar{\epsilon}\psi_\mu) + (\bar{\epsilon}\gamma_\mu\lambda) - 2\alpha(\bar{\epsilon}\psi_\mu)(F_{\rho\sigma})^2 - 8\alpha(\bar{\epsilon}\psi_\mu)(\bar{\psi}_\nu\tilde{\mathcal{R}}^\nu), \tag{2.5a}$$
$$\delta_Q \psi_\mu = +\epsilon\tilde{F}_\mu + 8\alpha\psi_\mu(\bar{\epsilon}\gamma^\rho\partial^\sigma\lambda)F_{\rho\sigma} + 2\alpha\epsilon[\bar{\lambda}\gamma_\mu\not{\partial}(\gamma_\nu\lambda F^\nu)] + 8\alpha\epsilon(\bar{\lambda}\not{\partial}\lambda)\tilde{F}_\mu, \tag{2.5b}$$
$$\delta_Q \lambda = -\tfrac{1}{2}(\gamma^{\mu\nu}\epsilon)F_{\mu\nu} = -(\gamma^\mu\epsilon)\tilde{F}_\mu. \tag{2.5c}$$

The λ-transformation rule is *not* modified at $\mathcal{O}(\alpha)$.

The field equations for our total action $I \equiv \int d^3x \mathcal{L}$ are

$$\frac{\delta\mathcal{L}}{\delta\bar{\psi}_\mu} = +\tilde{\mathcal{R}}^\mu - m(\gamma^\mu\lambda) - m(\gamma^{\mu\nu}\psi_\nu) - 2\alpha\epsilon^{\mu\rho\sigma}\psi_\rho\partial_\sigma(F_{\tau\lambda}^2) \doteq 0, \tag{2.6a}$$
$$\frac{\delta\mathcal{L}}{\delta\bar{\lambda}} = -\not{\partial}\lambda + m(\gamma^\mu\psi_\mu) - 2\alpha(\gamma^\mu\lambda)\partial_\mu(\tilde{F}_\nu^2) + 2\alpha\tilde{F}_\mu\gamma^\mu\not{\partial}(\gamma^\nu\lambda\tilde{F}_\nu) + 2\alpha\lambda(\partial_\mu\lambda)(\partial^\mu\lambda) \doteq 0, \tag{2.6b}$$
$$\frac{\delta\mathcal{L}}{\delta A_\mu} = -\partial_\nu F^{\mu\nu} + m\tilde{F}^\mu - 4\alpha\epsilon^{\tau\nu\rho}\partial_\sigma[F^{\sigma\mu}(\bar{\psi}_\tau\mathcal{R}_{\nu\rho})] + \epsilon^{\mu\rho\sigma}\partial_\rho[+4\alpha\tilde{F}_\sigma\tilde{F}_\tau^2 - 4\alpha(\bar{\lambda}\not{\partial}\lambda)\tilde{F}_\sigma + 2\alpha\bar{\lambda}\gamma_\sigma\not{\partial}(\gamma^\nu\lambda\tilde{F}_\nu)] \tag{2.6c}$$
$$\doteq -\partial_\nu F^{\mu\nu} + m\tilde{F}^\mu - 4\alpha\epsilon^{\mu\rho\sigma}\tilde{F}_\rho\partial_\sigma(\tilde{F}_\tau^2) + 4\alpha(\bar{\lambda}\gamma^{\mu\nu}\partial_\rho\lambda)\partial_\nu\tilde{F}^\rho + 4\alpha(\partial^\mu\bar{\lambda})(\partial_\nu\lambda)\tilde{F}^\nu - 4\alpha(\bar{\lambda}\partial^\mu\partial_\nu\lambda)\tilde{F}^\nu + \mathcal{O}(\alpha^2, \alpha m) \doteq 0. \tag{2.6d}$$

The $\mathcal{O}(\alpha m)$ is ignored, because we have confirmed our action invariance only up to $\mathcal{O}(\alpha^2)$ or $\mathcal{O}(\alpha m)$-terms.

The expression (2.6c) is directly from the Lagrangian variation, while (2.6d) is simplification by using field equations. In other words, they are 'on-shell' equivalent up to $\mathcal{O}(\alpha^2, \alpha m)$-terms. The $\mathcal{O}(\alpha^0)$-field equations that can be used for the α-terms are such as

$$\partial_{[\mu}\tilde{F}_{\nu]} \doteq \mathcal{O}(\alpha, m), \quad \not{\partial}\lambda \doteq \mathcal{O}(\alpha, m), \quad \mathcal{R}_{\mu\nu} \doteq \mathcal{O}(\alpha, m). \tag{2.7}$$

III. CONSISTENCY OF INTERACTIONS

As the consistency confirmation of our total system, we first investigate the supersymmetry transformations of ψ_μ and λ-field equations (2.6a) and (2.6b). The supersymmetric variation of the vector-spinor field equation is

[2] Here $[D/2]$ is the Gauss' symbol for the integer part of the real number $D/2$.

[3] Our metric is $(\eta_{\mu\nu}) \equiv \text{diag.}(-,+,+)$. Accordingly, we have $\epsilon^{012} = +1$, $\gamma^{\mu\nu\rho} = +\epsilon^{\mu\nu\rho}$, $\gamma^{\mu\nu} = +\epsilon^{\mu\nu\rho}\gamma_\rho$, $\gamma^\mu = -(1/2)\epsilon^{\mu\rho\sigma}\gamma_{\rho\sigma}$, $I = -(1/6)\epsilon^{\mu\rho\sigma}\gamma_{\mu\rho\sigma}$.

[4] This transformation rule will be modified by interaction terms at $\mathcal{O}(\alpha)$ later in (2.5).

$$0 \overset{?}{=} +\delta_Q\left(\frac{\delta\mathcal{L}}{\delta\bar{\psi}_\mu}\right) = \delta_Q[+\mathcal{R}^\mu - m(\gamma^\mu\lambda) - m(\gamma^{\mu\nu}\psi_\nu) - 2\alpha\epsilon^{\mu\rho\sigma}\psi_\rho\partial_\sigma(F^2_{\tau\lambda})]$$
$$\doteq +\epsilon^{\mu\rho\sigma}\partial_\rho[\bar{\epsilon}\tilde{F}_\sigma + m\epsilon\tilde{F}_\sigma + 8\alpha\psi_\sigma(\bar{\epsilon}\gamma^\tau\partial_\lambda\lambda)F^\lambda_\tau + 2\alpha\epsilon\bar{\lambda}\gamma_\sigma\not{\partial}(\gamma_\nu\lambda\tilde{F}^\nu)] + 4\alpha\epsilon^{\mu\rho\sigma}(\bar{\epsilon}\tilde{F}_\rho)\partial_\sigma(\tilde{F}^2_\tau)$$
$$- 4\alpha\epsilon^{\mu\rho\sigma}\psi_\rho[\partial_\sigma(-2\bar{\epsilon}\gamma_\tau\partial_\lambda\lambda)]F^{\tau\lambda} - 4\alpha\epsilon^{\mu\rho\sigma}\psi_\rho(-2\bar{\epsilon}\gamma_\tau\partial_\lambda\lambda)\partial_\sigma F^{\tau\lambda} + \mathcal{O}(\alpha^2, \alpha m). \quad (3.1)$$

Here we have used the field equations (2.7) up to $\mathcal{O}(\alpha^2, \alpha m)$-terms. For the A_μ-field equation used for the fist term in (3.1), we have to include $\mathcal{O}(\alpha^0)$, $\mathcal{O}(\alpha)$, and $\mathcal{O}(m)$-terms in (2.6d), while for other terms with α, we need only the field equations (2.7) at $\mathcal{O}(\alpha^0)$. After these manipulations, all the remaining terms cancel, as desired. Similarly, we get

$$0 \overset{?}{=} -\delta_Q\left(\frac{\delta\mathcal{L}}{\delta\bar{\lambda}}\right) \doteq \delta_Q[\{\not{\partial}\lambda - m(\gamma^\mu\psi_\mu)\} + 2\alpha(\gamma^\mu\lambda)\partial_\mu(\tilde{F}^2_\nu) - 2\alpha\tilde{F}_\mu\gamma^\mu\not{\partial}(\gamma^\nu\lambda\tilde{F}_\nu) - 2\alpha\lambda(\partial_\mu\bar{\lambda})(\partial^\mu\lambda)] \quad (3.2)$$

$$\doteq +[-4\alpha(\gamma_\rho{}^\sigma\epsilon)\tilde{F}^\rho\partial_\sigma(\tilde{F}^2_\tau) + 4\alpha(\gamma^\mu\epsilon)(\partial_\mu\bar{\lambda})(\partial_\nu\lambda)\tilde{F}^\nu - 4\alpha(\gamma_\mu\epsilon)(\bar{\lambda}\partial_\mu\partial_\nu\lambda)\tilde{F}^\nu + 4\alpha(\gamma^\mu\epsilon)(\bar{\lambda}\gamma_{\mu\nu}\partial_\rho\lambda)\partial^\nu\tilde{F}^\rho]$$
$$+[+2\alpha(\gamma^{\rho\sigma}\epsilon\tilde{F}_\rho)\partial_\sigma(\tilde{F}^2_\tau) - 2\alpha\epsilon\tilde{F}^\mu\partial_\mu(\tilde{F}^2_\nu) + 4\alpha(\gamma_\mu\epsilon)(\bar{\lambda}\partial^\mu\partial_\nu\lambda)\tilde{F}^\nu - 2\alpha\epsilon(\bar{\lambda}\gamma_\mu\partial_\nu\lambda)\partial_\mu\tilde{F}_\nu + 2\alpha(\gamma_\rho\epsilon)(\bar{\lambda}\gamma^{\mu\rho}\partial_\nu\lambda)$$
$$\times\partial_\mu\tilde{F}^\nu + 2\alpha(\gamma^\mu\epsilon)(\bar{\lambda}\partial_\nu\lambda)\partial_\mu\tilde{F}^\nu] + [+2\alpha(\gamma^{\rho\sigma}\epsilon)\tilde{F}_\rho\partial_\sigma(\tilde{F}^2_\tau) + 2\alpha\epsilon\tilde{F}_\mu\partial^\mu(\tilde{F}^2_\nu) - 4\alpha(\gamma^\mu\epsilon)(\partial_\mu\bar{\lambda})(\partial_\nu\lambda)\tilde{F}^\nu$$
$$- 2\alpha(\gamma^\mu\epsilon)(\partial_\nu\bar{\lambda})(\partial^\nu\lambda)\tilde{F}^\mu] + [+2\alpha(\gamma^\mu\epsilon)(\partial_\nu\bar{\lambda})(\partial^\nu\lambda)\tilde{F}_\mu - 2\alpha(\gamma_\mu\epsilon)(\bar{\lambda}\partial_\nu\lambda)\partial^\nu\tilde{F}^\mu + 2\alpha(\gamma^\rho\epsilon)(\bar{\lambda}\gamma_{\nu\rho}\partial_\mu\lambda)\partial^\mu\tilde{F}^\nu$$
$$+ 2\alpha\epsilon(\bar{\lambda}\gamma^\mu\partial_\nu\lambda)\partial_\mu\tilde{F}^\nu] + \mathcal{O}(\alpha^2, \alpha m), \quad (3.3)$$

where there are four pairs of square brackets, each of which represents the variation of the four terms in (3.2), respectively. After using the $\mathcal{O}(\alpha^0)$-field equations (2.7), all the terms in (3.3) cancel each other. A useful lemma here is

$$\delta_Q F_{\mu\nu} \doteq -2(\bar{\epsilon}\gamma_{[\mu}\partial_{\nu]}\lambda) + m(\bar{\epsilon}\gamma_{\mu\nu}\lambda) + 2m(\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]}) + \mathcal{O}(\alpha^2, \alpha m), \quad (3.4)$$

without $\mathcal{O}(\alpha)$-term. In other words, the δ_Q-transformation of $F_{\mu\nu}$ has *no* $\mathcal{O}(\alpha)$ modification, because essentially the transformation structure of the λ-field equation is exactly the same as the conventional $N = 1$ SDBI system [5].

We next study the consistency of A_μ and ψ_μ-field equations with divergences. The former is just the $U(1)$-gauge invariance, while the latter is similar to consistency for supergravity [9]. For the former, we use (2.6d):

$$0 \overset{?}{=} \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta A_\mu}\right) \doteq -4\alpha\epsilon^{\mu\rho\sigma}(\partial_\mu\tilde{F}_\rho)\partial_\sigma(\tilde{F}^2_\tau) + 4\alpha(\partial^2_\mu\bar{\lambda})(\partial_\nu\lambda)\tilde{F}^\nu + 4\alpha(\partial_\mu\lambda)(\partial^\mu\partial_\nu\lambda)\tilde{F}^\nu + 4\alpha(\partial_\mu\bar{\lambda})(\partial_\nu\lambda)\partial^\mu\tilde{F}^\nu$$
$$- 4\alpha(\partial_\mu\bar{\lambda})(\partial^\mu\partial_\nu\lambda)\tilde{F}^\nu - 4\alpha(\bar{\lambda}\partial^2_\mu\partial^\nu\lambda)\tilde{F}_\nu - 4\alpha(\bar{\lambda}\partial_\mu\partial_\nu\lambda)\partial^\mu\tilde{F}^\nu + 4\alpha(\partial_\mu\bar{\lambda})\gamma^{\mu\nu}(\partial_\rho\lambda)\partial_\nu\tilde{F}^\rho + 4\alpha(\bar{\lambda}\gamma^{\mu\nu}\partial_\mu\partial_\rho\lambda)\partial_\nu\tilde{F}^\rho$$
$$+ \mathcal{O}(\alpha^2, \alpha m). \quad (3.5)$$

$$0 \overset{?}{=} \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta\bar{\psi}_\mu}\right) = +\partial_\mu[+\tilde{\mathcal{R}}^\mu - m(\gamma^\mu\lambda) - m(\gamma^{\mu\nu}\psi_\nu) - 2\alpha\epsilon^{\mu\rho\sigma}\psi_\rho\partial_\sigma(\tilde{F}^2_{\tau\lambda})] \doteq +2\alpha\epsilon^{\mu\rho\sigma}\mathcal{R}_{\mu\rho}\partial_\sigma(\tilde{F}^2_\tau) \doteq \mathcal{O}(\alpha^2, \alpha m). \quad (3.6)$$

After using $\mathcal{O}(\alpha^0)$-field equations (2.7), we see that all the terms in (3.5) cancel each other, leaving only $\mathcal{O}(\alpha^2, m)$-terms. Despite a non-trivial interaction term in (2.6a) the consistency equation (3.6) is satisfied. Before the discovery of supergravity [9], the possible inconsistency for such divergences was known as Velo-Zwanziger disease [10]. From this viewpoint, it is quite nontrivial that our vector-spinor field equation (2.6a) explicitly satisfies the consistency conditions without local supersymmetry.

In supergravity in 3D [11,12], even though the massless gravitino or graviton field has *no* physical DOF, their field equations still play important roles, when coupled to matter multiplet [11,12]. In a similar fashion, our vector-spinor field plays a significant role, when coupled to the vector and gaugino, accompanied by SDBI interactions.

IV. SDBI INTERACTION IN TERMS OF VECTOR SPINOR

We have so far not counted the real DOF for the vector spinor. When a vector spinor is massless, its physical DOF is $(3 - 3) \times 1 = 0$, since three components for the index μ is to be subtracted. This is because the γ-trace component should be subtracted.

However, for a massive vector spinor, the counting should be $(3-2)\times 1 = 1$. Hence, there must be one physical degree of freedom carried by ψ_μ. We can understand this in terms of ψ_μ and λ-field equations (2.6a) and (2.6b). We first multiply (2.6a) by γ_μ or apply ∂_μ, getting two equations

$$+(\gamma_\mu\tilde{\mathcal{R}}^\mu) - 3m\lambda - 2m(\gamma^\nu\psi_\nu) \doteq \mathcal{O}(\alpha), \quad (4.1a)$$

$$+m(\not{\partial}\lambda) + m(\gamma^\mu\tilde{\mathcal{R}}_\mu) \doteq \mathcal{O}(\alpha). \quad (4.1b)$$

From these equations, we can eliminate $(\gamma_\mu \tilde{\mathcal{R}}^\mu)$, as

$$+ (\not\partial\lambda) + 3m\lambda + 2m(\gamma^\mu\psi_\mu) \doteq \mathcal{O}(\alpha). \qquad (4.2)$$

Adding (4.2) to (2.6b), we get

$$+ \lambda \doteq -(\gamma^\mu\psi_\mu) + \mathcal{O}(\alpha). \qquad (4.3)$$

This is also consistent with (2.5b) and (2.5c). In other words, ψ_μ becomes the fundamental field. Combining (4.3) with (4.2), we get

$$+ (\not\partial\lambda) + m\lambda \doteq \mathcal{O}(\alpha). \qquad (4.4)$$

Namely, $\lambda \doteq -(\gamma^\mu\psi_\mu)$ is one propagating DOF. Relevantly, (4.3) with (2.6a) implies that $\tilde{\mathcal{R}}_\mu \doteq -m\psi_\mu + \mathcal{O}(\alpha)$, the divergence of which leads to

$$0 \equiv \partial_\mu \tilde{\mathcal{R}}^\mu + m\partial_\mu\psi^\mu \doteq \mathcal{O}(\alpha, m) \longrightarrow \partial_\mu\psi^\mu \doteq \mathcal{O}(\alpha). \qquad (4.5)$$

Equation (4.5) is analogous to $\partial_\mu A^\mu \doteq 0$ for the massive vector field equation $\partial_\nu F^{\mu\nu} + m^2 A_\mu \doteq 0$.

Eventually, ψ_μ is now massive and propagating with $(3-2)\times 1 = 1$ DOF, instead of the massless case $(3-3)\times 1 = 0$. However, when the mass term is added, the one DOF of λ is transferred to the vector spinor ψ_μ, making it massive and propagating.

The conventional SDBI action in terms of the vector multiplet (A_μ, λ) has been completely rewritten in terms of the new multiplet $(\psi_\mu, A_\mu, \lambda)$. In particular, the vector-spinor field is consistent with $N = 1$ supersymmetry. Moreover, we can rewrite $\lambda = -(\gamma^\mu\psi_\mu) + \mathcal{O}(\alpha)$ everywhere in the field equations (2.6), still maintaining supersymmetry.

V. CONCLUDING REMARKS

In this brief report, we have shown how to introduce nontrivial interaction to the multiplet $(\psi_\mu, A_\mu, \lambda)$ with a vector spinor in 3D. We have seen that the SDBI terms can be accommodated into the multiplet consistently with global $N = 1$ supersymmetry.

We have seen highly nontrivial structure for the supersymmetry transformations of ψ_μ and λ-field equations, consistent with all other field equations under $N = 1$ supersymmetry. We have also confirmed the consistency for the divergence of the A_μ and ψ_μ-field equations *without* local supersymmetry.

It has been well known that a *massless* vector spinor has *no* DOF in 3D by simple counting $(3-3)\times 1 = 0$. However, as supergravity theories in 3D indicate [11], the gravitino field equation plays an important role for matter couplings. By the same token, our ψ_μ-field equation plays an important role, when coupled to the vector and gaugino with nontrivial SDBI interactions.

We have shown moreover that the original system can be reexpressed only in terms of (ψ_μ, A_μ), because of the on-shell relationship $\lambda \doteq -(\gamma^\mu\psi_\mu) + \mathcal{O}(\alpha)$, when mass terms are present. In other words, the original 1 DOF of λ is transferred to that of the massive vector spinor ψ_μ. Consequently, the conventional SDBI interactions in terms of (A_μ, λ) can be reexpressed in terms of (ψ_μ, A_μ).

The conventional wisdom tells us that once a vector spinor is introduced into a system, supergravity, or local supersymmetry with graviton is inevitable. Even though our success might be attributed to the special feature of 3D, we can also regard that our results indicate the encouraging aspect of 3D, where we can study the nontrivial couplings of vector spinor without introducing a spin 2 field.

Our system in 3D can serve as the testing ground for the study of nontrivial interactions of spin (3/2, 1) multiplet in 4D [1,2], where auxiliary field structure in component language is considerably involved.

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