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The Aharonov-Bohm-Casher problem is examined for a charged particle describing a circular path in the presence of a Lorentz-violating background that is nonminimally coupled to a spinor and a gauge field. The particle eigenenergies were evaluated, showing that the LV background is able to lift the original degenerescence in the absence of magnetic field even for a neutral particle. The Aharonov-Casher phase is used to impose an upper bound on the background magnitude. A similar analysis is accomplished in a space endowed with a topological defect, revealing that both the disclination parameter and the LV background are able to modify the particle eigenenergies. We also analyze a particular case where the particles interact harmonically with the topological defect and the LV background, with similar results.

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I. INTRODUCTION

The Standard Model Extension (SME) [1,2] is the natural framework for studying properties of physical systems with Lorentz-violation once it includes Lorentz-violating terms in all sectors of the minimal standard model, including gravitation [3]. The Lorentz-violation terms are generated as vacuum expectation values of tensors defined in a high energy scale. The SME is a theoretical framework which has inspired a great deal of investigations in this theme in recent years. Such works encompass several distinct proposals involving fermion systems [4], *CPT*-probing experiments [5], the electromagnetic *CPT*-and Lorentz-odd term [6,7], the 19 electromagnetic *CPT*-even and Lorentz-odd coefficients [8,9], topological defects [10], topological phases [11,12], cosmic rays [13], and other relevant aspects [14]. These many contributions have elucidated the effects induced by Lorentz-violation and serve to set up stringent upper bounds on the Lorentz-violating (LV) coefficients.

Further investigation on the influence of Lorentz symmetry violation on fermion systems has been accomplished in Ref. [11], where a Lorentz-violating background four-vector, v^μ , was taken in a nonminimal coupling with the gauge field and the fermion spinor (Ψ) in the Dirac equation, $(i\gamma^\mu D_\mu - m)\Psi = 0$. In this case, the nonminimal covariant derivative was defined as

$$D_\mu = \partial_\mu + eA_\mu + i\frac{g}{2}\epsilon_{\mu\lambda\alpha\beta}v^\lambda F^{\alpha\beta}, \quad (1)$$

where g is the constant that measures the strength of this coupling, and $\gamma^\mu = (\gamma_0, \gamma^i)$ are the usual gamma matrices. Here, it is supposed that the Lorentz-violating background v^μ appears as the vacuum expectation value of a tensor quantity generated in a spontaneous breaking of symmetry that takes place in a fundamental theory defined at a higher energy scale (a consequence of an anisotropic vacuum at Planck scale). The product gv^μ has a mass dimension equal to -1 . The analysis of the nonrelativistic regime of this theory revealed that such a coupling is able to induce Aharonov-Casher (AC) [15] geometrical phases on the wave function of an electron interacting with the gauge field. Moreover, the background induces a magnetic moment for neutral particles, which explains the generation of the AC phase for a chargeless particle (deprived from the magnetic moment) in this context. Next, still in connection with this particular nonminimal coupling, it was found that particles and antiparticles may develop opposite AC phases [12]. It is worthy to stress that the standard AC phase is currently interpreted as due to a Lorentz change in the observer frame. In our proposal, namely, in a situation where Lorentz symmetry is violated, it rather emerges as a phase whose origin is ascribed to the presence of a privileged direction in the space-time, set up by the fixed background. Since in this kind of model Lorentz invariance in the particle frame is broken, the AC effect could no any longer be obtained by a suitable Lorentz change in the observer frame. An additional study involving such a coupling was developed in Ref. [16], where the corrections carried out yielded a hydrogen spectrum, allowing the establishment of some upper bounds on the background magnitude.

The study of systems with topological phases was initiated with the pioneering work of Aharonov-Bohm [17],

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followed by a series of other relevant works discussing the conditions and properties of such effects [18]. The discovery of the Aharonov-Casher effect [15] revealed that the generation of geometrical phases could be triggered by an electric field, motivating a great number of interesting investigations [19]. The first experimental confirmations of the AC effect [20] reported phase measurements as small as 10^{-3} rad, with precision in the order of 10^{-4} rad.

The influence of geometrical phases on condensed matter mesoscopic systems started to be studied on one-dimensional (1D) conducting rings in the early 90's. This issue was initially addressed under the action of an external magnetic field, in which the AB effect is responsible by the Altshuler-Aronov-Spivak (AAS) conductance oscillations [21]. In 1992, it was proposed that the AC effect could engender conductance oscillations in semiconducting 1D rings [22] as well. Further developments revealed that the Rashba spin-orbit interaction [23] and the AC effect can be used to control the conductance on 1D mesoscopic rings [24], with experimental demonstrations [25]. This issue is also discussed in some books [26].

Another point of interest concerns the influence of topology (inherent to topological defects) on the usual AB and AC phases. These defects are characterized by a space-time metric with a Riemann-Christoffel curvature tensor which is null everywhere except on the defects. Here, this conical topological defect is identified with a cosmic string. The cosmic string is used to study the gravitational analogue of the bound-state AB effect. Instead of a magnetic flux that is considered by the curvature flux provided by a cosmic string [27], this object, geometrically, corresponds to a Minkowski space-time with a conical singularity, that is, a line element given by

$$ds^2 = -c^2 dt^2 + dr^2 + \alpha^2 r^2 d\phi^2 + dz^2, \quad (2)$$

where $r \geq 0$ and $0 \leq \phi \leq 2\pi$. This metric has a conelike singularity at $r = 0$. The parameter α , which effectively introduces an angular deficit $2\pi(1 - \alpha)$ in the Minkowski geometry, is related to the linear mass density μ of the string by $\alpha = 1 - 4G\mu/c^2$, where G is the gravitational constant and c is the speed of light.

The curvature tensor of the defect is given by

$$R_{12}^{12} = R_1^1 = R_2^2 = 2\pi \left(\frac{\alpha - 1}{\alpha} \right) \delta(\mathbf{r}). \quad (3)$$

This tensor characterizes a two-dimensional conical singularity labeled by the δ -function. This defect is symmetrical in the z axis, which implies that it is a linear defect. In quantum mechanics, when dealing with problems involving singularities, we can not impose that the wave function of the particle is regular in all space including the singular point [28]. Just as for the AB effect, a singularity is also regarded as a topological effect. Its physical properties can only be studied at the quantum level. The reason is because the particle does not have

access to the core of such singularity. Although the particle does not have access to this region, its wave function and energy spectrum are influenced by the core.

A useful method to study the behavior of wave functions for a particle bound in the region inaccessible (core) is the self-adjoint extension method [29]. The problem dealing with self-adjoint extensions of operators appears in various contexts in quantum physics [30]. In such works the family of self-adjoint extensions was found. Although the authors discussed the role of the extension parameters, we cannot see them expressed in terms of the physics problems. One way to self-select the parameter extension is to modulate the problem by boundary conditions, as proposed by Kay and Studer [31]. In fact, the physics of the problem must select such extension parameters [32]. Here, we will not address the problem by the self-adjoint extension method, because our main goal is to analyze some quantum properties of a particle moving in a plane, having excluded the origin region.

In this work, we analyze the effects of a LV background vector, nonminimally coupled to the gauge and fermion fields, on the Aharonov-Bohm-Casher (ABC) problem of a charged particle in a circular motion in a plane, in the presence of magnetic and electric fields. We also study the quantum dynamics of a charged particle in the presence of a conical singularity and the particular case where the particle interacts harmonically with a conical singularity. This paper is organized as follows. In Sec. II we formally introduce the nonminimal coupling and write the Hamiltonian of the system. In Sec. III we study the ABC problem in context of the LV nonminimal coupling. We determine the expressions for the energy eigenvalues, the geometric phase and discuss the role played by the nonminimal background on the eigenenergies and the AC phase. In Sec. IV we analyze the topological ABC problem in the presence of Lorentz-violation. We study the effects of topology and the LV background field ν on the energy and geometric phase, comparing the results with the ones of the original problem. In Sec. V the topological ABC interacting with a two-dimensional harmonic oscillator is studied, and expressions for the energy eigenfunctions and eigenvalues are found. In Sec. VI a brief conclusion is given.

II. THE NONMINIMAL COUPLING

The nonminimal coupling of fermions to a gauge field and a LV background four-vector was proposed and analyzed in Refs. [11,12]. In these works, after assessing the nonrelativistic regime, one has identified a generalized canonical momentum,

$$\mathbf{\Pi} = (\mathbf{p} - q\mathbf{A} + g\nu_0\mathbf{B} - g\mathbf{v} \times \mathbf{E}), \quad (4)$$

where the background is $\nu^\mu = (\nu_0, \mathbf{v})$. This result allows us to introduce this nonminimal coupling in an operational

way, that is, just redefining the vector potential and the corresponding magnetic field as indicated below:

$$\tilde{\mathbf{A}} = \mathbf{A} - \frac{g}{q} v_0 \mathbf{B} + \frac{g}{q} (\mathbf{v} \times \mathbf{E}), \quad (5)$$

$$\tilde{\mathbf{B}} = \nabla \times \mathbf{A} - \frac{g}{q} v_0 \nabla \times \mathbf{B} + \frac{g}{q} \nabla \times (\mathbf{v} \times \mathbf{E}), \quad (6)$$

so that the generalized canonical momentum recovers the usual form, $\mathbf{\Pi} = (\mathbf{p} - q\tilde{\mathbf{A}})$. Taking these prescriptions into account, we obtain the following interaction Hamiltonian density (written in natural units):

$$H = \frac{1}{2m} \mathbf{\Pi}^2 - \boldsymbol{\mu} \cdot \mathbf{B} + \frac{1}{2m} g v_0 \boldsymbol{\sigma} \cdot (\nabla \times \mathbf{B}) + \frac{g}{2m} \boldsymbol{\sigma} \cdot \nabla \times (\mathbf{v} \times \mathbf{E}) + qU, \quad (7)$$

where q , $\boldsymbol{\mu} = \frac{g}{2m} \boldsymbol{\sigma}$ are the charge and the magnetic momentum dipole of the particle, while U is the electrostatic potential due to linear charge distribution. For a situation where the magnetic field is constant and $\nabla \times (\mathbf{v} \times \mathbf{E}) = 0$, the Hamiltonian is

$$H = \frac{1}{2m} [\mathbf{p} - q\mathbf{A} + g v_0 \mathbf{B} - g \mathbf{v} \times \mathbf{E}]^2 - \boldsymbol{\mu} \cdot \tilde{\mathbf{B}} + qU. \quad (8)$$

The nonminimal coupling of the background with the electromagnetic field generates a magnetic dipole moment $g\mathbf{v}$ [11], even for a chargeless particle. In the next section, we will study the dynamics of a charged particle describing a circular path and governed by this Hamiltonian.

III. THE AHARONOV-BOHM-CASHER PROBLEM IN THE CONTEXT OF THE LV NONMINIMAL COUPLING

In this section, we reassess the motion of a particle in a circular path of mesoscopic dimension, subject to the action of an orthogonal magnetic field and a radial electric field. This system is ruled by the Schrödinger equation, once we are here disregarding the spin degree of freedom. This kind of system may have a connection with the physics of the 1D mesoscopic rings analyzed in Refs. [22–24].

The magnetic flux tube is confined into a solenoid orthogonal to the plane, being specified by

$$\mathbf{B} = \frac{\Phi_{(B)}}{r} \delta(r) \delta(\phi) \hat{z}, \quad \mathbf{A} = \frac{\Phi_{(B)}}{2\pi r} \hat{\phi}, \quad (9)$$

where $\Phi_{(B)}$ is the magnetic flux and the vector potential \mathbf{A} is aligned in the direction $\hat{\phi}$. The magnetic field is confined along the z axis. Moreover, let us admit that the solenoid has a linear charge density λ distributed uniformly along its length, which engenders the following electric field:

$$\mathbf{E} = \frac{\Phi_{(E)}}{2\pi r} \hat{r}. \quad (10)$$

In order to examine the role played by the nonminimal coupling (1) on this system, we take the Hamiltonian (8) with the fields configurations (9) and (10). The background field \mathbf{v} is chosen parallel to the magnetic field \mathbf{B} so that the term $(\mathbf{v} \times \mathbf{E})$ will contribute to the geometric phase. The particle is allowed to move in a region where the magnetic field, \mathbf{B} , is null. So the Hamiltonian (8) takes the form

$$H = \frac{1}{2m} [(\mathbf{p} - q\mathbf{A})^2 - 2g(\mathbf{v} \times \mathbf{E}) \cdot (\mathbf{p} - q\mathbf{A}) + g^2(\mathbf{v} \times \mathbf{E})^2] + qU. \quad (11)$$

As is well known, the kinetic part of Hamiltonian, $(\mathbf{p} - q\mathbf{A})^2/2m$, in polar coordinates, is written as

$$\frac{1}{2mr_0^2} \left[\frac{\Phi}{\Phi_0} + i \frac{\partial}{\partial \phi} \right]^2 - \frac{1}{2m} \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right], \quad (12)$$

and we have used $\hbar = 1$, $\nabla \cdot \mathbf{A} = 0$, $\Phi_0 = 2\pi/q$. Further, if the particle is constrained to move in a path with a constant radius, $r = r_0$ (with $\theta = \pi/2$), the wave function will depend only on the azimuthal angle ϕ , and the Schrödinger equation, $H\Psi = \mathcal{E}_n\Psi$, provides a linear second order differential equation with constant coefficients:

$$\frac{d^2\Psi}{d\phi^2} - 2i\beta \frac{d\Psi}{d\phi} + \xi\Psi = 0, \quad (13)$$

where

$$\beta \equiv \frac{q\Phi_{(B)} + g|\mathbf{v}|\Phi_{(E)}}{2\pi}, \quad \xi \equiv 2mr_0^2(\mathcal{E}_n - qU) - \beta^2. \quad (14)$$

Assuming that the eigenfunctions of Eq. (13) have the form

$$\Psi = C e^{i\ell\phi}, \quad (15)$$

and replacing it in Eq. (13), one achieves the following characteristic equation for ℓ :

$$\ell = \beta \pm \sqrt{\beta^2 + \xi}. \quad (16)$$

For the wave function Ψ to be single-valued, the parameter ℓ must be an integer. In this case,

$$\beta \pm \sqrt{\beta^2 + \xi} = n, \quad (17)$$

where $n = 0, \pm 1, \pm 2, \dots$. With this condition, the system eigenenergies assume discrete values:

$$\mathcal{E}_n = \frac{1}{2mr_0^2} \left[n - \frac{q\Phi_{(B)} + g\Phi_{(E)}|\mathbf{v}|}{2\pi} \right]^2 + qU, \quad (18)$$

which depend on the magnetic/electric fluxes, and on the Lorentz-violating background, whose role can be elucidated as follows. In the absence of electromagnetic fields, the particle energy would be $\mathcal{E}_n = n^2/2mr_0^2$, the same value for a particle moving clockwise or counterclockwise, implying a circulation degenerescence.

TABLE I. Bound state energies for possible combinations of n , q and $|v|$.

$\mathcal{E}_n = \frac{1}{2mr_0^2} [n - \frac{(q\Phi_{(B)} + g\Phi_{(E)} v)}{2\pi}]^2 + qU$	$n = 0, 1, 2, \dots$	$q > 0$	$\mathbf{v} = \mathbf{v} $
$\mathcal{E}_n = \frac{1}{2mr_0^2} [n + \frac{(q\Phi_{(B)} + g\Phi_{(E)} v)}{2\pi}]^2 - qU$	$n = 0, 1, 2, \dots$	$q < 0$	$\mathbf{v} = - \mathbf{v} $
$\mathcal{E}_n = \frac{1}{2mr_0^2} [n - \frac{(q\Phi_{(B)} + g\Phi_{(E)} v)}{2\pi}]^2 + qU$	$n = 0, -1, -2, \dots$	$q > 0$	$\mathbf{v} = \mathbf{v} $
$\mathcal{E}_n = \frac{1}{2mr_0^2} [n - \frac{(q\Phi_{(B)} - g\Phi_{(E)} v)}{2\pi}]^2 - qU$	$n = 0, -1, -2, \dots$	$q > 0$	$\mathbf{v} = - \mathbf{v} $
$\mathcal{E}_n = \frac{1}{2mr_0^2} [n - \frac{(q\Phi_{(B)} - g\Phi_{(E)} v)}{2\pi}]^2 + qU$	$n = 0, 1, 2, \dots$	$q > 0$	$\mathbf{v} = - \mathbf{v} $
$\mathcal{E}_n = \frac{1}{2mr_0^2} [n + \frac{(q\Phi_{(B)} - g\Phi_{(E)} v)}{2\pi}]^2 - qU$	$n = 0, 1, 2, \dots$	$q < 0$	$\mathbf{v} = \mathbf{v} $
$\mathcal{E}_n = \frac{1}{2mr_0^2} [n - \frac{(q\Phi_{(B)} - g\Phi_{(E)} v)}{2\pi}]^2 + qU$	$n = 0, -1, -2, \dots$	$q > 0$	$\mathbf{v} = - \mathbf{v} $
$\mathcal{E}_n = \frac{1}{2mr_0^2} [n + \frac{(q\Phi_{(B)} - g\Phi_{(E)} v)}{2\pi}]^2 - qU$	$n = 0, -1, -2, \dots$	$q < 0$	$\mathbf{v} = \mathbf{v} $

If the electric flux is taken as null in Eq. (18) (case without charge distribution along the solenoid), we are left only with the original AB result:

$$\mathcal{E}_n = \frac{1}{2mr_0^2} \left[n - \frac{q\Phi_{(B)}}{2\pi} \right]^2. \quad (19)$$

Note that this choice has eliminated the Lorentz-violating contribution, showing that the original AB effect cannot be influenced by the Lorentz-violating term in this context. The result (19) is the same one obtained in the presence of only a magnetic field [26], in which the dependence on $\Phi_{(B)}$ is enough for breaking the circulation degenerescence. Indeed, it is easy to note that a positive n yields lower energies (assuming q is positive) than the corresponding negative values, $-|n|$. This can then be interpreted as follows: the positive values of n are compatible with a particle traveling in the same sense as the current in the solenoid, while the negative values of n are compatible with a particle traveling in the opposite sense.

If now the magnetic flux is supposed to vanish, we retain only the AC effect, whose eigenvalues,

$$\mathcal{E}_n = \frac{1}{2mr_0^2} \left[n - \frac{g\Phi_{(E)}|v|}{2\pi} \right]^2 + qU, \quad (20)$$

are clearly affected by the background magnitude, $|v|$. Thus, we see that the LV background lifts the degenerescence even in the absence of the magnetic field (it occurs even for a neutral particle deprived from a magnetic moment). The degenerescence shift is proportional to the background magnitude, in such a way that the Aharonov-Casher effect can be used as a tool to impose upper bounds on the magnitude of the LV background, nonminimally coupled to fermions, as stipulated in Eq. (1). Below, this is done by taking the geometrical AC phase as a key point.

As is well known, the induced phase is

$$\varphi = q \int \tilde{\mathbf{A}} \cdot d\boldsymbol{\ell}, \quad (21)$$

with the potential $\tilde{\mathbf{A}}$ defined as in Eq. (5). The total induced phase

$$\varphi = q \left(\Phi_{(B)} + \frac{g}{q} \Phi_{(E)} |v| \right), \quad (22)$$

obviously is a sum of the AB and AC phases, proportional to $\Phi_{(B)}$ and $\Phi_{(E)}$, respectively. For a vanishing background, $|v| \rightarrow 0$, there remains only the original AB phase. For a vanishing magnetic field, the attained AC phase is $g\Phi_{(E)}|v|$. It is interesting to note that the nonminimal coupling is able to induce an AC phase even for a neutral particle without a magnetic moment ($q = 0$, $\mu = 0$), being that this *gedanken* situation is used to impose an upper bound on the magnitude of the product $g|v|$. The point is that, for this particle, no usual physics may explain the induction of a topological phase. Supposing that there is the experimental ability to measure geometrical phases as small as 10^{-4} rad [20], we can affirm that the theoretical AC phase induced for a neutral particle without a magnetic moment ($q = 0$, $\mu = 0$) cannot be larger than this value, that is,

$$2\pi r_0 |\mathbf{E}| g |v| < 10^{-4} \text{ rad}. \quad (23)$$

Taking $|\mathbf{E}| = 10^7$ Volt/m, $r_0 = 10^{-5}$ m (usual values of electric field and radius for 1D mesoscopic rings [24]), and working in the natural units system ($\hbar = c = 1$), wherein 1 Volt = 11.7 eV, Eq. (23) leads to the following upper bound:

$$g|v| < 10^{-8} (\text{eV})^{-1}. \quad (24)$$

Table I above shows the results of n , q and $|v|$ which provide discrete values for eigenenergy (18).

IV. THE AHARONOV-BOHM-CASHER PROBLEM IN THE PRESENCE OF A TOPOLOGICAL DEFECT AND LORENTZ VIOLATION

In this section, we study the effect of a topological defect (disclination) on the energy spectrum and wave function of a pointlike charge, which describes a circular path in a plane orthogonal to the magnetic field, as in Sec. III. We consider an infinitely long linear disclination disposed along the z axis, which is obtained by either removing (positive-curvature disclination) or inserting (negative-curvature disclination) a wedge of material [33]. If λ is

the angle that defines the wedge, the metric of the disclination medium is described by a non-Euclidean metric similar to Eq. (2) [34]:

$$ds^2 = dr^2 + \alpha^2 r^2 d\phi^2 + dz^2, \quad (25)$$

where $\alpha = 1 + \lambda/2\pi$. This metric corresponds to a locally flat medium with a conical singularity at the origin. The magnetic and electric fields are modified by the disclination, being given as

$$\mathbf{B} = \frac{\Phi_{(B)}}{\alpha r} \delta(r) \delta(\phi) \hat{z}, \quad \mathbf{E} = \frac{\Phi_{(E)}}{2\pi\alpha r} \hat{r}, \quad (26)$$

with the potential vector now written as $\mathbf{A} = (\Phi_{(B)}/2\pi\alpha r) \hat{\phi}$. Following the procedure developed in Sec. III, the time-independent Schrödinger equation in the metric (25) is given by

$$-\frac{1}{2m} \frac{1}{\alpha^2 r_0^2} \left[\frac{d^2}{d\phi^2} - 2i \left(\frac{q\Phi_{(B)} + g|\mathbf{v}|\Phi_{(E)}}{2\pi} \right) \frac{d}{d\phi} - \left(\frac{q\Phi_{(B)} + g|\mathbf{v}|\Phi_{(E)}}{2\pi} \right)^2 \right] \Psi(\phi) + qU\Psi(\phi) = \bar{\mathcal{E}}\Psi(\phi). \quad (27)$$

The expression (27) can be read as

$$\left[\frac{d^2}{d\phi^2} - 2i\beta \frac{d}{d\phi} - \beta^2 \right] \Psi(\phi) + \bar{\mathcal{E}}\Psi(\phi) = 0, \quad (28)$$

where β and ε are given by Eq. (14). The discrete values for the eigenenergies, in accordance with Eqs. (15) and (17), are given by

$$\bar{\mathcal{E}}_n = \frac{1}{2m\alpha^2 r_0^2} \left[n - \frac{q\Phi_{(B)} + g\Phi_{(E)}|\mathbf{v}|}{2\pi} \right]^2 + qU. \quad (29)$$

Here, we have the same result of Eq. (18). The only difference is the presence of the parameter α , which represents the disclination. This unique difference, however, is physically very important. The reason is that the topological defect affects the eigenenergies, even with the particle not accessing the defect core region. In the limit $\alpha \rightarrow 1$, when the curvature is zero, we recover the results in Table I.

If we impose the electric flux as null in Eq. (29), we obtain the eigenenergies associated with the topological AB effect:

$$\bar{\mathcal{E}}_n = \frac{1}{2m\alpha^2 r_0^2} \left[n - \frac{q\Phi_{(B)}}{2\pi} \right]^2, \quad (30)$$

with the same meaning of Eq. (19). Returning to Eq. (29), and taking $\Phi_{(B)} = 0$, we find the topological AC eigenenergies:

$$\bar{\mathcal{E}}_n = \frac{1}{2m\alpha^2 r_0^2} \left[n - \frac{g\Phi_{(E)}|\mathbf{v}|}{2\pi} \right]^2 + qU. \quad (31)$$

As in the foregoing section, the AC effect depends on the background, $|\mathbf{v}|$, lifting the particle degenerescence in the

absence of external magnetic field, a result that holds even for a neutral particle. In the limit of a vanishing curvature, $\alpha \rightarrow 1$, Eqs. (30) and (31) recover the AB and AC eigenenergies, given by Eqs. (19) and (20). We should still mention that the total induced geometrical phase for the topological ABC problem remains given by Eq. (22), with $\Phi_{(B)} = \alpha r_0 |\mathbf{B}|$, $\Phi_{(E)} = 2\pi\alpha r_0 |\mathbf{E}|$. With it, and in the absence of magnetic field, the upper bound (24) is rewritten as

$$\alpha g |\mathbf{v}| < 10^{-8} (\text{eV})^{-1}. \quad (32)$$

V. TOPOLOGICAL AHARONOV-BOHM-CASHER IN A TWO-DIMENSIONAL HARMONIC OSCILLATOR

In this section, we study the dynamics of a charged particle in a space endowed with a topological defect (disclination), an ABC potential, and the LV nonminimal coupling, and interacting with a two-dimensional harmonic oscillator. A simpler version of this problem was addressed in Ref. [33] to study the quantum dynamics of a particle interacting harmonically with conical singularities in different physical contexts; in Ref. [35] it was analyzed by the changes introduced on the spectrum of a bound particle confined to move in a plane with a disclination with a magnetic field. This type of system can be used to simulate the behavior of a charged particle in a continuous elastic medium with the topological defect and a magnetic field [36]. Here, we reassess this kind of situation in the presence of the nonminimally coupled Lorentz-violating background.

In this case, the metric is the same of Eq. (25). The Schrödinger equation, using (8), becomes

$$\left[\frac{1}{2m} (\mathbf{p} - q\tilde{\mathbf{A}})^2 + qU + \frac{1}{2} m\omega^2 r^2 \right] \Psi(r, \phi) = \tilde{\mathcal{E}}\Psi(r, \phi). \quad (33)$$

Inserting (9) and (10) into (33), we arrive at

$$-\frac{1}{2m} \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{\alpha^2 r^2} \frac{\partial^2}{\partial \phi^2} - \left(2iq \frac{\Phi_{(B)}}{2\pi\alpha r} + 2ig \frac{|\mathbf{v}|\Phi_{(E)}}{2\pi\alpha r} \right) \frac{1}{\alpha r} \frac{\partial}{\partial \phi} - 2qg \frac{|\mathbf{v}|\Phi_{(E)}\Phi_{(B)}}{(2\pi\alpha r)^2} - \left(\frac{\Phi_{(B)}}{2\pi\alpha r} \right)^2 \right] \times \Psi(r, \phi) + \left(qU + \frac{1}{2} m\omega^2 r^2 \right) \Psi(r, \phi) = \tilde{\mathcal{E}}\Psi(r, \phi). \quad (34)$$

Let us assume the eigenfunction of the form

$$\Psi(r, \phi) = e^{i\ell\phi} R(r), \quad (35)$$

which satisfies the usual asymptotic requirements and finiteness at the origin for a bound state. This leads to the radial equation

$$\left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{l^2}{\alpha^2 r^2}\right)R(r) - \gamma^2 r^2 R(r) - k^2 R(r) = 0, \quad (36)$$

where $l = \ell - q\beta\Phi_{(B)} - g\zeta|\mathbf{v}|\Phi_{(E)}$, $\gamma = m\omega$, $k^2 = 2m(qU - \tilde{\mathcal{E}})$ and $\zeta = 1/2\pi$. The radial Eq. (36) is well known and its solution is the degenerated hypergeometric function

$$R(r) = F\left(\eta, \frac{\alpha + |l|}{\alpha}; \gamma r^2\right), \quad (37)$$

with $\eta = [\alpha(2\gamma + k^2) + 2|l|\gamma]/4\gamma\alpha$. Moreover, if η is equal to 0 or a negative integer, the series terminates and the hypergeometric function becomes a polynomial of degree n . This condition guarantees that the hypergeometric equation has a regular singularity in the origin, which is essential for our treatment of the physical system considered since we have sources located in the origin. Therefore, the series in Eq. (37) must converge if we consider that

$$\eta = -n. \quad (38)$$

This condition also guarantees the normalization of the wave function. Using Eq. (38), we obtain the discrete eigenenergies as

$$\tilde{\mathcal{E}}_{n,\ell} = \omega\left(2n + \frac{1}{\alpha}|\ell - q\zeta\Phi_{(B)} - g\zeta|\mathbf{v}|\Phi_{(E)}| + 1\right) + qU, \quad (39)$$

where $n = 0, 1, 2, \dots$. We see that the presence of the parameter α , the magnetic flux, and the LV background, are able to break the degeneracy of the energy levels. In the absence of the background field \mathbf{v} reproduces, a result already known in [11]. The energy eigenfunction is given by

$$\Psi(r, \phi) = C_{n\ell} \gamma^{1/2((\alpha+|l|)/\alpha)} r^{|l|/\alpha} e^{-(1/2\gamma r^2)} e^{i\ell\phi} \times {}_1F_1\left(-n, \frac{\alpha + |l|}{\alpha}; \gamma r^2\right), \quad (40)$$

where $C_{n\ell}$ is a normalization constant. We note that the eigenfunctions are modified in a space with a disclination, even though the particle does not have access to the defect region. In the limit $\alpha \rightarrow 1$, we obtain the harmonic oscillator eigenenergies modified by the AB and AC contributions.

Let us analyze the result of Eq. (40) in the absence of the electric flux, for which the topological AB harmonic oscillator eigenenergies are

$$\tilde{\mathcal{E}}_{n,\ell} = \omega\left(2n + \frac{1}{\alpha}|\ell - q\beta\Phi_{(B)}| + 1\right). \quad (41)$$

Note that the energy depends only on the magnetic flux and on the disclination. The ground state of (41) is given by

$$\tilde{\mathcal{E}}_{0,0} = \omega\left(1 - \frac{|q|\zeta\Phi_{(B)}}{\alpha}\right). \quad (42)$$

We can see that the ground state is also affected by disclination. As a final analysis, we can obtain the

conventional harmonic oscillator making $\Phi_{(B)} \rightarrow 0$, $\Phi_{(E)} \rightarrow 0$ and $\alpha \rightarrow 1$, simultaneously.

Now, we examine the result of Eq. (40) in the absence of magnetic flux, for which the harmonic oscillator eigenenergies are

$$\tilde{\mathcal{E}}_{n,\ell} = \omega\left(2n + \frac{1}{\alpha}|\ell - g\zeta|\mathbf{v}|\Phi_{(E)}| + 1\right) + qU, \quad (43)$$

which depend on the parameter α , on the electric flux and the background magnitude, $|\mathbf{v}|$. Similar to the previous cases, we see that a LV background is able to modify the harmonic oscillator eigenenergies even for a chargeless particle without a magnetic moment ($q = 0, \mu = 0$). The corresponding ground state energy for Eq. (43), $n = \ell = 0$, is

$$\tilde{\mathcal{E}}_{0,0} = \omega\left(1 - \frac{g\zeta|\mathbf{v}|\Phi_{(E)}}{\alpha}\right) + qU. \quad (44)$$

VI. CONCLUSIONS

The ABC problem for a particle moving in a mesoscopic circular path was analyzed (a) in the presence of a LV background nonminimally coupled to the fermion and gauge fields, as proposed in Ref. [11], (b) in a space endowed with a topological defect (disclination) and considering the LV nonminimal coupling, (c) and for a particle in a continuous elastic medium with a topological defect and a LV background.

The background field vector $\vec{\mathbf{v}}$ is chosen in such a way that it contributes to the topological phase. For the case (a), we have shown that the discrete eigenenergies for the circling particle carry contributions coming from the magnetic and electric fluxes. We notice that the nonminimal coupling is able to lift the circulation degenerescence in the absence of a magnetic field even for chargeless particle without a magnetic moment ($\mu = 0$). The induced AC geometrical phase is used to impose a good upper bound, $g|\mathbf{v}| < 10^{-8} \text{ (eV)}^{-1}$, by means of a different route from the one in Ref. [16].

In the case (b), the particle eigenenergies obey the same pattern of case (a), except for the inclusion of the parameter α that characterizes the topological defect. The eigenenergies are modified by the topology of the space, even noting that the particle does not have access to the defect region. In the limit $\alpha \rightarrow 1$ we recover the results of the case (a). In the case (c), we verify that the energy spectrum and wave functions are modified by the presence of a disclination. It is important to remark that the region outside the defect has zero curvature. We have found that the ground state is also affected by the disclination. Finally, we notice that the topology plays a similar role to the magnetic field along the axis of the AB problem, once they both do not exist in the region of space where the particle moves.

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- [1] D. Colladay and V. A. Kostelecký, *Phys. Rev. D* **55**, 6760 (1997); **58**, 116002 (1998); S. Coleman and S. L. Glashow, *Phys. Rev. D* **59**, 116008 (1999).
- [2] V. A. Kostelecký and S. Samuel, *Phys. Rev. Lett.* **63**, 224 (1989); **66**, 1811 (1991); *Phys. Rev. D* **39**, 683 (1989); **40**, 1886 (1989); V. A. Kostelecký and R. Potting, *Nucl. Phys. B* **359**, 545 (1991); *Phys. Lett. B* **381**, 89 (1996); *Phys. Rev. D* **51**, 3923 (1995).
- [3] V. Alan Kostelecký, *Phys. Rev. D* **69**, 105009 (2004); V. A. Kostelecký, N. Russell, and J. D. Tasson, *Phys. Rev. Lett.* **100**, 111102 (2008); V. A. Kostelecký and J. D. Tasson, *Phys. Rev. Lett.* **102**, 010402 (2009); Q. G. Bailey and V. A. Kostelecký, *Phys. Rev. D* **74**, 045001 (2006); Q. G. Bailey, *Phys. Rev. D* **80**, 044004 (2009); V. A. Kostelecký and R. Potting, *Phys. Rev. D* **79**, 065018 (2009); Q. G. Bailey, *Phys. Rev. D* **82**, 065012 (2010); C. N. Ferreira, and J. A. Helayël-Neto, *Phys. Rev. D* **71**, 044018 (2005); J. L. Boldo, J. A. Helayël-Neto, L. M. de Moraes, C. A. G. Sasaki, and V. J. Vasquez Otoyá, *Phys. Lett. B* **689**, 112 (2010).
- [4] B. Altschul, *Phys. Rev. D* **70**, 056005 (2004); G. M. Shore, *Nucl. Phys. B* **717**, 86 (2005); D. Colladay and V. A. Kostelecký, *Phys. Lett. B* **511**, 209 (2001); O. G. Kharlanov and V. Ch. Zhukovsky, *J. Math. Phys. (N.Y.)* **48**, 092302 (2007); R. Lehnert, *Phys. Rev. D* **68**, 085003 (2003); V. A. Kostelecký and C. D. Lane, *J. Math. Phys. (N.Y.)* **40**, 6245 (1999); R. Lehnert, *J. Math. Phys. (N.Y.)* **45**, 3399 (2004); W. F. Chen and G. Kunstatter, *Phys. Rev. D* **62**, 105029 (2000); B. Goncalves, Y. N. Obukhov, and I. L. Shapiro, *Phys. Rev. D* **80**, 125034 (2009); V. A. Kostelecký and R. Lehnert, *Phys. Rev. D* **63**, 065008 (2001); S. Chen, B. Wang, and R. Su, *Classical Quantum Gravity* **23**, 7581 (2006).
- [5] R. Bluhm, V. A. Kostelecký, and N. Russell, *Phys. Rev. Lett.* **79**, 1432 (1997); R. Bluhm, V. A. Kostelecký, and N. Russell, *Phys. Rev. D* **57**, 3932 (1998); R. Bluhm, V. A. Kostelecký, C. D. Lane, and N. Russell, *Phys. Rev. Lett.* **88**, 090801 (2002); R. Bluhm and V. A. Kostelecký, *Phys. Rev. Lett.* **84**, 1381 (2000); R. Bluhm, V. A. Kostelecký, and C. D. Lane, *Phys. Rev. Lett.* **84**, 1098 (2000); V. A. Kostelecký and N. Russell, *Phys. Rev. Lett.* **82**, 2254 (1999); V. A. Kostelecký and C. Lane, *Phys. Rev. D* **60**, 116010 (1999).
- [6] C. Adam and F. R. Klinkhamer, *Nucl. Phys. B* **607**, 247 (2001); **657**, 214 (2003); A. A. Andrianov and R. Soldati, *Phys. Rev. D* **51**, 5961 (1995); *Phys. Lett. B* **435**, 449 (1998); A. A. Andrianov, R. Soldati, and L. Sorbo, *Phys. Rev. D* **59**, 025002 (1998); A. A. Andrianov, D. Espriu, P. Giacconi, and R. Soldati, *J. High Energy Phys.* **09** (2009) 057; J. Alfaro, A. A. Andrianov, M. Cambiaso, P. Giacconi, and R. Soldati, *Int. J. Mod. Phys. A* **25**, 3271 (2010); V. Ch. Zhukovsky, A. E. Lobanov, and E. M. Murchikova, *Phys. Rev. D* **73**, 065016 (2006).
- [7] H. Belich, M. M. Ferreira, Jr., J. A. Helayël-Neto, and M. T. D. Orlando, *Phys. Rev. D* **67**, 125011 (2003); *Phys. Rev. D* **69**, 109903(E) (2004); A. P. Baêta Scarpelli, H. Belich, J. L. Boldo, and J. A. Helayël-Neto, *Phys. Rev. D* **67**, 085021 (2003); H. Belich *et al.*, *Nucl. Phys. B, Proc. Suppl.* **127**, 105 (2004); M. B. Cantcheff, *Eur. Phys. J. C* **46**, 247 (2006); H. Belich *et al.*, *Rev. Bras. Ens. Fis.* **29**, 57 (2007); H. Belich, L. P. Colatto, T. Costa-Soares, J. A. Helayël-Neto, and M. T. D. Orlando, *Eur. Phys. J. C* **62**, 425 (2009).
- [8] V. A. Kostelecký and M. Mewes, *Phys. Rev. Lett.* **87**, 251304 (2001); *Phys. Rev. D* **66**, 056005 (2002); **80**, 015020 (2009); *Phys. Rev. Lett.* **97**, 140401 (2006); F. R. Klinkhamer and M. Risse, *Phys. Rev. D* **77**, 016002 (2008); **77**, 117901 (2008); **78**, 085026 (2008); B. Altschul, *Phys. Rev. Lett.* **98**, 041603 (2007).
- [9] B. Altschul, *Nucl. Phys. B* **796**, 262 (2008); *Phys. Rev. Lett.* **98**, 041603 (2007); C. Kaufhold and F. R. Klinkhamer, *Phys. Rev. D* **76**, 025024 (2007).
- [10] M. Lubo, *Phys. Rev. D* **71**, 047701 (2005); M. N. Barreto, D. Bazeia, and R. Menezes, *Phys. Rev. D* **73**, 065015 (2006); D. Bazeia, M. M. Ferreira, Jr., A. R. Gomes, and R. Menezes, *Physica D (Amsterdam)* **239**, 942 (2010); A. de Souza Dutra and R. A. C. Correa, *Phys. Rev. D* **83**, 105007 (2011).
- [11] H. Belich, T. Costa-Soares, M. M. Ferreira, Jr., and J. A. Helayël-Neto, *Eur. Phys. J. C* **41**, 421 (2005).
- [12] H. Belich, T. Costa-Soares, M. M. Ferreira, Jr., J. A. Helayël-Neto, and M. T. D. Orlando, *Phys. Lett. B* **639**, 675 (2006).
- [13] F. W. Stecker and S. L. Glashow, *Astropart. Phys.* **16**, 97 (2001); O. Gagnon and G. D. Moore, *Phys. Rev. D* **70**, 065002 (2004); J. W. Moffat, *Int. J. Mod. Phys. D* **12**, 1279 (2003); F. W. Stecker and S. T. Scully, *Astropart. Phys.* **23**, 203 (2005).
- [14] N. M. Barraz, Jr., J. M. Fonseca, W. A. Moura-Melo, and J. A. Helayël-Neto, *Phys. Rev. D* **76**, 027701 (2007); M. B. Cantcheff, *Eur. Phys. J. C* **46**, 247 (2006); M. B. Cantcheff, C. F. L. Godinho, A. P. Baeta Scarpelli, and J. A. Helayël-Neto, *Phys. Rev. D* **68**, 065025 (2003); H. Belich, T. Costa-Soares, J. A. Helayël-Neto, M. T. D. Orlando, and R. C. Paschoal, *Phys. Lett. A* **370**, 126 (2007); H. Belich, L. P. Colatto, T. Costa-Soares, J. A. Helayël-Neto, and M. T. D. Orlando, *Eur. Phys. J. C* **62**, 425 (2009); C. N. Ferreira, J. A. Helayël-Neto, and C. E. C. Lima, *New J. Phys.* **12**, 053029 (2010); H. Belich, M. A. De Andrade, and M. A. Santos, *Mod. Phys. Lett. A* **20**, 2305 (2005); A. P. Baeta Scarpelli and J. A. Helayël-Neto, *Phys. Rev. D*

- 73**, 105020 (2006); H. Belich *et al.*, *Phys. Lett. A* **370**, 126 (2007).
- [15] Y. Aharonov and A. Casher, *Phys. Rev. Lett.* **53**, 319 (1984); A. S. Goldhaber, *Phys. Rev. Lett.* **62**, 482 (1989).
- [16] H. Belich, T. Costa-Soares, M. M. Ferreira, Jr., J. A. Helayël-Neto, and F. M. O. Moucherek, *Phys. Rev. D* **74**, 065009 (2006).
- [17] Y. Aharonov and D. Bohm, *Phys. Rev.* **115**, 485 (1959).
- [18] Y. Aharonov and D. Bohm, *Phys. Rev.* **123**, 1511 (1961); **130**, 1625 (1963); T. H. Boyer, *Phys. Rev. D* **8**, 1679 (1973); S. Olariu and I. I. Popescu, *Rev. Mod. Phys.* **57**, 339 (1985).
- [19] T. H. Boyer, *Phys. Rev. A* **36**, 5083 (1987); X.-G. He and B. H. J. McKellar, *Phys. Lett. B* **264**, 129 (1991); **256**, 250 (1991); *Phys. Rev. A* **47**, 3424 (1993).
- [20] A. Cimmino, G. I. Opat, A. G. Klein, H. Kaiser, S. A. Werner, M. Arif, and R. Clothier, *Phys. Rev. Lett.* **63**, 380 (1989); K. Sangster, E. A. Hinds, S. M. Barnett, and E. Riis, *Phys. Rev. Lett.* **71**, 3641 (1993); A. Görlitz, B. Schuh, and A. Weis, *Phys. Rev. A* **51**, R4305 (1995).
- [21] B. L. Altshuler, A. G. Aronov, and B. Z. Spivak, *Pis'ma Zh. Eksp. Teor. Fiz.* **33**, IOI (1981) [*JETP Lett.* **33**, 94 (1981)]; A. G. Aronov and Y. V. Sharvin, *Rev. Mod. Phys.* **59**, 755 (1987).
- [22] H. Mathur and A. D. Stone, *Phys. Rev. Lett.* **68**, 2964 (1992); A. V. Balatsky and B. L. Altshuler, *Phys. Rev. Lett.* **70**, 1678 (1993); T.-Z. Qian and Z.-B. Su, *Phys. Rev. Lett.* **72**, 2311 (1994).
- [23] A. G. Aronov and Y. B. Lyanda-Geller, *Phys. Rev. Lett.* **70**, 343 (1993); D. Frustaglia and K. Richter, *Phys. Rev. B* **69**, 235310 (2004); X. F. Wang and P. Vasilopoulos, *Phys. Rev. B* **72**, 165336 (2005).
- [24] J. Nitta, F. E. Meijer, and H. Takayanagi, *Appl. Phys. Lett.* **75**, 695 (1999); F. E. Meijer, A. F. Morpurgo, and T. M. Klapwijk, *Phys. Rev. B* **66**, 033107 (2002); T. Choi, S. Y. Cho, C.-M. Ryu, and C. K. Kim, *Phys. Rev. B* **56**, 4825 (1997); D. Frustaglia and K. Richter, *Phys. Rev. B* **69**, 235310 (2004); S.-Q. Shen, Z.-J. Li, and Z. Ma, *Appl. Phys. Lett.* **84**, 996 (2004).
- [25] M. Konig, A. Tschetschetkin, E. M. Hankiewicz, J. Sinova, V. Hock, V. Daumer, M. Schäfer, and C. R. Becker, *Phys. Rev. Lett.* **96**, 076804 (2006); T. Kobayashi, Y. Sekine, and J. Nitta, *Phys. Rev. Lett.* **97**, 196803 (2006); J. Nitta and T. Bergsten, *New J. Phys.* **9**, 341 (2007).
- [26] T. Ihn, *Electronic Quantum Transport in Mesoscopic Semiconductor Structures* (Springer-Verlag, Berlin, 2004).
- [27] C. Filgueiras and F. Moraes, *Phys. Lett. A* **361**, 13 (2007); K. Bakke and C. Furtado, *Phys. Rev. D* **80**, 024033 (2009); *Eur. Phys. J. C* **69**, 531 (2010); M. Alvarez, *J. Phys. A* **32**, 4079 (1999); K. Bakke, A. M. de M. Carvalho, and C. Furtado, *Eur. Phys. J. C* **63**, 149 (2009); K. Bakke and C. Furtado, *Phys. Rev. D* **82**, 084025 (2010); K. Bakke, *J. Math. Phys. (N.Y.)* **51**, 093516 (2010); K. Bakke, C. Furtado, and S. Sergeenkov, *Europhys. Lett.* **87**, 30002 (2009).
- [28] C. R. Hagen and D. K. Park, *Ann. Phys. (N.Y.)* **251**, 45 (1996); C. R. Hagen, *Phys. Rev. D* **48**, 5935 (1993); **41**, 2015 (1990).
- [29] M. Reed and B. Simon, *Methods of Modern Mathematical Physics II* (Academy Press, New York, 1975).
- [30] E. Langmann, A. Laptev, and C. Paufler, *J. Phys. A* **39**, 1057 (2006); T. J. Allen, *Nucl. Phys.* **B360**, 409 (1991); V. R. Khalilov, *Phys. Rev. A* **71**, 012105 (2005); V. R. Khalilov and C.-L. Ho, *Ann. Phys. (N.Y.)* **323**, 1280 (2008); D. K. Park and J. G. Oh, *Phys. Rev. D* **50**, 7715 (1994); Ph. S. Gerbert, *Phys. Rev. D* **40**, 1346 (1989).
- [31] B. S. Kay and U. M. Studer, *Commun. Math. Phys.* **139**, 103 (1991).
- [32] C. Filgueiras, E. O. Silva, W. Oliveira, and F. Moraes, *Ann. Phys. (N.Y.)* **325**, 2529 (2010); C. Filgueiras and F. Moraes, *Ann. Phys. (N.Y.)* **323**, 3150 (2008).
- [33] C. Furtado and F. Moraes, *J. Phys. A* **33**, 5513 (2000).
- [34] M. O. Katanaev and I. V. Volovich, *Ann. Phys. (N.Y.)* **216**, 1 (1992).
- [35] S. Azevedo, *Phys. Lett. A* **293**, 283 (2002).
- [36] C. Furtado, B. G. C. da Cunha, F. Moraes, E. R. B. de Mello, and V. B. Bezerra, *Phys. Lett. A* **195**, 90 (1994).