R^n gravity and the chameleon

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The Solar System bounds on R^n gravity are often ignored in the literature by invoking the chameleon mechanism. We show that in order for the latter to work, the exponent n must be ridiculously close to unity and, therefore, these theories are severely constrained.

The acceleration of the cosmic expansion discovered with type Ia supernovae [1] still lacks a satisfactory explanation. The hypothetical dark energy which is supposed to drive this acceleration is an *ad hoc* explanation: it cannot be detected directly in the laboratory and is extremely exotic due to its negative pressure P. Its equation of state should be $P \simeq -\rho$ (where ρ is the comoving energy density) and phantom energy, which opens the door to much trouble with its instabilities and thermodynamical behavior, is not at all excluded by the observations. Much theoretical effort has gone into proposing an abundance of models for dark energy and to constrain it observationally (see [2] for a detailed discussion and for references). An alternative approach consists of dispensing with dark energy and postulating, instead, that Einstein's theory of general relativity (GR) fails at the largest scales and that, with the cosmic acceleration, we have detected departures from the expected GR behavior. This proposal [3,4] has led to a revival of $f(R)$ or "modified" gravity, described by the action

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(R) + S^{(m)}, \qquad (1)$$

where g is the determinant of the spacetime metric g_{ab}, R is the Ricci scalar, $\kappa = 8\pi G$, G is Newton's constant, and $S^{(m)}$ is the matter action. This class of theories, which reduces to GR for a linear function $f(R)$, comes in three versions: metric, Palatini, and metric-affine formalisms (see [5,6] for reviews and [7] for introductions). The more complicated metric-affine formalism [8] is not fully developed yet and has seen little use in cosmology. Inside matter the Palatini formalism, in which the metric and the connection are treated as independent variables, is riddled with problems unless its field equations get modified by higher order terms [9] and, therefore, we will discuss here only the metric formalism, in which the connection is the metric connection (the distinction between metric and Palatini formalisms is irrelevant for GR, but the two variations produce inequivalent field equations for nonlinear $f(R)$ functions).

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Metric $f(R)$ gravity contains a scalar degree of freedom, identified with $\phi \equiv f'(R)$. In fact, metric $f(R)$ gravity is a Brans-Dicke theory [10] with parameter $\omega = 0$ and a special potential for the Brans-Dicke field ϕ [11]. Starting from the action (1) and introducing a new field χ, the action

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g}[f(\chi) + f'(\chi)(R - \chi)] + S^{(m)} \quad (2)$$

is dynamically equivalent to (1). Variation with respect to χ yields $f''(\chi)(R - \chi) = 0$ and $\chi = R$ if $f''(R) \neq 0$, and the action (1) is reproduced. If we define the field $\phi \equiv f'(\chi)$ and set

$$V(\phi) = \chi(\phi)\phi - f(\chi(\phi)), \qquad (3)$$

the action becomes [12]

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g}[\phi R - V(\phi)] + S^{(m)}, \qquad (4)$$

an $\omega = 0$ Brans-Dicke theory [10].

Many choices for the function $f(R)$ have appeared in the literature, and there are viable ones which satisfy both theoretical viability criteria (such as correct cosmological dynamics, smooth transition between different cosmological eras, well-posed initial value problem, stability, correct weak-field limit and dynamics of cosmological perturbations) and experimental constraints [5,6]. There is a large body of literature ([14,15] and references therein) on the choice $f(R) = \alpha R^n$ (where $\alpha > 0$ has the dimensions of a mass squared and n is not restricted to be an integer), on which we focus. Let us be clear on the terminology here: often, the literature refers to the theory described by $f(R) = R + \alpha R^2$ motivated by quantum corrections to the Einstein-Hilbert Lagrangian as "R^2-gravity" (and, consequently, to $f(R) = R + \alpha R^n$ as "R^n-gravity"). This is not what we mean here: the term "R^n-gravity" in this paper refers strictly to the choice $f(R) = \alpha R^n$ and our considerations apply only to this class of theories (the prospects appear much better for $f(R) = R + \alpha R^n$ theories).

R^n gravity, like any $f(R)$ theory, is subject to experimental constraints: while, from the mathematical physics point of view, it is perfectly acceptable to study this theory as a toy model in order to obtain analytical or qualitative

insight on exact solutions, or on the role that the scalar degree of freedom $f'(R)$ may play in modifying GR, or to replace the full theory $f(R) = R + \alpha R^n$ with αR^n (which is mathematically easier to handle) for $n > 0$ in the strong gravity regime, the exponent n is not an entirely free parameter if the theory is meant to constitute a realistic alternative to dark energy.

Let us first consider two basic theoretical requirements associated with stability. First, avoiding the Dolgov-Kawasaki instability [16] leads to $f''(R) \geq 0$ [17,18] which corresponds to $n \leq 0$ or $n \geq 1$ for $f(R) = \alpha R^n$. Second, on time scales shorter than the Hubble time, one models the present universe as a de Sitter one, and de Sitter space is usually found to be a late-time attractor in $f(R)$ and dark energy models. Therefore, it is important that de Sitter space be stable, too, which provides the second criterion. The mass m of the scalar field $\phi = f'(R)$ in de Sitter space is given by

$$m^2 = \frac{1}{3}\left(\frac{f'_0}{f''_0} - R_0\right), \qquad (5)$$

where a zero subscript denotes quantities evaluated in the de Sitter space with Ricci scalar R_0. Equation (5) has been derived in a variety of ways, including the weak-field limit [19–21], gauge-invariant perturbation analyses of de Sitter space [22], and calculations of the propagator of $f(R)$ gravity in a locally flat background [23]. For $f(R) = \alpha R^n$, it is

$$m^2 = \frac{(2 - n)}{3(n - 1)}R_0, \qquad (6)$$

and the requirement that the field ϕ be nontachyonic is equivalent to $1 \leq n \leq 2$. We take the parameter n in the intersection of these two intervals $1 \leq n \leq 2$ (bounded from below by GR).

In order for $f(R) = \alpha R^n$ to provide a realistic alternative to dark energy, it also needs to satisfy the available experimental constraints. Writing $n \equiv 1 + \delta$, light deflection does not provide bounds [24–26] but the precession of Mercury's perihelion yields the stringent limits [24,27–29]

$$\delta = (2.7 \pm 4.5) \times 10^{-19}. \qquad (7)$$

This constraint is often ignored in studies of R^n gravity [15], based on the belief that the Solar System limits are circumvented because in the weak-field limit of general $f(R)$ gravity, the effective degree of freedom $\phi = f'(R)$ is endowed with a range which may be very small at Solar System densities and much larger at cosmological densities. This feature would enable effects on cosmological scales but would shelter ϕ from the experimental bounds in the Solar System (the chameleon mechanism at work, see below). This argument is misleading: let us examine how it applies to the weak-field limit of $f(R)$ gravity in general, and then discuss the specific R^n theory.

The weak-field limit of $f(R)$ gravity has been studied by various authors [21,30–32]. Based on the equivalence between metric $f(R)$ and $\omega = 0$ Brans-Dicke gravity and on the Cassini bound $|\omega| > 40\,000$ [33], early work dismissed all $f(R)$ theories as unviable [30]. However, the fact was missed that the Cassini limit only applies to a Brans-Dicke field with range larger than, or comparable to, the size of Solar System experiments, while the effective mass and range of the scalar field $\phi = f'(R)$ depend on the background curvature R, hence on the energy density of the environment. This is the chameleon mechanism originally discovered in quintessence models of dark energy [34], and later rediscovered in modified gravity [35]. The chameleon mechanism is not imposed to fine-tune the theory and evade the experimental limits: it is contained naturally in $f(R)$ gravity and whether it works or not depends on the specific theory considered.

In the weak-field limit of $f(R)$ theories [21,31], one considers a spherically symmetric, weakly gravitating, perturbation of mass M of a cosmological space. In an adiabatic approximation, the background is taken to be a de Sitter space (with constant curvature, $R_{ab} = R_0 g_{ab}/4$, and $R_0 = 12H_0^2$), which is a solution of $f(R)$ gravity subject to the conditions [5]

$$f'_0 R_0 = 2f_0. H_0 = \sqrt{\frac{f_0}{6f'_0}}. \qquad (8)$$

The weak-field line element is written as

$$\begin{aligned} ds^2 = &-[1 + 2\Psi(r) - H_0^2 r^2]dt^2 \\ &+ [1 + 2\Phi(r) + H_0^2 r^2]dr^2 + r^2 d\Omega^2, \end{aligned} \qquad (9)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ is the line element on the unit 2-sphere and $\Psi(r)$ and $\Phi(r)$ are post-Newtonian potentials. The goal is to compute these potentials by solving the linearized fourth order field equations and to obtain the parametrized post-Newtonian parameter $\gamma = -\Psi/\Phi$, which is subject to the Cassini bound [33]

$$|\gamma - 1| < 2.3 \times 10^{-5} \qquad (10)$$

(in GR, $\Psi = -\Phi$ is the Newtonian potential $-\kappa M/(8\pi r)$ and $\gamma = 1$). A linearized analysis assuming that $|\Psi(r)|$, $|\Phi(r)| \ll 1$, $H_0 r \ll 1$, $f(R)$ is analytical at R_0, and $mr \ll 1$ yields [21,31,32]

$$\Psi(r) = -\frac{\kappa M}{6\pi f'_0 r}, \qquad \Phi(r) = \frac{\kappa M}{12\pi f'_0 r}, \qquad \gamma = \frac{1}{2}, \qquad (11)$$

in gross violation of the Cassini bound. This result would spell the end for $f(R)$ gravity if it wasn't for the fact that the assumption of a light scalar field, $mr \ll 1$, is violated. In many $f(R)$ theories this happens naturally and the mass of ϕ is large at high (i.e., Solar System) densities and almost zero at cosmological densities [35]. But does this mechanism work for $f(R) = \alpha R^n$?

To answer this question, note that for $n = 2$ (the largest value of n allowed by theoretical stability) the mass (5) of the scalar $\phi = f'(R)$ in a de Sitter background is *exactly* zero and this field has infinite range independent of the density of the environment; therefore, it is certainly subject to Solar System constraints and R^2 gravity is ruled out experimentally.

At a first sight, it looks surprising that the mass m vanishes while the potential (3) turns out to be $V(\phi) = \frac{\phi^2}{4\alpha}$ for this theory. The solution to this apparent contradiction is that it is not $V(\phi)$, but rather the combination $\phi \frac{dV}{d\phi} - 2V(\phi)$ that enters the equation of motion for the Brans-Dicke scalar [10]

$$\Box \phi = \frac{1}{2\omega + 3}\left[8\pi T^{(m)} + \phi \frac{dV}{d\phi} - 2V \right], \quad (12)$$

where $T^{(m)}$ is the trace of the matter stress-energy tensor $T_{ab}^{(m)}$ (which, in the weak-field, slow-motion limit, reduces to $-\rho$) and $\phi \frac{dV}{d\phi} - 2V(\phi)$ vanishes identically for a purely quadratic potential [36].

Incidentally, the theory $f(R) = \alpha R^2$ with $\alpha > 0$ (in D spacetime dimensions, $f(R) = \alpha R^{D/2}$ [38]) has other peculiarities or theoretical problems [39,40]: it does not have the correct Newtonian limit [41] and Eq. (8) is satisfied for all, not for special, values of the Ricci curvature R, which leads to unpleasant consequences [42]. That something goes wrong in the weak-field limit can be seen in the post-Newtonian potentials (11) which, using $R_0 = 12H_0^2$, reduce to

$$\Psi = -2\Phi = -\frac{\kappa M H_0^{-1}}{864\pi\alpha}\frac{1}{H_0 r} = -\frac{1}{216}\frac{R_s c H_0^{-1}}{\alpha}\frac{c H_0^{-1}}{r}, \quad (13)$$

(restoring G and c) where $R_s = 2GM/c^2$ is the Schwarzschild radius of the mass M. Ψ and Φ are no longer guaranteed to be small in absolute value because $cH_0^{-1}/r \ll 1$ and it is not clear how to choose the parameter α. A more refined analysis including terms of order $H_0 r$ yields post-Newtonian potentials with Yukawa terms [25,35,43]

$$\Psi = -\frac{GM}{r}\left(1 - \frac{\delta e^{-ar}}{a^2 r}\right), \quad (14)$$

$$\Phi = \frac{GM}{r}\left[1 + \frac{\delta(1 + ar)e^{-ar}}{a^2 r}\right]. \quad (15)$$

In the limit $n \to 2^-$ in which $a \to 0$ and the range of the scalar becomes infinite, the Yukawa terms dominate the Newtonian ones and diverge. The range of ϕ *must* be kept small in order to recover even the Newtonian limit [44].

At the opposite range of values for n we have GR, which is viable and in agreement with all available Solar System experiments. Between the values $n = 1$ and $n = 2$, the
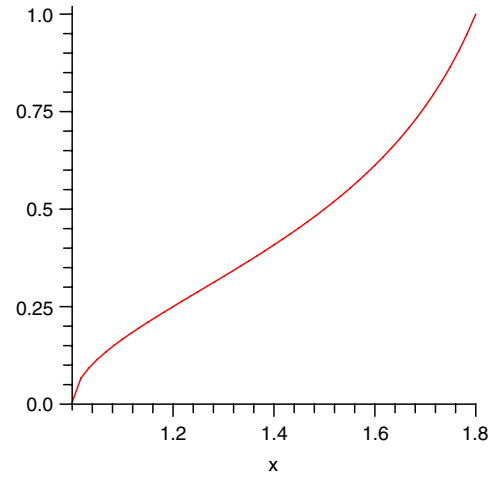

FIG. 1 (color online). The range $s(n)$ of the effective scalar degree of freedom $\phi = f'(R)$ (in units cH_0^{-1}) for the theory $f(R) = \alpha R^n$. The parameter n is in the range $1 \leq n \leq 2$ allowed by stability. The function $s(n)$ starts out with vertical tangent at $n = 1$.

range of the scalar field varies continuously but rapidly between zero and infinity. This range is given by the function

$$s(n) = \frac{1}{2}\sqrt{\frac{n-1}{2-n}} c H_0^{-1} \quad (16)$$

in the interval $[1, 2]$. This function varies continuously between $s(1) = 0$ and its limit $\lim_{s \to 2^-} s(n) = +\infty$, always increasing. The derivative $s'(n) = \frac{cH_0^{-1}}{2(n-1)^{1/2}(2-n)^{3/2}}$ is always positive and the tangent to the graph of $s(n)$ starts vertically at $n = 1$ and ends vertically as $n \to 2^-$, which means that the range of ϕ increases quickly as the R^n-theory departs very slightly from GR (see Fig. 1). Clearly, as long as the exponent n is very close to unity, the theory behaves as GR and passes the experimental tests while, approaching values of n closer to 2, the experimental bounds begin being violated, and disaster happens in the limit $n \to 2^-$. In conclusion, only for very small values of n it is possible to invoke the chameleon mechanism in the weak-field analysis. By imposing the range of the scalar to be less than 1 AU $(1.496 \times 10^{13}$ cm) and using the value $H_0 = 70$ km $\cdot$ s$^{-1} \cdot$ Mpc^{-1} for the Hubble parameter, one would obtain the requirement

$$0 \leq \delta \equiv n - 1 \leq 5 \times 10^{-30}. \quad (17)$$

Of course, realistic Solar System experiments do not have this level of precision, and the limit (7) applies instead. This renders R^n gravity a poor candidate for a realistic alternative to dark energy.

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