

Dynamics of the Laplace-Runge-Lenz vector in the quantum-corrected Newton gravity

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Recently it was shown that quantum corrections to the Newton potential can explain the rotation curves in spiral galaxies without introducing the dark matter halo. The unique phenomenological parameter $\alpha\nu$ of the theory grows with the mass of the galaxy. In order to better investigate the mass-dependence of $\alpha\nu$ one needs to check the upper bound for $\alpha\nu$ at a smaller scale. Here we perform the corresponding calculation by analyzing the dynamics of the Laplace-Runge-Lenz vector. The resulting limitation on quantum corrections is quite severe, suggesting a strong mass-dependence of $\alpha\nu$.

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I. INTRODUCTION

It is a common belief nowadays that the general relativity (GR) is not the ultimate theory of gravity. One reason for this is that the relevant solutions of GR, such as the spherically symmetric solution and the homogeneous and isotropic one, both manifest singular behavior in their extremes. In the first case, the space-time singularity is in the center of the black hole, and in the second case, it is in the initial instant of the universe “big bang.” In both cases, the singularity is surrounded by a very small space-time region with very high magnitudes of curvature tensor components. This makes it perfectly possible that the higher derivative terms in the gravitational action may change the geometry in such a way that the singularities should disappear. The importance of higher derivative terms in the gravitational action is due to the fact that they are requested for constructing a renormalizable theory of matter (including standard model) on curved background (see, e.g., [1] and further references therein). The effects of higher derivative terms on singularities were discussed in the cosmological (see, e.g., [2–4]) and black hole (see, e.g., [5,6]) settings and there are serious reasons to consider the possibility of erased singularity due to the higher derivative terms in the classical action and quantum corrections [7].

Despite the fact that the discussion of higher derivative terms and their possible effect on singularities is interesting, we are much more curious about possible modifications of gravity and, especially, about possible quantum effects at low-energy scale, where the observations are much more real. At low energies, the effect of higher derivative terms is usually assumed to be Planck-suppressed and one has to

deal with, e.g., quantum corrections to the Hilbert-Einstein action. For the higher derivative section of the theory, direct calculations of the low-energy quantum contributions were recently performed in [8,9] using the approximation of linearized metric on flat background. At the same time, it was demonstrated in [8] that this kind of calculational technique is useless for deriving quantum corrections for the cosmological and Hilbert-Einstein terms. Therefore, although this part is much more interesting from the physical point of view, here we have achieved much less. Since the subject looks quite relevant for applications, it is worthwhile to try some phenomenological approaches in this case.

In what follows, we will consider the low-energy quantum corrections to the Newton potential using the renormalization group technique and the identification of renormalization scale, which was recently proposed in [10] on the phenomenological basis and then justified theoretically in [11]. This identification of scale includes some uncertainty, which is measured by a dimensionless phenomenological parameter $\alpha\nu$. It was shown in [10] that values of $\alpha\nu$ of the order of 10^{-7} can provide the detailed and precise explanation for the rotation curves for a small but quite representative sample of spiral galaxies. It is remarkable that the mentioned parameter is steadily growing with the increase of the mass of the galaxy. Therefore, it is natural to expect a much smaller value for $\alpha\nu$ for much smaller astrophysical systems, such as stars. Our purpose is to set an upper bound on $\alpha\nu$ for the Solar System by using a very efficient approach based on the dynamics of the Laplace-Runge-Lenz (LRL) vector (see, for instance, [12] and references therein for the introduction; for an interesting historical review, see [13]).

This paper is organized as follows. In the next section, we present very general arguments about unique possible

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form of renormalization group running of Newton constant G in the IR region. In Sec. III, we offer a brief introduction and necessary information about the Laplace-Runge-Lenz vector in the almost Keplerian problem. The numerical estimates and the upper bound for $\alpha\nu$ from the Mercury precession are derived in Sec. IV. In Sec. V, we draw our conclusions.

II. QUANTUM EFFECTS AND NEWTON POTENTIAL

One of the most powerful techniques for evaluating quantum corrections is renormalization group. So, let us check out what is the renormalization group equation in the low-energy gravitational sector. The unique relevant parameter at the astrophysical scale is the Newton constant G , and at the quantum level it becomes a running parameter $G(\mu)$, where μ defines a scale. The problem of identifying μ with some physical quantity will be discussed later on, and now we concentrate on the dependence $G(\mu)$, which is always governed by the corresponding renormalization group equation $\mu(dG/d\mu) = \beta_G$.

Consider an *arbitrary* quantum theory with gravity. It can be, for instance, some quantum theory of the gravitational field or quantum theory of matter fields. Every kind of quantum theory can be characterized by the massive parameters m which define scale. There may be, of course, more than one such parameter, so let us consider, for the sake of generality, the whole set $\{m_i\}$. For example, the elements of the set $\{m_i\}$ can be the masses of all particles or fields which are present in the given quantum theory.

Let us present general arguments about the possible form of the running Newton constant, $G(\mu)$. Using dimensional arguments we can establish the unique possible form of the renormalization group equation

$$\begin{aligned} \mu \frac{dG^{-1}}{d\mu} &= \sum_{\text{particles}} A_{ij} m_i m_j = 2\nu M_p^2, \\ G^{-1}(\mu_0) &= G_0^{-1} = M_p^2. \end{aligned} \quad (1)$$

In particular, in the SM-like or GUT-like theory, at the one-loop level, one has

$$\sum_{\text{particles}} A_{ij} m_i m_j = \sum_{\text{fermions}} \frac{m_f^2}{3(4\pi)^2} - \sum_{\text{scalars}} \frac{m_s^2}{(4\pi)^2} \left(\xi_s - \frac{1}{6} \right), \quad (2)$$

where the fermion masses were denoted by m_f and ξ_s is the nonminimal parameter for the scalar with the mass m_s .

At the same time, it is important to stress that Eq. (1) is *not* just a one-loop equation, but it is valid at *any* loop order and in any theory which is capable to produce the renormalization group equation for G . In general, beyond the one-loop level, the coefficients A_{ij} in Eq. (1) depend on coupling constants which are present in the theory.

One can rewrite Eq. (1) as

$$\mu \frac{d(G/G_0)}{d\mu} = -2\nu(G/G_0)^2. \quad (3)$$

Solving this equation we obtain the universal form of the scale dependence for the Newton constant.

$$G(\mu) = \frac{G_0}{1 + \nu \ln(\mu^2/\mu_0^2)}. \quad (4)$$

Here the word *universal* means either that G is not running (and this means there are no quantum effects in the low-energy gravity sector) or that such running is given by Eq. (4). From this perspective, it is not a surprise that Eqs. (3) and (4) can be met by such qualitatively different approaches as higher derivative quantum gravity [14–17], quantum theory of matter fields on curved background [1,18], and quantum theory of conformal factor [19]. The same Eq. (4) shows up also in the phenomenological approach based on the hypothesis of the Appelquist and Carazzone—like decoupling for the cosmological constant and conservation law for the quantum-corrected gravitational action [20]. The reason behind these occurrences is that any other form of $G(\mu)$ would be in conflict with very simple (and hence very safe) dimensional considerations and also with the covariance arguments which play a very significant role here [7].

The next problem is how to identify μ . As usual, this identification depends on the physical problem under discussion and there is no universal solution. In the case of the gravitational field of a pointlike mass, the most natural choice is $\mu \sim 1/r$, where r is the distance from the mass position. This choice of the scale identification has been used in various publications [20–23]. In particular, this identification enables one to roughly explain the flat rotation curves for the pointlike model of the spiral galaxies [20–22] by directly using Eq. (4). Moreover, one can achieve very good and detailed description for the good sample of rotation curves by using Eq. (4) together with more sophisticated identification of the scale,

$$\frac{\mu}{\mu_0} = \left(\frac{\Phi_{\text{Newt}}}{\Phi_0} \right)^\alpha, \quad (5)$$

where the value of Φ_0 is irrelevant and Φ_{Newt} is the Newtonian potential computed with the boundary condition of it being zero at infinity. In Eq. (5), α is a phenomenological parameter which should be defined from fitting to the observational data. Let us note that the same dependence (5) can be also obtained from the regular scale-setting procedure [24], which was applied to the present case in [11].

It is easy to see from Eqs. (4) and (5) that α always shows up as a factor in the product $\alpha\nu$. It turns out that the fit with the observational data is perfect (for a sample of nine galaxies) if we assume that the product $\alpha\nu$ is about 10^{-7} and, moreover, this product grows up with the increasing of

the mass of the galaxy [10]. Indeed, this is a very nice feature, because then one may hope that the effect of “corrected” Newton law would be very weak at the scale of the Solar System, which has a mass of many orders of magnitude smaller than the one of a galaxy. The purpose of the present paper is to make the last statement quantitative, that is to set an upper bound on the value of $\alpha\nu$ inside the Solar System. The best available data here are about the precession of the perihelion of Mercury, so it is sufficient to deal with these data only.

III. LAPLACE-RUNGE-LENZ VECTOR

The method that we will use to calculate the precession in the orbit of the Mercury, due to quantum effect in the Newtonian potential, is based on the known Laplace-Runge-Lenz vector. Therefore we shall discuss it briefly and we will see if we can use it to calculate the precession velocity.

Consider first the motion of a particle of mass m for the nonperturbed Kepler’s problem, when there is a single Newtonian force acting on the particle,

$$\mathbf{F}_{\text{Newt}} = -\frac{k}{r^2}\hat{\mathbf{r}}, \quad (6)$$

where $k = G_0Mm$. The Laplace-Runge-Lenz vector is defined as

$$\mathbf{A} = \mathbf{p} \times \boldsymbol{\ell} - mk\hat{\mathbf{r}}, \quad (7)$$

where $\boldsymbol{\ell} = \mathbf{r} \times \mathbf{p}$ is the angular momentum vector. One can show that in the case of the nonperturbed Kepler’s problem, LRL is a constant of motion, namely,

$$\frac{d\mathbf{A}}{dt} = \frac{d\mathbf{p}}{dt} \times \boldsymbol{\ell} + \mathbf{p} \times \frac{d\boldsymbol{\ell}}{dt} - mk\frac{d\hat{\mathbf{r}}}{dt} = \mathbf{0}. \quad (8)$$

Furthermore, LRL has some important relationships with other constants of motion. For example, $\boldsymbol{\ell} \cdot \mathbf{A} = 0$, hence LRL always remains in the plane of the orbit. Another important relation concerns the total energy of the particle E and its angular momentum

$$\begin{aligned} \mathbf{A}^2 &= (\mathbf{p} \times \boldsymbol{\ell} - mk\hat{\mathbf{r}}) \cdot (\mathbf{p} \times \boldsymbol{\ell} - mk\hat{\mathbf{r}}) \\ &= m^2k^2\left(1 + \frac{2E\ell^2}{mk^2}\right). \end{aligned} \quad (9)$$

Remembering that for the Kepler problem the eccentricity of the orbit is related to the energy and angular momentum as

$$\varepsilon = \sqrt{1 + \frac{2E\ell^2}{mk^2}}, \quad (10)$$

we can write the modulo of the LRL vector as

$$|\mathbf{A}| = mk\varepsilon. \quad (11)$$

One can see that the magnitude of the LRL vector measures the eccentricity of the orbit. Moreover, one can arrive

at the equation of the orbit by taking scalar product of the LRL with the position vector, namely

$$r|\mathbf{A}| \cos(\varphi - \varphi_0) = \mathbf{r} \cdot (\mathbf{p} \times \boldsymbol{\ell} - mk\hat{\mathbf{r}}) = \ell^2 - mkr, \quad (12)$$

where φ_0 is the angle between \mathbf{A} and polar axis. After some simple manipulations we get

$$r = \frac{\ell^2/mk}{1 + \frac{|\mathbf{A}|}{mk} \cos(\varphi - \varphi_0)}. \quad (13)$$

One can see that \mathbf{A} is pointing to the direction of symmetry of the orbit, which can be limited or unlimited. Then it is convenient to choose the polar axis in the direction of the LRL vector \mathbf{A} , e.g., by choosing $\varphi_0 = 0$. Then $\mathbf{A} = mk\varepsilon\hat{\mathbf{x}}$ and

$$r = \frac{\ell^2/mk}{1 + \varepsilon \cos\varphi} = \frac{a(1 - \varepsilon^2)}{1 + \varepsilon \cos\varphi}. \quad (14)$$

where a is the major semiaxis of the ellipse.

Once we know the properties and the interpretation of the LRL vector, we are able to deal with the Kepler’s problem when a small perturbation is introduced. In this case the new orbit will be very similar to the old one, however there will be a precession. In other words, the particle has an approximately elliptical orbit, but in such a way that the major semiaxis slowly rotates. The velocity of this rotation is called precession velocity. A comparison between theoretical predictions and available experimental (observational) data may give valuable information such as, for instance, upper bounds on relevant parameters which are present in the perturbing force. In our case, we shall follow this line by using the well-known data for the Mercury precession.

Consider a particle of mass m which moves under the action of a total force

$$\mathbf{F} = -\frac{k}{r^2}\hat{\mathbf{r}} + \mathbf{f}. \quad (15)$$

Here \mathbf{f} is a small perturbation force ($|\mathbf{f}| \ll k/r^2$), which can be noncentral, in principle. Some formulas below are general, but in fact we are mainly interested in the central perturbed force $\mathbf{f} = -\text{grad}u(r)$, where the perturbing term for the potential energy can be derived from the Eqs. (4) and (5) to be [10,20]

$$\Phi = \Phi_{\text{Newt}} + u(r), \quad u(r) = \frac{mc^2}{2} \frac{\delta G}{G_0}, \quad (16)$$

where $\delta G = G(\mu) - G_0$.

Let us present, for the sake of completeness, the short review of the derivation of Eq. (16). One can start from the expression for the action with variable G

$$S_{\text{grav}} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \frac{R}{G(\mu)}, \quad (17)$$

where μ is supposed to depend on some energy features of the gravitational field. We assume that G has a very weak

deviation from the constant value $G_0 = 1/M_{\text{Pl}}^2$, namely $G = G_0(1 + \kappa)$. In what follows we will consider κ to be a very small quantity and hence keep only first-order terms in this parameter. Our purpose is to link the action (17) with the usual one of GR, with G_0 instead of $G(\mu)$. For this end we perform the conformal transformation [20] according to

$$\bar{g}_{\mu\nu} = \frac{G_0}{G} g_{\mu\nu} = (1 - \kappa)g_{\mu\nu}. \quad (18)$$

The derivatives of κ emerge in the transformed action, but only in the second power, hence they may be neglected. Then, in the linear order in κ , the metric $\bar{g}_{\mu\nu}$ satisfies Einstein equations with constant G_0 , and the nonrelativistic limit of the two metrics $\bar{g}_{\mu\nu}$ and $g_{\mu\nu}$ is related as

$$g_{00} = -1 - \frac{2\Phi}{c^2}, \quad \text{hence } \bar{g}_{00} = -1 - \frac{2\Phi_{\text{Newt}}}{c^2}. \quad (19)$$

Here Φ_{Newt} is the usual Newton potential and Φ is an apparent potential corresponding to the nonrelativistic solution of the modified gravitational theory (17). Because of (19), we arrive at [10,20]

$$\Phi = \Phi_{\text{Newt}} + \frac{c^2}{2}\kappa = \Phi_{\text{Newt}} + \frac{c^2 \delta G}{2G_0}, \quad (20)$$

which is nothing else but (16). Now we can come back to the analysis of this formula. For the Solar System case, we can use the relation $\Phi_{\text{Newt}} = -k/r$. Hence, the identification (5) effectively coincides with the $\mu \sim 1/r$ one of the Refs. [20–23].

One can easily prove the following statement: For the case of a central perturbation force $\mathbf{f} = -\text{grad}u(r)$, the magnitude of the LRL vector varies according to the relation

$$dA^2 = -2m\ell^2 du(r) = -2m\ell^2 u'(r)dr. \quad (21)$$

The last relation shows that the magnitude of the vector \mathbf{A} varies as the distance r varies, but in such a way that it takes equal values for equal values of r . If we restrict the discussion by quasi-elliptic orbits, which are restricted in space, we have $r_1 \leq r \leq r_2$, where r_1 and r_2 are the corresponding turning points, then the modulus of the Laplace-Runge-Lenz vector \mathbf{A} will assume the same value whenever the distance r is the same. Hence, the dynamics of \mathbf{A} are perfectly useful for evaluating the precession of the orbit in the quasi-Newtonian case. The variation of the magnitude $|\mathbf{A}|$ is of the first order of magnitude in the small perturbation $u(r)$. Therefore, we can completely neglect this variation when evaluating the precession of the LRL vector, because this precession is also of the first order in $u(r)$. In what follows we will assume, for the sake of simplicity, that $|\mathbf{A}|$ is constant even when the small perturbations are present.

The time variation rate of the LRL vector is given by

$$\frac{d\mathbf{A}}{dt} = \mathbf{f} \times \boldsymbol{\ell} + \mathbf{p} \times (\mathbf{r} \times \mathbf{f}). \quad (22)$$

As we have seen above, the LRL always points towards the symmetry axis of the orbit. In the Kepler problem, for elliptical case, this symmetry axis is the major semiaxis of the orbit. Therefore, we can calculate the velocity of precession of the orbit by simply computing the velocity of precession of the LRL vector. In the present work we are concerned with the quantum correction to the Newton gravitational potential. Such correction is expected to be a very small quantity, hence it is completely fair to employ a perturbative approach. With this consideration in mind, we compute the time average of the velocity of precession of the LRL vector for one period of the unperturbed orbit. One can recall that both $\boldsymbol{\ell}$ and \mathbf{A} are constants of motion for the unperturbed orbit. Therefore, for the sake of our calculation we can simply use $\boldsymbol{\ell}$ and \mathbf{A} , instead of $\langle \boldsymbol{\ell} \rangle$ and $\langle \mathbf{A} \rangle$. Taking, then, the time average of the time derivative of the LRL vector over the period of the unperturbed orbit, we obtain

$$\left\langle \frac{d\mathbf{A}}{dt} \right\rangle = \langle \mathbf{f} \times \boldsymbol{\ell} \rangle + \langle \mathbf{p} \times (\mathbf{r} \times \mathbf{f}) \rangle = \boldsymbol{\Omega} \times \mathbf{A}, \quad (23)$$

where \mathbf{A} is the LRL vector in the unperturbed case, and we denote by $\boldsymbol{\Omega}$ the time average value of the velocity of precession of the LRL vector. Let us note that, in the left-hand side of the Eq. (23), there is a quantity which is of the first order in the perturbing force. That is why the sign of averaging can not be omitted here.

In Eq. (23) the time-averaging of a function F means

$$\langle F \rangle = \frac{1}{\tau} \int_0^\tau F[r(t), \varphi(t)] dt, \quad (24)$$

where τ is the period of the nonperturbed motion (for the closed orbit case). However, since we are mainly interested in the trajectory, $r(\varphi)$, it is convenient to perform a change of variables and trade the time integration for the angular integration, namely,

$$\langle F \rangle = \frac{m}{\ell\tau} \int_0^{2\pi} r^2(\varphi) F[r(\varphi), \varphi] d\varphi. \quad (25)$$

We note that for a central perturbative force \mathbf{f} , the angular momentum $\boldsymbol{\ell}$ is constant and the second term in the right-hand side of Eq. (22) vanishes. Then one can write, in the Cartesian basis,

$$\left\langle \frac{d\mathbf{A}}{dt} \right\rangle = \langle \mathbf{f} \times \boldsymbol{\ell} \rangle = \langle f(r) \cos\varphi \rangle \hat{\mathbf{x}} \times \boldsymbol{\ell} + \langle f(r) \sin\varphi \rangle \hat{\mathbf{y}} \times \boldsymbol{\ell}. \quad (26)$$

Since f depends only on the distance r and the non-perturbed orbit is symmetric with respect to the major semiaxis, the second term in the right-hand side of (26) vanishes and we obtain

$$\left\langle \frac{d\mathbf{A}}{dt} \right\rangle = \boldsymbol{\Omega} \times \mathbf{A}, \quad \text{where } \boldsymbol{\Omega} = -\frac{\langle f(r) \cos \varphi \rangle}{mk\varepsilon} \boldsymbol{\ell}, \quad (27)$$

describing the average precession of the orbit.

IV. SOLAR SYSTEM TESTS FOR LOGARITHMIC TERM

Let us now apply the method developed in the previous section to the case where the disturbing force is due to quantum effects. As we have already seen in Sec. II, the corresponding additional term for the gravitational potential is given by

$$u(r) = \frac{mc^2}{2} \frac{\delta G}{G_0}, \quad (28)$$

where, according to Eqs. (4) and (5), we meet logarithmic dependence

$$G(\Phi_{\text{Newt}}) = \frac{G_0}{1 + \nu\alpha \ln(\Phi_{\text{Newt}}/\Phi_0)^2}. \quad (29)$$

From the previous equation, the perturbing force acting on the particle is given by

$$\mathbf{f} = -\frac{mc^2}{2G_0} \text{grad}G(\Phi_{\text{Newt}}) = -\frac{mc^2\nu\alpha\hat{\mathbf{r}}}{r[1 + \nu\alpha \ln(r_0^2/r^2)]^2}. \quad (30)$$

From Eqs. (27) and (30), we obtain the following average precession rate,

$$\begin{aligned} \boldsymbol{\Omega} &= \frac{\nu\alpha c^2 \boldsymbol{\ell}}{k\varepsilon} \left\langle \frac{\cos \varphi}{r} \left[1 + \nu\alpha \log\left(\frac{r_0^2}{r^2}\right) \right]^{-2} \right\rangle \\ &= \frac{\nu\alpha mc^2}{\tau k\varepsilon} \frac{\boldsymbol{\ell}}{\ell} \int_0^{2\pi} \frac{r(\varphi) \cos \varphi}{[1 + 2\nu\alpha \ln(r_0/r)]^2} d\varphi. \end{aligned} \quad (31)$$

One can remember that already at the typical galaxy scale $\nu\alpha \propto 10^{-7}$ and that the expected bound for the Solar System should be essentially smaller. Therefore it is justified to use an approximation $\nu\alpha \ll 1$ and keep only first-order terms in this parameter. Then, after some simple calculations, we arrive at the following expression for the absolute value of the precession velocity:

$$\begin{aligned} \Omega &\simeq \frac{\nu\alpha mc^2}{\tau k\varepsilon} \int_0^{2\pi} r(\varphi) \cos \varphi \left[1 - 2\nu\alpha \ln\left(\frac{r_0}{r}\right) \right] d\varphi \\ &\simeq \frac{\nu\alpha mc^2}{\tau k\varepsilon} \int_0^{2\pi} r(\varphi) \cos \varphi d\varphi \\ &= \frac{\nu\alpha mc^2 a(1 - \varepsilon^2)}{\tau k\varepsilon} \int_0^{2\pi} \frac{\cos \varphi}{1 + \varepsilon \cos \varphi} d\varphi. \end{aligned} \quad (32)$$

The integral in the above expression can be easily solved and as a result we find

$$\Omega = \frac{2\pi c^2 \alpha \nu [\sqrt{1 - \varepsilon^2} - (1 - \varepsilon^2)]}{G_0 M \tau \varepsilon^2}. \quad (33)$$

One can use Eq. (32) for deriving an upper bound for the parameter $\alpha\nu$ in the Solar System. For the case of the

precession of Mercury, the uncertainty in the measurement of the velocity of precession can be found, for instance, in Weinberg's book [25] and is given by $0.41''$ per century. After some simple calculations we arrive at the upper bound,

$$\nu\alpha < 10^{-17}, \quad (34)$$

where we have used the following values:

$$\begin{aligned} G &= 6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}, & M &= 1.98 \times 10^{30} \text{kg}, \\ c &= 3 \times 10^8 \text{ms}^{-2}, & a &= 6.97 \times 10^{10} \text{m}, \\ \tau &= 0.241 \text{ years} & \varepsilon &= 0.2056. \end{aligned} \quad (35)$$

Here τ , a , ε are the period of rotation of Mercury, the major semiaxis and eccentricity of its orbit.

V. CONCLUSIONS

We have considered the running of the Newton constant G in the framework of a general renormalization group approach. The covariance and dimensional arguments lead to a unique possible form of such running, which can take place in all loop orders. The beta-function for G has one arbitrary parameter ν , which depends on the details of the given theory. Vanishing ν means there is no running at all, this means there are no relevant quantum corrections in the low-energy sector (Hilbert-Einstein) of the gravitational action.

Assuming that ν is nonzero, one can try to derive upper bound for ν from different gravitational observations. In the recent paper [10] it was shown that the identification of the renormalization group scale $\mu \sim (\Phi_{\text{Newt}})^\alpha$ provides an excellent fit for the rotation curves of the galaxies, with the product $\alpha\nu$ being about 10^{-7} and moreover steadily growing with the increase of the mass of the galaxy under consideration. No dark matter is required for this fit. In the present work, we have derived an upper bound for $\alpha\nu$ in the Solar System using the method based on the dynamics of the Laplace-Runge-Lenz vector. Using experimental data for the precession of Mercury, the maximal possible value we have obtained for $\alpha\nu$ is 10^{-17} , which implies qualitatively the same form of a running that was predicted in [11] on theoretical basis. It is worth mentioning that, had we used the available experimental data for the precession of other planets of the Solar System, our upper bound for $\alpha\nu$ would not be improved. In fact, as pointed out by Weinberg [25], ‘‘accuracy available from the major planets degrades rapidly as we move away from the sun’’, so that the best constraint for the maximal value of $\alpha\nu$ is indeed obtained from the precession of Mercury, as discussed in the text.

Indeed, both values $\alpha\nu \propto 10^{-7}$ and 10^{-17} have to be seen as maximal ones, representing upper bounds. In case of rotation curves one can admit certain amount of a dark matter (with Ω_{DM}^0 smaller than usual), which should be helpful to explain other observations (large scale structure, cosmic microwave background radiation, baryon acoustic oscillations, etc.) Let us note that the first paper exploring

the possibility of an alternative concordance model with taking into account quantum corrections is in preparation [26]. In the case of a quasi-Newtonian potential in the Solar System we can also see the value 10^{-17} as an upper bound, such that the real value of $\alpha\nu$ can be much smaller than that. However, it is definitely remarkable that the two different observations produced results which are consistent with each other and also with the theoretical prediction of [11].

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