Generalized cosmological term from Maxwell symmetries

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By gauging the Maxwell spacetime algebra, the standard geometric framework of Einstein gravity with cosmological constant term is extended by adding six four-vector fields $A_\mu^{ab}(x)$ associated with the six Abelian tensorial charges in the Maxwell algebra. In the simplest Maxwell extension of Einstein gravity this leads to a generalized cosmological term that includes a contribution from these vector fields. We also consider going beyond the basic gravitational model by means of bilinear actions for the new Abelian gauge fields. Finally, an analogy with the supersymmetric generalization of gravity is indicated. In an appendix, we propose an equivalent description of the model in terms of a shift of the standard spin connection by the $A_\mu^{ab}(x)$ fields.

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I. INTRODUCTION

It is known (see e.g. [1,2]) that dark energy may be described by adding the cosmological constant term to the standard Einstein-Hilbert action. In a geometric framework leading to gravity, a cosmological term appears when the de Sitter spacetime algebra is gauged. This algebra contains (see e.g. [3]) noncommutative four-momenta generators P_a, $[P_a, P_b] = \frac{1}{R^2} M_{ab}$, where M_{ab} are the six Lorentz generators, R is the de Sitter radius and the cosmological constant is identified as $\lambda = \frac{1}{R^2}$, $[\lambda] = M^2$.

A similar noncommutative modification of the Poincaré Abelian four-momenta commutators also appears in the $D = 4$ 16-dimensional Maxwell algebra [4,5]. This is given by

$$[P_a, P_b] = \Lambda Z_{ab}, \tag{1}$$

where the six generators Z_{ab} ($a = 0, 1, 2, 3$) commute among themselves as well as with P_a and behave as an antisymmetric second-rank Lorentz tensor. The remaining Maxwell algebra commutators are

$$\begin{aligned} &[Z_{ab}, Z_{cd}] = 0 = [P_a, Z_{cd}], \\ &[M_{ab}, P_c] = -(\eta_{ca} P_b - \eta_{cb} P_a) = -\eta_{c[a} P_{b]}, \\ &[M_{ab}, Z_{cd}] = -(\eta_{c[a} Z_{b]d} - \eta_{d[a} Z_{b]c}), \end{aligned} \tag{2}$$

plus the standard Lorentz algebra commutators for M_{ab}. Thus, the Maxwell algebra has the semidirect sum structure $\mathcal{I} \oplus so(1,3)$, where the ideal $\mathcal{I} = \langle P_a, Z_{ab} \rangle$ is itself a central extension of the Abelian translation algebra $\langle P_a \rangle$ by $\langle Z_{ab} \rangle$. The constant Λ is dimensionful, $[\Lambda] = M^2$, and is the central charge that characterizes the extension. Clearly, $[M_{ab}] = M^0$, $[P_a] = M$ and $[Z_{ab}] = M^0$.

Our aim in this paper is to consider an alternative way of introducing the cosmological term. This term will appear in a generalized form, with a dependence on the additional gauge fields associated with the new generators Z_{ab}. In this paper we shall limit ourselves to providing the new geometric framework; its applications to realistic cosmological models will not be addressed here. We shall consider the local gauging of Maxwell algebra (1) and (2), to look for possible extensions of standard gravity. Because the noncommutativity of the four-momenta in de Sitter gravity leads to the appearance of a cosmological term, it is interesting to analyze the geometrical consequences of the noncommutativity expressed by Eq. (1) in a gauged Maxwell algebra approach to gravity. Further, since this includes six gauge vector fields A_μ^{ab} associated with the Abelian Z_{ab} generators, it is interesting to recall (see e.g. [6–8]) that inflation can also be driven by suitably coupled vector fields.

In this paper we introduce the geometric framework obtained by gauging of the Maxwell group. Besides the vierbein e_μ^a and the spin connection ω_μ^{ab}, our scheme includes six vector fields A_μ^{ab} which introduce a new set of curvatures. Besides the standard torsion T^a corresponding to the translational curvature, we have now two curvature tensors, the standard Lorentz curvature tensor $R_{\mu\nu}^{ab}$ and the new $F_{\mu\nu}^{ab}$ associated with the six Abelian gauge fields A_μ^{ab}. These two tensors will be the building blocks for constructing new gravity actions. Our basic choice of the action will provide a modification of the standard gravity, given by the Einstein action plus a generalized cosmological term. Our model will depend on three constants: the new central charge Λ in Eq. (1), the conventional Einstein gravitational constant κ ($[\kappa] = M^{-2}$), and the cosmological constant λ ($[\lambda] = M^2$) accompanying the standard cosmological term.

Additional gauge fields that describe the non-Riemannian part of a connection have been considered in analysis of metric affine gravity models (see [9], Sec. 3.11; [10]); the earliest example of a connection modified by an Abelian gauge field is the Weyl connection [11]. From these considerations it follows that one can use the one-forms $A^{ab} = A^{ab}_\mu dx^\mu$ by formally extending the Riemannian connection $\omega^{ab} = \omega^{ab}_\mu dx^\mu$ to a non-Riemannian one with torsion

$$\tilde{\omega}^{ab} = \omega^{ab} - \mu A^{ab}. \quad (3)$$

We shall show further that the dimensionless parameter μ occurring in (3) is, in fact, equal to $\frac{\lambda}{\Lambda}$. The antisymmetry $A^{ab}_\mu = -A^{ba}_\mu$ tells us that we are dealing with an Einstein-Cartan geometry with nonmetricity tensor equal to zero because $\tilde{\omega}^{(ab)} = 0$ (a symmetric part of $\tilde{\omega}^{ab}$ would define the nonmetricity tensor [10]). As a result, the gauging of the Maxwell group may also be considered as the specific extension to a non-Riemannian framework determined by the structure of the Maxwell algebra.

The plan of the paper is the following. In Sec. II we provide the differential and geometric aspects of the gauging of Maxwell algebra. In Sec. III we study the Einstein action supplemented with the new generalized cosmological term, which appears naturally in the present framework as a modification of the standard four-volume form. We shall consider further the field equations and calculate the torsion generated by the fields A^{ab}_μ as power series in the parameter $\alpha = \frac{\mu^2}{\lambda}$. In order to have A^{ab}_μ as dynamical fields we add an additional piece to the action for the new Abelian gauge fields, as briefly discussed in Sec. IV. To conclude, we shall outline in Sec. V some link between the structure of the Maxwell generalization of gravity and the superextension of gravity; we shall also comment on the Maxwell extension of supergravity. The dynamics of Maxwell gravity in terms of vierbein and the shifted spin connection $\tilde{\omega}^{ab}$ in (3) is given in Appendix A.

II. GAUGING THE MAXWELL ALGEBRA

Let us introduce the set of Maxwell algebra-valued Maurer-Cartan forms

$$h = h^A X_A = e^a P_a + \frac{1}{2}\omega^{ab} M_{ab} + \frac{1}{2} A^{ab} Z_{ab}, \quad (4)$$

where $a, b = 0, 1, 2, 3$ are tangent space indices raised and lowered with the constant Minkowski metric η_{ab}. The associated gauge fields $h^A_\mu(x) = (e^a_\mu(x), \omega^{ab}_\mu(x), A^{ab}_\mu(x))$ are defined by the $D = 4$ spacetime one-form fields

$$e^a = e^a_\mu dx^\mu, \qquad \omega^{ab} = \omega^{ab}_\mu dx^\mu, \qquad A^{ab} = A^{ab}_\mu dx^\mu, \quad (5)$$

where $(e^a_\mu, \omega^{ab}_\mu)$ are the vierbein and the spin connection and the A^{ab}_μ are the new Abelian gauge fields; $[e^a] = M^{-1}$, $[\omega^{ab}] = M^0$ and, since Z_{ab} is dimensionless, $[A^{ab}] = M^0$.

The generators $X_A = (P_a, M_{ab}, Z_{ab})$ satisfy the Maxwell algebra commutation relations, $[X_A, X_B] = f_{AB}{}^C X_C$. The generic curvature two-forms of the associated gauge fields are given by

$$\mathcal{R} = dh + h \wedge h = dh + \frac{1}{2}[h, h] \equiv \mathcal{R}^A X_A. \quad (6)$$

Denoting the components of $\mathcal{R}$ by $\mathcal{R}^A = (T^a, R^{ab}, F^{ab})$, Eqs. (6), (1), and (2) give

$$T^a = de^a + \omega^a{}_c \wedge e^c \equiv (De)^a, \quad (7)$$

$$R^{ab} = d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb} \equiv (D\omega)^{ab} = -R^{ba}, \quad (8)$$

$$F^{ab} = dA^{ab} + \omega^{[a}{}_c \wedge A^{c|b]} + \Lambda e^a \wedge e^b$$
$$\equiv (DA)^{ab} + \Lambda e^a \wedge e^b = -F^{ba}, \quad (9)$$

where D is the covariant derivative with respect to ω^{ab}. Equations (7) and (8), are the standard torsion and curvature; Eq. (9) gives the curvature $(DA)^{ab}$ of the Abelian gauge fields A^{ab} plus the vierbein two-form $\Lambda e^a \wedge e^b$.

Subsequently we obtain

$$(DT)^a := dT^a + \omega^a{}_c \wedge T^c = R^{ac} \wedge e_c, \quad (10)$$

$$(DR)^{ab} = (dR + \omega \wedge R - R \wedge \omega)^{ab} = 0, \quad (11)$$

$$(DF)^{ab} = R^{[a|c} \wedge A_c{}^{b]} + \Lambda T^{[a} \wedge e^{b]}. \quad (12)$$

Under a local gauge transformation with Maxwell algebra-valued parameter $\zeta(x)$,

$$\zeta(x) = \zeta^A(x) X_A$$
$$= \xi^a(x) P_a + \frac{1}{2}\lambda^{ab}(x) M_{ab} + \frac{1}{2}\rho^{ab}(x) Z_{ab}, \quad (13)$$

h in Eq. (4) transforms as

$$\delta_\zeta h^A = d\zeta^A + f_{BC}{}^A h^B \zeta^C \equiv (\mathcal{D}\zeta)^A. \quad (14)$$

Similarly, the curvatures in Eq. (6) transform by

$$\delta_\zeta \mathcal{R}^A = f_{BC}{}^A \mathcal{R}^B \zeta^C, \quad (15)$$

which leads to

$$\delta_\zeta e^a = (D\xi)^a + e^c \lambda_c{}^a, \qquad \delta_\zeta \omega^{ab} = (D\lambda)^{ab}, \quad (16)$$

$$\delta_\zeta A^{ab} = (D\rho)^{ab} + A^{[a}{}_c \lambda^{c|b]} + \Lambda e^{[a} \xi^{b]}. \quad (17)$$

and

$$\delta_\zeta T^a = R^a{}_c \xi^c + T^c \lambda_c{}^a, \qquad \delta_\zeta R^{ab} = R^{[a}{}_c \lambda^{c|b]}, \quad (18)$$

$$\delta_\zeta F^{ab} = F^{[a}{}_c \lambda^{c|b]} + R^{[a}{}_c \rho^{c|b]} + \Lambda T^{[a} \xi^{b]}. \quad (19)$$

Thus, the two-forms T^a, R^{ab} and F^{ab} behave under local Lorentz transformations $\lambda^{ab}(x)$ in a tensorial manner.

It follows from the above that dimensionless four-form Lagrangians invariant under diffeomorphism and the local Lorentz transformations of the Einstein-Cartan theory may be constructed as bilinears in R^{ab} and F^{ab},

$$\mathcal{L}_1 = \frac{1}{2}\varepsilon_{abcd} R^{ab} \wedge R^{cd}, \quad (20)$$

$$\mathcal{L}_2 = \varepsilon_{abcd} R^{ab} \wedge F^{cd}, \qquad \mathcal{L}_3 = \frac{1}{2}\varepsilon_{abcd} F^{ab} \wedge F^{cd}. \quad (21)$$

Further, we can consider as well

$$\mathcal{L}_4 = \frac{1}{2} R^{ab} \wedge R_{ab}, \tag{22}$$

$$\mathcal{L}_5 = R^{ab} \wedge F_{ab}, \qquad \mathcal{L}_6 = \frac{1}{2} F^{ab} \wedge F_{ab}. \tag{23}$$

The terms (20) and (22) are known in a standard gravity framework. The topological density $\mathcal{L}_1$ produces a surface term which, in fact, is proportional to the Euler characteristic. The term $\mathcal{L}_4$ is also topological and corresponds to the Chern-Pontrjagin class. Our basic model will be constructed out of the Lagrangian forms in (21).

III. EINSTEIN ACTION WITH GENERALIZED COSMOLOGICAL TERM

Let us recall first that the Einstein-Hilbert action is

$$\mathcal{L}_E = -\frac{1}{2\kappa} \varepsilon_{abcd} R^{ab} \wedge e^c \wedge e^d, \tag{24}$$

where κ is the Einstein gravitational constant, $[\kappa] = M^{-2}$. Then, it is seen that $\mathcal{L}_2$ in (21) is

$$-\frac{1}{2\kappa\Lambda} \mathcal{L}_2 = -\frac{1}{2\kappa\Lambda} \varepsilon_{abcd} R^{ab} \wedge (DA)^{cd} + \mathcal{L}_E. \tag{25}$$

Now, using the Bianchi identity (11), the first term in the right-hand side of Eq. (25) is a surface term in the action:

$$d(\varepsilon_{abcd} R^{ab} \wedge A^{cd}) = \varepsilon_{abcd} R^{ab} \wedge (DA)^{cd}. \tag{26}$$

As a result, $\frac{-1}{2\kappa\Lambda} \mathcal{L}_2$ is the Einstein-Hilbert Lagrangian up to a surface term.

Let us now consider the $\mathcal{L}_3$ in (21), which is the announced Maxwell extension of the cosmological term. The standard cosmological term is given by the four-form

$$\mathcal{L}_{\text{cosm}} = \frac{\lambda}{4\kappa} \varepsilon_{abcd} e^a \wedge e^b \wedge e^c \wedge e^d. \tag{27}$$

If we observe that the curvature F^{ab} is given by (9), we see that $\mathcal{L}_3$ in Eq. (21) includes the standard cosmological term plus two additional pieces depending on A^{ab},

$$\begin{aligned} \tilde{\mathcal{L}}_{\text{cosm}} &= \frac{\lambda}{2\kappa\Lambda^2} \mathcal{L}_3 \\ &= \frac{\lambda}{4\kappa\Lambda^2} \varepsilon_{abcd}((DA)^{ab} + \Lambda e^a \wedge e^b) \wedge ((DA)^{cd} \\ &\quad + \Lambda e^c \wedge e^d) \\ &= \mathcal{L}_{\text{cosm}} + \frac{\lambda}{4\kappa\Lambda^2} \varepsilon_{abcd} (DA)^{ab} \wedge (DA)^{cd} \\ &\quad + \frac{\lambda}{2\kappa\Lambda} \varepsilon_{abcd} (DA)^{ab} \wedge e^c \wedge e^d. \end{aligned} \tag{28}$$

Using Eqs. (21) and $\mu \equiv \frac{\lambda}{\Lambda}$, we now propose the following Lagrangian four-form for Maxwell gravity

$$\begin{aligned} \mathcal{L} &= \frac{\mu}{2\kappa\lambda}(-\mathcal{L}_2 + \mu \mathcal{L}_3) \\ &= \mathcal{L}_E + \mathcal{L}_{\text{cosm}} + \frac{\mu}{2\kappa} \varepsilon_{abcd} (DA)^{ab} \wedge e^c \wedge e^d \\ &\quad + \frac{\mu^2}{4\kappa\lambda} \varepsilon_{abcd} (DA)^{ab} \wedge (DA)^{cd}. \end{aligned} \tag{29}$$

Let us compute the field equations. The variation of the Lagrangian (29) with respect to ω^{ab} gives

$$\begin{aligned} \delta_\omega \mathcal{L} = \delta\omega^{ab} \wedge [L]_{\omega^{ab}} &= d\left[-\frac{1}{2\kappa} \varepsilon_{abcd} \delta\omega^{ab} \wedge e^c \wedge e^d\right] - \frac{1}{\kappa} \varepsilon_{abcd} \delta\omega^{ab} \wedge (De)^c \wedge e^d + \frac{\mu}{\kappa} \varepsilon_{abcd} \delta\omega^a{}_e \wedge A^{eb} \wedge e^c \wedge e^d \\ &\quad + \frac{\mu^2}{\kappa\lambda} \varepsilon_{abcd} \delta\omega^a{}_e \wedge A^{eb} \wedge (DA)^{cd} \\ &= \delta\omega^{ab}\left[-\frac{1}{\kappa} \varepsilon_{abcd} \wedge \left[(De)^c \wedge e^d - \frac{\mu^2}{\lambda} A^c{}_e \wedge \left((DA)^{ed} + \frac{\lambda}{\mu} e^e \wedge e^d\right)\right]\right]. \end{aligned} \tag{30}$$

We then obtain

$$[L]_{\omega^{ab}} = -\frac{1}{\kappa} \varepsilon_{abcd}\left[(De)^c \wedge e^d - \frac{\mu^2}{\lambda} A^c{}_e \wedge F^{ed}\right] = 0. \tag{31}$$

The Eq. (31) expressed in terms of the standard torsion $T^a = (De)^a$ is the following

$$T^{[a} \wedge e^{b]} + \frac{\mu^2}{\lambda} F^{[a}{}_c \wedge A^{c|b]} = 0. \tag{32}$$

It will be further used as the algebraic equation determining the spin connection as a function of the vierbein and the new gauge fields; $\omega^{ab}_\mu(e, A)$.

The variation of (29) with respect to e^a gives

$$\begin{aligned} \delta_e \mathcal{L} = \delta e^a \wedge [L]_{e^a} &= -\frac{1}{\kappa} \varepsilon_{abcd} R^{ab} \wedge e^c \wedge \delta e^d + \frac{\lambda}{\kappa} \varepsilon_{abcd} e^a \wedge e^b \wedge e^c \\ &\quad \wedge \delta e^d + \frac{\mu}{\kappa} \varepsilon_{abcd} (DA)^{ab} \wedge e^c \wedge \delta e^d \\ &= -\frac{1}{\kappa} \delta e^a \varepsilon_{abcd} \wedge [R^{bc} \wedge e^d - \lambda e^b \wedge e^c \wedge e^d \\ &\quad - \mu (DA)^{bc} \wedge e^d] \end{aligned} \tag{33}$$

so that, using (9),

$$[L]_{e^a} = -\frac{1}{\kappa}\varepsilon_{abcd}[R^{bc}\wedge e^d - \mu F^{bc}\wedge e^d] = 0. \quad (34)$$

The curvature satisfies the field equation

$$\varepsilon_{abcd}e^b\wedge(R^{cd} - \lambda e^c\wedge e^d - \mu(DA)^{cd}) = 0. \quad (35)$$

The variation of (29) with respect to A^{ab} gives

$$\begin{aligned}\delta_A\mathcal{L} &= \delta A^{ab}\wedge[L]_{A^{ab}}\\ &= d\Big[\frac{\mu}{2\kappa}\varepsilon_{abcd}\delta A^{ab}\wedge e^c\wedge e^d\\ &\quad + \frac{\mu^2}{2\kappa\lambda}\varepsilon_{abcd}\delta A^{ab}\wedge(DA)^{cd}\Big]\\ &\quad + \frac{\mu}{\kappa}\varepsilon_{abcd}\delta A^{ab}\wedge(De)^c\wedge e^d\\ &\quad + \frac{\mu^2}{2\kappa\lambda}\varepsilon_{abcd}\delta A^{ab}\wedge(DDA)^{cd},\end{aligned} \quad (36)$$

from which it follows that

$$[L]_{A^{ab}} = \frac{\mu}{\kappa}\varepsilon_{abcd}\Big[(De)^c\wedge e^d + \frac{\mu}{\lambda}R^c{}_e\wedge A^{ed}\Big] = 0. \quad (37)$$

Equation (37) can be written alternatively using the torsion as

$$T^{[a}\wedge e^{b]} + \frac{\mu}{\lambda}R^{[a}{}_e\wedge A^{e|b]} = 0. \quad (38)$$

A special solution of Eq. (35) is given by

$$R^{ab} = \mu(DA)^{ab} + \lambda e^a\wedge e^b = \mu F^{ab}. \quad (39)$$

If Eq. (39) holds, after using the Bianchi identities (11) and (12) one obtains Eq. (38), which can be rewritten as

$$(DF)^{ab} = 0. \quad (40)$$

Further, if we insert Eq. (39) in Eq. (38) we get Eq. (32). We see therefore that the set of equations of motion (32), (35), and (38), are satisfied if the Lorentz and gauge connections are related by (39).

Let us now solve Eq. (31)) or Eq. (32) by expressing ω^{ab} in terms of the vierbein and A^{ab}. First we note that Eqs. (31) are six three-form equations

$$\varepsilon_{abcd}\Big[(De)^c\wedge e^d - \frac{\mu^2}{\lambda}A^c{}_e\wedge\Big((DA)^{ed} + \frac{\lambda}{\mu}e^c\wedge e^d\Big)\Big] = 0, \quad (41)$$

depending linearly on the 24 unknowns ω^{ab}_μ. Since the number of equations and unknowns match, in principle Eq. (41) can be solved algebraically. We recall that in the standard gravity ($\mu = 0$) the equation

$$\varepsilon_{abcd}(De)^c\wedge e^d = 0, \quad\rightarrow\quad T^c = (De)^c = de^c + \omega^{cd}\wedge e_d = 0, \quad (42)$$

is solved assuming regularity of $e_\mu{}^a$ as

$$\begin{aligned}\omega_{ab} &= \omega^{(0)}_{ab} = \frac{1}{2}(W_{bc,a} + W_{ca,b} - W_{ab,c})e^c,\\ W_{ab,c} &\equiv e_a{}^\rho e_b{}^\sigma\partial_{[\rho}e_{\sigma]c}.\end{aligned} \quad (43)$$

Equation (41) is simpler if we use the shifted connection $\tilde\omega^{ab} = \omega^{ab} - \mu A^{ab}$ (see Eq. (3))

$$\varepsilon_{abcd}\Big(\tilde\omega^{ae}\wedge\Big(e_e\wedge e^b + \frac{\mu^2}{\lambda}A_{ef}\wedge A^{fb}\Big) + de^a\wedge e^b + \frac{\mu^2}{\lambda}dA^{ae}\wedge A_e{}^b\Big) = 0, \quad (44)$$

or, equivalently,

$$\frac{1}{2}\varepsilon_{abcd}(d\mathcal{K}^{ab} + \tilde\omega^{[ae}\wedge\mathcal{K}_e{}^{b]}) = 0, \quad (45)$$

where

$$\mathcal{K}^{ab} = e^a\wedge e^b + \frac{\mu^2}{\lambda}A^a{}_f\wedge A^{fb}. \quad (46)$$

We may now find a perturbative solution of Eq. (44) for ω^{ab}. First, we write $\tilde\omega_{ab} = \omega^{(0)}_{ab} + \alpha\omega^{(1)}_{ab} + \alpha^2\omega^{(2)}_{ab} + \ldots$ or, equivalently,

$$\omega_{ab} = \mu A_{ab} + \omega^{(0)}_{ab} + \alpha\omega^{(1)}_{ab} + \alpha^2\omega^{(2)}_{ab} + \ldots, \quad (47)$$

where $\alpha = \frac{\mu^2}{\lambda}$ and $\omega^{(0)}_{ab}$ is given in Eq. (43). Inserting (47) in Eq. (41) we find

$$\begin{aligned}&\varepsilon_{abcd}[(de^c + \omega^{ce}\wedge e_e)\wedge e^d + \alpha(dA^{ce} + \omega^{cf}\wedge A_f{}^e)\wedge A_e{}^d - \mu A^c{}_e\wedge e^e\wedge e^d]\\ &\quad = \varepsilon_{abcd}[(\omega^{(0)}\wedge e)^c\wedge e^d + de^c\wedge e^d] + \alpha\varepsilon_{abcd}[(\omega^{(1)}\wedge e)^c\wedge e^d + (dA\wedge A)^{cd} + (\omega^{(0)})^{ce}\wedge(A\wedge A)_e{}^d]\\ &\qquad + \sum_{i=2}^{\infty}\alpha^i\varepsilon_{abcd}[(\omega^{(i)}\wedge e)^c\wedge e^d + (\omega^{(i-1)})^{ce}\wedge(A\wedge A)_e{}^d] = 0.\end{aligned} \quad (48)$$

Requiring that the terms for different powers of α should vanish separately we can determine recursively $\omega^{(n)}$. This defines the standard torsion as follows

$$T^a = de^a + \omega^{ab}\wedge e_b = (\mu A + \alpha\omega^{(1)} + \alpha^2\omega^{(2)} + \ldots)^{ab}\wedge e_b. \quad (49)$$

The α^0 term in Eq. (48) vanishes if we choose $\omega^{(0)}_{ab}$ as given by Eq. (43); the other terms $\omega^{(j)}_{ab}$ $(j > 0)$ follow recursively and depend on the gauge fields A^{ab}_μ and their derivatives (see Appendix B).

By solving Eq. (32), we can eliminate the spin connection ω^{ab}_μ and move to a second-order formalism, with independent variables e^a_μ and A^{ab}_μ. At the next step the differential Eqs. (35) and (38) are solved for e^a_μ and A^{ab}_μ. It is worth noting that Eq. (35) adopts the form of a generalized Einstein equation for the shifted curvature

$$J^{ab} \equiv R^{ab} - \mu F^{ab}. \tag{50}$$

After expressing J^{ab} in local coordinates Eq. (35) takes the form

$$\mathcal{J}^\mu{}_\nu - \frac{1}{2}\delta^\mu{}_\nu \mathcal{J} = (R^\mu{}_\nu - \mu \mathcal{F}^\mu{}_\nu) - \frac{1}{2}\delta^\mu{}_\nu (R - \mu\mathcal{F}) = 0 \tag{51}$$

or, equivalently,

$$G^\mu{}_\nu = R^\mu{}_\nu - \frac{1}{2}\delta^\mu{}_\nu R = \mu T^\mu{}_\nu, \qquad T^\mu{}_\nu \equiv \mathcal{F}^\mu{}_\nu - \frac{1}{2}\delta^\mu{}_\nu \mathcal{F}. \tag{52}$$

Here

$$e_a{}^\mu e_b{}^\nu J^{ab} = \frac{1}{2}\mathcal{J}^{\mu\nu}{}_{\rho\sigma} dx^\rho \wedge dx^\sigma, \qquad \mathcal{J}^\mu{}_\rho \equiv \mathcal{J}^{\mu\nu}{}_{\rho\nu}, \qquad \mathcal{J} \equiv \mathcal{J}^\mu{}_\mu, \tag{53}$$

$$e_a{}^\mu e_b{}^\nu R^{ab} = \frac{1}{2} R^{\mu\nu}{}_{\rho\sigma} dx^\rho \wedge dx^\sigma, \qquad R^\mu{}_\rho \equiv R^{\mu\nu}{}_{\rho\nu}, \qquad R \equiv R^\mu{}_\mu, \tag{54}$$

$$e_a{}^\mu e_b{}^\nu F^{ab} = \frac{1}{2}\mathcal{F}^{\mu\nu}{}_{\rho\sigma} dx^\rho \wedge dx^\sigma, \tag{55}$$

$$\mathcal{F}^\mu{}_\rho \equiv \mathcal{F}^{\mu\nu}{}_{\rho\nu} = e_a{}^\mu e_b{}^\nu (D_{[\rho}A_{\nu]})^{ab} + 3\Lambda \delta^\mu{}_\rho, \tag{56}$$

$$\mathcal{F} \equiv \mathcal{F}^\mu{}_\mu = e_a{}^\mu e_b{}^\nu (D_{[\mu}A_{\nu]})^{ab} + 12\Lambda, \tag{57}$$

where $R^{\mu\nu}{}_{\rho\sigma}$, $R^\mu{}_\rho$, R are the Riemann, Ricci and scalar curvatures and D_μ is the covariant derivative with respect to ω^{ab}_μ, which are now given as functions of e^a_μ and A^{ab}_μ. Using Eq. (56), (57), and (52) may be written as the Einstein equation in de Sitter space with cosmological constant $\lambda = \mu\Lambda$,

$$R^\mu{}_\nu - \frac{R}{2}\delta^\mu{}_\nu - 3\lambda\delta^\mu{}_\nu = \mu(e_a{}^\mu e_b{}^\sigma (D_{[\nu}A_{\sigma]})^{ab} - \delta^\mu{}_\nu e_a{}^\rho e_b{}^\sigma (D_\rho A_\sigma)^{ab}) \tag{58}$$

with the source linear in new gauge fields.

We conclude this section by noting that in Appendix A we show that the action (29) and its equations of motion may be equivalently described by using shifted spin connection $\tilde{\omega}^{ab}$ and curvature J^{ab}.

IV. DYNAMICAL TERMS FOR NEW GAUGE FIELDS

The remaining Eq. (37), obtained by varying the action (29) with respect to the fields A^{ab}_μ, does not depend explicitly on the derivatives of A^{ab}_μ. In order to have dynamical gauge fields A^{ab}_μ, terms bilinear in their derivatives are needed. In the collection of diffeomorphism invariant geometrical actions (20)–(23) only the term $\mathcal{L}_6$ could be a candidate, but due to formula (9) its nontopological part is only linear in A^{ab}_μ. Thus, to get the free action for the new gauge fields a Maxwell-like term $\tilde{\mathcal{L}}_6 = -\frac{\beta}{2} F \wedge *F$ would have to be added; however it is less geometric since the Hodge star operator involves the metric $g_{\mu\nu} = \eta_{ab} e_\mu{}^a e_\nu{}^b$. It takes the form

$$\tilde{\mathcal{L}}_6 = -\beta \frac{\sqrt{-g}}{4} g^{\mu\nu} g^{\rho\sigma} F^{ab}_{\mu\rho} F_{\nu\sigma ab} d^4x, \tag{59}$$

where $g = \det(g_{\mu\nu})$.

The field equations following from the addition of (29) and (59) look as follows

$$\delta\omega^{ab}\colon\ -\frac{1}{\kappa}\varepsilon_{abcd}\Big[(De)^c \wedge e^d - \frac{\mu^2}{\lambda} A^c{}_e \wedge F^{ed}\Big] - \beta A_{c[a} \wedge (*F)_{b]}{}^c = 0, \tag{60}$$

$$\delta e^a\colon\ -\frac{1}{\kappa}\varepsilon_{abcd}[R^{bc} \wedge e^d - \mu F^{bc} \wedge e^d] - 2\beta\Lambda(*F)_{ab} \wedge e^b - \beta T_{Fa}{}^b * e_b = 0, \tag{61}$$

$$\delta A^{ab}\colon \frac{\mu}{\kappa}\varepsilon_{abcd}[(De)^c \wedge e^d + \frac{\mu}{\lambda} R^c{}_e \wedge A^{ed}] - \beta(D * F)_{ab} = 0, \tag{62}$$

where $T_{Fa}{}^b$ is

$$T_{Fa}{}^b = e_{\mu a} e_\nu{}^b \Big(\frac{g^{\mu\nu}}{4}(F^{\rho\sigma}F_{\rho\sigma}) - \frac{1}{2}F^{(\mu\rho}F^{\nu)}{}_\rho\Big). \tag{63}$$

Equation (60) modifies the torsion relation (Eq. (32)) and changes the expression for the spin connection ω^{ab}_μ in terms of e^a_μ and A^{ab}_μ (see Appendix B for the $\beta = 0$ case). Equation (61) modifies the energy-momentum tensor in Eq. (52). Finally, Eq. (62) produces a dynamical equation for A^{ab}_μ. If we use Bianchi identity, (11) and (12) and Eq. (40) is replaced by the following one

$$(DF)^{ab} = -\frac{\beta\kappa\Lambda}{2\mu}\varepsilon^{abcd}(D * F)_{cd}. \tag{64}$$

V. FINAL REMARKS

It is often thought that the cosmological constant problem may require an alternative approach to gravity. Here we have presented a new geometric framework, based on the $D = 4$ Maxwell algebra [12], which involves six new gauge fields associated with their Abelian tensorial generators, and described its simplest application: a generalization of the cosmological term.

There are some possible extensions of this work, as

(a) Using the analogy between the semidirect sum structure of the Maxwell and supersymmetry algebras,

$$\langle P_a, Z_{ab}\rangle \oplus so(1,3), \qquad \langle Q_\alpha, P_\mu\rangle \oplus so(1,3),$$

we can obtain the bosonic Maxwell counterpart of the superspace formulation of supergravity by enlarging spacetime with the Maxwell group variables associated with the Z_{ab} generators.

(b) Recently, the simplest Maxwell superalgebra was introduced in [14]. This algebra could be gauged following the approach presented in this paper to provide an extension of the standard $D = 4$ supergravity framework. Besides the fields $A_\mu^{ab}(x)$, such an approach would include two gravitino fields: the standard gravitino and an additional one, required by the second Weyl supercharge in the Maxwell superalgebra [14].

(c) An important step in extending the model presented here would consist in adding covariantly coupled matter fields as sources, which would appear as local currents on the right-hand side of the equations for the Maxwell gravity gauge fields. As it is known, the equation for the spacetime curvature has the energy-momentum tensor as its source, and the torsion is coupled to the local spin density. In order to introduce the new local currents describing the sources of the additional gauge fields A_μ^{ab}, we should couple these gauge fields to matter invariant under the Maxwell symmetry. The new local currents would define the local densities providing, after space integration, the conserved tensorial central charges Z_{ab}.

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APPENDIX A: MAXWELL GRAVITY IN TERMS OF SHIFTED RIEMANNIAN CONNECTION $\tilde{\omega}^{ab}$

We may add to the Lagrangian (29) the topological density in Eq. (20) as follows

$$\mathcal{L} = \frac{1}{2\kappa\lambda}(\mathcal{L}_1 - \mu\mathcal{L}_2 + \mu^2\mathcal{L}_3). \tag{A1}$$

Since $\mathcal{L}_1$ is a surface term, only the last two terms contribute to the field equations. Therefore, the $\mathcal{L}$ in Eq. (A1) may be expressed as a quadratic expression in the $R(\omega)$ curvature shifted by bilinear terms in the vierbein [3,13,15] and by the new gauge fields A^{ab},

$$\mathcal{L} = \frac{1}{4\kappa\lambda}\varepsilon_{abcd}J^{ab}\wedge J^{cd}, \tag{A2}$$

where J^{ab} is given in Eq. (50). Denoting $(A^2)^{ab} = A^a{}_c \wedge A^{cb}$, we get

$$J^{ab} = R^{ab}(\omega) - \mu F^{ab} = R^{ab}(\tilde{\omega}) - \lambda e^a\wedge e^b - \mu^2(A^2)^{ab}$$
$$\equiv \tilde{R}^{ab} - \lambda e^a \wedge e^b, \tag{A3}$$

where $\tilde{\omega}^{ab}$ is given in Eq. (3) and

$$R^{ab}(\tilde{\omega}) \equiv d\tilde{\omega}^{ab} + \tilde{\omega}^a{}_c\wedge\tilde{\omega}^{cb},$$
$$\tilde{R}^{ab} \equiv R^{ab}(\tilde{\omega}) - \mu^2(A^2)^{ab}. \tag{A4}$$

Note that it is $\tilde{R}^{ab}$ rather than $R^{ab}(\tilde{\omega})$ that is the "true" curvature of the shifted connection $\tilde{\omega}^{ab}$, since $\tilde{R}^{ab}$ does not contain (because the Z_{ab} are Abelian) the $\mu^2(A^2)^{ab}$ piece that is present in $R^{ab}(\tilde{\omega})$.

The Lagrangian $\mathcal{L}$ in (A2) may then be written in the following two equivalent forms

$$\mathcal{L} = \varepsilon_{abcd}\Big(\frac{1}{4\kappa\lambda}\tilde{R}^{ab}\wedge\tilde{R}^{cd} - \frac{1}{2\kappa}\tilde{R}^{ab}\wedge e^c\wedge e^d$$
$$+ \frac{\lambda}{4\kappa}e^a\wedge e^b\wedge e^c\wedge e^d\Big) \tag{A5}$$

and

$$\mathcal{L} = \frac{1}{4\kappa\lambda}\varepsilon_{abcd}R^{ab}(\tilde{\omega})\wedge R^{cd}(\tilde{\omega})$$
$$- \frac{1}{2\kappa}\varepsilon_{abcd}e^a\wedge e^b\wedge R(\tilde{\omega})^{cd}$$
$$+ \frac{\lambda}{4\kappa}\varepsilon_{abcd}e^a\wedge e^b\wedge e^c\wedge e^d$$
$$+ \frac{\mu^4}{4\kappa\lambda}\varepsilon_{abcd}(A^2)^{ab}\wedge(A^2)^{cd}$$
$$- \frac{\mu^2}{2\kappa\lambda}\varepsilon_{abcd}(R^{ab}(\tilde{\omega}) - \lambda e^a\wedge e^b)(A^2)^{cd}. \tag{A6}$$

The first term in (A6) is an exact form and will be ignored. The second piece of $\mathcal{L}$ is the Einstein-Hilbert action for the shifted connection $\tilde{\omega}$ and the third one is the standard cosmological term. The fourth term of $\mathcal{L}$ vanishes due to the identity

$$\varepsilon_{abc[d}(A^2)^{ab}\wedge A^c{}_{e]} = 0 \tag{A7}$$

that holds for any antisymmetric one-form A^{ab}. Finally, the last term is the remaining addition to the standard cosmological term. Thus, we can write

$$\mathcal{L} = \mathcal{L}_{EH}(\tilde{\omega}) + \mathcal{L}_{\rm cosm} + \mathcal{L}_A, \tag{A8}$$

$$\mathcal{L}_A = -\frac{\mu^2}{2\kappa\lambda}\varepsilon_{abcd}(R^{ab}(\tilde{\omega}) - \lambda e^a \wedge e^b) \wedge (A^2)^{cd}. \tag{A9}$$

Let us now consider the field equations, obtained by varying $I = \int \mathcal{L}$ with respect to $\tilde{\omega}^{ab}$, e^a and A^{ab}.

$$\delta\tilde{\omega}^{cd}\colon \varepsilon_{abcd}\left((\tilde{D}e)^a \wedge e^b + \frac{\mu^2}{\lambda}(\tilde{D}A)^{ae} \wedge A_e{}^b\right) = 0, \tag{A10}$$

$$\delta e^a\colon \varepsilon_{abcd}e^b \wedge (R^{cd}(\tilde{\omega}) - \lambda e^c \wedge e^d - \mu^2(A^2)^{cd}) = 0, \tag{A11}$$

$$\delta A^{de}\colon \varepsilon_{abc[d}(R^{ab}(\tilde{\omega}) - \lambda e^a \wedge e^b) \wedge A^c{}_{e]} = 0. \tag{A12}$$

Because of identity (A7), Eq. (A12) can be replaced by

$$\varepsilon_{abc[d}(R^{ab}(\tilde{\omega}) - \lambda e^a \wedge e^b - \mu^2(A^2)^{ab}) \wedge A^c{}_{e]} = 0. \tag{A13}$$

The Bianchi identity for $R(\tilde{\omega})^{ab}$, $(\tilde{D}R(\tilde{\omega}))^{ab} = 0$, shows

$$(\tilde{D}J)^{ab} = -\lambda(\tilde{D}e)^{[a} \wedge e^{b]} - \mu^2(\tilde{D}A)^{[a}{}_c \wedge A^{c|b]}. \tag{A14}$$

Using it in Eq. (A10) the set of equations of motion becomes

$$\delta\tilde{\omega}^{ab}\colon (\tilde{D}J)^{ab} = dJ^{ab} + \tilde{\omega}^{[a|c}J_c{}^{b]} = 0, \tag{A15}$$

$$\delta e^a\colon \varepsilon_{abcd}e^b J^{cd} = 0, \tag{A16}$$

$$\delta A^{de}\colon \varepsilon_{abc[d}J^{ab}A^c{}_{e]} = 0. \tag{A17}$$

They coincide with the equations of motion (32), (35), and (38), respectively.

Writing the forms in local coordinates (see also Eq. (54))

$$e_a{}^\mu e_b{}^\nu J^{ab} = \frac{1}{2}\mathcal{J}^{\mu\nu}{}_{\rho\sigma}dx^\rho \wedge dx^\sigma, \quad e_a{}^\mu e_b{}^\nu A^{ab} = \frac{1}{2}A^{\mu\nu}{}_\rho dx^\rho. \tag{A18}$$

After assuming the invertibility for the vierbein, we obtain

$$\mathcal{J}^{\mu\nu}{}_{\rho\sigma} = R^{\mu\nu}{}_{\rho\sigma}(\tilde{\omega}) - \lambda\delta^\mu{}_{[\rho}\delta^\nu_{\sigma]} - \mu^2 A^{\mu\lambda}{}_{[\rho}A_\lambda{}^\nu{}_{\sigma]}, \tag{A19}$$

$$\mathcal{J}^\mu{}_\rho \equiv \mathcal{J}^{\mu\nu}{}_{\rho\nu} = R^\mu{}_\rho(\tilde{\omega}) - 3\lambda\delta^\mu{}_\rho + \mu^2(A^{\mu\lambda}{}_\rho A^\nu{}_{\lambda\nu} - A^{\mu\lambda}{}_\nu A^\nu{}_{\lambda\rho}), \tag{A20}$$

$$\mathcal{J} = \mathcal{J}^\mu{}_\mu = R(\tilde{\omega}) - 12\lambda + \mu^2(A^{\mu\lambda}{}_\mu A^\nu{}_{\lambda\nu} - A^{\mu\lambda}{}_\nu A^\nu{}_{\lambda\mu}), \tag{A21}$$

where $R^{\mu\nu}{}_{\rho\sigma}(\tilde{\omega})$, $R^\mu{}_\rho(\tilde{\omega})$, $R(\tilde{\omega})$ are the Riemann, Ricci, and scalar tensors for the shifted spin connection $\tilde{\omega}$. By following the derivation of Einstein equation from the Einstein-Hilbert Lagrangian (24), we obtain the generalized Einstein equation Eq. (51), with $\mathcal{J}^\mu{}_\nu$ and $\mathcal{J}$ expressed by the formulae in (A20) and (A21).

An obvious solution of Eqs. (A15)–(A17) is $J^{ab} = 0$ (see also Eq. (39)), which in the formalism with the shifted spin connection, specifies the curvature through Eq. (A3) as

$$R(\tilde{\omega})^{cd} = \lambda e^c \wedge e^d + \mu^2(A^2)^{cd}. \tag{A22}$$

In such a case the new gauge fields are arbitrary, not restricted by Eq. (A17). If, however, $J^{ab} \neq 0$, the explicit solutions of the generalized Einstein Eq. (51) will then provide a restriction on the Abelian gauge fields A^{ab}_μ since Eq. (A17) will no longer be trivial.

We mention that to the Lagrangian (A1) one can add new terms by using the Lagrangian densities (22) and (23) as follows

$$\mathcal{L}' = \frac{a}{2\kappa\lambda}(\mathcal{L}_4 - \mu\mathcal{L}_5 + \mu^2\mathcal{L}_6), \tag{A23}$$

where a is a dimensionless constant. The total Lagrangian becomes

$$\mathcal{L} + \mathcal{L}' = \frac{1}{4\kappa\lambda}(\epsilon_{abcd}J^{ab} \wedge J^{cd} + aJ_{ab} \wedge J^{ab}), \tag{A24}$$

which leads to Eqs. (A15)–(A17) but written now with the tensor $\tilde{J}^{ab} = J^{ab} - \frac{a}{4}\varepsilon^{ab}{}_{cd}J^{cd}$. As mentioned in the main text, the Lagrangian (A24) does not contain a "free" term for the A^{ab} fields; this may be achieved by adding a $(F \wedge *F)$-type term, as in Sect. IV, which is not among the densities considered in Eqs. (20), (22), and (23).

APPENDIX B: EXPRESSION FOR THE HIGHER $\omega^{(j)ab}$

The explicit expression for the higher-order terms are determined recursively as follows. We write Eq. (48), for $j = 0, 1, 2, \ldots$, as

$$\varepsilon_{abcd}\omega^{(j)ce} \wedge e_e \wedge e^d + K^{(j)}_{ab} = 0, \tag{B1}$$

where

$$K^{(0)}_{ab} = \varepsilon_{abcd}de^c \wedge e^d,$$
$$K^{(1)}_{ab} = \varepsilon_{abcd}(dA^{ce} + \omega^{(0)[cf} \wedge A_f{}^{e]}) \wedge A_e{}^d,$$
$$K^{(i)}_{ab} = \varepsilon_{abcd}\omega^{(i-1)ce} \wedge (A \wedge A)_e{}^d, \qquad (i = 2, 3, \ldots). \tag{B2}$$

If we express the three-form $K^{(j)}_{ab}$ in terms of the three-forms $*e_c$ as

$$K^{(j)}_{ab} = K^{(j)}_{ab,c}(*e^c), \qquad e^a \wedge e^b \wedge e^c \equiv \varepsilon^{abcd}(*e_d), \tag{B3}$$

we find that $\omega^{(j)}_{ab}$ is given by

$$\omega_{ab}^{(j)} = \frac{1}{2}((K_{bc,a}^{(j)} + K_{ca,b}^{(j)} - K_{ab,c}^{(j)})e^c + K_{[ae}^{(j),e}e_{b]}) = -\omega_{ba}^{(j)}. \quad \text{(B4)}$$

For $j = 0$ this recovers (43). For $j > 0$, the $\omega_{ab}^{(j)}$ are found using

$$K_{ab}^{(1),h} = \varepsilon_{abcd}\varepsilon^{\mu\nu\rho\sigma}(D_\mu^{(0)}A_\nu)^c{}_e A_\rho^{ed} e^{-1} e_\sigma{}^h, \quad \text{(B5)}$$

$$K_{ab}^{(i),h} = \varepsilon_{abcd}\varepsilon^{\mu\nu\rho\sigma}\omega_\mu^{(i-1)cf} A_{\nu fe} A_\rho^{ed} e^{-1} e_\sigma{}^h, \quad (i = 2, 3, \ldots) \quad \text{(B6)}$$

where $e = \det(e_\mu{}^a)$ and $D^{(0)}$ is the covariant derivative with respect to the connection $\omega_{ab}^{(0)}$.

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