Electromagnetic power of merging and collapsing compact objects

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Understanding possible electromagnetic signatures of merging and collapsing compact objects is important for identifying possible sources of the LIGO signal. Electromagnetic emission can be produced as a precursor to the merger, as a prompt emission during the collapse of a neutron star and at the spin-down stage of the resulting Kerr-Newman black hole. For the neutron star–neutron star mergers, the precursor power scales as $L \approx B_{\rm NS}^2 G M_{\rm NS} R_{\rm NS}^8/(R_{\rm orb}^7 c)$, while for the neutron star–black hole mergers, it is $(GM/(c^2 R_{\rm NS}))^2$ times smaller. We demonstrate that the time evolution of the axisymmetric force-free magnetic fields can be expressed in terms of the hyperbolic Grad-Shafranov equation, and we formulate the generalization of Ferraro's law of isorotation to time-dependent angular velocity. We find an exact nonlinear time-dependent Michel-type (split-monopole) structure of magnetospheres driven by spinning and collapsing neutron stars in Schwarzschild geometry. Based on this solution, we argue that the collapse of a neutron star into a black hole happens smoothly, without the natural formation of current sheets or other dissipative structures on the open field lines; thus, it does not allow the magnetic field to become disconnected from the star and escape to infinity. Therefore, as long as an isolated Kerr black hole can produce plasma and currents, it does not lose its open magnetic field lines. Its magnetospheric structure evolves towards a split monopole, and the black hole spins down electromagnetically (the closed field lines get absorbed by the hole). The "no-hair theorem," which assumes that the outside medium is a vacuum, is not applicable in this case: highly conducting plasma introduces a topological constraint forbidding the disconnection of the magnetic field lines from the black hole. Eventually, a single random large scale spontaneous reconnection event will lead to magnetic field release, shutting down the electromagnetic black hole engine forever. Overall, the electromagnetic power in all the above cases is expected to be relatively small. We also discuss the nature of short gamma-ray bursts and suggest that if the magnetic field is amplified to $\sim 10^{14}$ G during the merger or the core collapse, the similarity of the early afterglow properties of long and short gamma-ray bursts can be related to the fact that in both cases a spinning black hole can retain a magnetic field for a sufficiently long time to extract a large fraction of its rotational energy and produce high energy emission via the internal dissipation in the wind.

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I. INTRODUCTION

Estimating a possible electromagnetic signature of merging and collapsing neutron stars is most desirable for the gravitation wave searches by LIGO and for identifying possible progenitors of short gamma-ray bursts (GRBs). Collapse of a neutron star into a black hole may proceed either through the accretion induced collapse (AIC) or during binary neutron star mergers. We expect that, at late stages, both processes proceed along a somewhat similar path: in the case of a merger, the two collapsing neutron stars form a transient supermassive neutron star which then collapses into a black hole. Both an accreting neutron star (in the case of an AIC) and the transient supermassive neutron star are expected to be magnetized. In addition, in the case of merging neutron stars the strong shearing of matter may increase the magnetic field well above the initial values.

In the case of a merger of compact stars electromagnetic power can be generated as a precursor to the merger due to either effective friction of the neutron star magnetospheres or a purely general relativistic effect; see Sec. II. Later, and in the case of the AIC, several types of electromagnetic emission can be foreseen. First, the electromagnetic power in vacuum may be generated directly, due to the changing magnetic moment of the collapsing star [1,2]. Even if the outside medium is highly conducting, electromagnetic emission may be generated via an effective (resistive) disconnection of the external magnetic fields, provided that the collapse naturally leads to the formation of a narrow dissipative current structure. Second, a pulsar-like electromagnetic power can be generated by the rotation of the neutron stars and extracted via the magnetic field. As we argue below, as long as the black hole can produce plasma via vacuum breakdown, it can self-generate electric currents, retain the magnetic fields, and spin-down electromagnetically for time periods much longer than the collapse time; see Sec. III.

Conventionally, in estimating the possible electromagnetic signatures it was first assumed that a fraction $R_{\rm NS}/R_G$ of the initial external magnetic energy (also built up by the collapse and compression of the magnetic field) is radiated away on a time scale of the order of the collapse time [3]. References [4,5] considered radiation from accelerated changes in the magnetic moment during collapse, producing energy $E \sim B_0^2(R_G R_{NS})^{3/2}$ (somewhat smaller than the energy of the magnetic field before the collapse). Along similar lines, Ref. [2] employed general relativistic MHD simulations and followed the collapse of a nonrotating neutron star into a black hole.

In our view the main limitation of these models is that the external medium was treated as a vacuum. Electrodynamically, a vacuum is a highly resistive medium, with an impedance of the order of $4\pi/c = 477\Omega$. As a result, nothing prevents magnetic fields from becoming disconnected from the star and escaping to infinity. We expect that the magnetic field dynamics would be drastically different if the external magnetosphere were treated as a highly conducting medium. This is a common consequence in relativistic astrophysical sources, since an ample supply of plasma is available through vacuum breakdown. For example, investigating the dynamics of the magnetic field in the simulations in Ref. [2] (see also [6]) shows that during the collapse the magnetic field becomes effectively disconnected from the star, at distances somewhat larger than the Schwarzschild radius. If the outside medium were treated as highly conducting plasma, such processes would be prohibited. The importance of resistive effects in the magnetosphere was stressed early on in the original paper [1], where it was pointed out that "for spherically symmetric collapse there is no energy released to the outside at all."

The magnetic field may still escape to infinity if the collapse naturally creates conditions favorable for reconnection, e.g., by forming narrow current sheets or leading to the overall breakdown of fluid approximation by creating regions where the electric field exceeds the magnetic field (the latter regions are naturally created both in pulsar magnetospheres [7] and near black holes moving through the external magnetic field [8]). In this paper we address the following question: "Does the collapse of a rotating magnetized neutron star naturally create a condition for efficient reconnection of magnetic field lines well before the footpoints cross the horizon?" We argue that this does not happen.

The plan of the paper is the following. In Sec. II we discuss possible types of precursor emission in neutron star–neutron star (NS-NS), neutron star–black hole (NS-BH), and black hole–black hole (BH-BH) mergers. In the main section, Sec. III, we find exact solutions for the structure of collapsing magnetospheres. Based on this solution we argue that, as long as the resulting black hole can produce plasma and currents by vacuum breakdown, it may produce electromagnetic power much longer than the collapse time.

II. PRECURSOR EMISSION IN MERGERS

For merging compact objects (NS-NS, BH-NS, BH-BH) a number of mechanisms can generate precursors or afterglow emission. In the case of merging neutron stars, one expects an electromagnetic precursor due to effective "friction" of the neutron stars' magnetospheres [9–11]. Qualitatively, a neutron star moving through a magnetic field generates an inductive potential drop, inducing real charges on the surface, which in turn produce a component of the electric field along the magnetic field and electric currents. The estimate of the corresponding power is

$$L_U \approx B_{NS}^2 R_{NS}^2 \beta^2 c = B_{NS}^2 GM \frac{R_{NS}^8}{R_{orb}^7 c}, \tag{1}$$

where B_{NS} is the surface magnetic field of a neutron star, R_{NS} is its initial radius, M is its mass, and $\beta = v/c$ is the dimensionless velocity of a neutron star. The last equality in Eq. (1) assumes a Keplerian orbit with radius R_{orb}. The estimate (1) can be derived by calculating the potential drop across the neutron star, $\Delta\Phi \approx \beta B_{NS} R_{NS}$, and assuming the resistance of the resulting electric circuit to be close to the vacuum inductance $\sim 4\pi/c$.

Just before contact, the unipolar power (1) is

$$L_{U,max} = 6 \times 10^{45}\ \mathrm{ergs}^{-1}\left(\frac{B_{NS}}{10^{12}\ \mathrm{G}}\right)^2 \tag{2}$$

for $M_{NS} = 1.4M_\odot$ and $R_{NS} = 10$ km.

The total electromagnetic energy produced by the unipolar induction mechanism can be found by integrating power (1) with the radius evolving due to radiation of gravitational waves, $R = R_{LC}(1 - (GM)^3 t/(c^5 R_{LC}^4))^{1/4}$, and a magnetic field scaling as $B = B_{NS}(R_{NS}/R)^3$ (the model becomes applicable when the magnetospheres of the neutron stars touch at the light cylinder distance R_{LC}; at earlier times the interaction is through winds and scales as a sum of the spin-down powers of the neutron stars) (see [11]),

$$E_{tot,U} = \int_{t(R_{LC})}^{t(R_{NS})} L_{U,GR}\, dt \approx B_{NS}^2 R_{NS}^3 \left(\frac{R_{NS}}{R_G}\right)^2 = 3 \times 10^{43}\ \mathrm{erg}\left(\frac{B_{NS}}{10^{12}\ \mathrm{G}}\right)^2. \tag{3}$$

In addition, there is a purely general relativistic effect, when the motion of the compact object across the magnetic field *in vacuum* generates a parallel electric field, which in turn leads to the generation of plasma and the production of electromagnetic outflows with power [8],

$$L_{U,GR} = \frac{(GM)^2 B_0^2 \beta^2}{c^3} = \frac{(GM)^3 B_0^2}{c^5 R_{orb}} \tag{4}$$

(see also [12,13]). This type of interaction is important for BH-NS and BH-BH mergers, in which case there are no real induced charges to produce the parallel electric field; the parallel electric field is a pure vacuum effect, resulting from the curvature of the space-time. This power is smaller than for NS-NS coalescence by a factor $(R_G/R_{NS})^2$, where $R_G = 2GM/c^2$ is the Schwarzschild radius.

Qualitatively, the power (4) can be estimated from the potential drop across the Schwarzschild horizon. There is an important difference between NS-NS and BH-NS electromagnetic interactions, though: in the case of the NS-NS system, the parallel electric field is produced by real surface charges [14], while in the case of the black holes the parallel electric field is a pure vacuum effect, resulting from the curvature of the space-time [8].

For a NS-BH system just before contact, the general relativistic unipolar power $L_{U,\mathrm{GR}}$ is

$$L_{U,\mathrm{GR}} = 3\times 10^{44}\ \mathrm{erg\,s}^{-1}\left(\frac{B_{\mathrm{NS}}}{10^{12}\ \mathrm{G}}\right)^2. \tag{5}$$

The total emitted energy is

$$E_{\mathrm{tot},U} = \int_{t(R_{\mathrm{LC}})}^{t(R_{\mathrm{NS}})} L_{U,\mathrm{GR}}\,dt \approx B_0^2 R_{\mathrm{NS}}^3 = 10^{42}\ \mathrm{erg}\left(\frac{B_{\mathrm{NS}}}{10^{12}\ \mathrm{G}}\right)^2. \tag{6}$$

[Relations (5) and (6) assume equal masses of the merging objects; it is straightforward to generalize them to unequal masses.] Thus, the total energy dissipated via the general relativistic unipolar induction mechanism is of the order of the magnetic energy of the neutron star. Note that the energy is taken from the linear motion of the neutron stars, and not from the energy of the magnetic field.

In addition, more involved electromagnetic signatures are expected due to the perturbations that the merging black holes induce in the possible surrounding gas [15–20].

III. MAGNETOSPHERES OF COLLAPSING NEUTRON STARS

A. Direct emission of electromagnetic waves during collapse

As a neutron star experiences a collapse, the frozen-in magnetic field evolves with time, generating an electric field and a possible electromagnetic signal. Historically, the first treatment of the electromagnetic fields of collapsing neutron stars was done in the quasistatic approach [1], in which case the electric field follows from the slow evolution of the magnetic field. The quasistatic approach was later demonstrated to give the incorrect asymptotic decay of the fields with time [21]. As the neutron star contracts, the magnetic moment decreases $\propto R_s$. The scaling of the decay of the fields on a black hole calculated in Ref. [21] was confirmed by [2], who performed numerical simulations of the neutron star collapse into a black hole and saw a predicted power-law decay of the electromagnetic fields.

Most of the power in the calculations done in Ref. [2] was emitted at times of the order of the collapse time, well before the predicted asymptotic limit. Overall, the simulations are dominated by heavy resistivity effects intrinsic to the vacuum approximation: the disconnection of the magnetic field lines from the star typically (except in the Kerr-Schild coordinates) occurs when the strong compression of the magnetic field against the horizon and the corresponding effects of the numerical resistivity become important.

The assumption of a highly conducting exterior changes the overall dynamics of the electromagnetic fields. As we argue below, the high conductivity of the external plasma would prevent the formation of disconnected magnetic surfaces, formally prohibiting the processes described in Ref. [2].

B. Force-free approximation in general relativity

There is a broad range of astrophysical problems where the magnetic fields play a dominant role, controlling the dynamics of the plasma [22]. The prime examples are pulsar and black hole magnetospheres; gamma-ray bursts and active galactic nuclei jets may also be magnetically dominated at some stage (e.g., [23]). If the magnetic field energy density dominates over the plasma energy density, the fluid velocity, enthalpy density, and pressure become small perturbations to the magnetic forces. The dynamics then can be described in a force-free approximation [24]. In the nonrelativistic plasma the notion of force-free fields is often related to the stationary configuration attained asymptotically by the system (subject to some boundary conditions and some constraints, e.g., conservation of helicity). This equilibrium is attained on time scales of the order of the Alfvén crossing times. In strongly magnetized relativistic plasma the Alfvén speed may become of the order of the speed of light c, so that the crossing times become of the order of the light travel time. But if plasma is moving relativistically its state is changing on the same time scale. This leads to a notion of dynamical force-free fields.

MHD formulation assumes (explicitly) that the second Poincare electromagnetic invariant $\vec{E}\cdot\vec{B} = 0$ and (implicitly) that the first electromagnetic invariant is positive, $B^2 - E^2 > 0$. This means that the electromagnetic stress energy tensor can be diagonalized and, equivalently, that there is a reference frame where the electric field is equal to zero, the plasma rest frame. This assumption is important since we are interested in the limit where the matter contribution to the stress energy tensor goes to zero; the possibility of diagonalization of the electromagnetic stress energy tensor distinguishes the force-free plasma and vacuum electromagnetic fields, where such diagonalization is generally not possible.

The equations of the force-free electrodynamics can be derived from Maxwell equations and a constraint $\vec{E}\cdot\vec{B} = 0$. This can be done in a general tensorial notation from the general relativistic MHD formulation in the limit of negligible inertia [25]. This offers an advantage that the system of equations may be set in the form of conservation laws [26]. A more practically appealing formulation involves the $3+1$ splitting of the equations of general relativity [27,28]. The Maxwell equations in the Kerr metric then take the form

$$\nabla \cdot \vec{E} = 4\pi\rho, \qquad \nabla \cdot \vec{B} = 0,$$
$$\nabla \times (\alpha\vec{B}) = 4\pi\alpha\vec{j} + D_t\vec{E}, \qquad \nabla \times (\alpha\vec{E}) = -D_t\vec{B}, \tag{7}$$

where $D_t = \partial_t - \mathcal{L}_{\vec{\beta}}$ is the total time derivative, including the Lie derivative along the velocity of the zero angular momentum observers, ∇ is a covariant derivative with the radial vector $\mathbf{e}_r = \alpha\partial_r$, and $\alpha = \sqrt{1-2M/r}$. Taking the total time derivative of the constraint $\vec{E}\cdot\vec{B} = 0$ and eliminating $D_t\vec{E}$ and $D_t\vec{B}$ using Maxwell equations, one arrives at the corresponding Ohm's law in the Kerr metric [8], generalizing the result of [24]:

$$\vec{j} = \frac{(\vec{B}\cdot\nabla\times(\alpha\vec{B}) - \vec{E}\cdot\nabla\times(\alpha\vec{E}))\vec{B} + \alpha(\nabla\cdot\vec{E})\vec{E}\times\vec{B}}{4\pi\alpha B^2}. \tag{8}$$

Note that this expression does not contain the shift function $\vec{\beta}$.

The generic limitation of the force-free formulation of MHD is that the evolution of the electromagnetic field leads, under certain conditions, to the formation of regions with $E > B$ (e.g., [7]), since there is no mathematical limitation on $B^2 - E^2$ changing sign under strict force-free conditions. In practice, the particles in these regions are subject to rapid acceleration through $\vec{E}\times\vec{B}$ drift, followed by a formation of pair plasma via various radiative effects and reduction of the electric field. Thus, regions with $E > B$ are necessarily resistive. This breaks the ideal assumption and leads to the slippage of magnetic field lines with respect to plasma. In addition, evolution of the magnetized plasma often leads to the formation of resistive current sheets, with a similar effect on the magnetic field. If such processes were to happen in the magnetospheres of the collapsing neutron star, this might potentially lead to disconnection of the magnetic field lines from the star and a magnetic field-powered signal. Below we argue that in the case of collapsing neutron stars this does not happen.

IV. THE RESTRICTED WAVE GRAD-SHAFRANOV EQUATIONS

Let us derive a dynamic equation that describes the temporal evolution of the force-free fields in special relativity under the assumption that the fields remain axially symmetric. Previously, the equations governing general time-dependent force-free motion have been written in [29,30].

In relativistic plasma the force-free condition is given by Ohm's law (8), where in this section we set $\alpha = 1$. Generally, any function can be represented as a sum of a gradient and a curl of a vector function. Under the assumption of axial symmetry and zero divergence for the magnetic field, we can express electric and magnetic fields as

$$\mathbf{B} = \frac{\nabla P \times \hat{e}_\phi}{r\sin\theta} - \frac{2I}{r\sin\theta}\hat{e}_\phi,$$
$$\mathbf{E} = -\nabla\Phi + \frac{\nabla K \times \hat{e}_\phi}{r\sin\theta} - \frac{2L}{r\sin\theta}\hat{e}_\phi, \tag{9}$$

where P is the magnetic flux function $P = A_\phi \varpi$, $\varpi = r\sin\theta$, A_ϕ is the electric potential, K and L are some arbitrary functions to be determined, and I is the poloidal current through a flux cross section divided by 2π. The Maxwell equation $\partial_t\mathbf{B} = -\nabla\times\mathbf{E}$ gives

$$L = \partial_t P/2, \tag{10}$$

$$\partial_t I = \frac{1}{2}\left(\partial_r^2 K + \frac{1}{r^2}\sin\theta\partial_\theta\frac{\partial_\theta K}{\sin\theta}\right) = \frac{1}{2}\Delta^* K,$$
$$\Delta^* = r^2\sin^2\theta\nabla\left(\frac{\nabla}{r^2\sin^2\theta}\right). \tag{11}$$

The ideal condition $\vec{E}\cdot\vec{B} = 0$ implies

$$2I\partial_t P = -(\nabla K + r\sin\theta\nabla\Phi\times\mathbf{e}_\phi)\cdot\nabla P. \tag{12}$$

Equations (10)–(12) highlight two separate types of nonstationarity: type (i) is due to the variations of the current $I(t)$ for a given shape of the flux function [Eq. (11)], and type (ii) is due to the variations of the shape of the flux function for a given current I [Eq. (12)].

A. Constant shape of flux functions, $\partial_t P = 0$, variable current

Let us first consider the case when $\partial_t P = 0$. Then Eq. (12) implies that $\nabla K_0 + r\sin\theta\nabla\Phi\times\mathbf{e}_\phi$ is orthogonal to ∇P (and is thus along the poloidal magnetic field). Above, K_0 denotes a particular case when P is constant in time. Thus

$$\nabla K_0 = -r\sin\theta\nabla\Phi\times\mathbf{e}_\phi + r\sin\theta\Omega\nabla P\times\mathbf{e}_\phi, \tag{13}$$

$$\vec{E} = -\Omega\nabla P = -v_\phi\mathbf{e}_\phi\times\vec{B},$$
$$\partial_t I = -\frac{r\sin\theta}{2}\nabla P\times\nabla\Omega\cdot\mathbf{e}_\phi = \frac{\varpi^2}{2}(\vec{B}\cdot\nabla\Omega), \tag{14}$$

where Ω is an arbitrary function, which can be identified with the angular velocity of the rotation.

The ϕ component of the induction equation then becomes the time-dependent Grad-Shafranov equations for the restricted case when the shape of the flux surfaces remains constant, but the angular velocity Ω and, thus, the poloidal current are time and space dependent:

$$\varpi^2\nabla\left(\frac{1-\varpi^2\Omega^2}{\varpi^2}\nabla P\right) + \frac{4I(\nabla P\cdot\nabla I)}{(\nabla P)^2} + \varpi^2\Omega(\nabla P\cdot\nabla\Omega) = 0. \tag{15}$$

This is a Grad-Shafranov equation for axisymmetric force-free structures that rotate with arbitrarily varying angular velocity, but keep the shape of the flux functions constant.

The poloidal components of the induction equation give

$$\partial_t \Omega = -\frac{2\nabla P \times \nabla I \cdot \mathbf{e}_\phi}{\varpi (\nabla P)^2} = \frac{2}{(\nabla P)^2}(\vec{B}\cdot\nabla I). \quad (16)$$

Note that Eqs. (14) and (16) involve only a poloidal magnetic field, which, under the assumption $\partial_t P = 0$, remains constant in time.

Equations (14) and (16) can be combined to determine the evolution of Ω:

$$\partial_t^2 \Omega = \frac{\varpi^2}{(\nabla P)^2}(\vec{B}\cdot\nabla(\vec{B}\cdot\nabla\Omega)) = \frac{\vec{B}\cdot\nabla(\vec{B}\cdot\nabla\Omega)}{B_p^2}, \quad (17)$$

where B_p is the poloidal magnetic field. Equation (17) is the generalization of Ferraro's law of isorotation to time-dependent angular velocity.

Equations (14)–(16) constitute a closed system of equations for variables P, I, Ω under the assumptions of time-dependent I and Ω and stationary P. Generally, it is not guaranteed that there is a physically meaningful solution of this system: recall that this system describes a restricted motion of force-free plasma, when the shape of the flux function remains constant. Naturally, in the constant Ω limit, Eq. (15) reduces to the conventional Grad-Shafranov equation, while Eqs. (16) and (17) then imply that the gradients of the electric current and the angular velocity are orthogonal to magnetic flux surfaces.

B. Variable shape of flux functions

By virtue of (12) and (14) variable shapes of the flux functions can be described by adding to ∇K_0 a term proportional to ∇P, $K = K_0 + F(P)$.

Let us first consider $K = F(P)$ separately, neglecting the cross terms in the electric field. The $\vec{E}\cdot\vec{B} = 0$ gives

$$\nabla F \cdot \nabla P = 2I\partial_t P \quad (18)$$

or, since $F = F(P)$,

$$F'(\nabla P)^2 = 2I\partial_t P. \quad (19)$$

The Maxwell equation $\partial_t \mathbf{B} = -\nabla \times \mathbf{E}$ gives

$$\partial_t I = \tfrac{1}{2}\Delta^* F = \tfrac{1}{2}(F'\Delta^* P + (\nabla P)^2 F''). \quad (20)$$

The ϕ component of the induction equation then gives the Grad-Shafranov equation

$$\Delta^* P - \partial_t^2 P + \frac{4I(\nabla P\cdot\nabla I)}{(\nabla P)^2} - 2\partial_t\left(\frac{I^2\partial_t P}{(\nabla P)^2}\right) = 0. \quad (21)$$

This is a wave (hyperbolic) Grad-Shafranov equation for nonrotating axisymmetric force-free structures that evolve with time. The current I here is determined from Eqs. (19) and (20).

The wave Grad-Shafranov equation can be written in a general case, when both the current and the flux function evolve with time (Appendix), but its overly complicated form makes it useless for practical purposes.

C. Michel's time-dependent split-monopole solution in flat space

Both in the case of accretion induced collapse and for neutron star-neutron star mergers, right before the final plunge the neutron star is expected to rotate with a spin close to the breakup limit of ~1 msec. As a result, the light cylinder is located close to the neutron star surface. The theory of pulsar magnetospheres predicts that outside the light cylinder the magnetic field structure resembles the split-monopole structure [31]. This is confirmed by direct numerical simulations [32]. In addition, if the inertia of the "wind particles" is taken into account, there is another monopole-type solution wherein the particles rigidly rotate up to the light cylinder and then turn into electromagnetic waves [33].

In Sec. IV we derived a hyperbolic wave Grad-Shafranov equation, describing time-dependent force-free electromagnetic fields. It may be verified directly that Michel's monopole solution for a rotating force-free magnetosphere [31] is valid for the time-dependent angular velocity Ω, surface magnetic field B_s, and neutron star radius R_s. For a monopole field, Eq. (17) gives a radially propagating nonlinear shear Alfvén wave [34],

$$\partial_t^2\Omega = \partial_r^2\Omega, \qquad \Omega = \Omega(r \pm t). \quad (22)$$

The flux conservation requires $B_s R_s^2 = \text{const} = B_{\rm NS} R_{\rm NS}^2$. Then the Grad-Shafranov equation (15) has a slit-monopole-type solution for electromagnetic fields of the collapsing neutron star:

$$\begin{aligned} B_r &= \left(\frac{R_s}{r}\right)^2 B_s, \\ B_\phi &= -\frac{R_s^2\Omega\sin\theta}{r}B_s, \\ E_\theta &= B_\phi j_r = -2\left(\frac{R_s}{r}\right)^2\cos\theta\,\Omega B_s, \\ P &= (1-\cos\theta)B_s R_s^2, \\ \Phi &= -P\Omega, \\ I &= -\frac{P(P-2B_sR_s^2)\Omega}{2B_sR_s^2} = \frac{1}{2}B_sR_s^2\Omega\sin^2\theta, \end{aligned} \quad (23)$$

where P is the flux function, Φ is the electric potential, and $\Omega = \Omega(r-t)$. It may be verified directly that Eq. (14) is satisfied.

Thus, we found exact solutions for time-dependent nonlinear relativistic force-free configurations. Though the configuration is nonstationary (there is a time-dependent propagating wave), the form of the flux surfaces remains constant.

V. ELECTRODYNAMICS OF NEUTRON STAR COLLAPSE

A. Force-free collapse in the Schwarzschild metric: an exception to the no-hair theorem

In the previous section we showed that during arbitrary evolution of the rotational angular velocity of a split-monopole-type magnetosphere, the structure of the magnetosphere remains the same. In this section we apply the solutions obtained in the previous section to the electrodynamics of neutron star collapse, taking into account general relativistic effects.

The split-monopole solution may be a good approximation for several reasons. First, the collapse is likely to induce strong shear of the surface footpoints. As a result, a strong electric current will be launched in the magnetosphere, strongly distorting it. Highly twisted magnetic field lines will tend to open up to infinity, so that the magnetosphere will resemble a monopolar solution at each moment corresponding to the changing angular velocity of the surface footpoints. For a general case of strongly sheared footpoints, a time-dependent angular velocity will break a force balance. Still, we expect that the overall dynamical behavior will be similar to Michel's time-dependent solution.

Second, as we argue below, the open field lines cannot slip off the horizon, while the closed field lines will quickly be absorbed by the black hole. Thus, the magnetosphere of the black hole will naturally evolve towards the split-monopole solution, Fig. 1. Finally, in a more restricted sense, the fully analytically solvable dynamics of the monopolar magnetosphere collapse can be used to estimate the physical effects occurring on the open field lines.

Michel's stationary solution has been generalized to the Schwarzschild metric [35] (BZ below). Extending the time-dependent solution (23) to the general relativistic case by the principle of minimal coupling (or the convention "comma becomes a semicolon"), Michel's solution (23) remains valid for arbitrary $\Omega(r_{\rm fast} - t)$ in general relativity. The argument of Ω should be evaluated at the position of a radially propagating fast mode in the Schwarzschild metric with $dr_{\rm fast}/dt = \alpha^2$,

$$\Omega \equiv \Omega(r - t + r(1-\alpha^2)\ln(r\alpha^2)). \tag{24}$$

The Michel solution in general relativity has the same flux function as in flat space [see Eq. (23)]; the poloidal magnetic field is derived from Φ using a covariant derivative, while the toroidal magnetic field and poloidal electric field change according to $B_\phi \rightarrow B_\phi/\alpha$ and $E_\theta \rightarrow E_\theta/\alpha$. Thus, the exact nonlinear general relativistic time-dependent force-free fields corresponding to the arbitrary solid-body rotation are

$$\begin{aligned} B_r &= \left(\frac{R_s}{r}\right)^2 B_s, \\ B_\phi &= -\frac{R_s^2\Omega\sin\theta}{\alpha r} B_s, \\ E_\theta &= B_\phi j_r = -2\left(\frac{R_s}{r}\right)^2 \frac{\cos\theta\Omega B_s}{\alpha} \end{aligned} \tag{25}$$

with Ω given by Eq. (24). In addition to the bulk current (25) there is a current sheet at $\theta = \pi$ containing the radial current separating the two split-monopole hemispheres. It may be verified by direct calculations that fields (25) satisfy the Maxwell equations (7) with Ohm's law (8).

As the surface of the neutron star approaches the black hole horizon, $R_s \rightarrow R_G$, $B_s \rightarrow (R_{\rm NS}/R_G)^2 B_{\rm NS}$, while its angular velocity approaches a finite limit which we estimate next. Let, initially, the neutron star rotate with angular velocity $\Omega_{\rm NS}$. The moment of inertia of a neutron star can be written

$$I_{\rm NS} = (2/5)\chi M_{\rm NS} R_{\rm NS}^2, \tag{26}$$

where $\chi \sim 0.1$–0.5 is an equation-of-state–dependent variable that describes how centrally condensed the star is [36]. The spin angular momentum is thus

$$S = (2/5)\chi M_{\rm NS} R_{\rm NS}^2 \Omega_{\rm NS}, \tag{27}$$

where $P_{\rm NS}$ is the initial spin period. The dimensionless Kerr parameter is then

$$a = (2/5)\chi(c/G)\frac{R_{\rm NS}^2\Omega_{\rm NS}}{M_{\rm NS}} = 0.04\chi_{-1}P_{{\rm NS},-3}^{-1}, \tag{28}$$

where $P_{{\rm NS},-3} = P_{\rm NS}/1$ msec. For merging neutron stars the Kerr parameter is expected to be much higher.

For a collapsing star, the time dilation near the horizon and the frame dragging of the horizon lead to the "horizon locking" condition: objects are dragged into corotation

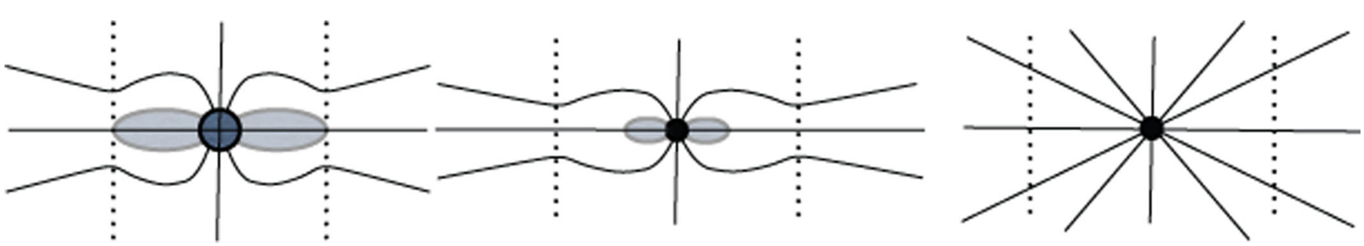

FIG. 1 (color online). Cartoon of the structure of magnetic fields around a collapsing rotating neutron star. Initially (left panel), the magnetic field is that of an isolated pulsar, with a set of field lines closing within the light cylinder (dashed vertical lines). Immediately after the collapse (central panel), the structure is similar. The closed field lines are absorbed by the black hole, while the open field lines remain attached to the black hole; the system relaxes to the monopole structure (right panel).

with the hole's event horizon, which has a frequency associated with it of

$$\Omega_H = a\frac{c^3}{2Gr_H} \approx \frac{\chi}{5}\frac{c^4 R_{\rm NS}^2}{(GM_{\rm NS})^2}\Omega_{\rm NS} = 2.9\times 10^3 \ {\rm rads}^{-1}\chi_{-1}P_{{\rm NS},-3}^{-1}, \qquad (29)$$

where $r_H = (1+\sqrt{1-a^2})GM/c^2 \approx R_G$ is the coordinate radius of the horizon of the Kerr black hole. (Note that for the chosen parameters the final spin is smaller than the initial spin, $\Omega_H/\Omega_{\rm NS} = 0.46\chi_{-1}$, due to the assumption of highly centrally concentrated initial mass distribution, $\chi \ll 1$.)

The electromagnetic power produced by Michel's rotator is then

$$L = \frac{2}{3}\frac{(B_s R_s^2)^2\Omega_H^2}{c} = \frac{2}{75}\chi^2\frac{c^7 B_{\rm NS}^2 R_{\rm NS}^8\Omega_H^2}{(GM_{\rm NS})^4} = 2\times 10^{44}\ {\rm ergs}^{-1}\chi_{-1}^2 B_{{\rm NS},12}^{-2}P_{{\rm NS},-3}^{-2}. \qquad (30)$$

It will lead to the black hole spin-down on a time scale

$$\tau = 6\frac{G^2 M_{\rm NS}^3}{c^3 B_{\rm NS}^2 R_{\rm NS}^4} = 2\times 10^7\ {\rm sec} B_{{\rm NS},12}^{-2} \qquad (31)$$

(Michel's solution corresponds to the spin-down index of $n = 1$, so that the spin evolution is described by a decaying exponential.) It is unlikely, though, that the assumptions of the model will be applicable for such a long time (see below).

In addition, the neutron star with a dipolar magnetic field has a net charge $Q = (1/3)B_{\rm NS}r_{\rm NS}^3\Omega_{\rm NS}/c$. As long as the assumptions of the model are satisfied (that the black hole produces a wind; see below), this charge is not canceled by the electrostatic attraction of charges from the surrounding medium. Thus, the black hole settles to the Kerr-Newman solution. The corresponding Newman parameter is small,

$$b = \frac{\sqrt{Q^2 G/c^4}}{R_H} = \frac{Q}{2\sqrt{G}M_{\rm NS}} = \frac{B_{\rm NS}R_{\rm NS}^3\Omega_{\rm NS}}{6cM_{\rm NS}} = 4\times 10^{-8} B_{{\rm NS},12}P_{{\rm NS},-3}^{-1}. \qquad (32)$$

As we argued above, the closed magnetic field lines will be quickly absorbed by the black hole, so that the magnetosphere will settle to the monopolar magnetic field structure with no electric charge; see Fig. 1.

B. How a neutron star collapse proceeds

To summarize the above discussion, first, the space-time of the collapsing neutron star temporarily passes through the Kerr-Newman solution with parameters given by (28) and (32); the electric charge is quickly lost due to the absorption of the closed field lines. (We stress that the loss of the electric charge is driven by the internal electrodynamics and not by the attraction of charges from the surrounding medium.) Second, and most importantly, we have demonstrated that the collapsing neutron star does not produce any narrow current structures or other dissipative/resistive structure that could become dissipative and "release" the overlaying magnetic field to infinity: the field always remains connected to the surface of the star.

The fate of the magnetic field lines connected to the surface of the star then depends on whether it is a closed magnetic field line or the one open to infinity. For closed loops, both footpoints are dragged toward the horizon and eventually absorbed by the black hole. On the other hand, the open magnetic field lines remain open and connected to the hole, without "sliding off the black hole," as long as the assumptions of the model remain satisfied. Thus, for a black hole surrounded by highly conducting plasma the open magnetic field lines never become disconnected from the black hole. As a result, the electromagnetic power emitted by the black hole may continue for times much longer than the immediate collapse time.

The key difference here from the conventional BZ mechanism is that in the latter case the magnetic field is assumed to be produced by the currents in an externally supplied accretion disk, while here the magnetic field is produced by the currents generated by the black hole itself. Also note that this result does not violate the no-hair theorem (e.g., [37]), which assumes that the outside is a vacuum. In our case the outside medium is assumed to be a highly conducting plasma all the way down to the black hole horizon. Under this assumption the magnetic field lines cannot be disconnected from the black hole.

There is a natural limit of applicability of the present model. The electric currents that support the magnetic field on the black hole are assumed to be self-produced by the black hole via the vacuum breakdown, and not supplied by the external current, like in the BZ case. The vacuum breakdown requires a sufficiently high electric potential. As the black hole spins down, the potential available for particle acceleration decreases. After some time, the black hole will not be able to break the vacuum. It will cross a death line (using pulsar terminology) after which no particles are produced anymore, the outside will become a vacuum, and by the no-hair theorem, the black hole will lose its magnetic field. Also, starting from this moment the black hole will be able to attract charges of opposite sign, canceling the internal charge.

In fact, a somewhat different scenario is likely to play out. Our experience with pulsars indicates that the plasma production in the magnetosphere is a highly nonstationary process. If there is an interruption in the plasma production for a sufficiently long time, the magnetic field will be able to slide off the black hole, shutting down the electromagnetic power forever.

VI. ON THE NATURE OF SHORT GAMMA-RAY BURSTS

The above results further highlight possible difficulties with the progenitors of short GRBs being the merging neutron stars [38]. On the one hand, numerical simulations indicate that the active stage of NS-NS coalescence typically takes 10–100 msec. Only a small amount, $\leq 0.1 M_{\odot}$, of material may be ejected during the merger, and it accretes on time scales of 1–10 secs, depending on the assumed α parameter of the resulting disk (e.g., [39–41]). Thus, there is not enough baryonic matter left outside the black hole to power a short GRB. Any energetically dominant activity on much longer time scales contradicts the NS-NS coalescence paradigm for short GRBs. This seem to contradict observations that some short GRBs have long-extended x-ray tails observed over time scales of tens to hundreds of seconds. The tail fluence can dominate over the primary burst (by a factor of 30 as in GRB080503 [42]). In addition, powerful flares appear late in the afterglows of both short and long GRBs (e.g., in the case of GRB050724, there is a powerful flare at 10^5 sec). In the standard forward shock model of afterglows this requires that at the end of the activity, lasting 10–100 msec, the source releases more energy than during the prompt emission in a form of low Γ shells, which collides with the forward shock after $\sim 10^6$ dynamical times, a highly fine-tuned scenario.

On the other hand, the expected electromagnetic powers estimated in the present paper are fairly low for all the discussed processes. Since the above results are based on the analytical Michel-type structure of the black hole magnetospheres, which for a given surface magnetic field and spin has the largest amount of open magnetic field lines and the largest electromagnetic power, the numerical estimates above can be considered as upper limits.

The only exception to the above could be that an efficient magnetic dynamo mechanism operates either during neutron star merger (for short GRBs) or during a core collapse of a massive star (for long GRBs), resulting in the formation of a millisecond magnetar-type object with a magnetic field reaching 10^{14} G [43,44]. Since, as we argue, the black hole can retain its magnetic field for a long period of time, the spindown time scale (31) may become sufficiently short, hundreds to thousands of seconds, so that the magnetic field can electromagnetically extract a large fraction of the total rotational energy of the black hole,

$$E_{\rm tot} = \frac{1}{2} I_{\rm NS} \Omega_{\rm NS}^2 = 2 \times 10^{51}\ {\rm erg} \chi_{-1} P_{\rm NS,-3}^{-2}. \qquad (33)$$

The fact that the electromagnetic extraction of the rotational energy of the black hole can operate both in long and short GRBs may explain a surprising observation that early afterglows of long and short GRBs look surprisingly similar, forming a continuous sequence, e.g., in the relative intensity of x-ray afterglows as a function of prompt energy [45]. This is surprising in a forward shock model: the properties of the forward shock do depend on the external density, while the prompt emission is independent of it. The difference between circumburst media densities in long GRBs (happening in star forming regions) and short GRBs (happening in low density galactic or even extragalactic medium) is many orders of magnitude. In defense of the forward shock model, one might argue that afterglow dynamics depends on $E_{\rm ejecta}/n$, both of which are orders of magnitude smaller for short GRBs than long GRBs. Yet, afterglows are very similar and, most importantly, form a continuous sequence.

We suggest that the similarity of the early afterglow properties of long and short GRBs, at times $\leq 10^5$ sec, can be related to the fact that in both cases a spinning black hole can retain a magnetic field for a sufficiently long time to power the prompt and early afterglow emission via internal dissipation in the wind [38].

VII. DISCUSSION

In this paper we discuss possible electromagnetic signatures of the merging and collapsing compact objects. At the inspiraling stage, in the case of a NS-NS system, the peak Poynting power is $L_{U,\max} = 6 \times 10^{45}$ ergs$^{-1} (B_{\rm NS}/10^{12}\ {\rm G})^2$, while for the black hole–neutron star systems it is an order of magnitude smaller. Both the peak power and the total energy of the precursor emission are fairly small (see Sec. II). Only for magnetar-type magnetic fields can the corresponding emission be observed at cosmological distances (see [11]).

We found a Michel-type solution for the structure of time-dependent force-free magnetospheres in general relativity. Based on this solution, we argued that, contrary to the previous estimates, the direct emission of the electromagnetic field, powered by the magnetic energy stored outside of the neutron star, does not produce a considerable electromagnetic signal: such a process is prohibited by the high conductivity of the surrounding plasma.

Most importantly, as long as the black hole is able to produce a highly conducting plasma via the vacuum breakdown, the magnetic field cannot "slide off" the black hole. As a result, a black hole can retain a magnetic field for a much longer time which is predicted by the no-hair theorem, producing an electromagnetic power for a long time after the collapse, without the need for an externally supplied magnetic field. The no-hair theorem does not apply here due to the assumed high conductivity of the plasma surrounding the black hole. (Pulsars produce plasma and currents all by themselves, without an external accretion disk.) In other words, under an ideal approximation the magnetic flux through a given surface remains fixed. Thus, as long as the ideal approximation remains valid, the total magnetic flux through the neutron star surface, which quickly approaches the horizon, remains constant. Therefore, as long as the assumptions of the model are satisfied (that the black hole produces a surrounding plasma), the black hole retains the magnetic field.

Since in the force-free limit the structures in the current sheet are flying away with the speed of light (cf., the corrugated current sheet solution in Ref. [46]), any magnetic field reconnection occurring beyond the light cylinder does not affect the global solution. On the other hand, the moment the black hole fails to produce the plasma (e.g., due to spontaneous reconnection within the light cylinder), it will quickly lose its magnetic field and stop producing any electromagnetic power. (It takes one malfunction to break the black hole electromagnetic engine.) It will likely be a random process, with no typical time scale, that will terminate the electromagnetic emission well before the black hole spins down.

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APPENDIX: HYPERBOLIC GRAD-SHAFRANOV EQUATION

Generally, we can write

$$\begin{aligned}\nabla K &= -r\sin\theta\nabla\Phi\times\mathbf{e}_\phi + r\sin\theta\Omega\nabla P\times\mathbf{e}_\phi + \nabla F(P),\\ \vec{E} &= -\Omega\nabla P - \frac{\partial_t P}{\varpi}\mathbf{e}_\phi + \vec{B}_p,\\ F'\vec{E}\cdot\vec{B} = 0 &\rightarrow F' = \frac{2I\partial_t P}{(\nabla P)^2}.\end{aligned} \tag{A1}$$

The ϕ component of the induction equation gives (the poloidal components are satisfied identically)

$$\partial_t I = \tfrac{1}{2}(\Delta^* F + \varpi(\nabla\Omega\times\nabla P)) = \tfrac{1}{2}(\Delta^* F + \varpi^2(\vec{B}\cdot\nabla\Omega)). \tag{A2}$$

The ϕ component of Ampere's law gives the hyperbolic wave Grad-Shafranov equation

$$\begin{aligned}&\varpi^2\nabla\left(\frac{1-\varpi^2\Omega^2}{\varpi^2}\nabla P\right) - \partial_t^2 P + \{-4(\Delta^* P)I^2 - 2IF''\partial_t P(\nabla P)^2 + IF'\partial_t(\nabla P)^2 4(\nabla P\cdot\nabla I)I\\ &\quad + 4\Delta^* P\varpi^2 I^2\Omega^2 - 2\varpi I\Omega^2(\nabla P\times\nabla(\partial_t P/\Omega))\cdot\mathbf{e}_\phi - 2(\Delta^* P)IF'\partial_t P + ((\nabla P)^2 + 8I^2)\varpi^2\Omega(\nabla P\cdot\nabla\Omega)\\ &\quad + 8\varpi I^2\Omega^2(\nabla P\cdot\nabla(r\sin\theta))\}\frac{1}{(\nabla P)^2 + 4I^2} = 0.\end{aligned} \tag{A3}$$

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