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Acoustics of tachyon Fermi gas

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We consider a Fermi gas of free tachyons as a continuous medium and find whether it satisfies the causality condition. There is no stable tachyon matter with the particle density below critical value n_T and the Fermi momentum $k_F < \sqrt{\frac{3}{2}}m$ that depends on the tachyon mass m. The pressure P and energy density E cannot be arbitrary small, but the situation $P > E$ is not forbidden. Existence of shock waves in tachyon gas is also discussed. At low density $n_T < n < 3.45 n_T$ the tachyon matter remains stable but no shock wave survives.

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I. INTRODUCTION

Tachyons, first introduced for the description of superluminal motion [1,2], are commonly known in the field theory as instabilities, whose energy spectrum is

$$\varepsilon_k = \sqrt{k^2 - m^2} \qquad k > m, \tag{1}$$

where m is the tachyon mass and relativistic units $c = \hbar = 1$ are used. The concept of tachyon fields plays a significant role in the modern research, and tachyons are considered as candidates for the dark matter and dark energy [3,4]; they often appear in brane theories [5] and cosmological models [6,7]. A system of many tachyons can be studied in the frames of statistical mechanics [8,9], and thermodynamical functions of ideal tachyon Fermi and Bose gases are calculated [10,11].

In the present paper our interest is focused on the bulk and acoustic properties of tachyon Fermi gas at zero temperature. Whether it concerns the Universe or a dense compact star, we consider the tachyon matter as continuous medium and analyze its stability to the causality condition [12,13]

$$c_s^2 = \frac{dP}{dE} \leq 1, \tag{2}$$

which implies that the sound perturbations must travel at a subluminal speed c_s. It is not evident whether the causality is satisfied at finite density of tachyon matter.

The shock wave propagation in tachyon medium is another problem that interests us. It is necessary to check the existence of stable shock waves and find relevant characteristics of tachyonic medium. Applied problems of astrophysics require more knowledge about collective features of tachyon matter, and it is important to outline its nontrivial behavior.

II. TACHYON FERMI GAS

Consider a system of free particles with the energy spectrum ε_k. The energy density of this system is defined as [14]

$$E = \frac{\gamma}{2\pi^2} \int \varepsilon_k f_k k^2 dk. \tag{3}$$

Its pressure is

$$P = \frac{\gamma}{6\pi^2} \int \frac{\partial \varepsilon_k}{\partial k} f_k k^3 dk \tag{4}$$

and the particle number density is

$$n = \frac{\gamma}{2\pi^2} \int f_k k^2 dk, \tag{5}$$

where γ is the degeneracy factor. The distribution function of fermions at zero temperature is approximated by the Heaviside step-function

$$f_k = \Theta(\varepsilon_F - \varepsilon_k), \tag{6}$$

where $\varepsilon_F \equiv \varepsilon_{k=k_F}$ is the Fermi energy level corresponding to the Fermi momentum k_F.

Substituting the energy spectrum of massive subluminal particles (bradyons)

$$\varepsilon_k = \sqrt{k^2 + m^2} \tag{7}$$

in Eq. (5) and (4) and integrating from 0 to k_F, we find the Fermi momentum

$$k_F = \left(\frac{6\pi^2 n}{\gamma}\right)^{1/3} \tag{8}$$

and the standard expressions for the energy density

$$E = \frac{\gamma}{8\pi^2} k_F^3 \varepsilon_F + \frac{1}{4} m n_s \tag{9}$$

and the pressure

$$P = \frac{\gamma}{24\pi^2} k_F^3 \varepsilon_F - \frac{1}{4} m n_s, \tag{10}$$

where

$$n_s = \frac{\gamma m}{4\pi^2}\left(k_F \varepsilon_F - m^2 \ln\frac{k_F + \varepsilon_F}{m}\right) \tag{11}$$

is the scalar density, and the Fermi energy is

$$\varepsilon_F = \sqrt{k_F^2 + m^2}. \tag{12}$$

Substituting the energy spectrum of tachyons (1) in Eq. (5) and integrating from m to k_F, we determine the Fermi momentum of tachyons

$$k_F = \left(\frac{6\pi^2}{\gamma} n + m^3\right)^{1/3}. \tag{13}$$

Substituting (1) in (3) and (4), we obtain the energy density of tachyons

$$E = \frac{\gamma}{2\pi^2}\int_m^{k_F} \sqrt{k^2 - m^2} k^2 dk = \frac{\gamma}{8\pi^2} k_F^3 \varepsilon_F - \frac{\gamma}{16\pi^2} m^2\left(k_F\varepsilon_F + m^2 \ln\frac{k_F + \varepsilon_F}{m}\right) \tag{14}$$

and the tachyon pressure

$$P = \frac{\gamma}{6\pi^2}\int_m^{k_F} \frac{k^4 dk}{\sqrt{k^2 - m^2}} = \frac{\gamma}{24\pi^2} k_F^3 \varepsilon_F + \frac{\gamma}{16\pi^2} m^2\left(k_F\varepsilon_F + m^2 \ln\frac{k_F + \varepsilon_F}{m}\right), \tag{15}$$

where the tachyon Fermi energy is

$$\varepsilon_F = \sqrt{k_F^2 - m^2}. \tag{16}$$

In the light of (9)–(11), we can define the tachyon scalar density as

$$n_s = \frac{\gamma}{4\pi^2} m\left(k_F\varepsilon_F + m^2 \ln\frac{k_F + \varepsilon_F}{m}\right) \tag{17}$$

and rewrite (14) and (15) in a brief form like (9) and (10). This tachyon scalar density n_s (17) will be incorporated in the energy density functional (14) when one considers a system of tachyons interacting via a scalar field. The interaction is not so important in the ultrarelativistic tachyon gas and $n_s \to n$ when $k \gg k_F$.

At low density $n \to 0$ the formula (13) is expanded so

$$k_F = m\left(1 + \frac{2\pi^2}{\gamma} n\right). \tag{18}$$

When $k_F \to m$ the tachyon Fermi gas can be treated as nonrelativistic, and its Fermi energy (16) is approximated by formula

$$\varepsilon_F \cong 2\pi\sqrt{\frac{n}{\gamma m}}. \tag{19}$$

Note that the relevant nonrelativistic approximation of (12) for the ordinary Fermi gas is written as

$$\varepsilon_F = m + \frac{1}{2m}\left(\frac{6\pi^2 n}{\gamma}\right)^{2/3}. \tag{20}$$

On the other hand, at high density

$$n \gg \frac{\gamma}{6\pi^2} m^3 \qquad k_F \gg m \tag{21}$$

the formula (13) coincides with (8) and the formulas for ordinary particles (9) and (10) and tachyons (14) and (15) yield the same ultrarelativistic equation of state (EOS)

$$P = \frac{E}{3}. \tag{22}$$

We can also present the formula (13) in the form

$$n = \frac{\gamma m^3}{6\pi^2}(\beta^3 - 1) \tag{23}$$

with a dimensionless momentum

$$\beta = \frac{k_F}{m}. \tag{24}$$

Substituting it in (16), we write the Fermi energy in a universal form

$$\varepsilon_F = m\sqrt{\beta^2 - 1}. \tag{25}$$

Substituting (24) and (25) in (14) and (15), we get universal formulas for the energy density and the pressure

$$E = \frac{\gamma m^4}{8\pi^2}\left[\beta^3\sqrt{\beta^2 - 1} - \frac{1}{2}\beta\sqrt{\beta^2 - 1} - \frac{1}{2}\ln(\beta + \sqrt{\beta^2 - 1})\right] \tag{26}$$

and

$$P = \frac{\gamma m^4}{8\pi^2}\left[\frac{1}{3}\beta^3\sqrt{\beta^2 - 1} + \frac{1}{2}\beta\sqrt{\beta^2 - 1} + \frac{1}{2}\ln(\beta + \sqrt{\beta^2 - 1})\right]. \tag{27}$$

III. PROPERTIES OF STABLE TACHYON MATTER

The tachyon gas reveals most peculiar behavior at low density (23), when $k_F \sim m$ ($\beta \sim 1$) and it may occur when $P > E$. However, the stable continuous medium must satisfy the causality condition (2). Otherwise, the matter will be unstable to sound perturbations and may appear in the form of droplets rather than continuous medium. For the ordinary Fermi gas (9) and (10) it is satisfied automatically because

$$c_s^2 = \frac{1}{3}\frac{k_F^2}{k_F^2 + m^2} \le \frac{1}{3} \le 1 \tag{28}$$

For the tachyon matter, in the light of (14) and (15), the causality condition (2) implies

$$c_s^2 = \frac{1}{3}\frac{k_F^2}{k_F^2 - m^2} \le 1 \tag{29}$$

or, in a dimensionless form,

$$c_s^2 = \frac{1}{3}\frac{\beta^2}{\beta - 1} \leq 1. \tag{30}$$

Of course, each single tachyon moves faster than light, but existence of a many-particle system of free tachyons does not contradict the causality (2). The constraint (29) implies that everywhere inside the tachyonic medium it must be

$$k_F \geq k_T = \sqrt{\frac{3}{2}}m \tag{31}$$

or, in terms of the dimensionless variable (24), it is

$$\beta \geq \beta_T = \sqrt{\frac{3}{2}} \cong 1.225. \tag{32}$$

Substituting the critical Fermi momentum k_T (31) in (16), we get the critical Fermi level

$$\varepsilon_F \geq \varepsilon_T = \frac{m}{\sqrt{2}}. \tag{33}$$

The Fermi energy of tachyon matter ε_F cannot be arbitrary small. The tachyon matter, satisfying the causality condition (29), is always relativistic matter, while the nonrelativistic approximation (19) could be applied only when $\varepsilon_F \to 0$ ($\beta \to 1$).

Substituting the critical Fermi momentum (32) in (23), we estimate the critical density

$$n \geq n_T = \frac{\gamma m^3}{6\pi^2}\left[\left(\frac{3}{2}\right)^{3/2} - 1\right] \cong 1.41 \times 10^{-2}\gamma m^3. \tag{34}$$

The causality condition (29) implies that the tachyon matter becomes unstable at low density $n < n_T$. The energy and pressure at the causal point $\beta = \beta_T$ (32) are calculated according to formulas (26) and (27) so

$$E_T = E(k_T) = \frac{\gamma m^4}{16\pi^2}\left[\sqrt{3} - \ln\left(\sqrt{\frac{3}{2}} + \frac{1}{\sqrt{2}}\right)\right] = 6.80 \times 10^{-3}\gamma m^4 \tag{35}$$

and

$$P_T = P(k_T) = \frac{\gamma m^4}{16\pi^2}\left[\sqrt{3} + \ln\left(\sqrt{\frac{3}{2}} + \frac{1}{\sqrt{2}}\right)\right] = 1.51 \times 10^{-2}\gamma m^4. \tag{36}$$

Hence, the tachyon pressure exceeds twice the energy density

$$P_T \cong 2.23 E_T. \tag{37}$$

It confirms the hypothesis of an EOS

$$P > E \tag{38}$$

that may not break the causality [12,13]. Equation

$$E_1 = E(k_1) = P(k_1) \tag{39}$$

determines

$$\beta_1 = \frac{k_1}{m} \cong 1.529 \tag{40}$$

while

$$\frac{E_1}{E_T} \cong 5.135. \tag{41}$$

According to the equation of state (Fig. 1) the tachyon material remains "hyperstiff" (38) when the Fermi momentum varies in the range of

$$k_T < k_F < k_1. \tag{42}$$

In the light of (39) we may expect that the sound speed at the point of "stiffness" $k = k_1$ should be $c_s = 1$. However, substituting (40) in (29) we find

$$c_s^2(k_1) = 0.58. \tag{43}$$

From Equations (23), (34), and (40), we also find the particle number density

$$n_1 = n(k_1) = 0.0434\gamma m^3 = 3.04 n_T. \tag{44}$$

The equation of state of tachyon Fermi gas, calculated according to formulas (26) and (27), is shown in Fig. 1. The tachyon matter behaves like ordinary ultrarelativistic Fermi gas (22) when its parameters are much greater than the critical values (34)–(36), namely, when

$$\bar{n} = \frac{n}{n_T} \gg 1 \qquad \bar{E} = \frac{E}{E_T} \gg 1 \qquad \bar{P} = \frac{P}{P_T} \gg 1. \tag{45}$$

The "stiffness" of tachyon matter immediately increases at low density: it becomes "absolute stiff" $E = P$ when

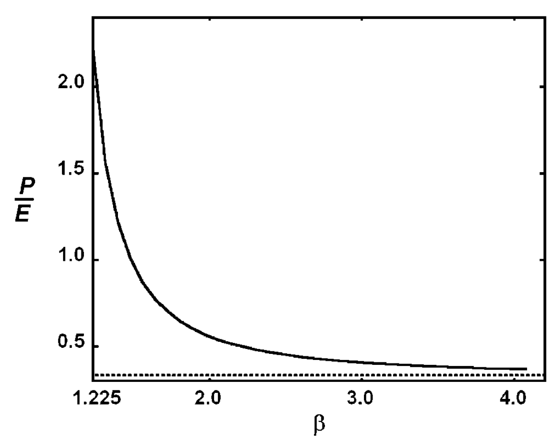

FIG. 1. The ratio of pressure to energy density P/E vs Fermi momentum of tachyon gas $\beta = k_F/m$.

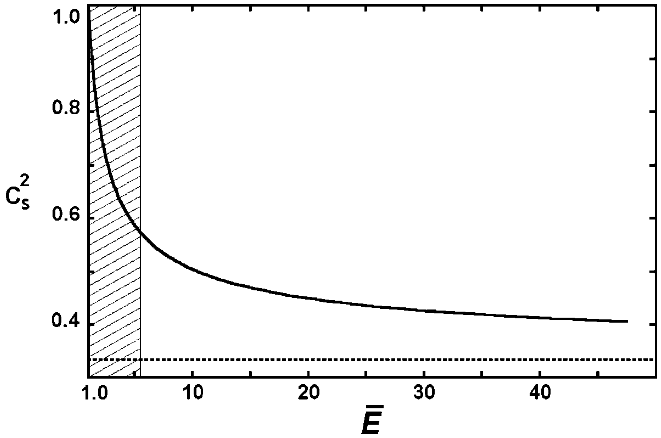

FIG. 2. Sound speed c_s^2 vs energy density of tachyon Fermi gas $\bar{E} = E/E_T$ [critical value E_T is defined in Eq. (35)].

$n = n_1 = 3.04 n_T$ (44), and in the range of densities $n_1 < n < n_T$ it is even "hyperstiff" (38). The sound speed (30) vs the energy density (26) is plotted in Fig. 2. It also reveals a rather peculiar behavior: it decreases at high E and tends to 1 when the energy density approaches the critical value $E \to E_T$ (35).

IV. SHOCK WAVES IN TACHYON MATTER

As soon as we have tested the causality condition in a tachyon-continuous medium, it is reasonable to consider shock waves. The shock wave velocity w_- and the velocity behind the shock w_+ are given by formulas [15]

$$v_-^2 = \frac{P_+ - P_-}{E_+ - E_-} \frac{E_+ + P_-}{E_- + P_+} \tag{46}$$

and

$$v_+^2 = \frac{P_+ - P_-}{E_+ - E_-} \frac{E_- + P_+}{E_+ + P_-}, \tag{47}$$

where index "$-$" indicates parameters before the shock wave, while index "$+$" corresponds to parameters behind it. Existence of stable shock waves is determined by the constraint called as the evolutionary condition [16]

$$v_- > c_- \tag{48}$$

$$v_+ < c_+. \tag{49}$$

For a shock wave of small amplitude

$$v_+^2 \to v_-^2 \to v_-^2 \to c_-^2 = \frac{dP}{dE} \tag{50}$$

we can apply a linear approximation and consider

$$c_+^2 = c_-^2 + \frac{dc_-^2}{dE} \Delta E + \ldots \tag{51}$$

where the increment

$$\Delta E = E_+ - E_- \to 0 \tag{52}$$

is small. Equations (46) and (47), then, yield, respectively

$$v_-^2 \simeq c_-^2 \left(1 + \frac{1 - c_-^2}{E_- + P_-} \Delta E\right) \tag{53}$$

and

$$v_+^2 \simeq c_-^2 \left(1 - \frac{1 - c_-^2}{E + P} \Delta E\right). \tag{54}$$

Substituting (53) in the evolutionary condition (48), we have to state that

$$\Delta E > 0. \tag{55}$$

Substituting (53) and (51) in (49), we get

$$1 - \frac{1 - c_-^2}{E_- + P_-} \Delta E < c_-^2 + \frac{dc_-^2}{dE} \Delta E. \tag{56}$$

In the light of (55), the constraint (56) implies that stable shock waves propagate in the matter if

$$\frac{E + P}{c_s^2 (1 - c_s^2)} \left(-\frac{dc_s^2}{dE}\right) < 1. \tag{57}$$

Here we omit index "$-$" for simplicity and put $c_- \equiv c_s$.

The condition (57) is automatically satisfied when

$$\frac{dc_s^2}{dE} > 0 \tag{58}$$

that corresponds to ordinary matter. According to our calculation in Fig. 2, the tachyon gas has always

$$\frac{dc_s^2}{dE} < 0. \tag{59}$$

Substituting (59) in (57), we have

$$\frac{E + P}{c_s^2 (1 - c_s^2)} \left|\frac{dc_s^2}{dE}\right| < 1. \tag{60}$$

In the light of (29) and (14) the inequality (60) is developed so

$$\frac{E + P}{c_s^2 (1 - c_s^2)} \left|\frac{dc_s^2}{dk_F}\right| \left(\frac{dE}{dk_F}\right)^{-1} < 1. \tag{61}$$

That is equivalent to

$$\frac{E + P}{c_s^2 (1 - c_s^2)} \left|\frac{dc_s^2}{d\beta}\right| \left(\frac{dE}{d\beta}\right)^{-1} < 1, \tag{62}$$

where the energy density E (26) and pressure P (27) are expressed in terms of dimensionless momentum $\beta = k_F/m$. Numerical simulation shows that the constraint (61) is satisfied when

$$k_F > k_D \cong 1.581 m. \tag{63}$$

The value k_D exceeds the critical Fermi momentum $k_T = 1.225$ (31). According to (23) and (34), the relevant particle number density

$$n_D = n(k_D) = 3.45 n_T \tag{64}$$

sufficiently exceeds the critical density n_T. The relevant energy density (26) is

$$E_D = E(k_D) = 6.25 E_T \tag{65}$$

and the relevant sound speed (29) is

$$c_s^2(k_D) = 0.56. \tag{66}$$

Formula (63)–(65) determine the band of shock wave instability in tachyonic-continuous medium

$$k_D > k_F > k_T \quad n_D > n > k_T \quad E_D > E > E_T \tag{67}$$

In Fig. 2 it is labeled by shading. No discontinuity is possible when the parameters of tachyon gas vary in the range (67). If, anyhow, an abrupt peak of density is created, it will be transformed in a smooth transition, rather than exist in the form of shock wave.

Substituting (55) and (59) in (51) we find that

$$c_+ < c_-. \tag{68}$$

This strange inequality (68) is also applied to discontinuities of the current in superconducting cosmic strings in the ''electric'' regime [17], while it is always

$$c_+ > c_- \tag{69}$$

for shock waves propagating in ordinary continuous medium which is characterized by inequality (58).

Another peculiar property of tachyon-continuous medium concerns the shock waves. The criterion of shock wave stability (48) and (49) is satisfied when the Fermi momentum exceeds the value $k_D = 1.581m$ (63). At the tachyon gas with $k_F > k_D$ is able to conduct shock waves. No shock wave can appear in the band of instability $k_D > k_F > k_T$. (67). If any discontinuity of density (pressure) is created artificially, it will be unstable and decay.

The tachyon equation of state (Fig. 1) is very ''soft'' ($P \simeq E/3$) at high energy density $E \gg E_T$ (when $\beta = k_F/m \gg 1$) but it becomes ''hyperstiff'' ($P > E$) as soon as the energy density approaches the critical value E_T (35) when $\beta \to 1.225$. Therefore, a star with tachyon content will have rather ''soft'' core and much more ''stiff'' envelope. Particularly, for the pi-meson mass $m = m_\pi = 138$ MeV and at $\gamma = 1$, the critical parameters of tachyon matter (34) and (35) are estimated as

$$n_T = 0.029 n_0 = 0.005\ \mathrm{fm}^{-3} \qquad E_T = 2 \times 10^{-3} \rho_0, \tag{70}$$

where $\rho_0 = 2.8 \times 10^{14}\ \mathrm{g \cdot cm^{-3}} = 159\ \mathrm{MeV \cdot fm^{-3}}$ is the normal nuclear density and $n_0 = 0.166\ \mathrm{fm}^{-3}$ is the relevant particle density. For the nucleon mass $m = m_p = 939$ MeV the critical parameters of tachyon matter are

$$n_T = 9.2 n_0 = 1.53\ \mathrm{fm}^{-3} \qquad E_T = 4.3 \rho_0. \tag{71}$$

It brings more intrigue to the problem. The latter value of critical energy E_T can be compared with the energy density E_* in the center of a neutron star with regular nuclear equation of state [18], where it is typically $E_* \sim 700\ \mathrm{MeV \cdot fm^{-3}} \sim 5\rho_0$ (corresponds to the nucleon number density $n_* \sim 8 n_0$).

It should be also noted that the star cannot contain tachyon matter if its central energy density is smaller than E_T (35). We may imagine a tachyon core only at $E_* > E > E_T$ because the tachyon gas cannot have free surface with zero pressure $P = 0$ and its pressure must be always higher than the critical pressure P_T (36). However, as soon as the energy density $E > E_1 = 5.13 E_T$ (41), the EOS of tachyon gas is even ''stiffer'' than the ''absolute stiff'' EOS $P = E$. Will the mass of tachyon star be great? Is it possible to form supermassive stellar objects? It is the subject for further research.

V. CONCLUSION

A system of many tachyons with energy spectrum (1) and the mass m, described in the frames of statistical thermodynamics and mechanics of continuous medium, is endowed with unusual properties. The EOS of tachyon Fermi gas at zero temperature is determined by Eqs. (14) and (15) or (26) and (27) that is shown in Fig. 1. The causality condition (2) is satisfied when the Fermi momentum of tachyon gas is not less than the critical value $k_F \geq k_T = \sqrt{3/2}m$ (31). It implies that the tachyon-continuous medium must have finite density $n > n_T$ and no stable matter exists below the critical density n_T whose value depends only on m (34). The peculiar behavior of tachyon gas is seen especially near the critical point $k_F \to k_T$, particularly, its EOS becomes ''hyperstiff'' $P > E$ (see Fig. 1), while the sound speed c_s decreases with the growth of energy density (Fig. 2). If the tachyon material is still expanded below the critical density n_T, it will lose stability to sound perturbations and, perhaps, will appear in the form of dense droplets rather than continuous substance.

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