Pulsar timing sensitivities to gravitational waves from relativistic metric theories of gravity

Márcio Eduardo da Silva Alves*
Instituto de Ciências Exatas, Universidade Federal de Itajubá, Itajubá, MG, 37500-903, Brazil

Massimo Tinto†
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109, USA
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Pulsar timing experiments aimed at the detection of gravitational radiation have been performed for decades now. With the forthcoming construction of large arrays capable of tracking multiple millisecond pulsars, it is very likely we will be able to make the first detection of gravitational radiation in the nano-Hertz band, and test Einstein's theory of relativity by measuring the polarization components of the detected signals. Since a gravitational wave predicted by the most general relativistic metric theory of gravity accounts for *six* polarization modes (the usual two Einstein's tensor polarizations as well as two vector and two scalar wave components), we have estimated the single-antenna sensitivities to these six polarizations. We find pulsar timing experiments to be significantly more sensitive, over their entire observational frequency band ($\approx 10^{-9}$–10^{-6} Hz), to scalar-longitudinal and vector waves than to scalar-transverse and tensor waves. At 10^{-7} Hz and with pulsars at a distance of 1 kpc, for instance, we estimate an average sensitivity to scalar-longitudinal waves that is more than two orders of magnitude better than the sensitivity to tensor waves. Our results imply that a direct detection of gravitational radiation by pulsar timing will result into a test of the theory of general relativity that is more stringent than that based on monitoring the decay of the orbital period of a binary system.

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I. INTRODUCTION

The pulsar timing technique for gravitational wave searches relies on the pulsar emission of highly periodic radio pulses, which are received at the Earth and cross-correlated against a template of the pulsed waveform. A gravitational wave (GW) propagating across the pulsar-Earth radio link introduces a time shift in the received pulses that is proportional to the amplitude of the GW. Since pulsars, and millisecond pulsars in particular, are the most stable clocks in the Universe over time scales of years, they provide a unique element for performing gravitational wave searches in the nano-Hertz band.

Pulsar timing sensitivities have improved significantly over the years, and with the advent of the forthcoming arraying projects it is very likely we will be able to make the first detection of gravitational radiation in the nano-Hertz band. The first unambiguous detection of a gravitational wave signal will also allow us to test Einstein's general theory of relativity by measuring the polarization components of the detected signals [1,2]. Since general relativity is the most restrictive among all the proposed relativistic metric theories of gravity [3], as it allows for only two of possible *six* different polarizations [4], by asserting that the spin-2 (''tensor'') polarizations are the only polarization components observed, we would make a powerful proof of the validity of Einstein's theory of relativity. Corroboration of polarization measurements with estimates of the propagation speed of the observed gravitational wave signal will provide further insight into the nature of the observed radiation and result into the determination of the mass of the graviton.

The advent of the ''dark energy and dark matter'' problem, the proposal of new inflationary scenarios, the theoretical attempts to quantize gravity, and the possible existence of extra dimensions [3], have all stimulated an increasing interest in theories of gravity that are alternatives to Einstein's theory of general relativity. Among the proposed theories, scalar-tensor theories predict GWs with one or two polarization modes with helicity $s = 0$ besides the two usual modes with helicity $s = \pm 2$ [4,5]. Also, theories that introduce geometrical corrections in the Einstein-Hilbert Lagrangian have appeared in the literature. The so-called $f(R)$ theories are of this kind and predict the existence of two additional scalar degrees of freedom [6]. Vector polarization modes, on the other hand, can appear in the ''quadratic gravity'' formulations [6], and in the context of theories for which the graviton has a finite mass (such as the Visser theory [7]). In fact, this last group of theories are the most general in that they allow for GWs with six polarization modes.

The problem of observing additional polarization modes with arrays of pulsar timing has been analyzed by Lee *et al.* [2] in the context of searches for an isotropic stochastic background of gravitational radiation. Their approach relied on cross-correlating pairs of time of arrival residuals (TOAR) from an array of pulsars. In what follows, we will

*alvesmes@unifei.edu.br
†Massimo.Tinto@jpl.nasa.gov

focus instead on the single-antenna pulsar timing sensitivity to GWs characterized by the presence of all possible polarization modes. This work is a follow up to our recent derivation of the Laser Interferometer Space Antenna (LISA) sensitivities to GWs predicted by the most general relativistic metric theory of gravity [8]. There we showed, in particular, that the LISA sensitivity to scalar-longitudinal waves is significantly better than that to tensor waves when the wavelength of the gravitational wave signal is shorter than the size of the detector. This is because the root-mean-squared of the one-way Doppler responses to waves with longitudinal components experience an amplification that is roughly proportional to the product between the frequency of the observed signal and the distance between the spacecraft. In the case of pulsar timing experiments, the resulting amplification to waves with longitudinal components over those purely transverse is rather significant due to their operational frequency bandwidth (10^{-9}–10^{-6} Hz) and typical pulsar-Earth distance (≈ 1 kpc) [2]. In the same spirit of [8], here we evaluate the pulsar timing sensitivities following the standard definition used for all other GW detectors: the strength of a sinusoidal gravitational wave signal, averaged over the sky and polarization states, required to achieve a given signal-to-noise ratio (SNR) over a specified integration time, as a function of Fourier frequency.

The paper is organized as follows. In Sec. II, we provide the expression for the TOAR response to GWs with helicity $s = 0$, ± 1 and ± 2. Since the time-derivative of the TOAR is equal to the Doppler response, we do not derive its expression here as this was presented in our earlier publication [8]. In Sec. III, after summarizing the main noise sources affecting pulsar timing experiments, we evaluate the sensitivities of these experiments to each of the different GW polarizations. We find pulsar timing experiments to be two to three orders of magnitude more sensitive, over their entire observational frequency band ($\approx 10^{-9}$–10^{-6} Hz), to scalar-longitudinal waves than to tensor waves. This result implies that a direct detection of gravitational radiation by pulsar timing will allow us to perform a dynamical test of the theory of general relativity that is more stringent than that based on monitoring the decay of the orbital period of a binary system (see for instance [9]). Finally in Sec. IV, we summarize our results and present out conclusions.

Throughout the paper, we will be using units such that $c = 1$ except where mentioned otherwise.

II. THE PULSAR TIMING RESPONSE

Since the time-derivative of the TOAR is equal to the relative frequency change of the radio pulses, in what follows we derive the pulsar timing sensitivity by relying on the one-way Doppler responses to the noises and to a GW signal containing all the six polarizations. Although the one-way Doppler response to a general GW was first derived by Hellings [10], a simpler and more compact expression for it was recently obtained by us [8], which is identical *in form* to that first obtained by Wahlquist [11] for GWs with $s = \pm 2$.

Our notation is summarized by Fig. 1: $\hat{\mathbf{n}}L$ is the vector oriented from Earth to the pulsar, which continuously emits radio pulses that are received at the ground station; $\hat{\mathbf{k}}$ is the unit wave vector of a GW propagating in the $+z$ direction, and θ and ϕ are the usual polar angles associated with $\hat{\mathbf{k}}$.

If we denote with $y_{\mathrm{GW}}(t)$ the relative frequency changes induced by a gravitational wave signal on the radio link between a pulsar and the Earth, its expression is equal to [8,12]

$$y_{\mathrm{GW}}(t) = (1 - \hat{k}\cdot\hat{n})[\Psi(t - (1 + \hat{k}\cdot\hat{n})L) - \Psi(t)], \quad (1)$$

where

$$\Psi(t) \equiv \frac{n^i h_{ij}(t) n^j}{2[1 - (\hat{k}\cdot\hat{n})^2]}. \quad (2)$$

In the above expression, $h_{ij}(t)$ are the spatial components of the GW metric perturbation corresponding to the following space-time line element

$$ds^2 = -dt^2 + (\delta_{ij} + h_{ij}(t - z))dx^i dx^j, \quad (3)$$

and $|h_{ij}| \ll 1$. In this way, the only restriction on $h_{\mu\nu}$ is that its temporal components are null, i.e., $h_{\mu 0} = 0$ and therefore it contains six degrees of freedom. A general GW perturbation can be written as a sum of six components in the following way:

$$h_{ij}(t - z) = \sum_{r=1}^{6} \epsilon_{ij}^{(r)} h_{(r)}(t - z), \quad (4)$$

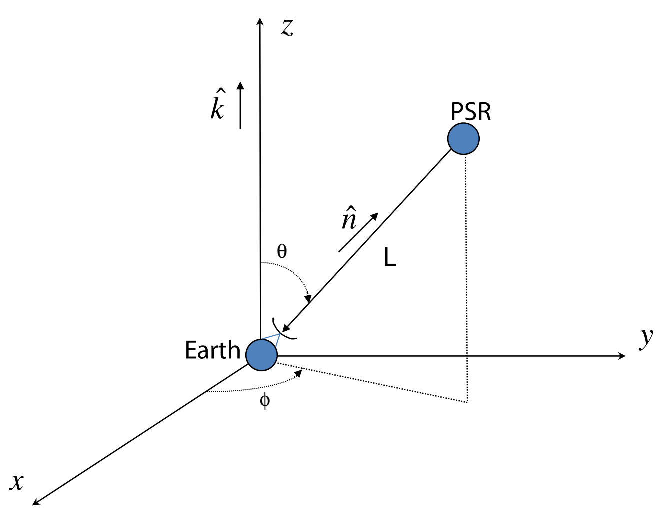

FIG. 1 (color online). The radio pulses emitted by the pulsar are received at Earth by a radio telescope. The gravitational wave train propagates along the z direction, and the two polar angles (θ, ϕ) describe the direction of propagation of the radio pulses relative to the wave. See text for a complete description.

where $\epsilon_{ij}^{(r)}$ are the six polarization tensors associated with the six waveforms of the gravitational wave signal. In the Cartesian coordinates system described in Fig. 1, the above polarization tensors assume the following matricial forms:

$$[\epsilon_{(1)}^{ij}] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad [\epsilon_{(2)}^{ij}] = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$
$$[\epsilon_{(3)}^{ij}] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad [\epsilon_{(4)}^{ij}] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{5}$$
$$[\epsilon_{(5)}^{ij}] = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad [\epsilon_{(6)}^{ij}] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

By analyzing the behavior of the above matrices under rotation around the z axis, we find that the polarization tensors $\epsilon_{ij}^{(1)}$ and $\epsilon_{ij}^{(6)}$ have helicity $s = 0$ and we call them scalar-longitudinal and scalar-transversal modes respectively; $\epsilon_{ij}^{(2)}$ and $\epsilon_{ij}^{(3)}$ have helicity $s = \pm 1$ and they are called vector-modes. Finally, the usual tensor modes are represented by the polarization tensors $\epsilon_{ij}^{(4)}$ and $\epsilon_{ij}^{(5)}$, which have helicity $s = \pm 2$.

III. SENSITIVITY

The sensitivity of a gravitational wave detector has been traditionally taken to be equal to (on average over the sky and polarization states) the strength of a sinusoidal gravitational wave required to achieve a given SNR over a specified integration time, as a function of Fourier frequency. Here, we will assume a SNR $= 1$ over an integration time of 10 years. Explicitly, we have computed the sensitivity using the following formula: $\sqrt{[S_y(f)B]}/(\text{rms of the GW response})$, where $S_y(f)$ is the spectrum of the relative frequency fluctuations due to the noises affecting the pulsar timing data, f is the Fourier frequency, and B is the bandwidth corresponding to an integration time of 10 years. The expression for the spectrum of the noise $S_y(f)$ we used in our sensitivity calculations relies on the noise-model discussed in [13]. In particular, we have assumed that: (i) multiple-frequency measurements can be implemented in order to adequately calibrate timing fluctuations due to the intergalactic and interplanetary plasma, and (ii) the tracked millisecond pulsars have frequency stabilities better than those of the operational ground clocks. A recent stability analysis of presently known millisecond pulsars [14] has shown that there might exist some with frequency stabilities superior to those displayed by the most stable operational clocks in the (10^{-9}–10^{-8} Hz) frequency band.

Under these assumptions, the expression for the noise spectrum, $S_y(f)$, is characterized by the contribution due the ground clock and a white-timing noise of 100 nsec. in a Fourier band $+/-$ 0.5 cycles/day (i.e. one sample per day). The 100 nsec. level is the current timing goal of leading timing array experiments as three pulsars are being timed to this level [14]. Following Jenet *et al.* [13] the expression for the noise spectrum $S_y(f)$ is equal to

$$S_y(f) = [4.0 \times 10^{-31} f^{-1} + 3.41 \times 10^{-8} f^2]\ \mathrm{Hz}^{-1}. \tag{6}$$

We have assumed the vector waves to be elliptically polarized and monochromatic, with their waveforms, ($h^{(2)}$, $h^{(3)}$), written in terms of a nominal amplitude, H, and the two Poincaré parameters, (Φ, Γ), in the following way

$$\begin{aligned} h^{(2)}(t) &= H \sin(\Gamma)\sin(\omega t + \Phi), \\ h^{(3)}(t) &= H \cos(\Gamma)\sin(\omega t). \end{aligned} \tag{7}$$

The sensitivity for elliptically polarized tensor components can be written in the same way as the vector modes by just replacing ($h^{(2)}$, $h^{(3)}$) with ($h^{(4)}$, $h^{(5)}$) in Eqs. (7). For scalar signals instead, the two wave functions, ($h^{(1)}$ and $h^{(6)}$), have been treated as independent since one is purely longitudinal and the other purely transverse to the direction of propagation of the GW.

Since we are interested in the sensitivity averaged over the sky, we calculated the averaged modulus squared of the Fourier transform of the GW response taken over the sky by assuming sources uniformly distributed over the celestial sphere; in the case of tensor and vector signals we also averaged over elliptical polarization states uniformly distributed on the Poincaré sphere for each source direction. Figure 2 shows the GW sensitivities corresponding to the noise spectrum given in Eq. (6) and after assuming a coherent integration of 10 years ($B = 1$ cycle/10 years).

The sensitivities of pulsar timing to vector and scalar-longitudinal polarized waves are significantly better than to tensor and scalar-transverse waves over the entire observational frequency band (10^{-9}–10^{-6} Hz). As explained in our earlier publication [8], where this effect was first noticed in the context of the LISA mission, the physical reason behind this sensitivity enhancement is due to the difference in the amount of time a "pulse" tensor wave and pulse wave with a longitudinal component affect the pulsar-Earth radio link. A tensor signal propagating orthogonally to the pulsar-Earth radio link (direction for which the one-way Doppler response can reach its maximum magnitude in this case) will only interact with the electromagnetic link for the time it takes the wave front to cross the radio link. On the other hand, if (for instance) a scalar-longitudinal wave propagates along the pulsar-Earth direction (over which the Doppler response will achieve its maximum in this case), the frequency of the radio link will be affected by the gravitational wave for the entire time L it takes the wave to propagate from the pulsar to Earth, resulting into an amplification of the Doppler response.

In mathematical terms, the argument can be explained in the following way. For the tensor modes, the maximum

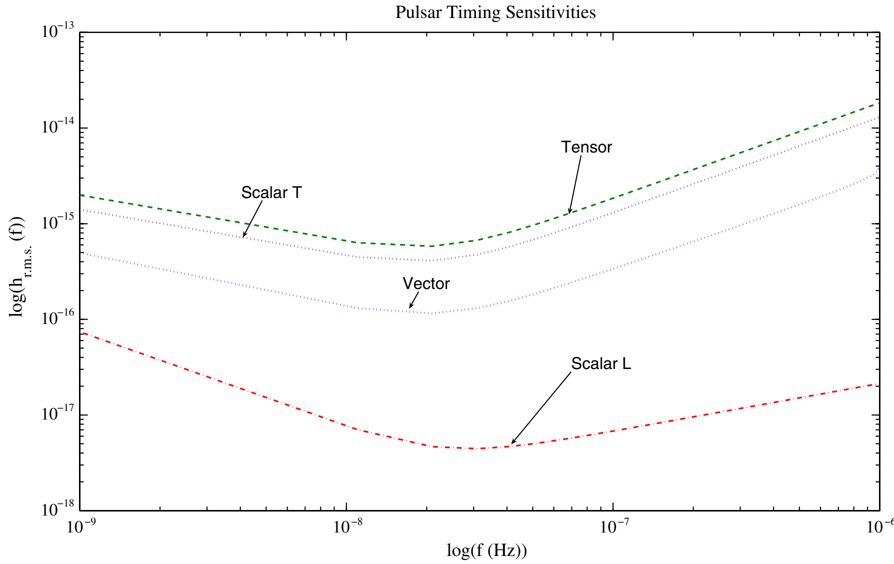

FIG. 2 (color online). Sensitivities of pulsar timing experiments to gravitational waves with tensor ($s = \pm 2$), vector ($s = \pm 1$), and scalar ($s = 0$) components. We may notice how more sensitive pulsar timing experiments are to vector and scalar-longitudinal signals than to tensor and scalar-transverse waves. This is true over the overall accessible frequency band, and more pronounced at higher frequencies. See text for more details.

response occurs when $\hat{k} \cdot \hat{n} \longrightarrow 0$ and it is proportional to $|\sin(\pi f L)|$. For the scalar-longitudinal mode instead, the maximum response is achieved when $\hat{k} \cdot \hat{n} \longrightarrow -1$ and is proportional to fL [15]. When the average over all possible directions and polarizations of the incoming waves is performed, the pulsar timing sensitivity to scalar-longitudinal waves is better than that to tensor waves by more than one order of magnitude at $f = 10^{-9}$ Hz and three orders of magnitude at $f = 10^{-6}$ Hz if we assume a pulsar out to a distance of 1 kpc. The same explanation applies to vector waves since they also affect space along their direction of propagation.

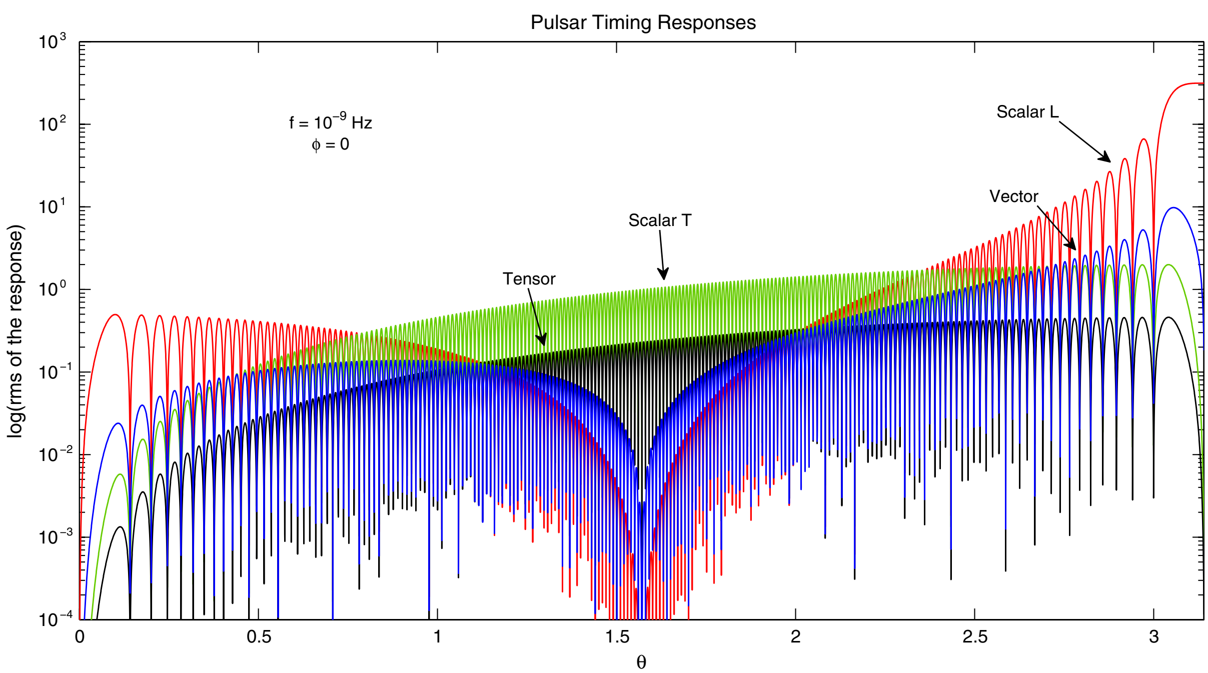

FIG. 3 (color online). Pulsar timing r.m.s. responses (over Poincaré's polarization angles) to tensor, vector, scalar-transversal and scalar-longitudinal gravitational waves. ϕ was taken to be zero as the r.m.s. vector and tensor polarizations are maxima for this value, while scalar-longitudinal and scalar-transverse responses do not depend on it. Note the rather large interval of θ values over which the response of the scalar-longitudinal polarization is better than the other polarizations. The plot was done by fixing $f = 10^{-9}$ Hz.

In order to estimate the interval for the angle θ over which the Doppler responses to longitudinal modes will experience an amplification over the transverse modes, we plot in Fig. 3 the root mean square (r.m.s.) of all the responses (taken over the Poincaré's polarization angles) as a function of the angle θ, and with $f = 10^{-9}$ Hz, $\phi = 0$. We find, for instance, the scalar-longitudinal Doppler r.m.s. response to be better than that of the tensor and vector polarizations over a fairly large interval of θ values. This shows that scalar-longitudinal waves will experience a Doppler response amplification over a significant fraction of the sky.

Since the measurements of the decay of the orbital period of the binary pulsars PSR B1913 + 16 and PSR B1534 + 12 were performed at an accuracy level of about 1% by comparing them against the expected energy loss due gravitational wave emission predicted by Einstein's theory of relativity, we infer that these measurements resulted into a 10% upper limit assessment of the amplitudes of the non-Einsteinian polarizations. This is because the energy of a GW signal is proportional to the square of its amplitude and we have also assumed that the mentioned accuracy can be related to the ability of this observations to access the nontensor polarizations. In this sense, the direct detection of a gravitational wave signal with pulsar timing could potentially achieve a much higher level of accuracy in testing the theory of relativity because of its enhanced sensitivity to non-Einsteinian modes. As an example, consider an hypothetic detection of a gravitational wave signal around the Fourier frequency 10^{-7} Hz. If an appropriate data analysis indicates that only the tensor components are present in the data, from our estimated sensitivities we will conclude that, on the average, the amplitude of a scalar-longitudinal wave must be smaller than that of the tensor wave by a factor of about or more. This translates into a test of the theory of relativity that is better than 0.3% in wave's amplitudes. Detections occurring at higher frequencies would result into more stringent tests of the theory, as implied by Fig. 2.

As a final comment, our estimated sensitivity enhancements experienced by the vector and scalar-longitudinal waves over tensor waves are independent of the assumptions underlining the spectrum of the noise used in our calculations.

IV. CONCLUSION

The main result of our work has been that pulsar timing experiments are more sensitive to scalar-longitudinal and vector signals than to scalar-transverse and tensor waves over their overall accessible frequency band. In particular, the fact that the sensitivities of pulsar timing experiments to waves with longitudinal components are better than to purely transverse signals by two to three orders of magnitude indicates that future experiments performed by forthcoming arraying projects will be able to assess the polarization of the detected waves. This will result in a dynamical test of Einstein's theory of relativity significantly more stringent than those performed to date based on measuring the decay of the orbital period of a binary system containing a pulsar.

A very important question we plan to answer in the near future is to estimate the measurement accuracies of the different polarizations of a detected gravitational wave background. Within the present results, we are not yet in a position to answer this question in general. However, if we detect a gravitational wave background whose amplitude is lower than that identified by our tensor and vector sensitivity curves but above that associated with the scalar-longitudinal mode, for instance, we can then state with certainty that the background we are observing is predominantly composed of scalar-longitudinal waves.

The ability of entirely reconstructing the waveforms of a detected signal will require the use of six or more millisecond pulsars [10], as it can be argued from a simple counting argument of the number of unknowns characterizing the most general gravitational wave signal. This task will be most suitably addressed by forthcoming pulsar arraying projects, and it will be the topic of our future investigation.

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