

Is there further evidence for spatial variation of fundamental constants?

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Indications of spatial variation of the fine-structure constant, α , based on study of quasar absorption systems have recently been reported [J. K. Webb, J. A. King, M. T. Murphy, V. V. Flambaum, R. F. Carswell, and M. B. Bainbridge, [arXiv:1008.3907](https://arxiv.org/abs/1008.3907)]. The physics that causes this α -variation should have other observable manifestations, and this motivates us to look for complementary astrophysical effects. In this paper we propose a method to test whether spatial variation of fundamental constants existed during the epoch of big bang nucleosynthesis and study existing measurements of deuterium abundance for a signal. We also examine existing quasar absorption spectra data that are sensitive to variation of the electron-to-proton mass ratio μ and $x = \alpha^2 \mu g_p$ for spatial variation.

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I. INTRODUCTION

The results of a very large study using data from both the Keck telescope and the Very Large Telescope (VLT) provide hints that there is a spatial gradient in the variation of the fine-structure constant, $\alpha = e^2/\hbar c$ [1,2]. In one direction α appears to have been smaller in the past, while in the other direction it appears to have been larger. Briefly, the method compares atomic spectra taken on Earth with spectra seen in absorption systems at high redshift. Any change in α results in well-understood differences between the two spectra. The use of many atoms and ions in a large number of systems ensures good control of systematics [3,4].

Astrophysical observations previously suggested that α may have been smaller in the past [5–8]. However, these studies all used spectra taken at the Keck telescope in Hawaii, at a latitude of 20° N. The recently observed gradient in α , which we will refer to as the “Australian dipole,” has a declination of around -60° , which explains why the Keck data, restricted mainly to the northern sky, originally suggested a time-varying α that was smaller in the past. Similar studies, using a much smaller sample from the VLT in Chile (latitude 25° S) at first showed a stringent null constraint [9]. More careful analysis of this sample suggested that the errors should be enlarged by a factor of 6 [10–12]. The recent study reported in [1,2] is the first large-scale analysis of VLT data for α -variation.

The indications of a spatial gradient in α motivates us to reinterpret, in terms of spatial variation, the existing studies of variation of different fundamental constants in other systems. In this paper we test whether further evidence for a dipole can be found in existing measurements of variation in electron-to-proton mass ratio, $\mu = m_e/m_p$, and combinations of fundamental constants such as $x = \alpha^2 \mu g_p$, where g_p is the proton g -factor. Finally, we consider possible variations in the relative primordial abundances of elements created during big bang nucleosynthesis. We develop a model for the detection of a dipole

in primordial abundances and its interpretation in terms of spatial variations of fundamental constants during the first few minutes after the big bang.

II. MODEL

We model the spatial variation of a fundamental constant X by

$$\frac{\delta X}{X_0} = d_X \Xi(\mathbf{r}), \quad (1)$$

where $\delta X/X_0 = (X(\mathbf{r}) - X_0)/X_0$ is the relative variation of X at a particular position \mathbf{r} in the Universe, d_X is the strength of the spatial variation, and $\Xi(\mathbf{r})$ describes its geometry. In this paper we examine potential spatial variation in astronomical data. We can only directly measure fundamental constants at the point where the spectrum originated; that is, in a past light cone centered on present-day telescopes at $\mathbf{r} = 0$. The exception to this is the primordial abundance data, which is indicative of conditions shortly after the big bang and will be discussed separately in Sec. VI. Note that in (1) we have assumed that $\delta X/X_0 = 0$ at zero redshift; however, this assumption should be tested for each system one measures using the same methods that are used at high redshift (e.g. by using absorbers within our own galaxy).

Since we are fitting a dipole, we use $\Xi(\mathbf{r}) = r \cos\psi$ where $r = ct$ is light-travel distance measured in gigalightyears (Gly), and ψ is the angle between the direction of the measurement and the axis of the dipole. ψ requires two parameters to specify the direction of the dipole: right ascension, ϕ , and declination, θ , in the equatorial coordinates. Then $\cos\psi$ is given in terms of the direction of the measurement and dipole axis

$$\begin{aligned} \Xi(\mathbf{r}) &= r \cos\psi(\phi_d, \theta_d), \\ \cos\psi &= \cos(\phi - \phi_d) \cos\theta \cos\theta_d + \sin\theta \sin\theta_d. \end{aligned} \quad (2)$$

We have additionally assumed that the effect of any spatial variation of fundamental constants increases with distance. This is model dependent for large redshifts, but at the level of accuracy required by our work we can assume the Universe is homogenous and isotropic. We therefore use the standard Λ_{CDM} cosmology parametrized by WMAP5 [13] to determine the distance r traveled by light along the past light cone,

$$r(z) = \frac{c}{H_0} \int_{1/(1+z)}^1 \frac{1}{\sqrt{\Omega_m a^{-3} + \Omega_\Lambda}} \frac{da}{a}. \quad (3)$$

In these coordinates the Australian dipole of α -variation found by [1] is

$$\frac{\delta\alpha}{\alpha} = (1.10 \pm 0.25) \times 10^{-6} \text{ Gly}^{-1} \cdot r \cos\psi, \quad (4)$$

where ψ is the angle between the direction of the measurement and the axis of the Australian dipole, [17.4 (0.6) h, $-58(6)^\circ$] in equatorial coordinates.

For any given set of data measuring variation of fundamental constants, one should ask whether a dipole model fits the data better than a monopole or a null hypothesis (that is, no variation). However, our goal here is to see if any of the existing data provide further evidence for the spatial variation of fundamental constants seen in [1]. Therefore it is also reasonable to question whether a dipole model with the axis specified by the Australian dipole provides a better fit to the data. There are some good theoretical justifications for such a procedure. For instance, the constants may vary because they are coupled to a (dimensionless) scalar field Φ which varies over space-time, such as the quintessence field Φ/c^2 or a dimensionless dilaton field. In this case the axis of the dipole is the direction of its gradient $\nabla\Phi$, and a fundamental constant X is coupled to its variation via

$$\frac{\delta X}{X_0} = k_X \delta\Phi, \quad (5)$$

where k_X is a dimensionless coupling coefficient. Our dipole model now requires $\delta\Phi(\mathbf{r}) \sim \Xi(\mathbf{r})$, but all constants will vary in the same direction (i.e. along the Australian dipole).

In this paper we discuss the constants α , μ , and the dimensionless mass ratio $X_q = m_q/\Lambda_{\text{QCD}}$, where m_q is the light-current quark mass and Λ_{QCD} is the position of the Landau pole in the logarithm of the running strong coupling constant, $\alpha_s \sim 1/\ln(\Lambda_{\text{QCD}}r/\hbar c)$. In the standard model the electron and quark masses are proportional to the vacuum expectation of the Higgs field, while the proton mass m_p is proportional to Λ_{QCD} (if we neglect the $\sim 10\%$ contribution of the quark masses). Relative variation of X_q is then approximately equal to the relative variation of m_q/m_p and $\mu = m_e/m_p$. We also mention systems that are sensitive to variation in the proton g -factor, g_p . This is

not a fundamental constant, but we can express its variation in terms of variation of light-quark mass using the relationship $\delta g_p/g_p \approx -0.1 \delta X_q/X_q$ [14] (accurate calculations for nuclear g -factors can be found in [15]).

We can relate the relative variation of different fundamental constants by equations like

$$k_\mu = R_\mu^\alpha k_\alpha, \quad (6)$$

where the R_X^α can be determined from observations and compared with theories of spatial variation.

III. H₂ ABSORPTION LINES

Limits on variation of μ at high redshift come from comparison of different rotational-electronic excitations in molecular hydrogen (H₂). The most recent determinations use the laboratory wavelengths and sensitivity coefficients presented in [16–18] to determine $\delta\mu/\mu$ in four different quasar absorption systems [19,20].

To determine which model of variation provides the best fit to the $\delta\mu/\mu$ data, we can use χ^2/ν as a goodness-of-fit parameter. Here ν is the number of degrees of freedom; for four data points (quasar absorption systems), $\nu = 4 - p$ where p is the number of parameters in the model. For example, fitting a monopole to the data reduces χ^2/ν from 1.21 (null hypothesis) to 1.08 (one parameter), which explains the mildly nonzero weighted mean, $\delta\mu/\mu = 3.4(2.7) \times 10^{-6}$.

Alternatively, we can test whether the data supports a dipole model in the direction of the Australian dipole. Using the one-parameter model (1) with the dipole direction fixed at (17.4 h, -58°) we obtain $d_\mu = 2.6(1.3) \times 10^{-6} \text{ Gly}^{-1}$, with a reduced $\chi^2/\nu = 0.33$. We can compare this to the null hypothesis using an F -test, which allows us to find the probability that χ^2 has improved by chance. We define F by

$$F(p_2 > p_1) = \frac{(\chi_1^2 - \chi_2^2)/(p_2 - p_1)}{\chi_2^2/(n - p_2)}, \quad (7)$$

where p_1 and p_2 are the number of parameters in the respective models and n is the number of data points (in the present case, $p_1 = 0$, $p_2 = 1$, and $n = 4$). Note that we cannot use this test to determine whether the dipole fit is better than the monopole (although χ^2/ν suggests that it is), since the models compared using the F -test must be “nested”: it must be possible to generate model 1 from model 2. In any case, because there is no detection the null hypothesis is really the better comparison. We obtain $F = 11.6$ and the probability that the dipole model is preferred due to chance is 4%.

Because of our obtained value of $\chi^2/\nu < 1$, one may argue that this anomalously small value is due to overfitting of the data, overestimated errors, or random fluctuations from sampling. We can see how likely it is that the result is spurious by using a bootstrap test, where we randomly

reassign the observed data values to different observed quasars and then refit the dipole model. In our first test we apply the data points without redraw: there are 24 possibilities for the four data points to be arranged on four quasar locations. Of these, three (12.5%) give values of χ^2 that are smaller than that of the real data. With such small numbers, this method suffers from “discretization” problems: there are simply not enough possibilities tested. For this reason we perform a second test where we generated 256 possibilities by allowing redraw when randomly reassigning data to quasars. Of these, 19 give smaller values of χ^2 than the real data, indicating an $\sim 8\%$ probability that the real value is due to random fluctuations.

Clearly the Australian dipole provides a good fit to the data, but what does the data alone say? To find the direction preferred by the data, we use the three parameter fit (1); the result is $d_\mu = 3.3(1.5) \times 10^{-6}$, $\phi_d = 16.7(1.5)$ h, $\theta_d = -62(5)^\circ$. This direction is consistent with the Australian dipole. With only one degree of freedom remaining, $\chi^2 = 0.02$. The errors for each model parameter are found by varying the parameter locally near the best-fit value until $\chi^2 = \chi_{\min}^2 + 1$, while keeping all other parameters fixed at their respective best-fit values. This ignores correlations between the parameters, which must be important; for example, if $d_\mu = 0$ (2 standard deviations from its best-fit value) then ϕ_d and θ_d are completely unspecified.

While the preceding paragraphs could be interpreted as tentative evidence for spatial variation of μ , a few things must be noted. Most obviously, we have a very small sample of data, which in principle can be improved although there are significant observational challenges in obtaining data of sufficient quality at high redshift [20,21]. Also of importance to this work is the lack of sky coverage represented in the sample. All four sources are below the equator, and three of them are within 25° of each other. Therefore, many more data points over a wider sky coverage are needed to obtain a significant detection. In any case, there is no clear evidence for a spatial variation in μ , and we certainly do not claim one. At best, we can say that our very limited data does not discount the possibility.

If, in the future, a detection is confirmed, we will be able to extract R_μ^α of Eq. (6). For example, with the d_μ suggested by this work and d_α given by (4), $R_\mu^\alpha = 2.6 \times 10^{-6} / 1.1 \times 10^{-6} = 2.4$. However, in Sec. V, we discuss stringent limits from studies of ammonia inversion spectra which, if confirmed, place very strong limits on cosmological variation of μ and hence R_μ^α .

IV. 21-CM RADIO VS UV LINES

The variation of a combination of fundamental constants is probed by comparing neutral hydrogen (HI) 21-cm radio and UV metal lines that have been redshifted into the optical band (see [22,23]). The ratio of these transition frequencies is proportional to $x = \alpha^2 \mu g_p$. Only a handful

of quasars have been found for which both optical and radio absorption are observed; the nine systems with good data are presented in [23]. These span the redshift range $z = 0.23$ to 2.35.

It was found in [23] that there is far more scatter in the data than would be expected from the statistical errors: the best monopole (single parameter) fit gives $\chi^2/\nu = 8.1$ ($\chi^2 = 64.6$), where ~ 1 would be expected if the errors were Gaussian and the model correct. Therefore, [23] concluded that there are large systematic errors, which they trace to the assumption that the 21-cm absorption component is at the same physical location as the UV component. It is supposed that the scatter seen in the data mainly comes from this assumption being incorrect; that is, the 21-cm and UV absorbing gases can be randomly offset in space and have quite different velocities (although see [24,25]).

Having enlarged the errors in order to force $\chi^2/\nu = 1$ for the monopole model, it is quite not surprising that [23] found no evidence for a dipole: the monopole model has been made to fit as well as could possibly be expected. Therefore, we have reanalyzed the data *without* the additional systematic to test whether any significant dipole exists. First, using the axis of the Australian dipole, we find χ^2 reduces from 110 (zero parameters) to 101 (one parameter). χ^2/ν is actually worse than for the monopole fit. Fitting the three parameter dipole (1) provides part of the explanation for why the Australian dipole is such a poor fit: the best-fitting dipole is at (7.3 h, 23°), an angle of 140° from the Australian dipole. For this model $\chi^2/\nu = 5.5$, which shows that even with the best-fit dipole systematics still dominate.

We conclude that the detection of a dipole in x of real significance will require much more data than the other systems discussed in this paper, due to the need to account for the large systematic errors caused by the velocity offset between the HI and metal absorption lines.

V. INTERPRETING SINGLE MEASUREMENTS

In this section we briefly discuss how individual measurements of combinations of fundamental constants can be compared with the dipole model presented in Sec. II. When there are only one or two samples, it is not possible to determine whether a dipole fit to the data is appropriate. However, one can still compare the measurement with the variation that the Australian dipole would lead us to expect. We summarize the results of this section in Table I.

There are two high-accuracy single-absorption-system α -variation results that use only Fe II transitions: at $z = 1.15$ towards HE0515-4414 [26] and at $z = 1.84$ towards Q1101-264 [27]. Using (4), we obtain an expected variation of $\delta\alpha/\alpha = (1.9 \pm 1.5) \times 10^{-6}$ for the former, which was measured to be $(-0.07 \pm 0.84) \times 10^{-6}$ [26]. The absorber towards Q1101-264 has a larger expected variation of $\delta\alpha/\alpha = (3.8 \pm 2.0) \times 10^{-6}$, and was

TABLE I. Comparison of expected variation, given by Eq. (4) and measured variation of fundamental constants in different systems. Each measurement corresponds to a single absorption system: the first four lines are measurements of quasistellar objects with $z > 1$, the remainder are from the five known molecular absorbers of radio-loud quasar spectra. $R_{g_p}^\alpha$ and R_μ^α are defined by equations like (6). As noted in Sec. II, $R_{g_p}^\alpha \approx -0.1R_q^\alpha$ and $R_\mu^\alpha \approx R_q^\alpha$ where R_q^α is the variation of the light-quark mass X_q relative to α -variation. Errors in the expected α -variation (the prefactors in the fifth column) are of order $\sim 1.5 \times 10^{-6}$.

System	Constant	z	$\cos\psi$	Expected variation ($\times 10^{-6}$)	Measurement ($\times 10^{-6}$)	Ref.
Fe II	α	1.15	0.21	1.9	-0.07 ± 0.84	[26]
		1.84	0.34	3.8	5.4 ± 2.5	[27]
HI 21-cm + CI	$\alpha^2\mu g_p$	1.36	0.11	$1.12(2 + R_\mu^\alpha + R_{g_p}^\alpha)$	$6.64 \pm 0.84_{\text{stat}} \pm 6.7_{\text{sys}}$	[28]
		1.56	-0.50	$-5.20(2 + R_\mu^\alpha + R_{g_p}^\alpha)$	$7.0 \pm 1.8_{\text{stat}} \pm 6.7_{\text{sys}}$	[28]
NH ₃ inversion	μ	0.685	-0.80	$-5.47R_\mu^\alpha$	$< 1.8(2\sigma)$	[29]
		0.886	0.17	$1.34R_\mu^\alpha$	$< 1.4(3\sigma)$	[30]
HI 21-cm + mol. rot.	$\alpha^2 g_p$	0.247	0.16	$0.50(2 + R_{g_p}^\alpha)$	-2.0 ± 4.4	[31]
		0.685	-0.80	$-5.47(2 + R_{g_p}^\alpha)$	-1.6 ± 5.4	[31]
HI 21-cm + OH 18-cm	$(\alpha^2/\mu)^{1.57} g_p$	0.765	-0.14	$-1.04(3.14 - 1.57R_\mu^\alpha + R_{g_p}^\alpha)$	$4.4 \pm 3.6_{\text{stat}} \pm 10_{\text{sys}}$	[32]
OH 18-cm	$(\alpha^2/\mu)^{1.85} g_p$	0.247	0.16	$0.50(3.70 - 1.85R_\mu^\alpha + R_{g_p}^\alpha)$	-11.8 ± 4.6	[33]

measured to be $(5.4 \pm 2.5) \times 10^{-6}$ [27]. The single system results are therefore seen to be consistent with the Australian dipole.

Two measurements comparing HI 21-cm with neutral carbon (CI) absorption lines at $z = 1.36$ and $z = 1.56$, along lines of sight to quasars Q2237-011 and Q0458-020, have been reported recently [28]. As discussed in Sec. IV, these measurements probe the combination of constants $\alpha^2\mu g_p$. In Table I we compare the results of these measurements with the variation expected according to the Australian dipole and the model presented in Sec. II. To obtain the model prediction, we calculate the expected variation of α according to (4) and leave the factors $R_{g_p}^\alpha$ and R_μ^α , defined by equations such as (6), for later determination. The type of analysis presented here could eventually yield values for the ratios $R_{g_p}^\alpha$ and R_μ^α , but a larger sample of measurements is required.

Measurements of μ -variation have been made by comparing inversion lines of ammonia, NH₃, with rotational lines of CO, HCO⁺, and HCN molecules. The inversion spectrum of ammonia has an enhanced sensitivity to variation of μ because it depends on the exponentially small probability of tunneling of the three hydrogen atoms through the potential barrier [34]. In Table I we present two measurements of μ variation based on this method.

The frequency of the hydrogenic hyperfine line is proportional to $\alpha^2\mu g_p$; molecular rotational frequencies are proportional to μ . Comparison of the two placed limits on variation of the parameter $\alpha^2 g_p$ [35] in two quasar absorption spectra. A similar analysis was repeated using more accurate data for the same objects [31], at $z = 0.247$ and at $z = 0.6847$, resulting in the limits shown in lines 7 and 8 of Table I, respectively. The object at $z = 0.6847$ is associated with the gravitational lens toward quasar B0218 + 357 and corresponds to lookback time ~ 6.2 Gyr.

Two other measurements are shown in Table I. The combination $(\alpha^2/\mu)^{1.57} g_p$ can be extracted by comparison of OH 18-cm and HI 21-cm lines; this has been performed for the gravitational lens toward PMN J0134-0931 at $z = 0.765$ [32]. A measurement of $(\alpha^2/\mu)^{1.85} g_p$, derived from comparison of conjugate-satellite OH 18-cm lines, has yielded a 2.6σ nonzero detection towards PKS 1413 + 135 [33]. We note that this absorber lies at $\sim 81^\circ$ to the Australian dipole and is at low redshift ($r \sim 2.9$ Gly); therefore, minimal variation in fundamental constants is expected.

VI. BIG BANG NUCLEOSYNTHESIS

In principle one can search for spatial variation in the primordial abundance of any element produced during BBN (the most common are D, ³H, ³He, ⁴He, ⁷Li). The best limits on primordial deuterium abundance are derived from isotope-shifted Ly- α spectra in quasar absorption systems, where the absorbers are at $z \sim 2.5$. By contrast ⁴He is observed in ionized (H II) regions of low-metallicity dwarf galaxies at $z \lesssim 0.01$, while the primordial ⁷Li abundance is determined from metal-poor Population II stars in our galaxy. Therefore, in practice, only the deuterium abundance is currently measured at large enough redshifts for one to expect to see any effect of spatial variation of fundamental constants.

In the case of primordial abundances we need to extract an average value from the data (monopole term) as well as any potential dipole. Therefore, we use the model

$$\log_{10}(a/H) = m_a + d_a \Xi(\mathbf{r}) = m_a + d_a r \cos\psi(\phi_d, \theta_d), \quad (8)$$

where $\log_{10}(a/H)$ is the primordial abundance of element a relative to hydrogen abundance at a particular place \mathbf{r} , m_a is the average primordial abundance, d_a is the strength of

the spatial variation, and $\Xi(\mathbf{r})$ describes the geometry of the spatial variation (2). An additional subtlety here is that all measurements represent physics within 10 min of the big bang. Therefore, our test really is a pure spatial variation, rather than a probe such as (1) of physics along the past light cone.

In the future one might hope to measure primordial abundances of the other elements at high redshift. The production of each of these elements has a different sensitivity to fundamental constants; therefore, a complete set of data can, in principle, simultaneously measure several fundamental constants at the time of nucleosynthesis. The dependence of primordial abundances on fundamental constants such as α and m_q/Λ_{QCD} (the ratio of light-quark mass to the pole in the running strong coupling constant) is the subject of current research [36–39]. An observed spatial variation in primordial abundance (nonzero d_a) can be related to the variation of a fundamental constant X at that position in space at the time of big bang nucleosynthesis using the relationships

$$d_a = \frac{\partial \log_{10}(a/H)}{\partial \Xi} = \frac{\partial \log_{10}(a/H)}{\partial \ln X} \frac{\partial \ln X}{\partial \Xi}, \quad (9)$$

where $\partial \log_{10}(a/H)/\partial \ln X$ is determined from theory.

We have performed a preliminary search for such a dipole using the seven best determinations of primordial deuterium abundance presented in [40]. It is known that the standard (monopole) model gives a minimum χ^2 of 19.13 rather than the ~ 6 that would be expected if the errors were purely statistical; therefore, there is at least a possibility that the dipole model of spatial variation could produce a better fit to the data. As in the cases presented in earlier sections, we can investigate this possibility using χ^2/ν as a measure of the fitness of our model. The results are shown in Table II. We find that the dipole model with direction fixed by the Australian dipole [1] is not significantly preferred over the simple monopole model, with χ^2/ν increasing very slightly. On the other hand, if we let the data *choose* the dipole direction, in effect solving for m , d , ϕ_d , and θ_d simultaneously, we only have three remaining degrees of freedom, and χ^2/ν increases substantially. The dipole is found to point in broadly the same direction as the Australian dipole (compare the red and orange 1σ regions in Fig. 1).

TABLE II. Calculated best fits to the primordial deuterium abundance data using various models. The model parameters, including m_D , d_D , and the direction of the dipole (right ascension and declination) from (1), as well as minimum χ^2 and χ^2 per degree of freedom (ν) are shown. The monopole model is the standard interpretation (m_D only). The dipole model was tested using fixed and varying dipole directions. The fixed dipole direction corresponds to the best fit of [1]: (17.4 h, -58°) in equatorial coordinates. Errors in each parameter are found ignoring correlations; see Sec. III of the text for details.

	m_D	d_D	R.A. (hours)	Decl. (deg)	χ^2	χ^2/ν
Monopole	-4.55(2)				19.13	3.19
Dipole (fixed direction)	-4.55(2)	0.0045(35)			17.50	3.50
Dipole	-4.56(2)	0.0054(29)	15.5(1.6) h	$-14(51)^\circ$	15.73	5.24

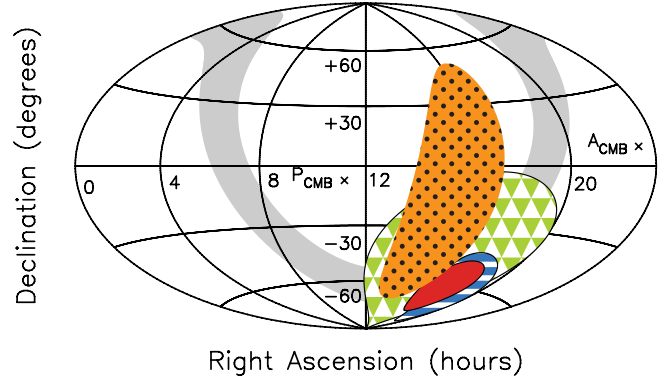


FIG. 1 (color online). 1σ confidence regions for the Keck (green triangles), VLT (blue stripes), and combined (plain red) Australian dipoles of α -variation in the Hammer-Aitoff projection, taken from [2]. The spotted orange region is the 1σ confidence interval for a dipole in deuterium abundance, where each point on the contour represents a separate minimization of m_D and d_D for a dipole in that direction. The galactic plane (grey) and CMB dipole direction (P_{CMB}) are also shown.

In Fig. 2 we show the measured abundances against $r \cos \psi$. The dipole fit (solid line) fits better than the monopole fit (dashed line). We clearly need more sky coverage: all of the data points lie at angles $55^\circ < \psi < 125^\circ$ from the dipole axis. The distances are similar for all the data points ($r \sim 11$ Gly); therefore, our results do not vary significantly if we remove the distance dependence and let $\Xi(\mathbf{r}) = \cos \psi$. We note in passing that if the two outlying points in Fig. 2 are removed and only five remain, $\chi^2/\nu = 2.00$ for the monopole fit and 0.44 for the dipole fit: $m = -4.55(2)$ and $d = 0.010(4)$.

A more reasonable approach to the χ^2 discrepancy is to increase the error bars to account for an unknown systematic. We have done this by adding a constant error σ_{sys} to all the statistical errors in quadrature: $\sigma_{\text{tot}}^2 = \sigma_{\text{stat}}^2 + \sigma_{\text{sys}}^2$. Increasing the errors such that the dipole fit has $\chi^2 = 5$ requires an additional systematic of $\sigma_{\text{sys}} = 0.11$ to be added to all the data points. An increase in the uncertainty of this magnitude destroys any justification for fitting a dipole to the data.

Finally, we wish to provide some guidance as to how one might interpret any detection (or nondetection) of a dipole

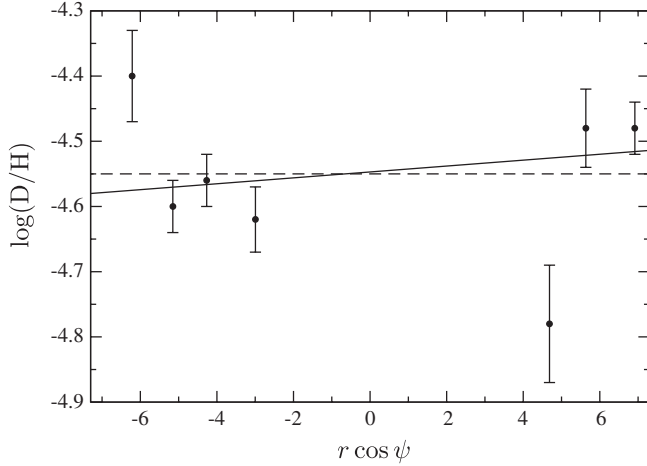


FIG. 2. Deuterium abundance vs $r \cos \psi$ (Gly) where ψ is the angle between the measurement direction and the dipole of Ref. [1]. Dashed line: monopole fit; solid line: dipole fit (fixed direction). Fit parameters are given in Table II.

in the primordial deuterium abundance in terms of variations in fundamental constants. Using the existing data (as we have seen, rather inadequate since it suffers from excess scatter), we obtain $d_D = 0.0045(35)$ and we may interpret this in terms of variation of α . Using the result $\partial \ln(D/H)/\partial \ln \alpha = 3.6$ [38] in Eq. (9) we obtain

$$\frac{\delta \alpha}{\alpha} = \frac{0.0045}{3.6/\ln 10} \Xi(\mathbf{r}) = (2.9 \pm 2.2) \times 10^{-3} \Xi(\mathbf{r}).$$

To clarify the meaning of this equation, a system 1 Gly in the direction of the dipole axis had a fine-structure constant different from the present-day laboratory value by $\delta \alpha/\alpha = 0.003$ at the time of big bang nucleosynthesis. Of course our data must be considered as consistent with zero variation.

An effect at the level 10^{-3} is much larger than that seen directly in quasar absorption spectra: $\delta \alpha/\alpha \sim 10^{-6} \Xi$ in [1]. Yet even if such a signal is found, there would be no inconsistency. The direct measurements of [1] probe physics occurring a few billion years ago, while the primordial abundance measurements probe the first few minutes after the big bang.

We chose the fine-structure constant since spatial variation of α has been reported. However, big bang nucleosynthesis is more sensitive to $X_q = m_q/\Lambda_{\text{QCD}}$, the dimensionless ratio of light-quark mass to the pole in the running coupling constant, and it is generally more reasonable to interpret the variation in terms of smaller shifts in these constants. For example, using the result $\partial \ln(D/H)/\partial \ln X_q = 7.7$ [37] we obtain

$$\frac{\delta X_q}{X_q} = 0.0013(10) \Xi(\mathbf{r}).$$

TABLE III. The sensitivity of relative variation in primordial abundances to relative variation in fundamental constants $\partial \ln Y_a/\partial \ln X$. Here $Y_a = (D/H, {}^3\text{He}/H, Y_p, {}^6\text{Li}/H, {}^7\text{Li}/H)$ are number ratios of primordial isotope abundances to hydrogen, except Y_p that is the mass ratio ${}^4\text{He}/H$.

X	D/H	${}^3\text{He}/H$	Y_p	${}^6\text{Li}/H$	${}^7\text{Li}/H$	
α	3.6	0.95	1.9	6.6	-11	[38]
X_q	7.7	—	-0.95	—	-50	[37]
η	-1.6	-0.57	0.04	-1.5	2.1	[38]
G	0.94	0.33	0.36	1.4	-0.72	[38]

Clearly the detection and confirmation of spatial variation in fundamental constants at the time of big bang nucleosynthesis requires a large improvement in the number of deuterium measurements and their coverage of the sky. However, it may also be possible to detect other elements produced in BBN such as ${}^3\text{He}$, ${}^4\text{He}$, ${}^6\text{Li}$, and ${}^7\text{Li}$ in high-redshift quasar absorption spectra and determine their primordial abundances. If the response of BBN to variation of constants is known, one can readily predict the spatial variation of their abundances given a model of fundamental constant variation. We reproduce some of the known response functions from the theory [37,38] in Table III.

We can see, for example, that a variation of 1% in α would amount to a 1.9% change in Y_p (the mass ratio ${}^4\text{He}/H$ —for all other elements the number ratio is presented) and a -11% change in ${}^7\text{Li}$ abundance. This provides an independent means of verifying a particular change in fundamental constants. The primordial ${}^7\text{Li}$ abundance evidently provides a particularly sensitive probe of fundamental constants. In Table III we also present the sensitivity of primordial abundances to the baryon-to-photon ratio η (relative to the WMAP value $\eta = 6.1 \times 10^{-10}$), which prior to WMAP was constrained chiefly by BBN data.

VII. CONCLUSION

The existing H_2 data from quasar absorption spectra shows weak hints that there may be a dipole in μ -variation with an axis corresponding to that of the gradient in α found in [1]. On the other hand $x = \alpha^2 \mu g_p$ variation, taken from HI 21-cm data, has a best-fit dipole whose axis does not correspond to that of the Australian dipole, although in this case systematics heavily dominate. We have demonstrated that it is possible to infer spatial variation of fundamental constants during big bang nucleosynthesis from high-redshift measurements of primordial abundances. Although the existing deuterium data does not support the dipole interpretation, with the significance of the dipole model being similar to that of the monopole model, it is interesting that the preferred axis is consistent with the direction of the Australian dipole. There is a

strong impetus now to perform measurements of relative primordial abundance at high redshifts of as many elements as possible in as many different spatial directions as possible.

Finally, we note that it may be possible to observe a spatial dipole in other cosmological systems [41]. For example, α -variation may be seen in the CMB anisotropy if a high enough sensitivity can be reached. Although the results of [1] (interpreted as strictly spatial variation) suggest that accuracy at the level 10^{-6} will be required, if the hints from BBN turn out to be real then there is an additional redshift (time) dependence that could increase the variation at the time of the CMB substantially. The observed spatial variation of α could also be connected to the

apparent accelerated expansion of the Universe, deduced from the redshift-luminosity distance relation seen for Type Ia supernovae and from other probes of cosmological history (e.g. [42–44]). If a common physical mechanism were responsible for both α -variation and acceleration, it might be possible to observe a dipole in the SNIa redshift-luminosity relation (see e.g. [45]) or in other probes sensitive to anisotropic expansion, e.g. [46].

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