The μ problem and the axion in gauge mediation

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We revisit the idea of generating the Higgs μ parameter through a spontaneously broken Peccei-Quinn (PQ) symmetry in a gauge-mediated supersymmetry breaking scenario. For the messenger scale of gauge mediation higher than the PQ scale, the setup naturally generates $\mu \sim m_{soft}$ and the Higgs soft parameter $B \leq \mathcal{O}(m_{soft})$ with the CP phase of B aligned to the phase of gaugino masses, while giving the PQ scale $v_{\rm PQ} \sim \sqrt{m_{\rm soft} \Lambda}$, where $m_{\rm soft}$ denotes the gauge-mediated gaugino or sfermion masses and Λ is the cutoff scale which can be identified as the Planck scale or the grand unified theory scale. The PQ sector of the model results in distinctive cosmology including a late thermal inflation. We discuss the issue of dark matter and baryogenesis in the resulting thermal inflation scenario, and find that a right amount of gravitino dark matter can be produced together with a successful Affleck-Dine leptogenesis, when the gravitino mass $m_{3/2} = \mathcal{O}(100)$ keV.

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I. INTRODUCTION

Weak scale supersymmetry (SUSY) is one of the most attractive candidates for new physics beyond the standard model (SM) at the TeV scale [[1](#page-6-0)]. It provides an appealing solution to the gauge hierarchy problem, and also the successful unification of gauge couplings at the scale $M_{\text{GUT}} \sim 2 \times 10^{16}$ GeV. On the other hand, the absence of unacceptably large flavor or CP violations requires a rather special type of supersymmetry breaking which yields flavor and CP conserving soft terms. Supersymmetry breaking through gauge mediation [\[2\]](#page-6-1) provides flavor conserving (possibly CP conserving also) soft terms in a natural manner as the structure of soft terms is determined mostly by the SM gauge interactions. One potential difficulty of the gauge mediation mechanism is the generation of the Higgs μ and B parameters having a right size for the electroweak symmetry breaking. If the Higgs sector communicates with the SUSY breaking sector to generate $\mu \sim m_{soft}$, where m_{soft} denotes the gaugino and sfermion masses in gauge mediation, one often finds $B \sim 8\pi^2 m_{\text{soft}}$, which is too large to achieve a successful electroweak symmetry breaking. There have been many attempts to solve the μ problem in gauge mediation, including those in Ref. [\[3](#page-6-2)].

As was noticed in the original work of Kim and Nilles [[4\]](#page-6-3), a satisfactory solution of the μ problem should provide a theoretical reasoning for the absence of the bare μ term with $\mu \sim \Lambda$, as well as a dynamical mechanism to generate $\mu \sim m_{soft}$ together with $B \sim m_{soft}$ at the weak scale, where Λ denotes the cutoff scale of the model which can be taken as either the reduced Planck scale $M_{\text{Pl}} \sim 2 \times 10^{18} \text{ GeV}$ or the grand unified theory (GUT) scale M_{GUT} 2×10^{16} GeV. It was further noticed in Ref. [[4\]](#page-6-3) that the Peccei-Quinn (PQ) symmetry solving the strong CP problem might play a crucial role for the μ problem as well. The $U(1)_{\text{PO}}$ symmetry might forbid the bare μ term, while allowing the following nonrenormalizable term in the superpotential:

$$
\frac{1}{\Lambda}X^2H_uH_d,\tag{1}
$$

where X is a PQ charged SM singlet field whose vacuum value breaks $U(1)_{\text{PO}}$ spontaneously. If the PQ sector of the model couples to the SUSY breaking sector to stabilize X at

$$
v_{\rm PQ} \equiv \langle X \rangle \sim \sqrt{m_{\rm soft} \Lambda},\tag{2}
$$

and the F component of the stabilized X satisfies $F^X/X \leq$ $\mathcal{O}(m_{\text{soft}})$, the resulting μ and B (at the weak scale) have a right size for successful electroweak symmetry breaking.

The Kim-Nilles mechanism was discussed originally in the context of gravity mediation with $m_{soft} \sim m_{3/2}$ [\[4–](#page-6-3)[6\]](#page-6-4). Later it was realized that the mechanism can be implemented also in gauge mediation [\[7\]](#page-6-5). However, the specific models discussed in [\[7\]](#page-6-5) involve $U(1)_{PQ}$ which is assumed to be an R symmetry. In such models, the nonzero vacuum value of the superpotential, which is required to tune the cosmological constant vanish, should appear as a consequence of the spontaneous breakdown of $U(1)_{\text{PO}}$, and this makes a complete realization of the setup quite complicated. In this paper, we revisit the Kim-Nilles mechanism to generate μ and B in gauge mediation, while focusing on the case that $U(1)_{\text{PO}}$ is not an R symmetry, but an ordinary anomalous global symmetry. It is noticed that such a class of models can have a distinctive cosmological feature such as a late thermal inflation triggered by the PQ sector [[8\]](#page-6-6). We then need a late baryogenesis after thermal inflation as well as a mechanism to produce a right amount of dark matter. We find that a right amount of gravitino dark matter can be produced after thermal inflation when $v_{\text{PO}} =$ $\mathcal{O}(10^9 - 10^{10})$ GeV and $m_{3/2} = \mathcal{O}(100)$ keV. With the nonrenormalizable term [\(1\)](#page-0-0) and also the seesaw term for light Majorana neutrino masses, the model can accommodate also a successful Affleck-Dine (AD) leptogenesis proposed in [[9](#page-6-7)[,10\]](#page-6-8).

This paper is organized as follows. In Sec. [II](#page-1-0), we discuss the Kim-Nilles mechanism to generate $\mu \sim m_{soft}$ and $B \le$ $\mathcal{O}(m_{\text{soft}})$ with a spontaneously broken $U(1)_{\text{PQ}}$ in gauge mediation. Section [III](#page-3-0) discusses the cosmological aspects of the model, including the mechanisms to generate the right amount of dark matter and baryon asymmetry, and the conclusion will be given in Sec. [IV.](#page-5-0)

II. THE KIM-NILLES MECHANISM IN GAUGE MEDIATION

In the (minimal) gauge mediation scenario, SUSY breaking is mediated by SM gauge-charged messengers Φ , Φ^c which couple to the SUSY breaking field $Z = M +$ $F\theta^2$ in the superpotential. Then sfermions and gauginos in the minimal supersymmetric standard model (MSSM) get soft SUSY breaking masses

$$
m_{\text{soft}} \sim \frac{g^2}{16\pi^2} \frac{F}{M} \tag{3}
$$

through the loops involving the messenger fields Φ , Φ^c . In order to implement the Kim-Nilles mechanism to generate the μ term, we introduce an additional SM singlet but PQ charged superfields which break $U(1)_{PQ}$ spontaneously, and also extra vectorlike quark superfields¹ which have a Yukawa coupling to the $U(1)_{\text{PO}}$ -breaking fields. With $U(1)_{\text{PO}}$, one can forbid the renormalizable superpotential term of the $U(1)_{\text{PO}}$ -breaking fields, while allowing a nonrenormalizable term suppressed by the cutoff scale Λ of the model. Then due to the SUSY breaking effects mediated through the Yukawa coupling to extra quark superfields, the $U(1)_{\text{PO}}$ -breaking fields are destabilized at the origin. On the other hand, the supersymmetric scalar potential originating from the nonrenormalizable superpotential prevents the runaway of the $U(1)_{\text{PO}}$ -breaking fields, and stabilize them at an intermediate scale $v_{\rm PQ} \sim \sqrt{m_{\rm soft} \Lambda}$. With Λ presumed to be the GUT scale or the Planck scale, this scenario naturally generates a QCD axion scale $v_{\text{PO}} =$ $\mathcal{O}(10^9-10^{11})$ GeV, as well as a correct size of $\mu \sim$ $v_{\text{PQ}}^2/\Lambda \sim m_{\text{soft}}$ via the Kim-Nilles mechanism. Furthermore, in this setup one can easily obtain the Higgs B parameter at the weak scale which is (at most) comparable to m_{soft} and has a CP phase aligned to the phase of gaugino masses.

As a specific model to realize this scenario, we consider the superpotential

$$
W = y_u Q H_u u^c + y_d Q H_d d^c + y_e L H_d e^c + \frac{y_\nu}{M_N} L H_u L H_u,
$$

+ $\lambda_X X \Psi \Psi^c + \frac{\kappa_1}{6\Lambda} X^3 Y + \frac{\kappa_2}{2\Lambda} X^2 H_u H_d + \lambda_Z Z \Phi \Phi^c,$ (4)

where the first line denotes the usual Yukawa couplings between the Higgs fields and the quarks and/or leptons, including the term for small neutrino masses which might be generated by the seesaw mechanism [[11](#page-6-9)] with a righthanded neutrino mass M_N far above the weak scale. Here the flavor indices are omitted, and y_u , y_d , y_e , and y_v , should be understood as 3×3 matrices. As we will see, the above model has a variety of interesting cosmological features, including a late thermal inflation associated with the PQ phase transition in the early universe. Although we do not specify the origin of M_N here, an interesting possibility is that M_N is generated as a consequence of $U(1)_{\text{PO}}$ breaking, which would give $M_N \sim \langle X \rangle$, so that the seesaw scale is identified as the PQ scale² [[12](#page-6-10)]. As we will see in the next section, such a setup can be useful also for a successful Affleck-Dine leptogenesis after thermal inflation.

The second line of the superpotential [\(4\)](#page-1-1) is the PQ sector generating the Higgs μ parameter through the Kim-Nilles mechanism, while providing a QCD axion to solve the strong CP problem [[13](#page-6-11)]. The third line is for the minimal gauge mediation of SUSY breaking, where Z is the SUSY breaking field with

$$
\langle Z \rangle = M + F\theta^2. \tag{5}
$$

Note that we can always make λ_X , κ_1 , κ_2 , and $\lambda_Z\langle Z \rangle$ all real and positive through appropriate field redefinitions, and we will take such field basis in the following discussion. To be specific, we also assume that the cutoff scale Λ is around the GUT scale

$$
\Lambda \sim M_{\text{GUT}} = 2 \times 10^{16} \text{ GeV.}
$$
 (6)

Although a different choice of Λ would change the value of the PQ scale, the μ parameter obtained by the Kim-Nilles mechanism is independent of Λ and always of the order of m_{soft} as long as the dimensionless parameters κ_1 and κ_2 have a similar size. Note that the superpotential [\(4\)](#page-1-1) takes the most general form (up to dim $=$ 4 terms) allowed by the SM gauge symmetries, R-parity, and $U(1)_{PQ}$, where the $U(1)_{\text{PO}}$ charges are given as follows:

¹To keep the successful unification of gauge couplings in the MSSM, these extra vectorlike quarks can be extended to form a full GUT multiplet.

²Of course, then the Yukawa couplings between H_u and the left- and right-handed neutrinos should have appropriately small values to produce the observed neutrino mass-square differences and mixing angles.

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- ∠	\mathbf{r}	\sim H_u	H_{d}	Ou^c	d^c	$L_{\rho}c$	$\Psi \Psi$	$\Phi\Phi^c$
							$\overline{}$	

Let us now discuss the vacuum configuration of the $U(1)_{\text{PO}}$ -breaking fields X and Y. As the gravitino mass is much smaller than the weak scale, we can safely ignore the supergravity effects. Then the scalar potential of X , Y can be well approximated by the global SUSY potential including the soft SUSY breaking terms induced by radiative corrections:

$$
V(X, Y) = V_{\text{soft}}(X) + \frac{\kappa_1^2}{36\Lambda^2} |X|^6 + \frac{\kappa_1^2}{4\Lambda^2} |X|^4 |Y|^2, \tag{7}
$$

where

$$
V_{\text{soft}}(X, Y) = m_X^2 |X|^2 + m_Y^2 |Y|^2 + \left(A_{\kappa_1} \frac{\kappa_1}{6\Lambda} X^3 Y + \text{H.c.}\right).
$$

If the messenger scale M_{Φ} of gauge mediation is above the PQ threshold scale

$$
M_{\Phi} \equiv \lambda_Z \langle Z \rangle \gtrsim \lambda_X \langle X \rangle, \tag{8}
$$

which is in fact necessary to generate $\mu \sim m_{soft}$ independently of the value of Λ , the soft mass m_X at scales below M_{Φ} is generated mostly by the renormalization group (RG) running triggered by the Yukawa coupling $\lambda_X X \Psi \Psi^c$. The RG equation for m_X^2 is given by

$$
\frac{dm_X^2}{d\ln\mu^2} = \frac{3\lambda_X^2}{16\pi^2} (m_{\tilde{\Psi}}^2 + m_{\tilde{\Psi}^c}^2 + m_X^2 + |A_{X\Psi\Psi^c}|^2), \quad (9)
$$

where the factor 3 is the color factor, and $m_{\tilde{\Psi}}$, $m_{\tilde{\Psi}^c}$ and $A_{X\Psi\Psi^c}$ are the gauge-mediated soft scalar masses and trilinear scalar coupling, respectively, for the squark components of Ψ , Ψ^c . As m_X and $A_{X\Psi\Psi^c}$ at the messenger scale are negligible compared to $m_{\tilde{\Psi}} \sim m_{soft}$, the soft mass m_X at the lower renormalization point $\langle X \rangle$ is determined as

$$
m_X^2(|X|) \simeq -(m_{\tilde{\Psi}}^2 + m_{\tilde{\Psi}^c}^2) \frac{3\lambda_X^2}{8\pi^2} \ln \frac{M_\Phi}{|X|},\tag{10}
$$

where we have ignored higher powers of $\frac{1}{8\pi^2} \ln(M_\Phi / |X|)$ with the assumption that M_{Φ} is not so far above $\langle X \rangle$. Since the PQ breaking scale $\langle X \rangle$ is constrained to be of $\mathcal{O}(10^9 - 10^{12})$ GeV, while the messenger scale should be lower than $\mathcal{O}(10^{15})$ GeV in order for the gauge mediation to give dominant contribution to soft terms, the value of $ln(M_{\Phi}/|X|)$ cannot be so large, and therefore our assumption is justified.

In our approximation, $m_{\tilde{\Psi}}$ and $m_{\tilde{\Psi}}$ in [\(10\)](#page-2-0) can be regarded as the soft squark masses at the messenger scale M_{Φ} , which are given by

$$
m_{\tilde{\Psi}}^2 = m_{\tilde{\Psi}^c}^2 = \frac{8N_{\Phi}}{3} \left[\frac{g_s^2}{16\pi^2} \frac{F}{M} \right]^2 \approx m_{\text{soft}}^2,
$$
 (11)

where N_{Φ} is the number of messenger pairs in the fundamental representation. Minimizing the scalar potential [\(7\)](#page-2-1) with the tachyonic m_X^2 given by ([10](#page-2-0)), we find

$$
\upsilon_{\rm PQ}^2 \equiv \langle |X|^2 \rangle \simeq \frac{3\lambda_X \sqrt{\ln(M_\Phi/\nu_{\rm PQ})}}{\pi \kappa_1} m_{\tilde{\Psi}} \Lambda = \mathcal{O}(m_{\rm soft} \Lambda) \tag{12}
$$

and the resulting Higgs μ parameter

$$
\mu \simeq \frac{3\kappa_2 \lambda_X \sqrt{\ln(M_\Phi/v_{\rm PQ})}}{2\kappa_1 \pi} m_{\tilde{\Psi}} = \mathcal{O}(m_{\rm soft}),\tag{13}
$$

where we assumed that λ_X , κ_1 , and κ_2 are all of order unity for the order of magnitude estimate in the last step. Note that μ is independent of the precise value of the cutoff scale Λ , while the PQ scale has a mild dependence on Λ .

The vacuum expectation value of X in [\(12\)](#page-2-2) generates an effective mass of Y through the term $\propto |X|^4 |Y|^2$ in the scalar potential:

$$
\frac{\kappa_1^2}{4\Lambda^2} \langle |X|^4 \rangle |Y|^2 = 3|m_X|^2 |Y|^2. \tag{14}
$$

It also generates an effective tadpole of Y through the A term $\propto X^3Y$ in the scalar potential, which is generated by the RG evolution of the wave function factor of X . We then find

$$
\frac{A_{\kappa_1}(|X|)}{m_{\tilde{\Psi}}} \simeq -\frac{3\sqrt{3N_{\Phi}}\lambda_X^2}{4\sqrt{2}\pi^2} \frac{g_s^2}{8\pi^2} \left(\ln\frac{M_{\Phi}}{|X|}\right)^2,\tag{15}
$$

and therefore

$$
\frac{\langle |Y| \rangle}{\langle |X| \rangle} = \frac{|A_{\kappa_1}|}{3\sqrt{3}|m_X|} \simeq \frac{\sqrt{N_{\Phi}}}{2\sqrt{6}} \frac{g_s^2 \lambda_X}{8\pi^3} \left(\ln \frac{M_{\Phi}}{\langle |X| \rangle} \right)^{3/2} . \tag{16}
$$

With the above results, one can compute the Higgs B parameter around the messenger scale, which is given by

$$
B(M_{\Phi}) = \frac{B\mu}{\mu} \bigg|_{M_{\Phi}} = 2\left(\frac{F^{X}}{X}\right) = -2\left(\left(\frac{\partial_{X}W}{X^{*}}\right)^{*} + \frac{A_{\kappa_{1}}}{3}\right) \approx 0,
$$
\n(17)

upon ignoring the gravity mediated contribution of $\mathcal{O}(m_{3/2})$. Note that the equation of motion of Y leads to the cancellation between the two contributions to $B(M_{\Phi})$, making $B(M_{\Phi})$ even smaller than $\mathcal{O}(A_{\kappa_1})$. As the B parameter at M_{Φ} is negligible compared to m_{soft} , its low energy value is determined by the RG running from M_{Φ} to the weak scale. In our case, the messenger scale M_{Φ} is required to be higher than the PQ scale $v_{PQ} \sim \sqrt{m_{\text{soft}} M_{GUT}} = \mathcal{O}(10^9 - 10^{10})$ GeV. As a result, a sizable value of B can be induced at the weak scale, giving $tan \beta = 10-20$, and furthermore its CP phase is automatically aligned to the phase of gaugino masses.

With the PQ sector stabilized as above, we can identify the mass eigenstates of the PQ sector fields and compute their mass eigenvalues. First of all, the PQ sector provides a QCD axion having a decay constant v_{PQ} and thus a mass $m_a \sim f_\pi m_\pi / v_{\text{PQ}}$, which corresponds mostly to the phase degree of freedom of X. It contains also three real scalars with a mass comparable to m_{soft} , i.e., the saxion x which is mostly the modulus of X and two others from $Y =$ $y_1 + iy_2$, and two Majorana fermions \tilde{a}_i (i = 1, 2) which form approximately a Dirac axino $\tilde{a} = (\tilde{a}_1, \tilde{a}_2)$. It is then straightforward to find

$$
m_x \approx 2|m_X|
$$
, $m_{y_1} \approx m_{y_2} \approx m_{\tilde{a}} \approx \sqrt{3}|m_X|$, (18)

where m_X is given by [\(10\)](#page-2-0).

In summary, in our model the messenger scale of gauge mediation is assumed to be higher than the PQ scale, and then the $U(1)_{PQ}$ -breaking fields X and Y are destabilized from the origin due to the tachyonic soft mass of X and the scalar A term associated with the nonrenormalizable superpotential term $\kappa_1 X^3 Y/6\Lambda$. These soft SUSY breaking terms of X and Y are induced by the combined effects of gauge-mediated SUSY breaking and the Yukawa coupling $\lambda_X X \Psi \Psi^c$. The supersymmetric scalar potential from the nonrenormalizable superpotential prevents the runaway of X and Y , and stabilizes them as

$$
\langle |X| \rangle \sim \left(\frac{\lambda_X}{\kappa_1}\right)^{1/2} \left(\ln \frac{M_\Phi}{\langle |X|\rangle}\right)^{1/4} \sqrt{m_{\text{soft}}\Lambda},
$$

$$
\langle |Y| \rangle \sim \frac{g_s^2 \lambda_X}{32\pi^3} \left(\ln \frac{M_\Phi}{\langle |X|\rangle}\right)^{3/2} \langle |X| \rangle,
$$
 (19)

where the messenger scale $M_{\Phi} > v_{\text{PQ}} \equiv \langle |X| \rangle$ and Λ is the cutoff scale of the model. If we assume that λ_X , κ_1 , and κ_2 are all of order unity and $\Lambda \sim M_{\text{GUT}} = 2 \times 10^{16} \text{ GeV}$, while M_{Φ} is not so far above $v_{\rm PO}$, the mass scales of the model are estimated as

$$
\mu \sim m_{\text{soft}} \sim m_{\text{PQ}} = \mathcal{O}(10^2 - 10^3) \text{ GeV},
$$

\n
$$
v_{\text{PQ}} \sim \sqrt{m_{\text{soft}} M_{\text{GUT}}} = \mathcal{O}(10^9 - 10^{10}) \text{ GeV},
$$
\n(20)

where m_{PO} stands for the masses of the PQ sector fields (other than the QCD axion), including the saxion and axino masses. The *B* parameter at the messenger scale is negligible compared to m_{soft} , and therefore its weak scale value is determined by the RG evolution below the messenger scale, making its CP phase automatically aligned to the phase of gaugino masses.

III. COSMOLOGY OF THE MODEL

The model described in Sec. [II](#page-1-0) has a variety of interesting cosmological implications. Because the PQ preserving field configuration $X = Y = 0$ is a local minimum of the effective potential at high temperature $T \gg m_{soft}$, it is quite plausible that X is settled down at the origin after the primordial inflation. Then the early universe experiences a late thermal inflation [[8](#page-6-6)[,14\]](#page-6-12) before the PQ phase transition occurs, which might be useful to eliminate (or dilute) potentially dangerous cosmological relics such as light moduli or gravitinos. 3 Since this thermal inflation will erase out any primordial baryon asymmetry, we need a baryogenesis mechanism operating after thermal inflation is over. As for the dark matter in our model, one can consider two possible candidates, QCD axion with a mass $m_a \sim f_{\pi} m_{\pi}/v_{\text{PQ}}$ and light gravitino with a mass $m_{3/2} \sim F/M_{\text{Pl}}$. However, the PQ scale of our model is determined as $v_{\text{PQ}} \sim \sqrt{m_{\text{soft}} M_{\text{GUT}}} \sim \mathcal{O}(10^9 \text{--} 10^{10}) \text{ GeV},$ which might be too low to give a QCD axion constituting the major fraction of the observed dark matter [\[13\]](#page-6-11). This leads us to focus on the possibility of gravitino dark matter.

In the following, we briefly discuss the cosmological features of our model, while leaving more complete discussions for a separate paper [[15](#page-6-13)]. As we will see, for the gravitino mass range

$$
m_{3/2} = O(100) \text{ keV}, \tag{21}
$$

a right amount of gravitino dark matter can be produced after thermal inflation, together with a successful Affleck-Dine leptogenesis. As the gravitino mass is given by

$$
m_{3/2} \sim \frac{F}{M_{\text{Pl}}} \sim \frac{16\pi^2}{g^2} \frac{M}{M_{\text{Pl}}} m_{\text{soft}}
$$
 (22)

for the SUSY breaking spurion $Z = M + F\theta^2$, this range of $m_{3/2}$ suggests that the messenger scale of gauge mediation, i.e., $M_{\Phi} = \lambda_Z M$, which is presumed to be higher than the PQ scale v_{PQ} , should be somewhat close to $v_{\rm PO} \sim 10^9$ –10¹⁰ GeV.

A. Thermal inflation

Near the origin, the finite-temperature effective potential of the flat direction $|X|$ is given by

$$
V(X) = V_0 + (\beta_X^2 T^2 - |m_X^2(0)|)|X|^2 + \dots,
$$
 (23)

where $V_0 = O(m_X^2 v_{\text{PQ}}^2)$ is the potential energy at the origin, which is set to make the cosmological constant at true vacuum vanish, and β_X comes from the couplings to thermal bath. Once the Universe was in a radiation dominated period with $T > V_0^{1/4}$ after the primordial inflation is over, thermal inflation begins at the temperature

$$
T_{\rm b} \sim V_0^{1/4} \sim 10^6 \text{ GeV}
$$
 (24)

and ends when $|X|$ is destabilized from the origin at the critical temperature

$$
T_{\rm c} = \frac{|m_X(0)|}{\beta_X}.
$$
 (25)

³Note that a UV completion of the model within the framework of supergravity or string theory might contain cosmologically harmful light moduli causing the so-called moduli problem.

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Soon after the end of thermal inflation, the Universe is dominated for a while by the coherent oscillation of $|X|$ around its true minimum $\langle |X| \rangle = v_{\text{PO}}$, which eventually decays into lighter particles to reheat the Universe. Since the saxion mass $m_x = \mathcal{O}(10^2 - 10^3)$ GeV in our model [see [\(18\)](#page-3-1)], it can decay dominantly to the light Higgs boson pair $h + h^*$ through the coupling of the form $\mu^2 h h^* \delta x / v_{\text{PO}}$, where δx denotes the saxion fluctuation around its vacuum. Assuming that m_x is heavier than $2m_h$ for the light Higgs boson mass $m_h \approx 120$ GeV, the decay rate of |X| is estimated as

$$
\Gamma_X \sim \frac{1}{4\pi} \frac{\mu^4}{m_x v_{\text{PQ}}^2},\tag{26}
$$

and then we find the reheat temperature is given by

$$
T_{\rm RH} = \left(\frac{\pi^2}{15}g_*(T_{\rm RH})\right)^{-1/4} \Gamma_X^{1/2} M_{\rm Pl}^{1/2},\tag{27}
$$

$$
\approx 1 \text{ TeV} \left(\frac{300 \text{ GeV}}{m_x}\right)^{1/2} \left(\frac{\mu}{600 \text{ GeV}}\right)^2
$$

$$
\times \left(\frac{3 \times 10^9 \text{ GeV}}{v_{\text{PQ}}}\right),\tag{28}
$$

where $g_*(T_{\rm RH}) \sim 100$ is the number of light degrees of freedom at $T = T_{\text{RH}}$. The total number of e-foldings of this thermal inflation is estimated to be about 10 and the dilution factor due to the entropy release in the decay of |X| is about $\mathcal{O}(10^{10})$. This would be large enough to remove, for instance, the gravitinos produced before thermal inflation [\[16\]](#page-6-14).

Our model has two other oscillating scalar fields which are mostly $y_1 = \text{Re}(Y)$ and $y_2 = \text{Im}(Y)$. Although they have a mass comparable to m_x [see ([18](#page-3-1))], their energy densities are suppressed by $\langle |Y|^2 \rangle / \langle |X|^2 \rangle =$ densities are suppressed by $\langle |Y|$ $\binom{2}{|X|^2}$ = $\mathcal{O}(g_s^4 \lambda_X^2/(8\pi^3)^2)$ compared to that of |X|, and therefore they do not give a significant impact on the cosmological evolution after thermal inflation.

B. Affleck-Dine leptogenesis

Thermal inflation erases preexisting baryon asymmetry. One may think that an AD baryogenesis before thermal inflation with a very large initial value of AD field can produce enough baryon asymmetry which would survive after thermal inflation [\[17\]](#page-6-15). However, it is known that the formation of Q-balls makes it difficult to realize such scenario [[18](#page-6-16)]. Fortunately, our model can realize the latetime AD leptogenesis proposed in Refs. [\[9](#page-6-7)[,10](#page-6-8)[,19–](#page-6-17)[23\]](#page-6-18).

In order for the AD leptogenesis to work, the MSSM flat direction LH_u is required to have a nonzero value at certain stage. In our case, this initial condition can be achieved as LH_u has a tachyonic soft mass-square $-m_{\text{LH}_{u}}^2$ in the limit $\mu = 0$, so unstable at the origin if the temperature drops below its critical temperatures $T_{\text{LH}_{u}} = m_{\text{LH}_{u}} / \beta_{\text{LH}_{u}}$ $\sqrt{2}m_{\text{LH}_{u}}$ and X is still staying at the origin. This requires

$$
T_{\rm c} = \frac{|m_X(0)|}{\beta_X} < \sqrt{2} m_{\rm LH_{u'}} \tag{29}
$$

and thus

$$
\beta_X^2 > \left(\frac{m_X(0)}{m_{\text{LH}_u}}\right)^2 \simeq \frac{6\lambda_X^2}{\pi^2} \left(\frac{m_{\tilde{\Psi}}}{m_{\text{LH}_u}}\right)^2 \ln\left(\frac{M_{\Phi}}{\nu_{\text{PQ}}}\right),\tag{30}
$$

where we have used $m_X(0) \approx 4m_X(v_{\text{PQ}})$ together with the result [\(10\)](#page-2-0) for $m_X(v_{\text{PQ}})$. In our model, β_X^2 receive a contribution from the exotic quark superfields Ψ , Ψ^c , giving

$$
\Delta \beta_X^2 = \frac{3}{4} \lambda_X^2. \tag{31}
$$

It turns out that it is difficult to satisfy [\(30\)](#page-4-0) only with $\Delta \beta_X^2$ for typical parameter values of our model. However this difficulty can be easily avoided if the field X couples to the right-handed neutrinos N to generate the seesaw scale [[12\]](#page-6-10). Then there will be an additional contribution to β_X^2 from the Yukawa coupling $\lambda_N XNN$ [\[24\]](#page-6-19),

$$
\beta_X^2 = \frac{1}{4} \left(\sum_N \lambda_N^2 + 3\lambda_X^2 \right),\tag{32}
$$

with which the condition ([30](#page-4-0)) can be satisfied with a reasonable value of λ_N .

Once the key condition [\(30\)](#page-4-0) for AD leptogenesis is satisfied, the AD field LH_u rolls down to nonzero value at the temperature $T \sim m_{\text{LH}_{u}}$. If T drops further down to T_c , |X| rolls away from the origin to generate nonzero μ , and then LH_u gets a positive mass-square due to the contribution from μ^2 . As a result, LH_u rolls back to the origin with an angular motion generated by CP-violating terms in the scalar potential. The lepton asymmetry associated with the angular motion of AD field is finally converted to baryon asymmetry through the sphaleron process. The resulting baryon asymmetry at present is estimated as [\[10\]](#page-6-8)

$$
\frac{n_B}{s} \sim \frac{3}{8} \frac{n_L T_{\rm RH}}{n_x m_x},\tag{33}
$$

where $n_x \sim m_x v_{\text{PQ}}^2$ is the saxion number density for the coherently oscillating saxion field $|X|$, and n_L is the lepton number density associated with the angular motion of the AD field. In fact, n_L depends on many details of the full scalar potential, including the terms associated with the lepton number violating neutrino mass term in the superpotential. It depends, for instance, on the initial displacement of the AD field from the origin, curvature of the potential in angular direction, CP phase, etc. Using the results of $[10,20,21,23]$ $[10,20,21,23]$ $[10,20,21,23]$ $[10,20,21,23]$, we find that a value of $n_L = \mathcal{O}(10^{11} - 10^{12})$ GeV³ can be achieved under a reasonable assumption on the involved model parameters, and therefore the AD leptogenesis after thermal inflation can produce the observed value of $n_B/s \sim 10^{-10}$ within the uncertainties in the involved parameters.

C. Dark matter

In our model the lightest supersymmetric particle (LSP) is the gravitino, because all other supersymmetric particles including axinos have mass of order m_{soft} , and the gravitino mass is much smaller than m_{soft} . Thus, light gravitino is the prime candidate of the dark matter. On the other hand, thermal inflation also dilutes preexisting gravitino relics. After that, by decay of $|X|$, the Universe is reheated with temperature $T_{\text{RH}} \sim 1$ TeV as in Eq. ([27](#page-4-1)). At this temperature, most of the MSSM fields are thermalized and will produce light gravitinos. We can divide this process into two parts. One is thermal (TH) production in which gravitinos are produced by scatterings and decays of the MSSM fields in thermal bath. The other is nonthermal (NTH) production in which gravitinos are produced by out of equilibrium decays of the frozen relics such as the next LSP which is the ordinary LSP (OLSP) in the MSSM sector or the axino. The corresponding relic density of the gravitino can be represented by

$$
\Omega_{3/2}h^2 = \Omega_{3/2}^{TH}h^2 + \Omega_{3/2}^{NTH}h^2
$$

\n
$$
\approx 2.8 \times 10^4 \left(\frac{m_{3/2}}{100 \text{ keV}}\right) (Y_{3/2}^{TH} + Y_{3/2}^{NTH}), \quad (34)
$$

 $\frac{3}{2}$

where $Y_{3/2}^{(N)TH} = n_{3/2}^{(N)TH}(T)/s(T)$ is the yield of the gravitino which is produced by a (non-)thermal process. At $T_{\text{RH}} \sim 1 \text{ TeV}$, gravitinos from thermal production can provide a right amount of cold dark matter, $\Omega_{3/2}^{TH}h^2 \simeq 0.1$, if the mass is [\[16\]](#page-6-14)

$$
m_{3/2} \sim 100 \text{ keV.}
$$
 (35)

For the NTH production of the gravitinos, the contribution from the OLSP decay is small enough in the above range of $m_{3/2}$, but we have to pay attention to the production from the axino decays. Although the axino couplings to the MSSM particles are suppressed by $1/v_{\text{PO}}$, they are still large enough to generate a significant axino abundance from the thermal bath. If the axino is stable, one needs $T_{\text{RH}} \ll m_{\text{soft}}$ in order to suppress its relic density suffi-ciently [[25](#page-6-22)]. In our case of $T_{\text{RH}} \sim m_{\text{soft}}$, thermally generated axinos may produce a large number of gravitinos from their decays. If axinos decay only to gravitinos, the nonthermal relic density of gravitinos turns out to be too large. Thus, the axino decay to the gravitino must be suppressed. For this, let us now consider the following axino mass range:

$$
m_{\chi} + m_h < m_{\tilde{a}} < \mu - m_h \tag{36}
$$

where χ is the OLSP. Then the dominant production and decay channels of the axino come from the higgs (h) higgsino (h) -axino (\tilde{a}) coupling:

$$
\int d^2\theta \frac{\kappa_2}{2\Lambda} X^2 H_u H_d = \frac{\mu}{v_{\rm PQ}} h \tilde{h} \tilde{a} + \dots \tag{37}
$$

That is, axinos are produced thermally by the process $h \rightarrow h\tilde{a}$, and then they decay mainly through $\tilde{a} \rightarrow h\chi$. Denoting the decay rate of higgsino to axino as $\Gamma(h \rightarrow$ $h\tilde{a}$, one finds the thermal axino abundance as follows:

$$
Y_{\tilde{a}}^{\text{TH}} \approx \frac{135\zeta(3)}{8\pi^4 g_*} \frac{\Gamma(\tilde{h} \to h\tilde{a})}{H} \bigg|_{T=\mu}, \tag{38}
$$

where $g_* \sim 200$ is the relativistic degrees of freedom and $H \approx 0.33 \sqrt{g_*} T^2 / M_P$ is the Hubble parameter at the temperature T. Then, the nonthermal abundance of the gravitino is

$$
Y_{3/2}^{\text{NTH}} = \frac{\Gamma(\tilde{a} \to \psi_{3/2} a)}{\Gamma(\tilde{a} \to \chi h)} Y_{\tilde{a}}^{\text{TH}},\tag{39}
$$

in which partial decay rates of axinos are given by $\Gamma(\tilde{a} \rightarrow$ $\psi_{3/2}a$) = $m_{\tilde{a}}^5/96\pi m_{3/2}^2 M_P^2$ [\[26\]](#page-6-23) with $\psi_{3/2}$ being the gravitino and $\Gamma(\tilde{a} \to \chi h) = \theta^2 \mu^2 m_{\tilde{a}}/8 \pi v_{\text{PQ}}^2$. Here θ parametrizes the OLSP fraction in the Higgsino component. For our estimation, we will use $\theta \sim m_Z s_W/\mu$ which is valid in the limit of the large Higgsino and small gaugino masses. Combining ([38](#page-5-1)) and [\(39\)](#page-5-2), we get

$$
Y_{3/2}^{\text{NTH}} \approx 1.5 \times 10^{-7} \left(\frac{m_{\tilde{a}}}{350 \text{ GeV}}\right)^4 \left(\frac{100 \text{ keV}}{m_{3/2}}\right)^2 \left(600 \frac{\text{GeV}}{\mu}\right),\tag{40}
$$

which shows that the nonthermal gravitino relic density can be safely neglected. Finally, let us remark that the axino decays well before the OLSP freezes out, that is, $\Gamma(\tilde{a} \rightarrow \chi h) > H(T_f)$ for the OLSP freeze-out temperature $T_f \le 20$ GeV for our choice of parameters. Therefore, the OLSPs from the axino decay are thermalized.

IV. CONCLUSION

In this paper, we have examined a model to generate the Higgs μ parameter with a spontaneously broken $U(1)_{\text{PO}}$ symmetry in a gauge mediation scenario. The PQ sector of the model contains $U(1)_{\text{PO}}$ breaking fields which have a Yukawa coupling to extra quarks. The $U(1)_{\text{PO}}$ breaking fields also have a nonrenormalizable superpotential suppressed by the cutoff scale Λ which might be identified as the Planck scale or the GUT scale. For the messenger scale higher than the PQ scale, the $U(1)_{PQ}$ breaking fields are destabilized at the origin due to the soft SUSY breaking terms induced by the combined effects of gauge-mediated SUSY breaking and the Yukawa coupling to extra quarks. They are then stabilized by the supersymmetric scalar potential from nonrenormalizable superpotential at an intermediate scale $v_{\text{PQ}} \sim \sqrt{m_{\text{soft}}\Lambda}$, generating $\mu \sim$ $v_{\text{PQ}}^2/\Lambda \sim m_{\text{soft}}$ in a natural manner. The B parameter at the messenger scale is predicted to be negligible, and therefore B at the weak scale is determined by the RG evolution below the messenger scale.

The model has a variety of interesting cosmological features associated with the PQ phase transition. In particular, a late thermal inflation is a natural possibility, which would require a late baryogenesis mechanism. We find that a successful Affleck-Dine leptogenesis after thermal inflation can be implemented within the model. We also find that a right amount of gravitino dark matter can be produced after thermal inflation when $v_{\text{PO}} =$ $\mathcal{O}(10^9 - 10^{10})$ GeV and $m_{3/2} = \mathcal{O}(100)$ keV, for which the messenger scale of gauge mediation is required to be not far above v_{PO} . A more complete discussion of the cosmological aspects of the model will be presented elsewhere.

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