

General rotating charged Kaluza-Klein AdS black holes in higher dimensions

Shuang-Qing Wu (吴双清)*

*Institute of Theoretical Physics, China West Normal University, Nanchong, Sichuan 637002, People's Republic of China
and College of Physical Science and Technology, Hua-Zhong Normal University, Wuhan, Hubei 430079, People's Republic of China
(Received 27 February 2011; published 15 June 2011)*

I construct exact solutions for general nonextremal rotating, charged Kaluza-Klein black holes with a cosmological constant and with arbitrary angular momenta in all higher dimensions. I then investigate their thermodynamics and find their generalizations with the Newman-Unti-Tamburino charges. The metrics are given in both Boyer-Lindquist coordinates and a form very similar to the famous Kerr-Schild ansatz, which highlights its potential application to include multiple electric charges into solutions yet to be found in gauged supergravity. It is also observed that the metric ansatz in $D = 4$ dimensions is similar to those previously suggested by Yilmaz and later by Bekenstein.

DOI: 10.1103/PhysRevD.83.121502

PACS numbers: 04.50.Gh, 04.20.Jb, 04.50.Cd, 04.70.Dy

I. INTRODUCTION

It is generally accepted that an appropriate ansatz for the metric and the gauge potential plays a crucial role in finding an exact solution to Einstein and Einstein-Maxwell field equations. A well-known example is provided by the Kerr solution [1], which was first derived via the Kerr-Schild ansatz [2]. This ansatz was then used by Myers and Perry [3] in 1986 to successfully obtain higher-dimensional vacuum generalizations of the Kerr solution. Several years ago, the Kerr-Schild form was adopted again by Gibbons, *et al* [4,5] to include a cosmological constant in the vacuum Myers-Perry's solution in all higher dimensions.

With the discovery of the remarkable anti-de Sitter/conformal field theory (AdS/CFT) correspondence, it is of considerable interest to generalize the above-mentioned neutral rotating solutions to charged ones in arbitrary dimensions. One of the major reasons is that rotating charged black holes with a cosmological constant in higher dimensions can provide new important gravitational backgrounds for the study of the microscopic entropy of black holes and for testing the AdS/CFT correspondence within the string theory framework. In the case of ungauged supergravity, research on string dualities has revealed that some global symmetries can be used as solution-generating transformations to obtain new solutions from old ones. Therefore, it is straightforward to employ a solution-generating procedure to generate charged solutions from neutral ones. In general, the generated solutions are very complicated and typically characterized by multiple electromagnetic charges, in addition to the mass and angular momenta. (For an earlier review, see [6] and references therein.)

However, the situation is quite different for the gauged cases. There is no longer a solution-generating technique available for deriving the nonextremal charged black holes

from neutral solutions in the gauged supergravity, since the presence of a cosmological constant breaks down the corresponding global symmetries of ungauged supergravity. One has little option but to resort to brute-force calculations [7,8], starting from a guessed ansatz to verify that all the equations of motion are completely satisfied. At present, almost all the previously-known solutions of rotating charged AdS black holes in higher dimensions have been obtained in this way. What is more, they are limited to very special cases either with some charges equal, or with equal rotation parameters [9].

As far as the simplest case with only one charge is concerned, the currently-known charged nonextremal rotating black hole solutions within the Kaluza-Klein supergravity theory are as follows. The first rotating charged black hole in the four-dimensional Kaluza-Klein theory was derived in [10] via the boost-reduction procedure, and its extension with a Newman-Unti-Tamburino (NUT) charge was obtained recently in [11]. Generalizations to all higher dimensions were presented in [12]. In the case of Kaluza-Klein gauged supergravity, a single-charged solution with only one rotation parameter nonvanishing in five dimensions was found for the first time in [13], and the general rotating charged solution with only one charge nonzero and with two unequal rotation parameters was then announced in [9]. Inspired by the work [13], Chow [14] recently found a solution describing a rotating charged AdS(-NUT) black hole in four-dimensional Kaluza-Klein gauged supergravity. General solutions that describe rotating charged Kaluza-Klein-AdS (KK-AdS) black holes in $D \geq 6$ dimensions are not yet known explicitly until the present work. For the most general case with three unequal charges and with two independent rotation parameters, the explicit form of charged rotating AdS₅ solutions has remained unknown up to now.

So far, all the previously-obtained solutions for rotating charged AdS black holes were not derived via a universal method other than via a combination of guesswork and trial and error, followed by explicit verification of the field

*Electronic address: sqwu@phy.ccnu.edu.cn

equations. A natural question is, can one develop an effective method somewhat like the Kerr-Schild ansatz to overcome the difficulty in the construction of rotating black holes with multiple charges in gauged supergravity? The answer seems likely to be definitive. The purpose of this article is to present a clue to resolve this dilemma. As a first step towards this direction and for simplicity, I shall be mainly concerned with the single-charge case in Kaluza-Klein supergravity.

In this paper, I begin by presenting the general solutions for rotating, charged KK-AdS black holes with a single electric charge and with arbitrary angular momenta in all higher dimensions. Then I calculate the conserved charges that obey the first law of thermodynamics, and make a generalization to include the NUT charges. After these, a primary analysis of the metric structure is given, which sheds new light on constructing the most general rotating, multiple-charged AdS black hole solutions yet unknown in gauged supergravity theories.

II. GENERAL KK-ADS SOLUTION

To present the general exact solutions, conventions are adopted as those in [4]. Let the dimension of spacetime be $D = 2N + 1 + \epsilon \geq 4$, with $N = [(D - 1)/2]$ being the number of rotation parameters a_i , and $2\epsilon = 1 + (-1)^D$. Let ϕ_i be the N azimuthal angles in the N orthogonal spatial 2-planes, each with period 2π . The remaining spatial dimensions are parametrized by a radial coordinate r and by $N + \epsilon = n = [D/2]$ “direction cosines” μ_i subject to the constraint $\sum_{i=1}^{N+\epsilon} \mu_i^2 = 1$, where $0 \leq \mu_i \leq 1$ for $1 \leq i \leq N$, and (for even D) $-1 \leq \mu_{N+1} \leq 1$, $a_{N+1} = 0$.

The general nonextremal rotating, charged KK-AdS solutions can be elegantly written in a unified form very similar to the Kerr-Schild metric ansatz [2] by

$$d\bar{s}^2 = H^{1/(D-2)} \left(d\bar{s}^2 + \frac{2m}{UH} K^2 \right), \quad (1)$$

$$A = \frac{2ms}{UH} K, \quad \Phi = \frac{-1}{D-2} \ln(H), \quad (2)$$

where the anti-de Sitter metric $d\bar{s}^2$ and the timelike 1-form K are given by

$$\begin{aligned} d\bar{s}^2 = & -(1 + g^2 r^2) W d\bar{t}^2 + F dr^2 \\ & + \sum_{i=1}^{N+\epsilon} \frac{r^2 + a_i^2}{\chi_i} d\mu_i^2 + \sum_{i=1}^N \frac{r^2 + a_i^2}{\chi_i} \mu_i^2 d\bar{\phi}_i^2 \\ & - \frac{g^2}{(1 + g^2 r^2) W} \left(\sum_{i=1}^{N+\epsilon} \frac{r^2 + a_i^2}{\chi_i} \mu_i d\mu_i \right)^2, \end{aligned} \quad (3)$$

$$K = cW d\bar{t} + \sqrt{f(r)} F dr - \sum_{i=1}^N \frac{a_i \sqrt{\Xi_i}}{\chi_i} \mu_i^2 d\bar{\phi}_i, \quad (4)$$

in which the functions (U, W, F, H) and $f(r)$ are defined to be

$$\begin{aligned} U = r^\epsilon \sum_{i=1}^{N+\epsilon} \frac{\mu_i^2}{r^2 + a_i^2} \prod_{j=1}^N (r^2 + a_j^2), \quad W = \sum_{i=1}^{N+\epsilon} \frac{\mu_i^2}{\chi_i}, \\ F = \frac{r^2}{1 + g^2 r^2} \sum_{i=1}^{N+\epsilon} \frac{\mu_i^2}{r^2 + a_i^2}, \quad H = 1 + \frac{2ms^2}{U}, \\ f(r) = c^2 - s^2(1 + g^2 r^2), \quad \Xi_i = c^2 - s^2 \chi_i, \\ \chi_i = 1 - g^2 a_i^2. \end{aligned}$$

In the above and below, the shorthand notations $c = \cosh \delta$ and $s = \sinh \delta$ are used.

One may transform the above solutions to a frame in terms of generalized Boyer-Lindquist coordinates by

$$\begin{aligned} d\bar{t} = dt + \frac{2mc\sqrt{f(r)} dr}{(1 + g^2 r^2)[V(r) - 2mf(r)]}, \\ d\bar{\phi}_i = d\phi_i + \frac{2ma_i\sqrt{\Xi_i}\sqrt{f(r)} dr}{(r^2 + a_i^2)[V(r) - 2mf(r)]}, \end{aligned} \quad (5)$$

where the function $V(r)$ is defined by

$$V(r) \equiv \frac{U}{F} = r^{\epsilon-2} (1 + g^2 r^2) \prod_{i=1}^N (r^2 + a_i^2).$$

The general KK-AdS solutions then have the form

$$\begin{aligned} ds^2 = H^{1/(D-2)} \left[-(1 + g^2 r^2) W dt^2 + \frac{U dr^2}{V(r) - 2mf(r)} \right. \\ \left. + \sum_{i=1}^{N+\epsilon} \frac{r^2 + a_i^2}{\chi_i} d\mu_i^2 + \sum_{i=1}^N \frac{r^2 + a_i^2}{\chi_i} \mu_i^2 d\phi_i^2 \right. \\ \left. - \frac{g^2}{(1 + g^2 r^2) W} \left(\sum_{i=1}^{N+\epsilon} \frac{r^2 + a_i^2}{\chi_i} \mu_i d\mu_i \right)^2 \right. \\ \left. + \frac{2m}{UH} \left(cW dt - \sum_{i=1}^N \frac{a_i \sqrt{\Xi_i}}{\chi_i} \mu_i^2 d\phi_i \right)^2 \right], \end{aligned} \quad (6)$$

$$A = \frac{2ms}{UH} \left(cW dt - \sum_{i=1}^N \frac{a_i \sqrt{\Xi_i}}{\chi_i} \mu_i^2 d\phi_i \right), \quad (7)$$

which is in a frame nonrotating at infinity. The gauge potential has been changed modulo a radial gauge transformation.

In the uncharged case ($\delta = 0$), the above metric (6) reduces to those found in [4,5]. On the other hand, if the cosmological constant is set to zero, it degenerates to those derived in [12]. In particular, the KK-AdS solutions in the $D = 4, 5, 7$ nonrotating case correspond to the supergravity black hole solutions [15] but with only one charge. I have directly and explicitly checked that the general solutions (6) and (7) obey the field equations derived from the Lagrangian of the Einstein-Maxwell-dilaton system ($\mathcal{F} = dA$)

$$\begin{aligned} \mathcal{L} = \sqrt{-g} \left\{ R - \frac{1}{4} (D-1)(D-2) (\partial\Phi)^2 - \frac{1}{4} e^{-(D-1)\Phi} \mathcal{F}^2 \right. \\ \left. + g^2 (D-1)[(D-3)e^\Phi + e^{-(D-3)\Phi}] \right\}, \end{aligned} \quad (8)$$

for the $D = 4, 5, 6, 7$ cases. Since the dimension of space-time is not distinguished in any way in the general expressions for the solutions, it is confident that the solutions are valid in all dimensions. In particular, the $D = 4$ solution reproduces the one recently found by Chow [14]. In the $D = 5$ case with only one rotation parameter and when one sets $\Xi_a = \Xi_b = 1$ here, the single-charged solution then coincides with the special solution found in [13] after setting $w = 1$ there. However, in the case of a rotating single-charged KK-AdS₅ solution with two unequal rotation parameters, no simple relation has been found to make the above solution contact with the general solution declared in [9].

III. THERMODYNAMICS

The KK-AdS black holes have Killing horizons at $r = r_+$, the largest positive root of $V(r_+) = 2mf(r_+)$. On the horizon, the Killing vector

$$l = \frac{\partial}{\partial t} + \sum_{i=1}^N \frac{(1 + g^2 r_+^2) a_i \sqrt{\Xi_i}}{(r_+^2 + a_i^2) c} \frac{\partial}{\partial \phi_i}$$

becomes null and obeys $l^\mu l_{\nu;\mu} = \kappa l_\nu$, where the surface gravity is given by

$$\kappa = \frac{r_+(1 + g^2 r_+^2) \sqrt{f(r_+)}}{c} \left[\sum_{i=1}^N \frac{1}{r_+^2 + a_i^2} + \frac{\epsilon - 2}{2r_+^2} + \frac{g^2 c^2}{(1 + g^2 r_+^2) f(r_+)} \right], \quad (9)$$

hence the Hawking temperature $T = \kappa/(2\pi)$ is

$$T = \frac{\sqrt{f(r_+)} [V'(r_+) - 2mf'(r_+)]}{4\pi r_+^{\epsilon-2} c \prod_{i=1}^N (r_+^2 + a_i^2)}. \quad (10)$$

The entropy of the outer horizon is easily evaluated as

$$S = \frac{\mathcal{V}_{D-2} r_+^{\epsilon-1} c}{4\sqrt{f(r_+)}} \prod_{i=1}^N \frac{r_+^2 + a_i^2}{\chi_i} = \frac{\mathcal{V}_{D-2} m r_+ c \sqrt{f(r_+)}}{2(1 + g^2 r_+^2) \prod_{i=1}^N \chi_i}, \quad (11)$$

where I denote the volume of the unit $(D - 2)$ -sphere as

$$\mathcal{V}_{D-2} = \frac{2\pi^{(D-1)/2}}{\Gamma[(D-1)/2]}. \quad (12)$$

On the horizon, the angular velocities and the electrostatic potential $\Phi_+ = \ell^\mu A_\mu|_{r_+}$ are given by

$$\Omega_i = \frac{(1 + g^2 r_+^2) a_i \sqrt{\Xi_i}}{(r_+^2 + a_i^2) c}, \quad \Phi_+ = \frac{s}{c} (1 + g^2 r_+^2). \quad (13)$$

I then adopt the procedure that was used in [16,17] to calculate the conserved charges as follows,

$$\begin{aligned} M &= \frac{\mathcal{V}_{D-2} m}{8\pi \prod_{j=1}^N \chi_j} \left[c^2 \left(\sum_{i=1}^N \frac{2}{\chi_i} + \epsilon - 2 \right) + 1 \right], \\ J_i &= \frac{\mathcal{V}_{D-2} m a_i c \sqrt{\Xi_i}}{4\pi \chi_i \prod_{j=1}^N \chi_j}, \\ Q &= \frac{(D-3) \mathcal{V}_{D-2} m c s}{8\pi \prod_{j=1}^N \chi_j}, \end{aligned} \quad (14)$$

and explicitly verify that they satisfy the differential and integral first laws of thermodynamics

$$dM = TdS + \sum_{i=1}^N \Omega_i dJ_i + \Phi_+ dQ - Pd\mathcal{V}, \quad (15a)$$

$$\frac{D-3}{D-2} (M - \Phi_+ Q) = TS + \sum_{i=1}^N \Omega_i J_i - P\mathcal{V}, \quad (15b)$$

where I have introduced the generalized pressure

$$P = \frac{g^{D-2} m}{4\pi(D-2) \prod_{j=1}^N \chi_j} \left[c^2 \left(\sum_{i=1}^N \frac{1}{\chi_i} + \frac{D-3+\epsilon}{2} - \frac{D-2}{1+g^2 r_+^2} \right) - \frac{D-3}{2} s^2 g^2 r_+^2 \right], \quad (16)$$

which is conjugate to the volume $\mathcal{V} = \mathcal{V}_{D-2} g^{2-D}$ of the $(D - 2)$ -sphere with the AdS radius $1/g$. The results presented above include those given in [12,18] as special cases when $g = 0$ and $\delta = 0$, respectively.

IV. INCLUSION OF THE NUT CHARGES

To include the NUT charges, it is convenient to adopt the Jacobi-Carter coordinates used in [19]. In doing so, I find that the general nonextremal KK-AdS-NUT solutions can be cast into the following compact form:

$$\begin{aligned} ds^2 &= H^{1/(D-2)} \left\{ - \prod_{\beta=1}^n \frac{1 - g^2 x_\beta^2}{\chi_\beta} dt^2 + \sum_{\alpha=1}^n \frac{U_\alpha}{X_\alpha} dx_\alpha^2 \right. \\ &\quad + \sum_{i=1}^{n-\epsilon} \frac{\tilde{\mu}_i^2}{\chi_i} d\phi_i^2 + \sum_{\alpha=1}^n \frac{2m_\alpha (-x_\alpha)^\epsilon}{U_\alpha} K_\alpha^2 \\ &\quad \left. - \frac{s^2}{H} \left[\sum_{\alpha=1}^n \frac{2m_\alpha (-x_\alpha)^\epsilon}{U_\alpha} K_\alpha \right]^2 \right\}, \end{aligned} \quad (17)$$

$$A = \frac{s}{H} \sum_{\alpha=1}^n \frac{2m_\alpha (-x_\alpha)^\epsilon}{U_\alpha} K_\alpha, \quad (18)$$

$$\Phi = \frac{-1}{D-2} \ln(H), \quad (19)$$

with n independent 1-forms

$$K_\alpha = \frac{c}{1 - g^2 x_\alpha^2} \prod_{\beta=1}^n \frac{1 - g^2 x_\beta^2}{\chi_\beta} dt - \sum_{i=1}^{n-\epsilon} \frac{a_i \tilde{\mu}_i^2 \sqrt{\Xi_i}}{(a_i^2 - x_\alpha^2) \chi_i} d\phi_i. \quad (20)$$

In the above, I denote

$$\begin{aligned}\tilde{\mu}_i^2 &= \frac{\prod_{\alpha=1}^n (a_i^2 - x_\alpha^2)}{\prod_{k=1}^n (a_i^2 - a_k^2)}, & U_\alpha &= \prod_{\beta=1}^n (x_\beta^2 - x_\alpha^2), \\ H &= 1 + s^2 \sum_{\alpha=1}^n \frac{2m_\alpha (-x_\alpha)^\epsilon}{U_\alpha}, & f(x_\alpha) &= 1 + g^2 s^2 x_\alpha^2, \\ X_\alpha &= \bar{X}_\alpha + 2m_\alpha (-x_\alpha)^\epsilon f(x_\alpha), \\ \bar{X}_\alpha &= \frac{1 - g^2 x_\alpha^2}{x_\alpha^2} \prod_{i=1}^n (a_i^2 - x_\alpha^2),\end{aligned}$$

where the prime on the product symbol in the definition of $\tilde{\mu}_i^2$ or U_α indicates that the vanishing factor (i.e. when $k = i$ or $\alpha = \beta$) is to be omitted. In odd dimensions, one has $m_n = m$; and for even dimensions, $m_n = im$ and $a_n = 0$. In all dimensions, one sets $x_n = ir$.

One may transform the above KK-AdS-NUT metrics via the following coordinate transformations,

$$\begin{aligned}d\bar{t} &= dt + c \sum_{\alpha=1}^n \frac{2im_\alpha (-x_\alpha)^\epsilon \sqrt{f(x_\alpha)}}{(1 - g^2 x_\alpha^2) X_\alpha} dx_\alpha, \\ d\bar{\phi}_j &= d\phi_j + a_j \sqrt{\Xi_j} \sum_{\alpha=1}^n \frac{2im_\alpha (-x_\alpha)^\epsilon \sqrt{f(x_\alpha)}}{(a_j^2 - x_\alpha^2) X_\alpha} dx_\alpha,\end{aligned}\quad (21)$$

into a form like the multi-Kerr-Schild ansatz [20]. The expected line element and the gauge potential can be rewritten as follows

$$\begin{aligned}ds^2 &= H^{1/(D-2)} \left\{ d\bar{s}^2 + \sum_{\alpha=1}^n \frac{2m_\alpha (-x_\alpha)^\epsilon}{U_\alpha} \bar{K}_\alpha^2 \right. \\ &\quad \left. - \frac{s^2}{H} \left[\sum_{\alpha=1}^n \frac{2m_\alpha (-x_\alpha)^\epsilon}{U_\alpha} \bar{K}_\alpha \right]^2 \right\}, \\ A &= \frac{s}{H} \sum_{\alpha=1}^n \frac{2m_\alpha (-x_\alpha)^\epsilon}{U_\alpha} \bar{K}_\alpha,\end{aligned}\quad (22)$$

where the NUT-AdS metric $d\bar{s}^2$ and the timelike 1-forms \bar{K}_α read

$$d\bar{s}^2 = - \prod_{\beta=1}^n \frac{1 - g^2 x_\beta^2}{\chi_\beta} d\bar{t}^2 + \sum_{\alpha=1}^n \frac{U_\alpha}{\bar{X}_\alpha} dx_\alpha^2 + \sum_{i=1}^{n-\epsilon} \frac{\tilde{\mu}_i^2}{\chi_i} d\bar{\phi}_i^2, \quad (24)$$

$$\begin{aligned}\bar{K}_\alpha &= \frac{c}{1 - g^2 x_\alpha^2} \prod_{\beta=1}^n \frac{1 - g^2 x_\beta^2}{\chi_\beta} d\bar{t} + i \sqrt{f(x_\alpha)} \frac{U_\alpha}{\bar{X}_\alpha} dx_\alpha \\ &\quad - \sum_{i=1}^{n-\epsilon} \frac{a_i \sqrt{\Xi_i}}{(a_i^2 - x_\alpha^2) \chi_i} \tilde{\mu}_i^2 d\bar{\phi}_i.\end{aligned}\quad (25)$$

Reinterpreted in $D + 1$ dimensions, the general KK-AdS-NUT solutions can be obtained via the standard dimension reduction along the z -direction from the $(D + 1)$ -dimensional line element

$$d\hat{s}^2 = dz^2 + d\bar{s}^2 + \sum_{\alpha=1}^n \frac{2m_\alpha (-x_\alpha)^\epsilon}{U_\alpha} (\bar{K}_\alpha + s dz)^2. \quad (26)$$

V. METRIC STRUCTURE AND MODIFIED NEWTONIAN DYNAMICS

Expressed in the language of tensors, the general non-extremal KK-AdS solutions (1) and (2) have a beautiful structure as follows

$$\begin{aligned}g_{\mu\nu} &= H^{1/(D-2)} \left(\bar{g}_{\mu\nu} + \frac{2m}{UH} K_\mu K_\nu \right), \\ g^{\mu\nu} &= H^{(-1)/(D-2)} \left(\bar{g}^{\mu\nu} - \frac{2m}{U} K^\mu K^\nu \right), \\ A_\mu &= \frac{2ms}{UH} K_\mu,\end{aligned}\quad (27)$$

with the vector $K^\mu = \bar{g}^{\mu\nu} K_\nu$ being raised by the background metric tensors $\bar{g}^{\mu\nu}$. Moreover, the vector K_μ is a timelike geodesic congruence, satisfying

$$\bar{g}^{\mu\nu} K_\mu K_\nu = -s^2, \quad K^\mu \bar{\nabla}_\mu K_\nu = K^\mu \bar{\nabla}_\nu K_\mu = 0. \quad (28)$$

In the presence of $(n - 1)$ NUT charges and the mass parameter, with the help of the relations: $\bar{g}_{\mu\nu} \bar{K}_\alpha^\mu \bar{K}_\alpha^\nu = \bar{g}_{\mu\nu} \bar{K}_\alpha^\mu \bar{K}_\beta^\nu = -s^2$, the inverse metric is found to be

$$g^{\mu\nu} = H^{(-1)/(D-2)} \left[\bar{g}^{\mu\nu} - \sum_{\alpha=1}^n \frac{2m_\alpha (-x_\alpha)^\epsilon}{U_\alpha} \bar{K}_\alpha^\mu \bar{K}_\alpha^\nu \right]. \quad (29)$$

The metric structure (27) naturally reduces to the notable Kerr-Schild ansatz [2] in the uncharged case, therefore it is likely the unique suitable generalization to that of the Kaluza-Klein gauged and ungauged supergravity theory in the case with only one electric charge. It has already been checked that with some further modifications, the ansatz (27) can be extended to large classes of already-known black hole solutions with multiple pure electric charges, both in the cases of rotating charged black holes in ungauged supergravity and in the cases of nonrotating AdS black holes in gauged supergravity. Guided by the generalized ansatz, in principle, one is able to construct the expected new exact gauged solutions. Therefore, it would be highly expected that the generalized ansatz (27) can open a new way towards constructing the most general rotating charged AdS black hole solutions with multiple pure electric charges in gauged supergravity theory.

Alternatively, one may use the dilaton scalar to reexpress the metric tensors and the gauge potential as

$$\begin{aligned}g_{\mu\nu} &= e^{-\Phi} \bar{g}_{\mu\nu} + [e^{-\Phi} - e^{(D-3)\Phi}] s^{-2} K_\mu K_\nu, \\ g^{\mu\nu} &= e^\Phi \bar{g}^{\mu\nu} + [e^\Phi - e^{-(D-3)\Phi}] s^{-2} K^\mu K^\nu, \\ A_\mu &= [1 - e^{(D-2)\Phi}] s^{-1} K_\mu.\end{aligned}\quad (30)$$

It is apparent that the metric of the four-dimensional KK-AdS black holes resembles those proposed by Yilmaz [21] and by Bekenstein [22]. Given the same matter contents of these two tensor-vector-scalar theories,

it deserves a lot of deeper investigations of the relation between them and the astrophysical implications of the four-dimensional Kaluza-Klein-AdS theory as another kind of modified Newtonian dynamics. For instance, it is an interesting question as to whether the experimental test of effects of the four-dimensional Kaluza-Klein black hole on our solar system can explore the existence of extra spatial dimensions or put some constraints on the size of extra fifth dimension.

VI. CONCLUSIONS

In this paper, I have found the general nonextremal rotating, charged KK-AdS black holes with arbitrary angular momenta in all higher dimensions, and extended them to include the $(n - 1)$ NUT charges. The conserved charges are given explicitly and shown to obey the differential and integral first laws of black hole thermodynamics. I then have explained that the general nonextremal KK-AdS solutions have a beautiful structure similar to the Kerr-Schild ansatz, which highlights its promising application to include multiple electric charges into solutions yet to be discovered in gauged supergravity. In addition, it is also observed that the generalized ansatz in the $D = 4$ case can be expressed as a form like the one

previously suggested by Yilmaz and later by Bekenstein in his tensor-vector-scalar theory.

ACKNOWLEDGMENTS

S.-Q. Wu is supported by the NSFC under Grant Nos. 10975058 and 10675051. The computation within this work has been done by using the GRTENSOR-II program based on MAPLE 7.

Notes added.—While this paper was being under review (and finally rejected by PRL), a generalized form of the ansatz proposed in this work was used to successfully construct a new exact rotating charged solution with two unequal rotation parameters and with two different electric charges, but the third charge being set to zero, in five-dimensional $U(1)^3$ gauged supergravity. Along the same line, the present author now succeeds in constructing the most general charged rotating AdS₅ solution with three unequal charges and with two independent rotation parameters, which is the most interesting solution previously unknown in $D = 5$ $U(1)^3$ gauged supergravity. As such, it is believed that the ansatz (27) proposed in this paper and its generalized form would bring about an important breakthrough in the method of constructing new exact gauged supergravity solutions.

-
- [1] R. P. Kerr, *Phys. Rev. Lett.* **11**, 237 (1963).
 - [2] R. P. Kerr and A. Schild, *Proc. Symp. Appl. Math.* **17**, 199 (1965).
 - [3] R. C. Myers and M. J. Perry, *Ann. Phys. (N.Y.)* **172**, 304 (1986).
 - [4] G. W. Gibbons, H. Lü, D. N. Page, and C. N. Pope, *Phys. Rev. Lett.* **93**, 171102 (2004).
 - [5] G. W. Gibbons, H. Lü, D. N. Page, and C. N. Pope, *J. Geom. Phys.* **53**, 49 (2005).
 - [6] D. Youm, *Phys. Rep.* **316**, 1 (1999).
 - [7] Z. W. Chong, M. Cvetič, H. Lü, and C. N. Pope, *Phys. Rev. Lett.* **95**, 161301 (2005).
 - [8] Z. W. Chong, M. Cvetič, H. Lü, and C. N. Pope, *Nucl. Phys.* **B717**, 246 (2005).
 - [9] Z. W. Chong, M. Cvetič, H. Lü, and C. N. Pope, *Phys. Lett. B* **644**, 192 (2007).
 - [10] V. Frolov, A. Zelnikov, and U. Bleyer, *Ann. Phys. (Leipzig)* **499**, 371 (1987).
 - [11] A. N. Aliev, H. Cebeci, and T. Dereli, *Phys. Rev. D* **77**, 124022 (2008).
 - [12] J. Kunz, D. Maison, F. Navarro-Lerida, and J. Viebahn, *Phys. Lett. B* **639**, 95 (2006).
 - [13] Z. W. Chong, M. Cvetič, H. Lü, and C. N. Pope, *Phys. Rev. D* **72**, 041901 (2005).
 - [14] D. D. K. Chow, *Classical Quantum Gravity* **28**, 032001 (2011).
 - [15] M. Cvetič *et al.*, *Nucl. Phys.* **B558**, 96 (1999).
 - [16] J. I. Koga, *Phys. Rev. D* **72**, 064010 (2005).
 - [17] W. Chen, H. Lü, and C. N. Pope, *Phys. Rev. D* **73**, 104036 (2006).
 - [18] G. W. Gibbons, M. J. Perry, and C. N. Pope, *Classical Quantum Gravity* **22**, 1503 (2005).
 - [19] W. Chen, H. Lü, and C. N. Pope, *Classical Quantum Gravity* **23**, 5323 (2006).
 - [20] W. Chen and H. Lü, *Phys. Lett. B* **658**, 158 (2008).
 - [21] W. C. dos Santos, *Astrophys. Space Sci.* **331**, 295 (2011).
 - [22] J. D. Bekenstein, *Phys. Rev. D* **70**, 083509 (2004).