

Is super-Planckian physics visible? Scattering of black holes in 5 dimensions

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It may be widely believed that probing short distance physics is limited by the presence of the Planck energy scale above which scale any information is cloaked behind a horizon. If this hypothesis is correct, we could observe quantum behavior of gravity only through a black hole of Planck mass. We numerically show that in a scattering of two black holes in the 5-dimensional spacetime, a visible domain, whose curvature radius is much shorter than the Planck length, can be formed. Our result indicates that super-Planckian phenomena may be observed without an obstruction by horizon formation in particle accelerators.

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I. INTRODUCTION

It is well known that if quantum effects are taken into account, a threshold energy scale, over which general relativity loses its validity, emerges. This threshold is called the Planck scale. For the 4-dimensional spacetime, the Planck energy is defined by $E_{\text{pl}} := \sqrt{\hbar c^5/2G} = 1.1 \times 10^{19}$ GeV, where c , \hbar , and G are the speed of light, Dirac constant, and Newton's gravitational constant, respectively. The circumferential radius of a spherically symmetric black hole of mass $E_{\text{pl}}c^{-2}$ is equal to its reduced Compton wavelength $\hbar c E_{\text{pl}}^{-1}$, and hence such a black hole will behave as a gravitating quantum object whose behavior is unpredictable in the framework of general relativity. The Planck energy, E_{pl} , is much larger than the electroweak scale (≈ 100 GeV), and this fact is recognized as the hierarchy problem in the elementary particle physics.

The large extra-dimension scenario was proposed as a solution for the hierarchy problem [1,2]. This scenario is inspired by superstring theories, and in this scenario, the fundamental Planck energy, E_P , may be as low as 10^3 GeV scale. The peculiarity of this scenario is that the length scales of the compactification of extra-dimensions can be much larger than the fundamental Planck length $\hbar c E_P^{-1}$. Hence, the gravity can be described by a higher-dimensional classical theory (e.g., higher-dimensional general relativity) for the distance scale smaller than the compactification scale and larger than $\hbar c E_P^{-1}$. Also in this scenario, the classical theory of gravity will lose its validity in the super-Planckian domains.

For nongravitational interactions, a length scale (e.g., Compton wavelength), which is usually explored by a particle scattering, decreases with increasing its energy measured at the center-of-mass frame. However, this relation may not hold for the phenomena in which gravity plays an important role. It is widely believed that collisions

of particles with the super-Planckian energy scale would produce black holes, and hence, physical processes characterized by the length scale shorter than $\hbar c E_P^{-1}$ are hidden inside black holes [3]. This is a kind of the cosmic censorship hypothesis for the super-Planckian domain, but it is not trivial whether this hypothesis is correct. The original cosmic censorship hypothesis claims that, roughly speaking, naked singularities are not formed in our Universe [4]. However, we have to note that even if the cosmic censorship hypothesis is correct, it does not necessarily imply that the super-Planckian physics is hidden inside horizons. In the framework of general relativity or in the large extra-dimension scenario, the super-Planckian curvature is not a spacetime singularity as long as it is finite, and thus the presence of the super-Planckian domains may be irrelevant to the cosmic censorship. Indeed, Nakao, Harada, and Miyamoto (NHM) recently suggested, by a simple dimensional analysis, a possibility that visible super-Planckian domains may be produced in high-energy particle collisions, if the spacetime dimension is larger than 4 [5].

To theoretically explore the phenomena in a higher-dimensional gravity, numerical relativity is probably the unique approach. In the past few years, several implementations for the higher-dimensional numerical relativity have been developed [6–10], and now, it is feasible to explore the nonlinear physics, such as high-velocity collision in higher dimensions as in 4 dimensions [11–13]. In this paper, we numerically show that in a scattering of two black holes in the framework of 5-dimensional (5D) general relativity, super-Planckian domains may be indeed visible.

Hereafter, we adopt the natural units $c = \hbar = 1$ and the abstract index notation: Latin indices, except for w , x , y and z , denote a tensor type, and Greek indices denote a component with respect to some basis vectors [14]. In this paper, we define the fundamental Planck energy E_P such that 5D Einstein's equation is written as

$$G_{ab} = 3\pi^2 E_P^{-3} T_{ab}. \quad (1)$$

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II. SUPER-PLANCKIAN DOMAIN

We define a super-Planckian domain as a region where 5D general relativity loses its validity. In the case that quantum effects on a spacetime geometry are not very large, quantum corrections to the Einstein-Hilbert action might be given in the form of scalar polynomials of the Riemann tensor. Hence, we adopt the square root of the absolute value of the Kretschmann invariant $|R^{abcd}R_{abcd}|^{1/2}$ as a reference quantity.

To determine a reasonable threshold value of $|R^{abcd}R_{abcd}|^{1/2}$ over which the domain becomes super-Planckian, we adopt, as a reference, the 5D Schwarzschild-Tangherlini (ST) black hole which is a spherically symmetric vacuum solution in 5D general relativity. By the definition of E_P through Eq. (1), the circumferential radius of the ST black hole with mass M is equal to $(M/E_P^3)^{1/2}$. Thus, the ST black hole with $M = E_P$ will behave as a gravitating quantum object which actually cannot be described by general relativity, because the reduced Compton wavelength of this black hole agrees with its circumferential radius. $|R^{abcd}R_{abcd}|^{1/2}$ at the event horizon of this black hole is equal to $6\sqrt{2}E_P^2$. Hence, by introducing a dimensionless reference quantity $\mathcal{K} \equiv (6\sqrt{2}E_P^2)^{-1}|R^{abcd}R_{abcd}|^{1/2}$, we can define a super-Planckian domain \mathcal{A} by

$$\inf_{\mathcal{A}} \mathcal{K} \geq 1. \quad (2)$$

Note that the above condition will be one of sufficient conditions for the appearance of a super-Planckian domain, because a domain, in which one of scalar quantities defined from the Riemann tensor exceeds an appropriately determined critical value, should be regarded as a super-Planckian one [5].

III. INITIAL DATA

Hereafter, we consider a scattering of two nonrotating black holes with identical mass M . A procedure for setting initial data of the scattering problem with negligible junk radiation was presented in [12], which we follow.

When the distance between two black holes is much larger than their gravitational radii $R_g := (E_P^{-3}M)^{1/2}$, the metric near each black hole in each rest frame is well approximated by that of the ST black hole spacetime,

$$ds^2 = -\alpha^2(r_0)dt_0^2 + \psi^2(r_0)(dw_0^2 + dx_0^2 + dy_0^2 + dz_0^2), \quad (3)$$

where

$$\psi(r) = 1 + \left(\frac{R_g}{2r}\right)^2 \quad \text{and} \quad \alpha(r) = \frac{2 - \psi(r)}{\psi(r)}, \quad (4)$$

and $r_0 = \sqrt{w_0^2 + x_0^2 + y_0^2 + z_0^2}$. To obtain an approximate metric in the vicinity of each black hole in the *center-of-mass frame* of the two-black hole system, we perform coordinate transformations for the above metric twice.

First, a boost transformation, $t = \Gamma(t_0 \mp vt_0)$, $w = \Gamma(w_0 \mp vt_0)$, $x = x_0$, $y = y_0$, and $z = z_0$, is performed, where the velocity v is a positive constant less than unity, and $\Gamma = 1/\sqrt{1 - v^2}$ is the Lorentz factor. Next, a spatial translation, $w \rightarrow w \mp \ell_w$ and $x \rightarrow x \mp \ell_x$, is performed, where ℓ_w and ℓ_x are positive constants, respectively. As a result, we obtain two coordinate systems in which the world line of the black hole (puncture) is given by $w = \pm(\ell_w - vt)$, $x = \pm\ell_x$, $y = z = 0$, and the line element is

$$ds_{\pm}^2 = -\Gamma^2(\alpha_{\pm}^2 - v^2\psi_{\pm}^2)dt^2 \pm 2\Gamma^2v(\alpha_{\pm}^2 - \psi_{\pm}^2)dt dw_{\pm} + \psi_{\pm}^2(B_{\pm}^2 dw^2 + dx^2 + dy^2 + dz^2), \quad (5)$$

where $\alpha_{\pm} = \alpha(r_{\pm})$, $\psi_{\pm} = \psi(r_{\pm})$, and $B_{\pm}^2 := \Gamma^2(1 - v^2\alpha_{\pm}^2\psi_{\pm}^{-2})$ with $r_{\pm} := \sqrt{\Gamma^2(w \mp \ell_w \pm vt)^2 + (x \mp \ell_x)^2 + y^2 + z^2}$. The components of the extrinsic curvature of the spacelike hypersurface labeled by t are

$$K_{ww}^{\pm} = \mp \frac{v\Gamma^3(w \mp \ell_w \pm vt)B_{\pm}}{r_{\pm}} \times \left[2\alpha'_{\pm} - \frac{\alpha_{\pm}}{2} \{\ln(\psi_{\pm}^2 - v^2\alpha_{\pm}^2)\}' \right], \quad (6)$$

$$K_{xx}^{\pm} = K_{yy}^{\pm} = K_{zz}^{\pm} = \mp \frac{\Gamma v(w \mp \ell_w \pm vt)\alpha_{\pm}\psi'_{\pm}}{B_0\psi_{\pm}r_{\pm}}, \quad (7)$$

$$K_{wx}^{\pm} = \mp \frac{\Gamma v(x \mp \ell_x)B_{\pm}}{r_{\pm}} \left[\alpha'_{\pm} - \frac{\alpha_{\pm}}{2} \{\ln(\psi_{\pm}^2 - v^2\alpha_{\pm}^2)\}' \right], \quad (8)$$

$$K_{wy}^{\pm} = \mp \frac{\Gamma v y B_{\pm}}{r_{\pm}} \left[\alpha'_{\pm} - \frac{\alpha_{\pm}}{2} \{\ln(\psi_{\pm}^2 - v^2\alpha_{\pm}^2)\}' \right], \quad (9)$$

$$K_{wz}^{\pm} = \mp \frac{\Gamma v z B_{\pm}}{r_{\pm}} \left[\alpha'_{\pm} - \frac{\alpha_{\pm}}{2} \{\ln(\psi_{\pm}^2 - v^2\alpha_{\pm}^2)\}' \right], \quad (10)$$

and the other components vanish. Here, the prime denotes the ordinary derivative with respect to r_{\pm} . Based on the above results, we write the initial data for a scattering of two black holes with initial velocities $\pm v$. The metric of the spacelike hypersurface is written in the following form:

$$dl^2 = (\Psi + \Phi)^2(B^2 dw^2 + dx^2 + dy^2 + dz^2), \quad (11)$$

where

$$\Psi = 1 + \left(\frac{R_g}{2r_+}\right)^2 + \left(\frac{R_g}{2r_-}\right)^2, \quad (12)$$

$$B^2 = \Gamma^2 \left[1 - \frac{v^2}{\Psi^4} (1 - 2\Psi)^2 \right]. \quad (13)$$

The extrinsic curvature is written by

$$K_{ab} = K_{ab}^+ + K_{ab}^- + \delta K_{ab}. \quad (14)$$

Finally, we set $t = 0$. The unknown functions Φ in Eq. (11) and δK_{ab} in Eq. (14) should be determined from

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the conditions that the Hamiltonian and momentum constraints are satisfied. However, if the coordinate separation between two black holes, $2\sqrt{\ell_w^2 + \ell_x^2}$, is much larger than R_g , $|\Phi|$ and $|\delta K_{\mu\nu}|$ are much smaller than $|\Psi|$ and $|K_{\mu\nu}^\pm|$ [12]. We choose the initial separation to be sufficiently large and set the small corrections to be zero. Such approximation is acceptable for our present purpose.

IV. NUMERICAL RESULTS

Numerical simulations were performed using SACRA-ND code reported in Ref. [6], in which the so-called Baumgarte-Shapiro-Shibata-Nakamura formalism [15] together with the ‘‘moving puncture’’ approach [16] and adaptive mesh refinement algorithm [17] are employed. The numerical accuracy is monitored by computing L_2 -norm of the Hamiltonian and momentum constraints. The convergence of the numerical solution was tested varying the grid resolutions with the grid spacing as $\Delta/R_g = 15/320$, $12/320$, and $10/320$. We confirmed a reasonable convergence behavior (see below).

Numerical simulations were systematically performed varying v and an impact parameter defined by $b \equiv 2\ell_x$. For a small velocity $v \leq 0.6$, we were always able to determine a critical value of the impact parameter, $b = b_{\text{crit}}$, for the merger of two black holes. Namely, for $b < b_{\text{crit}}$, two black holes merge to a single spinning black hole whereas for $b > b_{\text{crit}}$, two black holes go apart to infinity after one scattering. The zoom-whirl orbit was never found in 5 dimensions in contrast to the 4D case [12], because the law of the gravitational force is modified. Note that in the Newtonian limit in 5 dimensions, both gravitational and centrifugal forces are proportional to r^{-3} where r is the separation of two objects.

For a high velocity with $v > 0.6$, by contrast, we were not able to determine the value of b_{crit} using our current code. The reason is that for $b_B < b < b_C$, the numerical simulation crashed soon after the scattering of two black holes occurs. Figure 1 plots b_B and b_C as functions of v (note that for $v \leq 0.6$, $b_B = b_C = b_{\text{crit}}$). However, we were able to confirm that for $b < b_B$, two black holes

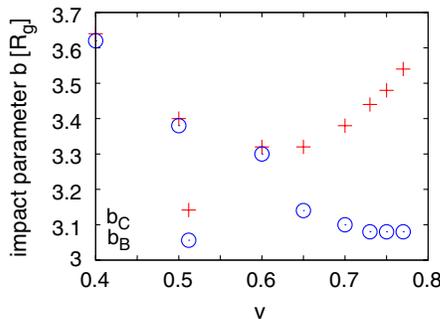


FIG. 1 (color online). Impact parameters b_B and b_C are plotted as functions of the initial speed v of each black hole. For $v < 0.6$, b_B and b_C agree with the critical impact parameter b_{crit} .

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merges, while for $b > b_C$, the merger does not happen and two black holes merely go apart to infinity after the scattering. In this paper, we focus on the case $b \geq b_C$.

Figure 2 plots the time variation of \mathcal{K} in a scattering process with $v = 0.7$ and $b = 3.38R_g$. The value of \mathcal{K} is shown in the unit of $E_p M^{-1}$ in the w - x plane. In this figure, the apparent horizon of each black hole is denoted by the solid circles. When the separation between two black holes is much larger than R_g , no super-Planckian domain emerges outside the apparent horizons as long as the mass of each black hole M is larger than E_p [see Fig. 2(a)]. By contrast, when the separation is equal to $2-3R_g$, a highly elongated domain with a large value of $\mathcal{K} \gg E_p/M$ is formed between two black holes [see Fig. 2(b)]. Finally, two black holes are scattered away, and the domain with a large value of \mathcal{K} disappears [see Fig. 2(c)].

Figure 3 plots \mathcal{K}^2 at the center-of-mass $w = x = y = z = 0$ as a function of the coordinate separation between two

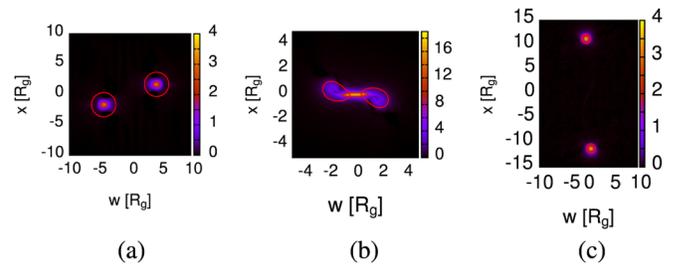


FIG. 2 (color online). Color maps of \mathcal{K} in the scattering of two black holes with $v = 0.7$ and $b = 3.38R_g$, (a) before the scattering, (b) during the scattering, and (c) after the scattering. At the stage (b), a highly elongated domain with a large value of $\mathcal{K} \gg E_p/M$ appears between two black holes. The solid distorted circles denote the apparent horizon of the black holes.

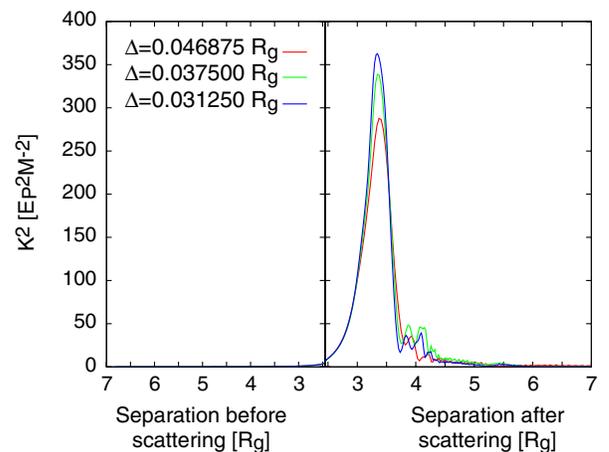


FIG. 3 (color online). \mathcal{K}^2 at the center-of-mass as a function of the coordinate separation between two black holes. The parameters v and b are the same as those for Fig. 2. The time proceeds from left to right. The results with three grid resolutions are plotted.

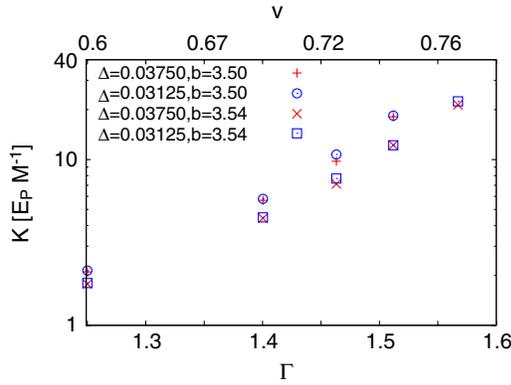


FIG. 4 (color online). The maximum value of \mathcal{K} calculated in the scattering of $b = 3.5R_g$ and $b = 3.54R_g$ at the center-of-mass as a function of v . The results only for $b > b_C$ are plotted. The pluses and the crosses show the results of $\Delta/R_g = 3/80$, and the circles and the squares show the results of $\Delta/R_g = 1/32$.

two black holes. In this figure, the time elapses from left to right. Initial speed of each black hole and the impact parameter are the same as those for Fig. 2. As the separation between two black holes becomes small, the value of \mathcal{K} steeply increases. Then, after the black holes slightly goes through the periastron, the maximum value of \mathcal{K} is reached. The maximum value \mathcal{K}_{\max} in the best resolution run is

$$\mathcal{K}_{\max} \simeq 19 \left(\frac{E_P}{M} \right). \quad (15)$$

In this scattering process, the event horizons of these black holes do not merge with each other, because these black holes go away toward infinity separately after this scattering. Hence, by the symmetry of this system, the center-of-mass is not enclosed by the event horizon. This fact implies that even if each black hole is a classical object before the scattering a visible super-Planckian domain can emerge in the vicinity of the center-of-mass in 5D general relativity.

Figure 4 plots the maximum values of \mathcal{K} for the scattering of a fixed impact parameter at the center-of-mass as a function of v . The results only for $b > b_C$ are plotted. This shows that the maximum value of \mathcal{K} increases steeply with v , and thus, the super-Planckian domain appears to be always visible in high-velocity collisions. This also suggests that for $v \rightarrow 1$, the maximum value of \mathcal{K} would be much larger than E_P/M .

V. SUMMARY AND DISCUSSION

Probing the short distance physics would be limited if scattering processes at energies well above the Planck scale

were hidden behind a black hole horizon. If this hypothesis is correct, possible productions of small black holes in particle colliders, such as the CERN Large Hadron Collider, could probably be the unique opportunity for exploring the nature of the quantum gravity [3,18]. However, as shown in this paper, a visible super-Planckian domain can be generated even through a scattering process of two classical black holes in 5D spacetime. We may say in the practical sense that the cosmic censorship does not hold in 5D spacetime, because a super-Planckian domain should be regarded as an effective singularity in the classical gravity.

The visible super-Planckian domain is formed even for $v \sim 0.7$, i.e., in a mildly relativistic scattering, and visible super-Planckian domains can be generated through gravitational scatterings more efficiently than a guess by NHM. It should be emphasized that such a domain could emerge even in the absence of special spacetime symmetry or special assumption of spacetime geometry. The present result implies that it is necessary to study quantum gravitational effects in the super-Planckian domain of no horizon. Although, at present, we do not know what happens in the super-Planckian domain, we expect that the semiclassical particle creation in the sub-Planckian region could occur around the visible super-Planckian domain [19].

Finally, we comment on the scatterings for $b_B(v) < b < b_C(v)$ with $v > 0.6$ for which our numerical simulations do not keep sufficient accuracy probably due to the emergence of a very large spacetime curvature. We performed simulations for $b > b_C$ with several grid resolutions. The maximum value of \mathcal{K} becomes larger for the finer grid resolution for a given value of b . It also increases with approaching b_C . These facts suggest that ultrahigh spacetime curvature, which is not encompassed in a horizon, may be realized at some impact parameter for $b_B < b < b_C$, i.e., the formation of a naked singularity. We would like to explore this issue in more detail elsewhere.

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